SAUDER
School of Business

Real Estate Division

Introduction to the Hewlett-Packard (HP) 10BII Calculator and Review of Mortgage Finance Calculations

Real Estate Division
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and
Review of Mortgage Finance Calculations

LEARNING OBJECTIVES

After studying this supplement, a student should be able to:

1. understand how to use the HP 10BII calculator to solve basic mathematical problems;
2. differentiate between nominal and periodic rates of interest;
3. calculate future and present values for lump sums;
4. calculate payments for mortgage loans;
5. calculate outstanding balances for mortgage loans;
6. calculate principal and interest portions of mortgages with varying payment frequencies and terms;
7. calculate the market value of a fully or partially amortized vendor take-back mortgage;
8. calculate the market value of a fully or partially amortized assumed loan;
9. calculate the market value of a loan which has an interest rate buydown;
10. understand the impact of all forms of beneficial financing on the value of real property.

INTRODUCTION

The purpose of this supplement is to provide an introduction to real estate finance: how to perform calculations for investments in real estate (and other assets). These concepts are necessary knowledge for all real estate practitioners and also provide a good foundation for the more complex analyses to follow in later courses. This supplement is provided as a review for students who have covered this material already in previous Real Estate Division courses or for students from other educational programs who require reference or practice materials for mortgage finance.

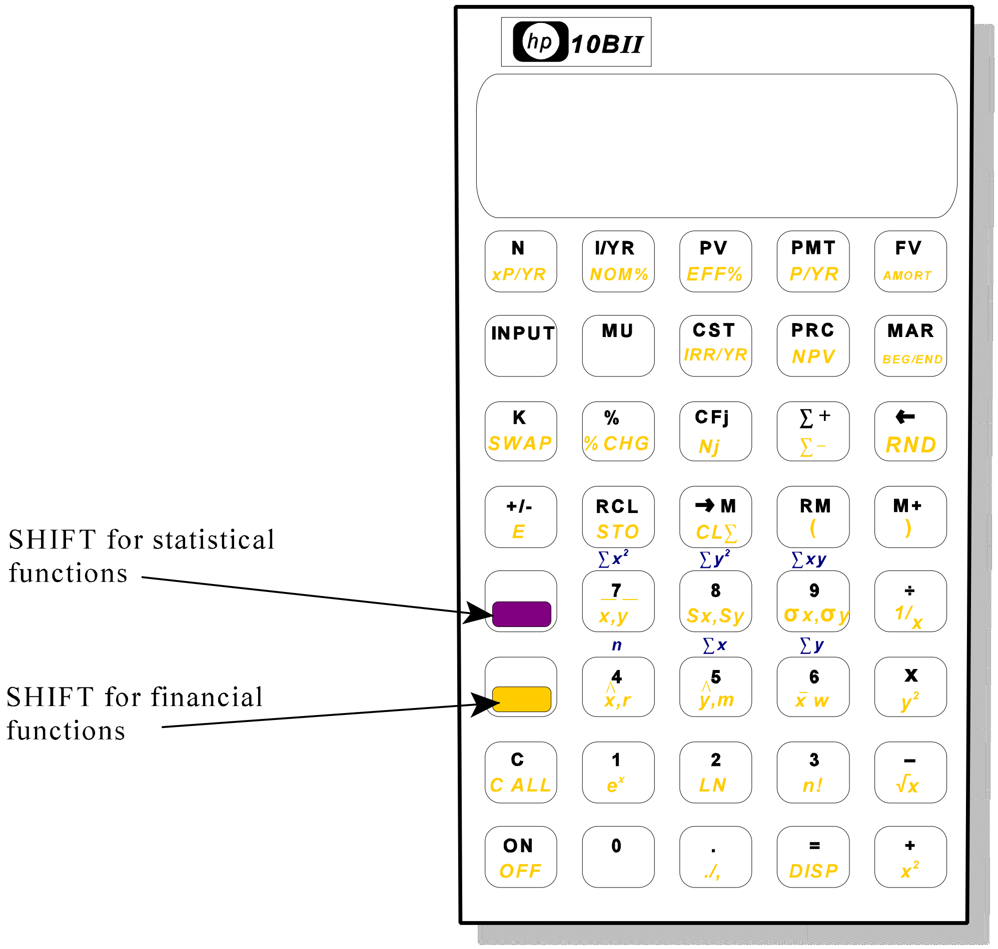
The first section of this supplement will provide a brief overview of the Hewlett Packard (HP) 10BII calculator. While you are not limited to this particular calculator and may use any calculator which is not both programmable and alphanumeric, if you elect to use a calculator other than the HP 10BII, it is strongly recommended that you ensure that the alternate calculator will perform all necessary calculations. It will then be up to you to consult the owner's manual to determine how the calculator of your choice operates.

The second section of this supplement, titled "Mortgage Financing Techniques - Part I," introduces a number of basic mortgage finance calculations including: conversion of an interest rate to an equivalent rate with a different compounding frequency; solving for constant payments; and calculating outstanding balances. However, these techniques do not cover the full range of analysis required of real estate professionals.

The third section of this supplement, titled "Mortgage Financing Techniques - Part II," builds upon these real estate financing techniques and discusses alternative financing arrangements.

INTRODUCTION TO THE HEWLETT PACKARD (HP) 10BII CALCULATOR

A. SHIFT Keys



The HP10BII has two ■ (shift) keys. One is yellow (for financial functions), the other is purple (for statistical functions). To access the financial functions, students should always use the yellow ■ key. All functions that are activated by the yellow shift key are located in the lower half of each of the calculator keys, and are also labelled in yellow.

B. BEG/END

Please be aware that the Hewlett-Packard 10BII calculator has both Begin and End modes. The Begin mode is needed for annuity due calculations, or those which require payments to be made "in advance". For example, lease payments are generally made at the beginning of each month, not at the end. On the other hand, interest payments are almost always calculated at the end of each payment period, or "not in advance". These types of calculations each require a different setting on the calculator. When your calculator is set in Begin mode, the bottom of the display screen will show "BEGIN". If "BEGIN" is not on your display screen, your calculator must be in End mode, as there is no annunciator for this mode.

To switch between modes, press ■ [BEG/END] .

C. Setting a Floating Decimal Place

To ensure your answers will be the same as those shown throughout this supplement, you should use a floating decimal place for your calculator. Under this setting, your calculator will display the maximum number of decimal places possible.

Press	Display
■ [DISP] [•]	0

Example 1 - The Floating Decimal

With the calculator set to a floating decimal, the calculation 7 ÷ 6 should result in the following:

Press	Display
7 [÷] 6 [=]	1.16666666667

Now see what the calculator displays when the decimal place is fixed at 2 places.

Example 1, Continued - Fixed Decimal

Press	Display
■ [DISP] 2	1.17

Notice that the calculator has now rounded the answer to the second decimal place (i.e. to the nearest cent). In mortgage finance calculations you should always use the floating decimal setting so that your calculations are as accurate as possible. Once the calculation is made, you can apply the appropriate rounding rule if necessary. Now set the calculator back to a floating decimal.

Press	Display
■ [DISP] [•]	1.16666666667

Notice in the example above that the calculator rounded the display to 1.17, but kept the original more accurate number in its memory.

D. Basic Arithmetic Calculations

Example 2 - Addition

There are 36 students in one classroom and 57 in another. What is the total number of students in the two classrooms?

36 + 57 = ? or

$$\begin{array}{r} 36 \\ +\ \underline{57} \\ ? \end{array}$$

Press	Display
36 [+] 57 [=]	93

Example 3 - Subtraction

Your bank account balance was $327.00, and you have just written a cheque in the amount of $129.48. What will be your new balance?

$327 - 129.48 = ?$ or

$$\begin{array}{r} 327 \\ -\ \underline{129.48} \\ ? \end{array}$$

Press	Display
327 [−] 129.48 [=]	197.52

Example 4 - Multiplication

You are buying 2.9 metres of fabric, priced at $6.49 per metre. What is the total cost of your purchase?

$2.9 \times 6.49 = ?$ or

$$\begin{array}{r} 2.9 \\ \times\ \underline{6.49} \\ ? \end{array}$$

Press	Display
2.9 [×] 6.49 [=]	18.821

Example 5 - Division

You have ordered a number of boxes containing 125 envelopes each. The total number of envelopes ordered was 7,875. How many boxes were ordered?

$7{,}875 \div 125 = ?$ or

$$\begin{array}{r} 7{,}875 \\ \div\ \underline{125} \\ ? \end{array}$$

or

$$\frac{7{,}875}{125}$$

Press	Display
7875 [÷] 125 [=]	63

E. Negative Numbers

The [+/−] key is merely a sign change key. You can press it numerous times and it will do nothing other than change the sign of the displayed number to negative or positive. To enter a negative number in your calculator, you must first enter the number and once it is showing on the display screen, you must press the [+/−] key to change it to a negative number.

Example 6 - Addition of Negative Numbers

$-10{,}862 + (-31{,}765) = ?$ or

$$\begin{array}{r} -10{,}862 \\ + \underline{(-31{,}765)} \\ ? \end{array}$$

Press	Display
10862 [+/−] [+] 31765 [+/−] [=]	-42,627

Example 7 - Subtraction of Negative Numbers

$-20 - (-76) = ?$ or

$$\begin{array}{r} -20 \\ -\underline{(-76)} \\ ? \end{array}$$

Press	Display
20 [+/−] [−] 76 [+/−] [=]	56

Example 8 - Multiplication of Negative Numbers

$-32 \times (-61) = ?$ or

$$\begin{array}{r} -32 \\ \times \underline{(-61)} \\ ? \end{array}$$

Press	Display
32 [+/−] [×] 61 [+/−] [=]	1,952

Example 9 - Division of Negative Numbers

$-150 \div (-35) = ?$ or

$$\begin{array}{r} -150 \\ \div \underline{(-35)} \\ ? \end{array}$$

Press	Display
150 [+/−] [÷] 35 [+/−] [=]	4.28571428571

F. Use of the [C] Key

To clear an unintentional numerical entry.

Example 10 - Clearing a Numerical Entry Mistake

You entered 567 by mistake. You meant to enter 568.

Press	Display
567 [C]	0
568	568

Another method of clearing an unintentional numerical entry is by using the [←] key. This erases the last character entered.

Example 11 - Erasing Last Character Entered

Press	Display
567	567
[←]	56
8	568

Example 12 - Clearing a Numerical Entry Mistake in an Arithmetic Calculation

You wish to add 720 plus 543. You have entered 720 plus 573 by mistake. To clear the 573 from the calculation, press the [C] key once. This only removes the mistaken numerical entry so the calculation can be continued.

Press	Display
720 [+] 573	573
[C]	0
543 [=]	1,263

Note that you could also correct this data entry error using the [←] key.

Example 13 - Clearing an Arithmetic Calculation

You enter the calculation for 5 ÷ 22. You actually meant to multiply 5 by 22 but hit the wrong key. Press [C] twice to remove the calculation so you can start again.

Press	Display
5 [÷] 22	22
[C] [C]	0
5 [×] 22 [=]	110

If you attempt a calculation which the calculator is incapable of performing, the word "Error" will show up on your display screen. Most likely the error occurred when the data was entered and you will just have to try the calculation again. However, you must first remove the Error message by pressing the [C] key.

Example 14 - Clearing an "Error" Message

You wish to divide 50 by 10. By mistake you enter the calculation for 50 divided by 0. Since this is an impossible calculation, you will get an error message on your display screen. The Error message displayed depends on the type of miscalculation. There are a total of eight different Error display messages, each one giving a partial description of the problem. You must clear the error message before you attempt the calculation again.

Press	Display
50 [÷] 0 [=]	Error - Func
[C] [C]	0
50 [÷] 10 [=]	5

Example 15 - Clearing All Stored Values

To clear all the stored values in the calculator:

Press	Display
■ [C ALL]	0

This removes all the values stored in any of the calculator's function keys, as well as the memory function.

G. Arithmetic Equations

Example 16 - Series of Additions

389.7 + 208.52 + 73.1978 = ?

Press	Display
389.7 [+] 208.52 [+] 73.1978 [=]	671.4178

Note that the [=] key could be pressed after each part of these types of calculations, but it is not necessary and means extra work!

Example 17 - Series of Subtractions

912.83 − 82.71 − 653.58 − 289.76 = ?

Press	Display
912.83 [−] 82.71 [−]	
653.58 [−] 289.76 [=]	-113.22

Example 18 - Series of Additions and Subtractions

287.99 + 112.05 − 232.65 + 38.7 − 56.98 = ?

Press	Display
287.99 [+] 112.05	112.05
[−] 232.65 [+] 38.7	38.7
[−] 56.98 [=]	149.11

H. Memory Keys

This calculator has a "constant memory." This means that whatever is stored in memory remains there until expressly changed (even when the calculator is turned off), unless the ■ [C ALL] function is used.

The keys are as follows:

- [→M] - stores the number showing on the display screen in memory.
- [RM] - recalls a number from memory and displays it. (The number remains stored in memory.)
- [M+] - adds the number on the display screen to the number that is already stored in memory (the sum is retained in memory).

Example 19 - Storing a Value in Memory

(7 ÷ 2) + (13 × 6) = ?

Press	Display
7 [÷] 2 [=]	3.5
[→M]	3.5
13 [×] 6 [=]	78
[+] [RM]	3.5
[=]	81.5

Example 20 - Recalling the Value Stored in Memory

Using the value stored in the previous calculation:

$(7 \div 2) + (13 \times 5) = ?$

Press	Display
13 [×] 5 [=]	65
[+] [RM]	3.5
[=]	68.5

Example 21 - Summing a Displayed Value and a Value in Memory

Using the value stored in the previous calculation:

$(7 \div 2) + (13 \times 5) = ?$

Press	Display
13 [×] 5 [=]	65
[M+] [RM]	68.5

This total sum will now be stored in memory.

I. Rules for Sequence of Arithmetic Calculations (Order of Operations)

1. Perform operations in brackets first.
2. Perform multiplication and division calculations from left to right.
3. Perform addition and subtraction calculations from left to right.

Solve using the steps above:

Example 22 - Calculating an Equation (Method 1)

$(3 \times 2) + (18 \div 6) - 7 = ?$

Press	Display
3 [×] 2 [=]	6
[→M]	6
18 [÷] 6 [=]	3
[M+]	3
[RM] [−] 7	2

Example 23 - Calculating an Equation (Method 2)

An alternate method to using the memory function is using the bracket functions. Performing the same calculation as the previous example, the calculator steps are as follows:

Press	Display
■ [(] 3 [×] 2 ■ [)]	6
[+] ■ [(] 18 [÷] 6 ■ [)]	3
[−] 7 [=]	2

Example 24 - Calculating an Equation

$(42.615 + 61.03) \times 4.352 + 127.8 \div 6.3$

Press	Display
42.615 [+] 61.03 [=]	103.645
[×] 4.352 [=] [→M]	451.06304
127.8 [÷] 6.3 [=]	20.2857142857
[+] [RM] [=]	471.348754286

J. Converting Fractions to Decimals

The line in a fraction means "divided by" (i.e., ½ means 1 [÷] 2).

$\frac{3}{4}$	3 [÷] 4 [=]	0.75
$\frac{1}{8}$	1 [÷] 8 [=]	0.125
$\frac{13}{16}$	13 [÷] 16 [=]	0.8125
$\frac{7}{13}$	7 [÷] 13 [=]	0.538461538

In this last calculation, a calculator set to a floating decimal will show 5.38461538**E-1**. The last part of the display, E-1, is scientific notation and means "move the decimal point one place to the left." This notation is used when a number is too large or too small to fit in the display ("E" stands for "exponent of ten").

Example 25 - Converting a Fraction to a Decimal

A bank quotes you an interest rate of $9\frac{7}{8}$ % per annum, compounded annually.

Press	Display
7 [÷] 8 [=]	0.875
[+] 9 [=]	9.875

K. Additional Function Keys

You may find numerous other function keys of use. For instance, the [%] key converts a number expressed as a percent (e.g., 60%) to its decimal equivalent (i.e., 0.60).

Example 26 - Using the [%] Key

What is 7.5% of 37?

Press	Display
7.5 [%]	0.075
[×] 37 [=]	2.775

Another key which may be useful is the reciprocal key [1/*x*].

Example 27 - Using the [1/*x*] Key

Convert $\frac{1}{6}$ to its decimal equivalent.

Press	Display
6 ■ [1/*x*]	1.66666667E-1

L. Automatic Shutoff

If your calculator is left on for several minutes without being used, it will shut off automatically.

MORTGAGE FINANCING TECHNIQUES - PART I

I. THE BASIS OF INTEREST RATE CALCULATIONS

Interest is, essentially, rent charged for the use of borrowed funds (i.e., the principal amount). A loan contract will specify that interest will be charged at the end of a specified time period; for example, interest might be charged at the end of each month that the borrower has had the use of the funds. These interest periods are referred to as "compounding" periods; if interest is charged monthly, the loan is said to have monthly compounding. The amount of interest charged at the end of a compounding period is some specified percentage of the amount of principal the borrower has had use of during the *entire compounding period.* The percentage is referred to as the periodic interest rate or the interest rate per compounding period. The amount of interest charged at the end of the compounding period is equal to the amount of principal outstanding during the compounding period multiplied by the interest rate expressed as a decimal.

Borrowers and lenders are concerned with the interest rate per compounding period, and how often these payment periods occur (or the length of the compounding periods). For example, if $1,000 is borrowed at 1.5% per compounding period, the borrower will pay more interest in a year if this 1.5% is charged monthly than if it is charged semi-annually (that is, monthly compounding rather than semi-annual compounding). When analyzing a financial arrangement, whether it is a credit card balance, a demand loan, or a mortgage, one must know both the interest rate per compounding period and the frequency of compounding. Borrowers and lenders may agree on any interest rate, frequency of compounding or frequency of payment. However, in Canada, a provision of the Interest Act requires the rate of interest to be quoted in a mortgage contract with either annual or semi-annual compounding. This provision has resulted in semi-annual compounding becoming the industry rule for mortgages[1].

The basic concept of valuation of financial assets focuses upon the relationship between when interest must be paid, and when principal must be repaid. In the case of simple interest, interest is charged and payable only once during the life of the mortgage – at the end of the term of the loan when the principal (upon which the interest was charged) is also repaid. If a mortgage contract specifies that interest be charged more than once during the life of the loan, whether the interest is actually paid at the time it is charged (interest only) or is added to the debt (interest accruing), the contract implies compound interest (discussed in a later section).

A. Nominal and Periodic Interest Rates

The annual interest rate generally quoted for compound interest is referred to as the "nominal interest rate per annum". The nominal rate is represented mathematically as "j_m" where:

j_m	=	nominal interest rate compounded "m" times per year
m	=	number of compounding periods per annum
i	=	interest rate per compounding period

The nominal rate of interest compounded "m" times per year (j_m) is equal to the periodic interest rate per compounding period (i) times the number of compounding periods per year (m).

[1] The reason semi-annual compounding is quoted rather than annual compounding is because it results in interest rates which appear to be lower than those based on annual compounding.

$$j_m = i \times m$$

or $$i = \frac{j_m}{m}$$

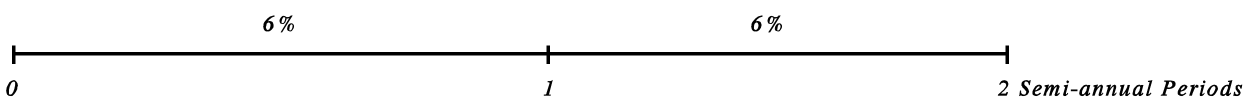
The nominal rate (j_m) is always expressed as a certain percentage per annum compounded a specific number of times per annum (m). Consider the nominal rate of 12% per annum, compounded semi-annually, not in advance[2]. This would be expressed as:

$$j_2 = 12\%$$

and $$i = \frac{j_2}{2} = \frac{12\%}{2} = 6\%$$

Thus, the statement that interest is 12% per annum, compounded semi-annually (not in advance) tells the analyst that there are two (m) compounding periods per annum and that interest is to be 6% ($i = j_m \div m$) per semi-annual compounding period.

This can be illustrated using a "time diagram"[3] as shown below:



j_2	= 12%	nominal interest rate (per year)
m	= 2	2 semi-annual periods per year
i	= 6%	periodic interest rate (semi-annual period)

Just as the nominal interest rate per annum (j_m) has an indicated frequency of compounding (m), it is also necessary to specify the frequency of compounding for periodic rates. The following shorthand notation is used in this supplement to indicate the frequency of compounding that is intended for periodic rates:

"i_d" represents an interest rate per daily compounding period
"i_w" represents an interest rate per weekly compounding period
"i_{mo}" represents an interest rate per monthly compounding period
"i_q" represents an interest rate per quarterly compounding period
"i_{sa}" represents an interest rate per semi-annual compounding period
"i_a" represents an interest rate per annual compounding period

2 "Not in advance" refers to the fact that the amount of interest accruing over the compounding period is calculated at the end of the compounding period, so that the borrower pays the interest at the end (or, not in advance) of the compounding period. Almost all rates of interest are calculated "not in advance". Therefore, the statement "not in advance" is frequently not used, and the interest rate would be quoted as 12% per annum, compounded semi-annually. Unless it is explicitly stated to be otherwise, students may assume that all interest rates are "not in advance".

3 Time diagrams are shown as a horizontal line representing time. The present value is at the left (time 0) and the future value is at the right. In financial arrangements, time is measured by compounding periods, and so 2 semi-annual compounding periods are shown along the "time" line.

For example:

i_d	=	$j_{365} \div 365$	i_q	=	$j_4 \div 4$
i_w	=	$j_{52} \div 52$	i_{sa}	=	$j_2 \div 2$
i_{mo}	=	$j_{12} \div 12$	i_a	=	$j_1 \div 1$

The interest rate j_1, which is the nominal rate per annum, compounded annually, is also known as the effective annual interest rate.

Completion of Illustration 1 should provide an increased familiarity with periodic interest rates, compounding frequency, nominal rates and the interrelationship between them.

Illustration 1

The tables below represent a survey of interest rates quoted by financial institutions on 5 year term deposits. Complete the tables by entering the appropriate values for the question marks for either the periodic rate, the number of compounding periods or the nominal rate.

Question	Periodic Rate i	Number of Compounding Periods per Year (m)	Nominal Rate ($j_m = i \times m$)
SAMPLE	6.5%	2	j_2 = 13%
(a)	1.0625%	12	j_{12} = ?
(b)	3.275%	4	j_4 = ?
(c)	0.0356164384%	m = ?	j_m = 13%

Question	Nominal Rate j_m	Number of Compounding Periods per Year (m)	Periodic Rate ($i = j_m \div m$)
SAMPLE	j_2 = 13%	2	i_{sa} = 6.5%
(d)	j_{12} = 18%	12	i_{mo} = ?
(e)	j_{365} = 8%	365	i_d = ?
(f)	j_m = 15%	m = ?	i_{mo} = 1.25%

Solution:

(a) j_{12} =12.75%
(b) j_4 = 13.1%
(c) m = 365
(d) i_{mo} = 1.5%
(e) i_d = 0.0219178082%
(f) m = 12

B. Compound Interest Calculations

As an introduction to the nature of compound interest calculations, consider Illustration 2:

Illustration 2

A commercial enterprise has arranged for an interest accruing loan whereby the $10,000 amount borrowed is to be repaid in full at the end of one year. The borrower has agreed, in addition, to pay interest at the rate of 15% per annum, compounded annually on the borrowed funds. Calculate the amount owing at the end of the one year term of the loan.

Solution:

Given that the borrower owes $10,000 throughout the year, the amount of interest owing at the end of the one year term is calculated as follows:

Interest Owing	=	Principal Borrowed × interest rate per interest calculation period (in this example, interest is calculated per *annual* compounding period)
	=	$10,000 × 15%
	=	$10,000 × 0.15

Thus, the amount of interest owing at the end of the one year term is $1,500. The total amount owing at the end of the one year term of this interest accruing loan would be the principal borrowed ($10,000) plus the interest charged ($1,500) or $11,500.

This example introduces a number of very important definitions and concepts. Financial analysts use short form abbreviations for the loan amount, interest rates and other mortgage items. In this shorthand notation, the following symbols are used:

PV	=	Present Value: the amount of principal owing at the beginning of an interest calculation period;
FV	=	Future Value: the amount of money owing in the future;
i	=	interest rate per compounding period; the fraction (or percentage) used to calculate the dollar amount of interest owing;
I	=	interest owing, in dollars, at the end of an interest calculation (compounding) period; and
n	=	number of compounding periods contracted for.

In Illustration 2, calculation of the amount of interest owing would be carried out as follows:

$$I = PV \times i$$
$$I = \$10,000 \times 15\%$$
$$I = \$10,000 \times 0.15$$
$$I = \$1,500$$

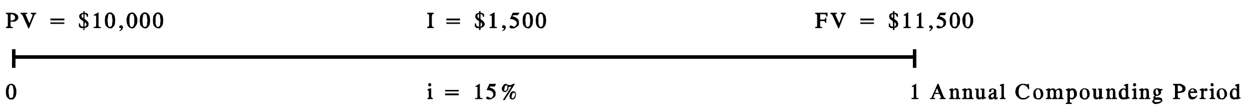
The total amount owing at the end of the one year term would be:

$$FV = PV + I$$
$$FV = \$10,000 + \$1,500$$
$$FV = \$11,500$$

Calculation

Press	Display	Comments
10000 [×] .15 [=]	1,500	Equals interest owing
[+] 10000 [=]	11,500	Equals total owing

This example can be illustrated on a time diagram:



ALERT!

Throughout the mortgage finance review, "step-by-step" instructions are presented to aid in the use of the financial calculator. In each case, it is assumed that the calculator is programmed to display a "floating decimal point." This is accomplished by turning the calculator ON, pressing the ■ then [DISP] and then the [●] key. Please do this now to avoid problems later in this supplement.

It was noted earlier that the amount owing at the end of year one could be determined by, first, calculating the amount of interest at the end of the first (annual) compounding period and, then, adding this amount to the principal borrowed. The formulae used to perform these calculations were $I = PV \times i$ and $FV = PV + I$. By using these relationships in a repetitive fashion, the amount owing at the end of the term of an interest accruing loan could be determined. However, there is a faster way to do these calculations.

In determining the amount owing at the end of the term of an interest accruing loan, the principal amount originally borrowed can be multiplied by one plus the rate of interest per compounding period (expressed as a decimal) for the number of compounding periods during the contract term. To simplify the analysis even further, standard mathematical notation can be used which represents the value of one plus the rate of interest per compounding period as $(1+i)$.

Illustration 3

Assume that the commercial borrower in Illustration 2 arranged for another loan which was similar in all respects, except that the contract specified a term of three years. Calculate the amount owing at the end of the three year term of the interest accruing loan.

Solution:

Amount owing at end of year one	=	$10,000 × 1.15	=	$11,500
Amount owing at end of year two	=	$11,500 × 1.15	=	$13,225
Amount owing at end of year three	=	$13,225 × 1.15	=	$15,208.75

A simpler calculation recognizes that for each annual compounding period the principal outstanding is multiplied by 1.15:

$$FV = \$10{,}000 \times 1.15 \times 1.15 \times 1.15$$
$$FV = \$15{,}208.75$$

A superscript indicates that a number has been raised to a power (or multiplied by itself some number of times) and the relationship may be restated as follows:

$$FV = \$10{,}000 \times (1.15)^3$$

or in more general terms:

$$FV = PV \times (1 + i)^n$$

Where FV = Future value (or amount owing in the future)
PV = Present value (or original amount borrowed)
i = interest rate per compounding period expressed as a decimal
n = number of compounding periods in the loan term

The HP 10BII calculator is able to do repetitive multiplications as outlined above; i.e, the calculator is preprogrammed for exponential calculations. The steps below show how the calculator can be used to determine the amount owing on the loan by using the exponential function.

Calculation

Press	**Display**	**Comments**
1.15 ■ [y^x] 3 [=]	1.520875	1.15 raised to the power of 3
[×] 10000 [=]	15,208.75	Total amount owed at end of year 3

The use of the exponential key [y^x] reduces the number of repetitive calculations required in analyzing interest accruing loans. One need only determine the value of $(1 + i)^n$ at the appropriate rate of interest (expressed as a decimal) and for the appropriate number of compounding periods and then multiply the result by the principal amount borrowed. Since this type of financial analysis is commonly needed in the real estate and finance industries, financial calculators have been preprogrammed with the underlying mathematical relationship developed above.

When using the HP 10BII calculator, the above formula, $FV = PV \times (1 + i)^n$, must be slightly modified to consider nominal interest rates. Recall that a periodic rate is equal to the nominal rate divided by the compounding frequency. Thus the formula becomes:

$$FV = PV \times (1 + j_m/m)^n$$

where j_m = nominal interest rate per annum
m = compounding frequency
n = number of compounding periods in the loan term

This modified version is necessary because this calculator only works with nominal interest rates. This formula for "interest accruing" loans has been preprogrammed into the mortgage finance keys of the HP 10BII calculator.

These keys are:

I/YR Nominal interest rate per year (j_m) - entered as a percent amount (not as a decimal)

■ P/YR "Periods per year" (m) - this indicates the compounding frequency of the nominal rate in I/YR and is located on the lower half of the PMT key

N Number of compounding or payment periods in the financial problem - this number will be expressed in the same frequency as P/YR (in other words, if P/YR is 12, then N will represent the number of months)

PV Present value

FV Future value after N periods

PMT Payment per period - this is expressed in the same frequency as P/YR and N (i.e. if N is months, PMT represents the payment per month)

The internal operation of the HP 10BII calculator requires that all financial calculations have at least one positive and one negative cash flow. This means that at least one of the PV, FV, and PMT keys will have to be shown as a negative amount. Generally, money invested is shown as a negative amount and money withdrawn is shown as positive. In other words, cash flowing **out** is negative, while cash flowing **in** is positive. For example, from a borrower's perspective, cash borrowed is a positive amount and cash paid back is negative. Similarly, from an investor's perspective, the initial investment is negative and the money received in the future is a positive amount. This distinction will be made clearer by the following example.

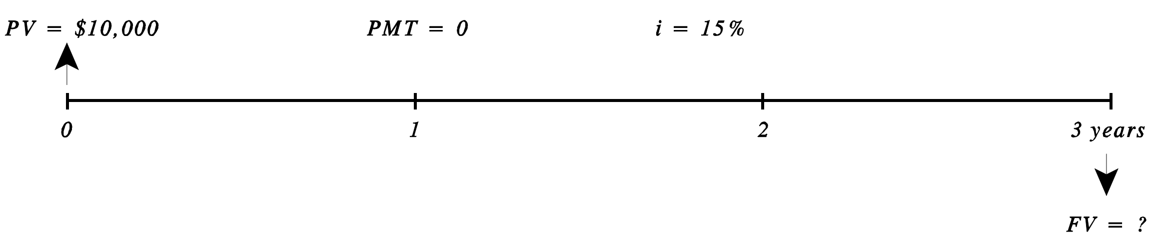


Illustration 3 is illustrated above with a "time diagram", but with a new feature added. The cash flows are placed along the horizontal line with an arrow representing positive or negative cash flows. An "up arrow" represents a positive cash flow (money received), while a "down arrow" represents a negative cash flow (money paid out).

To solve this problem with the HP 10BII calculator, a number should be entered and then "labelled" appropriately. For example, the loan in this example has a three year term, so "3" should be entered and then "N" pressed in order to enter a value of 3 as the number of compounding periods during the term. By entering a number and then labelling it, you can *enter the information in any order*.

Calculation

Press	Display	Comments
1 ■ [P/YR]	1	Enter compounding frequency
3 [N]	3	Enter number of compounding periods
15 [I/YR]	15	Enter nominal interest rate per year
0 [PMT]	0	No payments during term
10000 [PV]	10,000	Enter present value (the borrower receives the cash, so it is entered as a positive amount)
[FV]	-15,208.75	Computed future value (this will be paid out by the borrower, so it is a negative amount)

This is the same answer as that calculated with either of the two approaches shown earlier, but with much less work needed.

ALERT!

Note that if you enter an incorrect number on the screen, it can be cleared by pushing [C] once. If you enter an incorrect number into any of the six financial keys, [N] [I/YR] [PMT] [PV] [FV] [P/YR], it can be corrected by re-entering the desired number into that key. You can verify what information is stored in each of the above financial keys by pressing [RCL] and then the corresponding financial key you are interested in. For example, if you obtained an incorrect solution for the example above, you can check what is stored in [N] by pressing [RCL] [N]; [I/YR] by pressing [RCL] [I/YR], etc.

C. Equivalent Interest Rates

The basis upon which interest rate calculations are performed is stated as follows:

> *Two interest rates are said to be equivalent if, for the same amount borrowed, over the same period of time, the same amount is owed at the end of the period of time.*

One particular equivalent interest rate, the equivalent rate with annual compounding (j_1), is called the effective annual rate. By convention, the effective rate is used to standardize interest rates to allow borrowers and lenders to compare different rates on a common basis. The financial calculator also uses the effective annual interest rate to convert between equivalent nominal interest rates. A more useful variation of the above statement follows:

> *If two interest rates accumulate the same amount of interest for the same loan amount over the same period of time, they have the same effective annual interest rate. Therefore, two interest rates are said to be equivalent if they result in the same effective annual interest rate.*

Illustration 4

Assume that a bank agrees to give a loan at an interest rate of 14% per annum, compounded monthly. In order to determine the rate the bank must disclose under the Interest Act, calculate the nominal rate per annum with semi-annual compounding which is equivalent to j_{12} = 14%.

This problem can be solved using mathematical formulae, but this involves complex and time-consuming algebra. The alternative, and equally valid, approach to calculate equivalent interest rates is to use the financial keys of a business calculator. The process involves entering the nominal interest rate provided and converting it to its effective annual equivalent. Then, the desired compounding frequency is entered, which is usually the number of payment periods per year. The final step is to solve for the equivalent nominal rate with the desired compounding frequency. It is important to note that the HP 10BII works with nominal interest rates in the financial keys (some financial calculators work with periodic interest rates). To solve for a periodic rate, one must divide the nominal rate by its compounding frequency.

There are two other financial keys of the HP 10BII which have not yet been introduced, but are needed for interest rate conversion problems. These are:

■ [NOM%]	Nominal interest rate per year (j_m)
■ [EFF%]	Effective interest rate (j_1) which is calculated based on the nominal rate (j) in [NOM%] and the compounding frequency (m) entered in [P/YR]

Solution:

Enter the given nominal rate and the stated number of compounding periods per year (12, in this case). Solve for the effective annual rate (the nominal rate with annual compounding). Then, enter the desired compounding periods (2, in this case). Solve for the equivalent nominal rate. The calculator steps are as follows:

Calculation

Press	**Display**	**Comments**
14 ■ [NOM%]	14	Enter stated nominal rate[4]
12 ■ [P/YR]	12	Enter stated compounding frequency
■ [EFF%]	14.9342029207	Compute effective annual interest rate
2 ■ [P/YR]	2	Enter desired compounding frequency
■ [NOM%]	14.4147410237	Compute equivalent nominal rate with desired compounding frequency

The nominal rate per annum with semi-annual compounding equivalent to j_{12} = 14% is j_2 = 14.4147410237%. If it were necessary to calculate the periodic rate per semi-annual period, this could be done by dividing the nominal rate (j_2 = 14.4147410237%) by the number of compounding periods per year (2) to get the periodic rate (i_{sa} = 7.20737051185%).

[4] In the interest rate conversions illustrated in this supplement, the first step shown is to enter the stated nominal rate using ■ [NOM%] Students may notice that similar results can also be achieved by pressing [I/YR] alone.

Illustration 5

Assume that a bank agrees to give a loan at an interest rate of 9% per annum, compounded semi-annually. Calculate the equivalent nominal rate per annum with monthly compounding.

Solution:

Enter the given nominal rate and the stated number of compounding periods per year (2, in this case). Solve for the effective annual rate (the nominal rate with annual compounding). Then, enter the desired compounding periods (12, in this case). Solve for the equivalent nominal rate. The calculator steps are as follows:

Calculation

Press	**Display**	**Comments**
9 ■ [NOM%]	9	Enter stated nominal rate
2 ■ [P/YR]	2	Enter stated compounding frequency
■ [EFF%]	9.2025	Compute effective annual interest rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	8.83574763	Compute equivalent nominal rate with desired compounding frequency

The nominal rate per annum with monthly compounding equivalent to j_2 = 9% is j_{12} = 8.83574763%. If it were necessary to calculate the monthly periodic rate, this could be done by dividing the nominal rate (j_{12} = 8.83574763%) by the number of compounding periods per year (12) to get the periodic rate (i_{mo} = 0.7363123025%).

Practice Exercise

The following table is comprised of three columns:

(1) the first column specifies a nominal rate of interest with a given compounding frequency;
(2) the second column provides the desired compounding frequency;
(3) the third column presents an equivalent nominal interest rate with the desired frequency of compounding.

You should ensure that you are able to use the nominal rates of interest and desired frequencies of compounding shown in the first two columns to calculate the equivalent nominal interest rate given in the third column. This skill is critical to completing all of the following mortgage finance calculations!

Nominal Interest Rate	Desired number of compounding periods per annum	Equivalent nominal interest rate with desired compounding frequency
j_{12} = 12%	1	j_1 = 12.6825030132%
j_2 = 10%	12	j_{12} = 9.79781526228%
j_4 = 8%	2	j_2 = 8.08%
j_1 = 9%	365	j_{365} = 8.618787046%
j_4 = 7.5%	12	j_{12} = 7.45360726045%
j_1 = 6%	12	j_{12} = 5.84106067841%

II. FUTURE VALUE AND PRESENT VALUE FOR LUMP SUMS

One type of compound interest calculation that is frequently encountered relates to the future and present values of single payment or lump sum amounts.

A. Calculation of Future Value

As discussed earlier, the basic relationship between the present value and a future value of a lump sum, based on compound interest, can be expressed as follows:

$$FV = PV(1 + i)^n$$

Illustration 6

Assume you arrange an investment of $20,000 yielding interest at 11% per annum, compounded annually. What is the future value of this investment after 15 months?

Solution:

$$FV = PV(1 + i)^n$$

where

PV = $20,000;
j_1 = 11% (11% per annum, compounded annually);
n = 15 months; and
FV = ?

In the absence of information, it is assumed that payments are zero. Note that in this illustration, "n" is expressed in months and the interest rate is compounded annually. Therefore, the first step is to find the equivalent nominal rate, compounded monthly.

Calculation

Press	Display	Comments
11 ■ [NOM%]	11	Enter stated nominal rate
1 ■ [P/YR]	1	Enter stated compounding frequency
■ [EFF%]	11	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	10.4815125883	Compute equivalent nominal rate with desired compounding frequency.
15 [N]	15	Enter number of months
20000 [+/−] [PV]	-20,000	Enter amount of investment
0 [PMT]	0	No payments
[FV]	22,786.8198649	Computed future value

The future value of this investment after 15 months would be $22,786.82.[5]

ALERT!

When calculating monetary amounts, numbers will have to be rounded off, since it is impossible to pay or receive an amount less than one cent. When rounding "one-time only" monetary amounts (e.g. present value or future value), normal rounding rules are applied. This is the common mathematical rule which states:

- If the third decimal is 5 or greater, the number is rounded up: e.g. 8,955.436 would be rounded UP to $8,955.44 (because the third decimal is a 6).
- If the third decimal is less than 5, the number is rounded down: e.g. 8,955.433 would be rounded DOWN to $8,955.43 (because the third decimal is a 3).

However, as discussed in a later section, this rule does not apply for payments on amortizing loans. Periodic payments on amortized loans are ALWAYS rounded UP to the next higher cent, the next higher dollar, the next higher ten dollars, etc.

B. Calculation of Present Value

In order to calculate the present value of a single future value, we need to rearrange the basic relationship between the present value and a future value of a lump sum, based on compound interest. This can be expressed as follows:

$$PV = FV(1 + i)^{-n}$$

[5] Students may notice that this problem can be solved without needing to do this interest rate conversion. If the 15 months are entered into N as 1.25 years and the P/YR is entered as 1, then the I/YR can be entered as 11 and the same future value will result.

This is the normal expression for calculating the present value of a future lump sum. However, with modern calculators we need only be concerned about entering the known data and computing the unknown value.

Illustration 7

You are offered an investment that will produce $350,000 in 10 years. If you wish to earn 9% compounded semi-annually, how much should you offer to pay for the investment today?

Solution:

$PV = FV\ (1 + i)^{-n}$

where

$FV = \$350,000$
$j_2 = 9\%$ (9% per annum, compounded semi-annually)
$n = 10$ years

The solution requires you to calculate the present value based on the desired yield. In this case, the investment term is expressed in years (n) and the interest rate is compounded semi-annually. To solve, we need to calculate the equivalent nominal rate, compounded annually.

Calculation

Press	**Display**	**Comments**
9 ■ [NOM%]	9	Enter stated nominal rate
2 ■ [P/YR]	2	Enter stated compounding frequency
■ [EFF%]	9.2025	Compute equivalent effective annual rate
1 ■ [P/YR]	1	Enter desired compounding frequency
■ [NOM%]	9.2025	Compute equivalent nominal rate with desired compounding frequency.
350000 [FV]	350,000	Enter expected future value
10 [N]	10	Enter number of years
0 [PMT]	0	No payments
[PV]	-145,125.00089	Computed present value

You should offer $145,125 for the investment today.[6]

[6] As in the previous illustration, this problem can be solved without needing to do an interest rate conversion. If the 10 years are entered into N as 20 semi-annual periods and the P/YR is entered as 2, then the I/YR can be entered as 9 and the same present value will result.

III. ANNUITY CALCULATIONS

Up to this point we have been doing calculations involving only one-time lump sum cash flows. In order to do calculations involving recurring payments, we can use the PMT key on the calculator. In order to use the PMT key, payments must be in the form of an annuity. An annuity is a stream of equal payments which are spread evenly over time. An example of an annuity is the stream of payments of a constant payment mortgage, which is the most common application of the PMT key. Another example of an annuity would be monthly deposits to a bank account to accumulate some amount in the future.

ALERT!

Please be aware that the Hewlett-Packard 10BII calculator has both Begin and End modes. The Begin mode is needed for annuity due calculations, or those which require payments to be made "in advance". For example, lease payments are generally made at the beginning of each month, not at the end. On the other hand, interest payments are almost always calculated at the end of each payment period, or "not in advance". These types of calculations each require a different setting on the calculator. When your calculator is set in Begin mode, the bottom of the display screen will show "BEGIN". If "BEGIN" is not on your display screen, your calculator must be in End mode, as there is no annunciator for this mode.

In this supplement, there are no calculations which require your calculator to be in Begin mode, so your calculator should be in End mode at all times. You should not see the BEGIN annunciator on your calculator's display.

To switch between modes, press ■ BEG/END

A. Recurring Payments

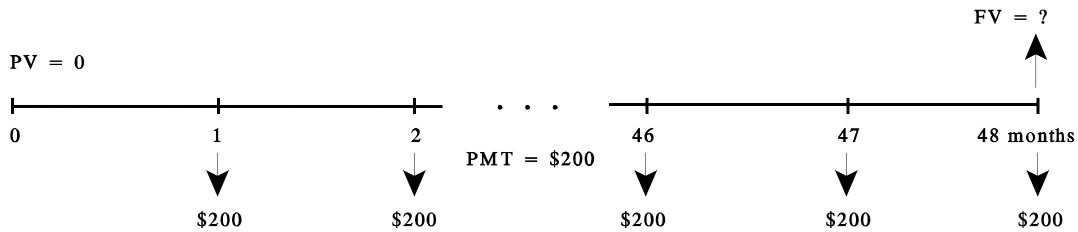
Illustration 8

An individual would like to put aside some money into a savings account to accumulate money to buy a boat. If she can put aside $200 at the end of every month, and the savings account earns interest at j_{12} *= 6%, how much money will have accumulated in the savings account by the end of the 4th year?*

Solution:

In order to calculate the amount in the savings account at the end of the 4th year, we must enter the information into the financial keys of the calculator. As before, we must enter information in all but one of the financial keys in order to calculate the final piece of information. The information given in the problem is as follows:

PV	= 0
N	= 48 compounding periods ($4 \times 12 = 48$)
j_{12}	= 6%
PMT	= $200 (paid out, so they will be negative amounts)
FV	= ? (cash received, so it will be a positive amount)



As the frequencies of payment and compounding correspond (both are monthly), this problem may be solved without needing to do an interest rate conversion. The solution is as follows:

Calculation

Press	Display	Comments
6 [I/YR]	6	Enter nominal interest rate
12 ■ [P/YR]	12	Enter compounding frequency
48 [N]	48	Enter number of payments
0 [PV]	0	No money in the account at beginning
200 [+/−] [PMT]	-200	Enter amount of payment
[FV]	10,819.5664438	Computed future value

By depositing $200 into a savings account at the end of each month for 48 months, the individual will accumulate $10,819.57 at the end of four years (48 payments).

A stream of cash flows such as the one in the above illustration, where regular payments are being set aside to accumulate money for some specific purpose in the future is known as a "sinking fund." Sinking funds are often used by businesses to accumulate money to repay a bond, or to replace worn machinery or equipment.

ALERT!

Notice that in the previous calculation, the frequency of compounding of the interest rate and the frequency of the payments matched. When using the financial keys, and the [PMT] key in particular, it is vital that the [I/YR], [N], and [PMT] keys all use the same frequency. For example, if payments were made semi-annually, the interest rate would have to be entered in the calculator as a j_2 rate ([I/YR] is a j_2, [P/YR] is 2), [N] would be the number of semi-annual payments, and [PMT] would be the amount of the semi-annual payments.

B. Calculations For Constant Payment Mortgages

The HP 10 BII financial calculator is preprogrammed to calculate loan amounts (PV), future values (FV), payments (PMT), amortization periods (N), and interest rates (I/YR). By entering any four of these variables (PV, FV, PMT, N, and I/YR), the calculator can then determine the fifth variable.

The following conditions must occur in order to use the calculator to analyze a constant payment mortgage:

(1) The present value must occur at the *beginning* of the first payment/compounding period.

(2) The payments must be equal in amount, occur at regular intervals, and be made at the *end* of each payment period.

(3) The rate of interest must be stated as, or *converted to*, a nominal rate with compounding frequency matching the payment frequency.

Illustration 9

A local trust company has been approached by a real estate investor desiring mortgage money. The investor will pay $4,000 per month over a 15 year period. What size of loan will the trust company advance if it desires a yield (or interest rate) of $j_2 = 14\%$*?*

Solution:

When a financial arrangement has a different frequency of compounding and payment, it is necessary to convert the given nominal rate of interest with the stated compounding frequency to an equivalent nominal interest rate for which the compounding frequency matches the payment frequency. In the above illustration, the lender demands a return on investment of 14% per annum, compounded semi-annually. The borrower, on the other hand, is making payments on a monthly basis.

The first step to solve for the maximum allowable loan amount involves calculating the nominal rate of interest with monthly compounding that is equivalent to $j_2 = 14\%$.

Calculation

Press	**Display**	**Comments**
14 ■ [NOM%]	14	Enter stated nominal rate
2 ■ [P/YR]	2	Enter stated compounding frequency
■ [EFF%]	14.49	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	13.6083121618	Compute equivalent nominal rate with desired compounding frequency.

The borrower will make 180 monthly payments (15 years × 12 payments per year) of $4,000, and the rate of interest is 13.6083121618% per annum, compounded monthly. Since the rate of $j_{12} = 13.6083121618\%$ is already entered as the nominal interest rate with monthly compounding, it does not have to be entered again. Equivalent interest rates should *not* be "keyed" into the calculator. Instead, they should be calculated and used directly to avoid errors in re-entering the number.

After determining the nominal rate, the maximum loan amount would be calculated as follows:

Calculation (cont.)

Press	Display	Comments
	13.6083121618	j_{12} rate displayed from previous calculation
4000 [+/−] [PMT]	-4,000	Payment per month
15 × 12 = [N]	180	Number of monthly payments
0 [FV]	0	Indicates to calculator FV is not to be used (because all of the loan is totally repaid at the end of 180 months)
[PV]	306,389.050169	Present value or loan amount

The lender, desiring to earn 14% per annum, compounded semi-annually, would be willing to advance $306,389.05 in exchange for the borrower's promise to pay $4,000 per month for 180 months.

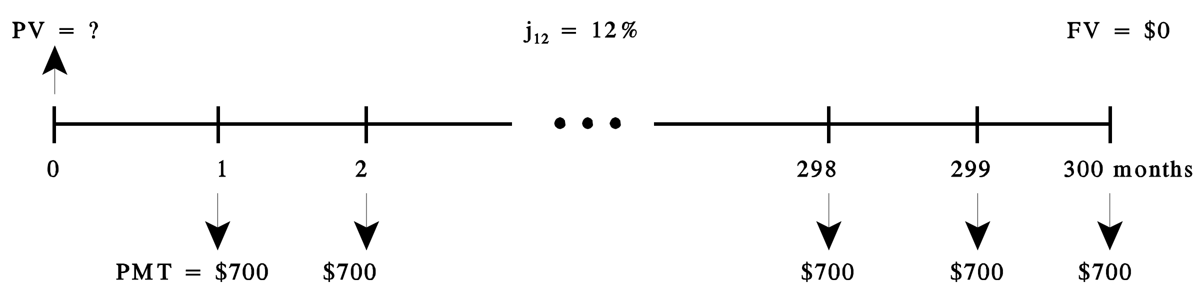
Illustration 10

An individual is thinking of buying a residential condominium but wants to limit mortgage payments to $700 per month. If mortgage rates are 12% per annum, compounded monthly, and the lender will permit monthly payments to be made over a 25-year amortization period, determine the maximum allowable loan.

Solution:

The financial terms of the proposed loan may be summarized as follows:

PV = ?
PMT = $700.00 (per month)
N = 25 × 12 = 300 (months)
j_{12} = 12%



As the frequencies of payment and compounding correspond (both are monthly), the problem may be solved directly as follows:

Calculation

Press	Display	Comments
700 [+/−] [PMT]	-700	Monthly payment[7]
25 × 12 = [N]	300	Months in amortization period
0 [FV]	0	This calculation will not use a future value amount so zero must be entered[8]
12 [I/YR]	12	Nominal rate with monthly compounding
12 ■ [P/YR]	12	Number of payments per year
[PV]	66,462.5858784	Present value (or loan amount)

The maximum loan based on the interest rate, payments and amortization period specified, is $66,462.59.

Illustration 11

If the loan above called for interest at the rate of 15% per annum, compounded monthly, determine the maximum loan amount.

Solution:

N = 300; j_{12} = 15%; PMT = $700; PV = ?
Because PMT, N, P/YR, and FV are already stored and do not require revision, the calculation is:

Calculation

Press	Display	Comments
15 [I/YR]	15	Nominal rate with monthly compounding
[PV]	54,652.0354807	Loan amount at j_{12} = 15%

Thus, increasing the interest rate from j_{12} = 12% to j_{12} = 15% has the effect of decreasing the maximum allowable loan by almost $12,000 (from $66,462.59 to $54,652.04).

---

[7] Most of the calculations in the remainder of this supplement are for mortgage loans. In these problems the borrower receives loan funds at the beginning of the loan term (cash in, so a positive amount) and makes periodic payments during the loan term and an outstanding balance payment at the end of the loan term (cash out, so negative amounts). In these examples, PV will be shown as positive, while PMT and FV will be shown as negatives.

[8] A future value amount is not used in this problem because at the end of 300 months the entire principal amount (or outstanding balance) has been repaid, making the future value of the loan zero.

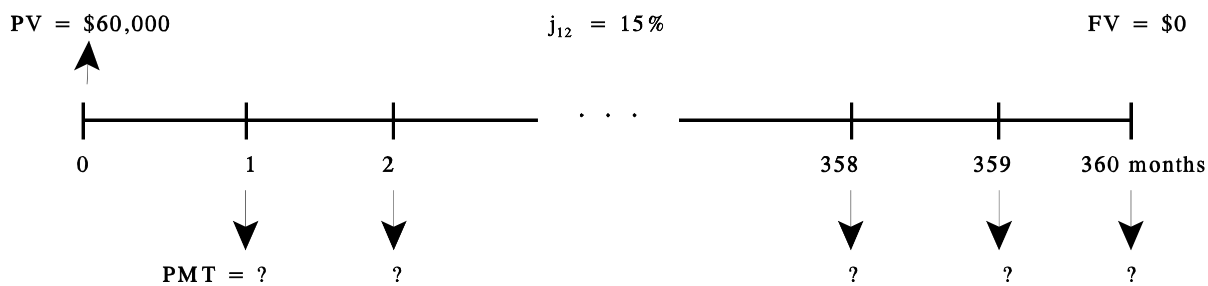
From the preceding examples, it is clear that the rate of interest charged on a loan has a large impact on the size of the loan a *fixed* series of payments will support. With constant payment mortgage loans, a large portion of each of the early payments is allocated to the payment of interest. Increased interest rates reduce the amount of each payment available for principal repayment, making a very large impact on an individual's ability to borrow a given amount. These examples are assuming there are no other borrower qualifications which is not typically the case in mortgage lending.

Illustration 12

A mortgage loan for $60,000 is to be repaid by equal monthly payments over a 30-year period. The interest rate is 15% per annum, compounded monthly. Calculate the size of the required monthly payments.

Solution:

j_{12} = 15% N = 12 × 30 = 360 PV = $60,000.00 PMT = ?



Calculation

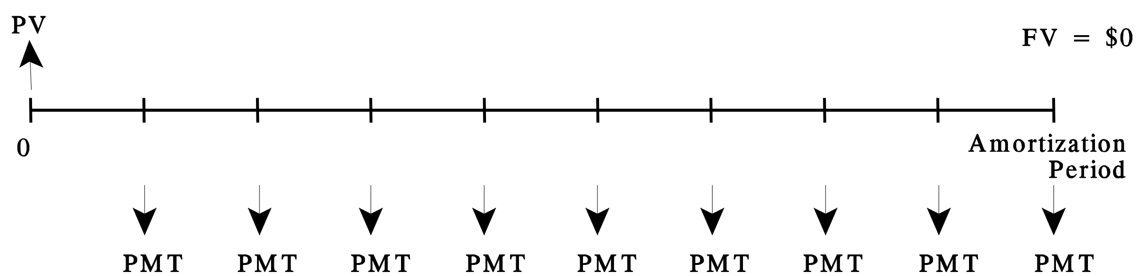
Press	Display	Comments
12 ■ [P/YR]	12	Enter payment frequency
15 [I/YR]	15	Enter nominal rate compounded monthly (same as compounding frequency, so no conversion needed)
30 × 12 = [N]	360	Enter amortization period in months
60000 [PV]	60,000	Enter present value
0 [FV]	0	Indicates that FV will not be used (loan is fully repaid at the end of 360 months)
[PMT]	-758.666412939	Size of required monthly payments

The calculated monthly payments are $758.666412939. Since borrowers cannot make payments which involve fractions of cents, the payments must be rounded to at least the next higher cent. It is standard practice, on a series of level payments, to round the payment amount *up* to the next higher cent regardless of which cent the payment is closer to (i.e. $78.422 would be rounded up to $78.43 even though it is closer to $78.42). Therefore, the payments on this loan would be $758.67 when rounded to the next higher cent. This rounding shortens the actual amortization period of the loan slightly, which results in a reduced number of payments required and/or more typically a final payment on the loan which is smaller than the other payments. Loan payments can be rounded up to any amount specified in a loan contract, for example to the next higher dollar, ten dollars, or even hundred dollars. **The most important point is that the payment must *always* be rounded *up*. In most cases, loan payments are rounded up to the next higher cent.**

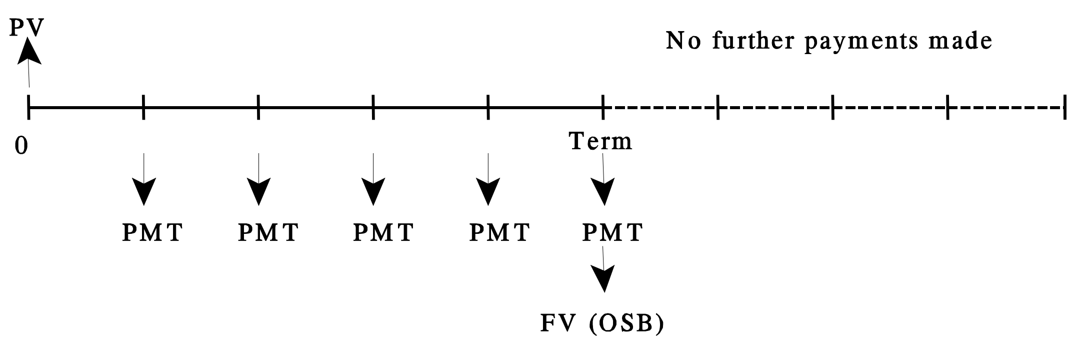
IV. CALCULATION OF AN OUTSTANDING BALANCE (OSB)

It is important to know how to calculate the outstanding balance of a loan, or the amount of principal owing at a specific point in time, for several reasons. Most vendors wish to know how much they will receive from the sale of their property after they have repaid the outstanding balance on their mortgage. While mortgage payments are calculated using the amortization period, the actual length of the mortgage contract may be different than the amortization period. The length of the mortgage contract is called the term. If the mortgage term and amortization period are the same length of time, the mortgage is said to be fully amortized. If the mortgage term is shorter than the amortization period, the mortgage is said to be partially amortized. Since mortgages are typically partially amortized with one to five year contractual terms, the amount of money which the borrower owes the lender when the contract expires must be calculated.

Payments are first calculated based on the full amortization period:



Outstanding Balance is calculated at the end of the loan term:



As shown below, the outstanding balance can be calculated quickly on your calculator.

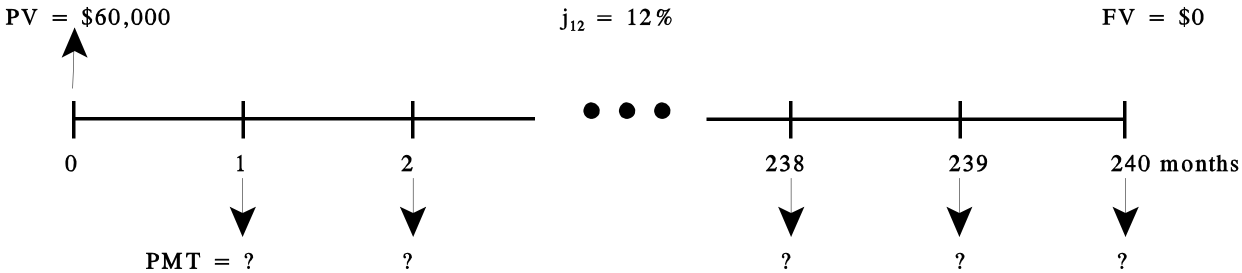
Illustration 13

A $60,000 mortgage loan, written at $j_{12} = 12\%$*, has a 3 year term and monthly payments based upon a 20 year amortization period. Payments were rounded up to the next higher cent. What is the outstanding balance of the mortgage at the end of its term? i.e., what is the outstanding balance just after the 36th payment (*OSB_{36}*) has been made?*

Solution:

(i) Calculate the size of the required monthly payments:

Amortization Period = 20 × 12 = 240 months
PV = $60,000 ; j_{12} = 12%

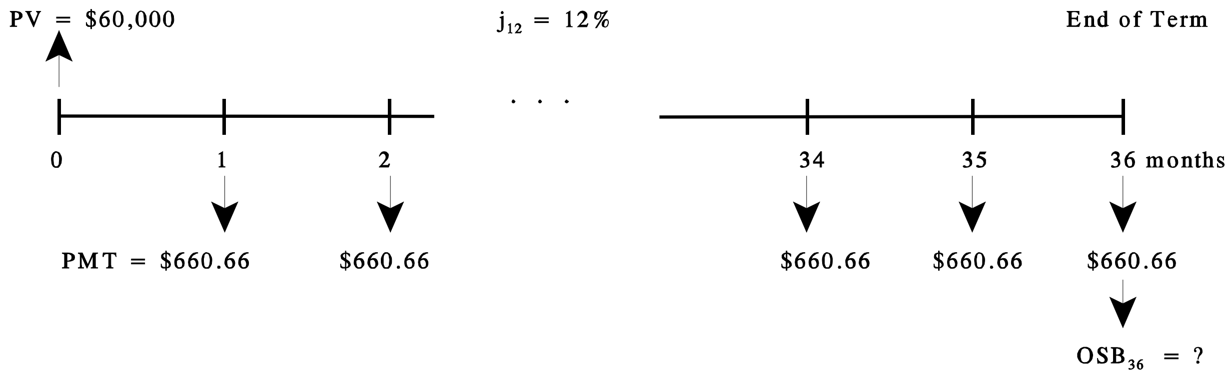


Calculation

Press	**Display**	**Comments**
12 I/YR	12	Enter nominal rate with monthly compounding (no conversion needed)
12 ■ P/YR	12	Enter compounding frequency
60000 PV	60,000	Enter loan amount
20 × 12 = N	240	Enter number of payments
0 FV	0	240 monthly payments would fully amortize loan
PMT	-660.651680142	Calculate payment

Actual payments rounded up to the next higher cent = $660.66 per month.

(ii) Calculate the outstanding balance due immediately after (with) the 36th monthly payment:



The HP 10BII calculator has a preprogrammed function that calculates outstanding balances. Before the outstanding balance can be calculated, the rounded payment must be entered into the calculator. The $660.66 payment is slightly higher than the $660.651680142 payment required to repay the loan as calculated using the initial information. Since the loan amount and interest rate are set by contract, the increase in the size of the payment causes more principal to be repaid in each month than is required to amortize the loan. This results in a faster repayment of the loan amount and, consequently, reduces the number of full payments needed to amortize the loan. When the new (larger) payment is entered, the preprogrammed function of your calculator revises the amortization period as part of the outstanding balance calculation:

Calculation (cont.)

Press	**Display**	**Comments**
	-660.651680142	Payment from previous calculation (not yet rounded up)
660.66 [+/-] [PMT]	-660.66	Enter rounded payment
[N]	239.987480651	Recompute amortization period given higher payment[9]
36 [INPUT] ■ [AMORT]	PEr 36 – 36	
= = =	57,386.9565587	Outstanding Balance after 36th monthly payment

If this loan was fully amortized, it would take 239.987480651 months to pay off the loan: 239 payments of $660.66 plus a 240th payment which is less than $660.66. However, this loan is partially amortized and the borrower will instead make 36 monthly payments of $660.66 and pay the remaining balance of $57,386.96 at the end of the 36th month.

Readers may have noticed that while pressing the "=" sign three times, three different numbers appeared on the screen. The first number that appears is the principal paid in the 36th payment, the second number is the interest paid in the 36th payment, and the final number is the outstanding balance owing immediately after the 36th payment. These functions will be explained further in the next section.

[9] This step is included for illustrative purposes and is not required in outstanding balance calculations. Recomputing the amortization period will not affect further calculations such as the outstanding balance, as long as the rounded payment has been re-entered.

V. CALCULATION OF PRINCIPAL AND INTEREST COMPONENTS OF PAYMENTS

In addition to the outstanding balance, it is often necessary to calculate the principal and interest components of payments on constant payment mortgages. These calculations are important because interest on payments can sometimes be deducted as an expense for income tax purposes. As well, borrowers like to know how much principal they have paid off in a single payment or over a series of payments. The calculation of principal and interest components of payments is done using the same keys on the calculator as the outstanding balance calculation shown above. This function will be explained using the following illustration.

Illustration 14

Three years ago, Tom and Nancy bought a house with a mortgage loan of $175,000, written at j_2 = 9.5%, with a 25 year amortization, monthly payments rounded up to the next higher dollar, and a three year term. Tom and Nancy are about to make their 36th monthly payment, the last one in the loan's term, and want to know the following information:

(a) *How much principal will they be paying off with their 36th payment?*
(b) *How much interest will they be paying with their 36th payment?*
(c) *What will be the amount they will have to refinance after the 36th payment? (What will be the outstanding balance immediately following the 36th payment?)*
(d) *How much principal did they pay off during the three year term?*
(e) *How much interest did they pay over the entire three year term?*

Solution:

In order to answer any of these questions, it is necessary to first find the monthly payments under the mortgage. This is done in the following calculation.

Calculation

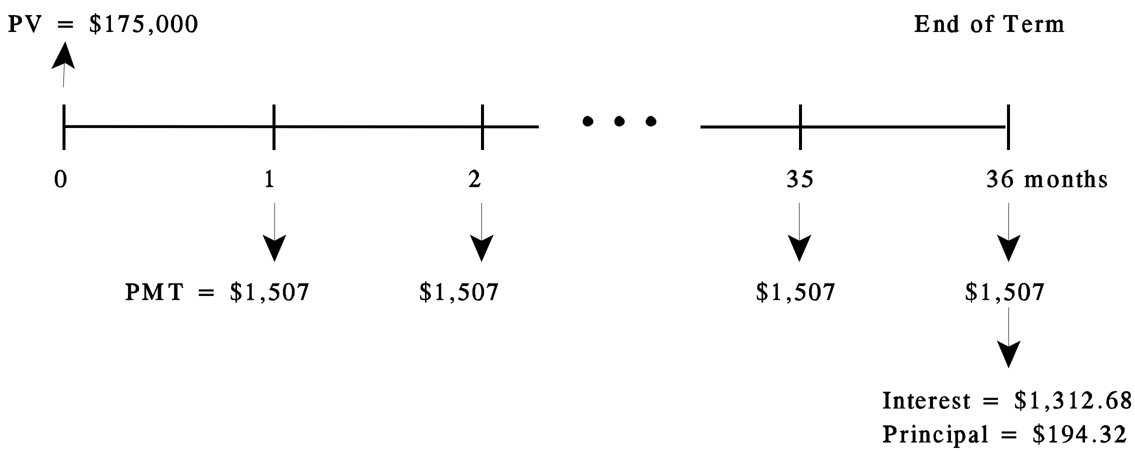
Press	**Display**	**Comments**
9.5 ■ [NOM%]	9.5	Enter stated nominal rate
2 ■ [P/YR]	2	Enter stated compounding frequency
■ [EFF%]	9.725625	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	9.31725980166	Compute nominal rate with monthly compounding
175000 [PV]	175,000	Enter present value
25 × 12 = [N]	300	Enter amortization period in months
0 [FV]	0	Payment is calculated to fully amortize loan
[PMT]	-1,506.79835525	Compute the monthly payment
1507 [+/−] [PMT]	-1,507	Enter rounded payment (to the next higher dollar)

The above calculation shows that the monthly payment on the loan will be $1,507. The answers to questions (a), (b), and (c) can all be found using the calculator's preprogrammed amortization function as in the following calculation.

Calculation (cont.)

Press	**Display**	**Comments**
	-1,507	Payment displayed from previous calculation
36 [INPUT] ■ [AMORT]	PEr 36 – 36	
=	-194.316609109	Principal portion of 36th payment
=	-1,312.68339089	Interest portion of 36th payment
=	168,870.41944	Outstanding balance immediately following 36th payment (OSB_{36})

From the above calculation, questions (a), (b), and (c) can be answered. The amount of interest paid in the 36th payment is $1,312.68. The amount of principal paid off in the 36th payment is $194.32. The outstanding balance immediately following the 36th payment (OSB_{36}) is $168,870.42. As expected, the principal paid and interest paid in the 36th payment total $1,507, which is the amount of the monthly payment.

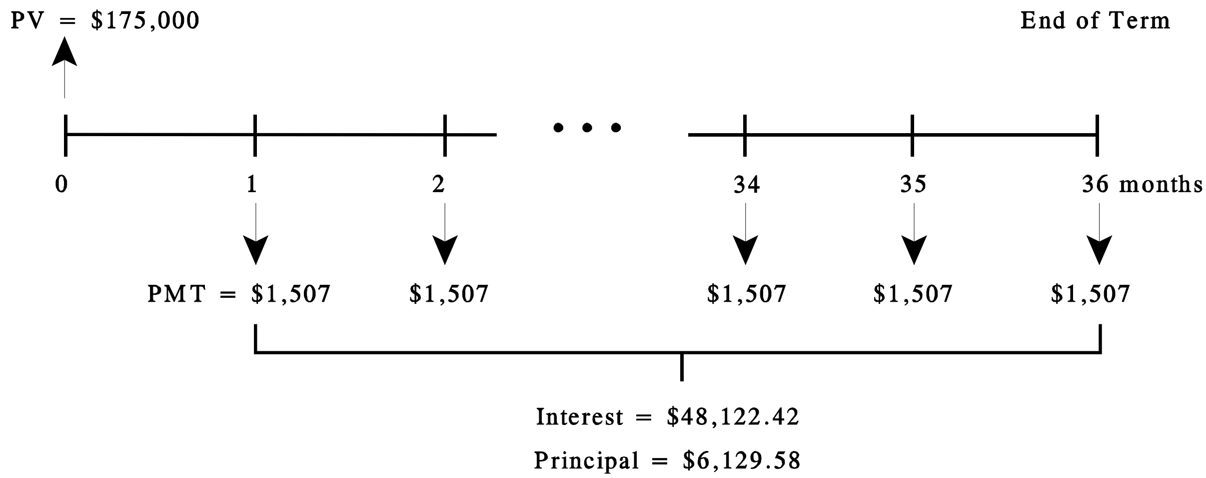


The same preprogrammed amortization function can be used to find the amounts of principal and interest paid over a series of payments, as in questions (d) and (e). In this case, the series of payments will be the entire 36 month loan term, although the amortization function can be used over any series of payments. Questions (d) and (e) can be answered using the following calculation.

Calculation (cont.)

Press	**Display**	**Comments**
1 [INPUT] 36	36	Enter desired series of payments
■ [AMORT]	PEr 1 – 36	Series of payments being amortized
=	-6,129.58055995	Total principal paid off in payments 1 - 36
=	-48,122.41944	Total interest paid in payments 1 - 36
=	168,870.41944	OSB_{36}

The answer to questions (d) and (e) are given by the above calculation. The total amount of interest paid over the term of the loan was $48,122.42. The total amount of principal paid off during the loan term was $6,129.58.



REVIEW PROBLEMS

1. Convert each of the following interest rates to the nominal or periodic interest rate requested.

(a) j_2 = 13% i_{sa} = ____________
(b) j_{12} = 16% i_{mo} = ____________
(c) j_4 = 12.5% i_q = ____________
(d) j_{365} = 14% i_d = ____________
(e) j_{52} = 6% i_w = ____________
(f) i_a = 9.5% j_1 = ____________
(g) i_d = 0.03% j_{365} = ____________
(h) i_{mo} = 1.692% j_{12} = ____________
(i) i_{sa} = 4.8% j_2 = ____________
(j) i_w = 0.18% j_{52} = ____________

2. The table below represents investment data collected by an investor for several properties he is considering buying. For each property, the investor has identified three important financial items. Assist the investor by calculating the missing piece of information for each property. Assume in each case that no payments are made (or costs incurred) during the investment horizon and ignore any possible income tax implications.

Property	Purchase Price (PV)	Estimated Future Value (FV)	Holding Period (N)	Interest Rate Earned (j_m)
(a)	$100,000.00	$185,000.00	6 years	? (j_1)
(b)	$100,000.00	$185,000.00	6 years	? (j_{12})
(c)	$ 20,000.00	?	24 months	j_{12} = 15%
(d)	?	$187,921.00	5 years	j_1 = 24%
(e)	$ 55,000.00	$145,931.37	? (years)	j_4 = 20%
(f)	$ 1,600.00	$139,294.08	? (years)	j_{12} = 18%
(g)	$ 27,500.00	$ 27,500.00	5 years	? (j_{52})
(h)	$ 10,000.00	$ 20,000.00	1 year	? (j_1)
(i)	$ 10,000.00	$ 20,000.00	2 years	? (j_1)
(j)	$ 55,179.00	?	20 years	j_{12} = 12%
(k)	$ 55,179.00	?	20 years	j_{12} = 24%

3. Calculate the missing piece of information for each of the following:

Loan	Purchase Price (PV)	Estimated Future Value (FV)	Payment (PMT)	Holding Period (N)	Interest Rate Paid (j_m)
(a)	$ 3,576.24	?	$ 0	6 years	j_1 = 13%
(b)	$ 4,775.00	$ 9,061.57	$ 0	5 years	? (j_1)
(c)	?	$ 0.00	$ 200	18 months	j_{12} = 16.5701%
(d)	$ 1,450.00	$ 17,980.00	$ 0	? (semi-annual periods)	j_2 = 11.8087582%
(e)	$ 895.75	$ 5,000.00	$ 0	300 months	? (j_{12})
(f)	?	$ 2,023.73	$ 0	22 quarters	j_4 = 10.5%
(g)	$ 650.23	?	$ 0	3 days	j_{365} = 16.5%
(h)	$ 0.00	?	$ 1,500	520 weeks	j_{52} = 13.25%

4. (a) Your client is considering the purchase of a property at the listed price of $179,000. She wishes to earn a minimum of 25.5% per annum, compounded annually. Assume that revenues will equal costs during the holding period. For what minimum price must this property be sold at the end of the five years in order to realize the 25.5% yield?

   (b) If you advise your client the selling price will more likely be $450,000 at the end of five years, what is the maximum price she should pay today in order to still be able to earn the 25.5% yield?

   (c) If the client were to sell the property at the end of four years for $450,000, what would be her yield if she paid the listed price?

   (d) If this same property could be sold in two parcels, one in two years' time at $150,000 and in five years' time at $200,000 - and the required yield was 27.25%, what is the maximum price that the investor should pay for this property?

5. Colleen has borrowed $10,000 from Lifetime Trust Company. The loan is interest accruing on which interest is to be charged at a rate of 16% per annum, compounded quarterly (or 4% charged at the end of each 3 month period). How much will Colleen owe at the end of the 5 year term?

6. Consider an interest-only mortgage of $100,000 with an interest rate of 10% per year, compounded monthly (i.e. 0.8333333% per month), a term of 10 years and monthly payments. Round payments up to the next higher cent.

   (a) How much principal is outstanding after the first year (i.e. after 12 monthly payments)?

   (b) What is the 13th monthly payment?

   (c) What is the 17th monthly payment?

   (d) How much principal is owing after the 18th monthly payment?

7. Mick Bumner wants to buy a boat and spend the rest of his days sailing the South Pacific with his new bride. He figures he can set aside $500 per month towards this goal and that he will need to accumulate $200,000.

   (a) If he can earn interest of 16% per annum, compounded monthly, on his monthly deposits, will he have accumulated enough to go at the end of 5 years?

   (b) If he decides he wants to go in 7 years, how much will he have to deposit per month?

8. Convert each of the following interest rates to an equivalent monthly rate.

	Equivalent j_{12} Rate
(a) j_2 = 13%	___________
(b) j_2 = 16%	___________
(c) j_2 = 12.5%	___________
(d) j_2 = 14%	___________
(e) j_2 = 20 1/8%	___________

9. Convert each of the following interest rates to the equivalent periodic rate requested.

(a) j_2	= 11%	Equivalent nominal rate with daily compounding =	___________
		Equivalent daily periodic rate =	___________
(b) j_2	= 19.25%	Equivalent nominal rate with annual compounding =	___________
		Equivalent annual periodic rate =	___________
(c) j_2	= 10%	Equivalent nominal rate with weekly compounding =	___________
		Equivalent weekly periodic rate =	___________
(d) j_4	= 15.6%	Equivalent nominal rate with monthly compounding =	___________
		Equivalent monthly periodic rate =	___________
(e) j_{12}	= 14.64%	Equivalent nominal rate with monthly compounding =	___________
		Equivalent monthly periodic rate =	___________
(f) j_2	= 12.8%	Equivalent nominal rate with daily compounding =	___________
		Equivalent daily periodic rate =	___________
(g) j_{12}	= 13%	Equivalent nominal rate with daily compounding =	___________
		Equivalent daily periodic rate =	___________
(h) j_4	= 15%	Equivalent nominal rate with monthly compounding =	___________
		Equivalent monthly periodic rate =	___________
(i) j_{365}	= 10%	Equivalent nominal rate with annual compounding =	___________
		Equivalent annual periodic rate =	___________
(j) j_2	= 19%	Equivalent nominal rate with annual compounding =	___________
		Equivalent annual periodic rate =	___________

10. Calculate the required monthly payment for each of the following loans. (All calculations are to be based on a 25-year amortization period).

	Loan Amount	Interest Rate	Monthly Payment
(a)	$ 110,000	j_2 = 19.25%	_________
(b)	$ 6,500	j_2 = 18%	_________
(c)	$ 37,588	j_2 = 17.5%	_________
(d)	$ 68,275	j_2 = 15%	_________
(e)	$ 55,000	j_2 = 16.5%	_________

11. Complete the following table:

	Mortgage Amount	j_2	Amortization Period (Years)	Monthly Payment
(a)	$ 105,000.00	19%	25	_______
(b)	$ 42,000.00	20.5%	10	_______
(c)	$ 72,500.00	14¼%	30	_______
(d)	$ 72,712.20	12%	__	$ 786.00
(e)	$ 12,250.00	17.2%	__	$ 284.35
(f)	_________	15.5%	12	$ 250.00
(g)	_________	13⅛%	20	$ 1,800.00
(h)	$ 125,575.00	____	25	$ 2,000.00
(i)	$ 8,750.00	____	8	$ 175.00

12. Calculate the outstanding balance at the end of the term for each of the following loans. (All calculations are to be based on monthly payments, a 25-year amortization period, and a 5-year term.)

	Loan Amount	Interest Rate	Outstanding Balance (OSB_{60})
(a)	$ 4,200	j_2 = 13.25%	_________
(b)	$75,000	j_2 = 10.5%	_________
(c)	$83,975	j_2 = 12%	_________
(d)	$92,100	j_2 = 19%	_________
(e)	$18,400	j_2 = 9%	_________

13. Ted Jones obtained a $12,000 second mortgage at an interest rate of 17% per annum, compounded semi-annually. Monthly payments were to be amortized over 20 years, but the outstanding balance was to be paid in full at the end of 5 years. Calculate the outstanding balance at the end of the term if:

(a) Payments are rounded up to the next higher dollar
(b) Payments are $175.00 per month
(c) Payments are $180.00 per month

14. A vendor has agreed to provide private financing to a purchaser to help the sale of the property. Under this mortgage, the vendor will lend the purchaser $80,000, with a nominal interest rate of 9% per annum, compounded semi-annually. The required monthly payments are $711.36. What is the exact amortization period of this loan?

15. Dan Davidson feels he can afford $2,000 per month in mortgage payments. He inquires at the River Bank about a mortgage loan in the amount of $117,500. The bank tells him that their current rate is 11.75% per annum compounded semi-annually not in advance on five-year term mortgage loans, amortized over 25 years.
    (a) What is the minimum required loan payment on the bank's terms?
    (b) What will the outstanding balance be at the end of the term?
    (c) If the River Bank allows Dan to make payments of $2,000 per month (instead of the payment calculated in Part (a)), what will the outstanding balance be at the end of the term?
    (d) If Dan made a prepayment of $10,000 at the end of the second year and payments of $2,000 per month for the full five years, what would his outstanding balance be at the end of the term? You may presume that his contract has no penalty for prepayment. (Hint: Find OSB_{24}, deduct $10,000, enter result as new loan amount (PV), then find OSB_{36})

16. Sally can afford a maximum of $850 per month in mortgage payments. She has approached two lending institutions.
    (a) Joe's Bank is willing to lend mortgage funds to be fully amortized by monthly payments over a 20-year amortization period at an interest rate of 14.25% per annum, compounded semi-annually. Given the maximum payment Sally can afford, what is the maximum loan she could receive from Joe's Bank?
    (b) Sam's Credit Union will lend funds at an interest rate of 14% per annum, compounded monthly, to be fully amortized by monthly payments over a 30-year amortization period. What is the maximum loan amount which Sam's Credit Union could offer Sally?

17. A borrower has arranged a mortgage loan in the amount of $75,000 with a 30-year amortization period and a three-year term. The interest rate on the loan is 12.5% per annum compounded semi-annually. Monthly payments are to be rounded up to the next higher dollar.

    Calculate:
    (a) The required monthly payment.
    (b) The outstanding balance at the end of the term.

18. A borrower has arranged for a $64,500 mortgage loan. The interest rate will be 12.375% per annum, compounded quarterly, the payments will be made monthly and rounded to the next higher cent and the amortization period will be 20 years.

    Calculate the outstanding balance if the loan has:
    (a) a two-year term
    (b) a four-year term
    (c) a five-year term

19. Erin Baxter took out a mortgage loan which had a five-year term. She borrowed $63,500 at an interest rate of 9.25% per annum, compounded semi-annually; made monthly payments based on a 25-year

amortization; and made each payment on its due date. Calculate the total interest paid and the total principal paid off over the five year term.

SOLUTIONS

1.
(a) i_{sa} = 6.5%
(b) i_{mo} = 1.33333333333%
(c) i_q = 3.125%
(d) i_d = 0.0383561644%
(e) i_w = 0.115384615%
(f) j_1 = 9.5%
(g) j_{365} = 10.95%
(h) j_{12} = 20.304%
(i) j_2 = 9.6%
(j) j_{52} = 9.36%

2.
(a) j_1 = 10.7971581852%
(b) j_{12} = 10.2970214782%
(c) FV = $26,947.02
(d) PV = $64,101.31
(e) N = 20 quarters = 5 years
(f) N = 300 months = 25 years
(g) j_{52} = 0%
(h) j_1 = 100%
(i) j_1 = 41.4213562373%
(j) FV = $601,040.22
(k) FV = $6,394,624.52

3.
(a) FV = $7,445.56
(b) j_1 = 13.6700405061%
(c) PV = $3,168.26
(d) N = 43.8879853851 semi-annual periods
(e) j_{12} = 6.89787698794%
(f) PV = $1,144.41
(g) FV = $651.11
(h) FV = $1,622,312.00

4.
(a) $557,277.69
(b) $144,541.94
(c) 25.9185954395%
(d) $152,578.48

5. $21,911.23

6.
(a) $100,000.00
(b) $833.34
(c) $833.34
(d) $100,000.00

7.
(a) No, he will only have $45,517.76
(b) $1,305.75 per month

8.
(a) 12.6612887764%
(b) 15.4913483562%
(c) 12.1863869431%
(d) 13.6083121618%
(e) 19.3296698983%

9.
(a) j_{365} = 10.7097242865%
i_d = .0293417104%
(b) j_1 = 20.17640625%
i_a = 20.17640625%
(c) j_{52} = 9.76719425636%
i_w = .187830659%
(d) j_{12} = 15.4014829273%
i_{mo} = 1.28345691061%
(e) j_{12} = 14.64%
i_{mo} = 1.22%
(f) j_{365} = 12.4091871295%
i_d = .033997773%
(g) j_{365} = 12.9323783072%
i_d = .0354311734%
(h) j_{12} = 14.8163112033%
i_{mo} = 1.23469260028%
(i) j_1 = 10.5155781616%
i_a = 10.5155781616%
(j) j_1 = 19.9025%
i_a = 19.9025%

10.
(a) $ 1,715.05
(b) $ 95.32
(c) $ 537.29
(d) $ 850.81
(e) $ 745.66

11.
(a) $1,617.58
(b) $802.67
(c) $850.12
(d) 20.000003779
(e) 5.49979326298
(f) $16,641.26
(g) $155,718.89
(h) 19.7008945984%
(i) 19.1793277426%

12.
(a) $ 4,040.57
(b) $70,794.51
(c) $80,162.48
(d) $90,627.51
(e) $17,133.18

13. (a) $11,380.47
    (b) $11,012.02
    (c) $10,551.45

14. N = 239.992246177

15. (a) $ 1,191.98
    (b) $111,971.77
    (c) $ 46,905.73
    (d) $ 32,820.60

16. (a) $68,979.93
    (b) $71,737.72

17. (a) $783.00
    (b) $74,077.64

18. (a) $62,795.19
    (b) $60,619.80
    (c) $59,316.36

19. Principal Paid Off = $4,226.61;
    Interest Paid = $27,947.19

MORTGAGE FINANCING TECHNIQUES - Part II

I. MARKET VALUE OF AN OFFER TO PURCHASE

A vendor who receives an offer which requests that the purchaser be permitted to spread the payment of the purchase price over a period of time, is being asked to act as a lender. If such an offer to purchase involves financing below (or, as happens occasionally, above) the current or "market" rate of interest, the offer will actually be worth an amount less than (or greater than) the stated offer price. Before accepting an offer requiring the provision of financing at a rate of interest other than the "market" rate, the vendor should be aware of the *cash-equivalent* price of that offer; i.e., the amount of cash which could be accepted in lieu of the offer.

The method used to calculate the cash-equivalent price of an offer is to add the amount of the down payment to the market value of the mortgage(s):

Downpayment Amount
\+ Market Value of Mortgage
= Market Value of the Offer (or Cash-Equivalent Price of the Offer)

The obvious question to ask is "what is the market value of a mortgage?" The market value of a mortgage is the present value of the future mortgage payments calculated at the market rate of interest.

When a vendor accepts an offer to purchase which contains a vendor-supplied mortgage, the borrower (purchaser) agrees to make a series of mortgage payments. The amount of these payments will be governed by the contract signed between the two parties and will be calculated at the rate of interest stated in their contract (the *contract* rate).

The value of this mortgage can then be calculated by determining what amount of money an outsider to the transaction would pay, today, in order to purchase the mortgage contract from the vendor and thereby own the right to receive the future mortgage payments from the purchaser. The cash the vendor would receive from selling the mortgage contract to such an outside investor is the cash-equivalent, or the market value, of the mortgage.

However, the outside investor has choices about where to invest his money and, in order to be persuaded to purchase (invest in) the mortgage under consideration, the investor must earn the *market* rate of interest; otherwise, these funds will be used to originate a new mortgage with a different borrower. Because the dollar value and number of future payments the investor would purchase is set by the original contract, the market rate must be applied to these set payments to calculate the maximum *present value* the investor would pay. This process is referred to in financial language as "*discounting* the stream of payments at the market rate of interest."

As you will see from examples in this supplement, the relationship between a stated offer price and its market value (or cash-equivalent price) can be summarized as shown below:

Table 1
Comparison of the Market Value of an Offer and Its Stated Price

Offer Involving Below Market Rate Financing

If the contract interest rate on the mortgage is less than the market rate, then the market value of the offer will be less than the stated offer price.

Offer Involving Market Rate Financing

If the contract interest rate on the mortgage is the same as the market rate, then the market value of the offer will be the same as the stated offer price.

Offer Involving Above Market Rate Financing

If the contract interest rate on the mortgage is greater than the market rate, then the market value of the offer will be greater than the stated offer price.

A. Vendor-Supplied Mortgage

Illustration 1

Assume that a prospective vendor listed his property for sale at $245,000, and indicated he might provide some financing to a "qualified purchaser".

Several days later, his real estate agent received a telephone call from a prospective purchaser who wished to view the property immediately. A viewing was arranged and the result was a "full price" offer to purchase the property for $245,000, subject to the vendor taking back a first mortgage for $165,000 at 7% per annum, compounded semi-annually, fully amortized with monthly payments over 25 years. The agent contacted her principal and presented the offer. However, the agent suggested that the offer not be accepted and that it be countered with a similar proposal, except that the mortgage be partially amortized with a three-year term and a 25-year amortization period.

Upon hearing the details of the offer and the agent's advice to counter offer rather than accept, the vendor was confused. The offer was for the full price, there was a large downpayment of $80,000, and the prospective purchaser's income and credit rating were also acceptable. The vendor, therefore, asked the agent to explain her reasons for suggesting the counter offer rather than accepting the purchaser's offer. The agent responded with the following detailed analysis:

Analysis of Market Value of the Offer to Purchase (Fully Amortized Loan)

***Proposed Offer Price*:**

	Amount Offered:	$ 245,000.00
−	Cash Down Payment:	−$ 80,000.00
	Vendor Mortgage:	$ 165,000.00

***Terms of Proposed Vendor Mortgage*:**

Face Value:	$165,000.00
Interest Rate:	j_2 = 7%
Amortization:	25 years
Contractual Term:	25 years
Payments:	Monthly

Solution:

10. Calculate the Loan Information with the Contract Rate

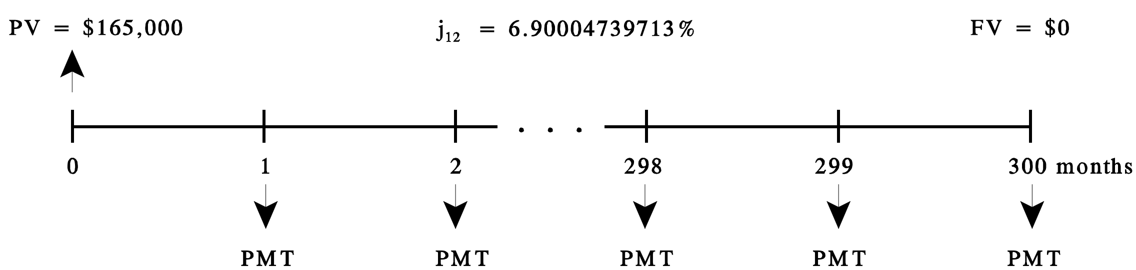
(i) Calculate the Equivalent Nominal Rate with Monthly Compounding

Calculation

Press	Display	Comments
7 ■ NOM%	7	Enter stated nominal rate
2 ■ P/YR	2	Enter stated compounding frequency
■ EFF%	7.1225	Compute equivalent effective annual rate
12 ■ P/YR	12	Enter desired compounding frequency
■ NOM%	6.90004739713	Compute nominal rate with monthly compounding

(ii) Calculate Monthly Payment

j_{12} = 6.90004739713% PV = $165,000.00 N = 300 PMT = ?



Calculation

Press	Display	Comments
	6.90004739713	j_{12} already stored
165000 [PV]	165,000	Actual loan amount
25 × 12 = [N]	300	Enter amortization period in months
0 [FV]	0	Payments should fully amortize loan over 300 months
[PMT]	−1,155.68599428	Compute monthly payment

The monthly mortgage payments, rounded up to the next higher cent, will be $1,155.69.

11. Calculate the Market Value of the Proposed Vendor Mortgage with the Market Rate

In a survey of local lenders, the agent finds that mortgages are available to qualified borrowers at interest rates of 15.5% - 16% per annum, compounded semi-annually. As the prospective borrower desires a fully amortized loan, a rate of j_2 = 16% is used to determine the market value of the vendor mortgage.[10]

(i) Calculate the Equivalent Nominal Rate with Monthly Compounding

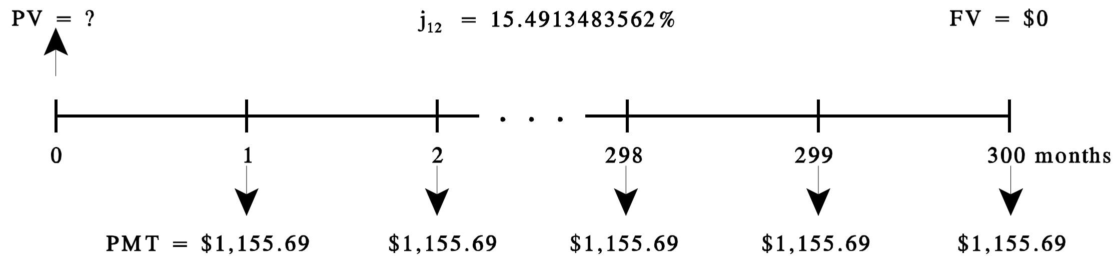
Calculation

Press	Display	Comments
16 ■ [NOM%]	16	Enter stated nominal rate
2 ■ [P/YR]	2	Enter stated compounding frequency
■ [EFF%]	16.64	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	15.4913483562	Compute nominal rate with monthly compounding

[10] As fully amortized loans are very rarely granted in practice, it would be difficult (if not impossible) to justify the interest rate used in this example. The rate chosen here is only for illustrative purposes and is not intended to suggest that fully amortized mortgages are readily available.

(ii) Calculate the Market Value of the Mortgage

j_{12} = 15.4913483562%   PMT = $1,155.69   N = 300   PV = ?



Calculation (continued)

Press	**Display**	**Comments**
	15.4913483562	j_{12} already stored from previous calculation
1155.69 [PMT] [+/−]	−1,155.69	Enter rounded payment
[PV]	87,614.0082863	Market value of payment stream

Thus, the payment stream that would result from the proposed vendor mortgage has a present value of $87,614.01 at the rate of interest paid on similar investments. Note that because all of the other loan information was already entered in the financial keys, once the new interest rate is calculated and the rounded payment is entered, the present value can be calculated without re-entering any other information.

12. Calculate the Market Value of the Offer

If the vendor had accepted the initial offer he would have received proceeds with a market value (or cash-equivalent price) of $167,614.01 rather than the $245,000 indicated by the stated offer price:

Cash	$ 80,000.00
+ Market value of the Mortgage	+ $ 87,614.01
Market Value of the Offer	$167,614.01

If you refer back to Table 1, you will see that this example illustrates the first relationship. Here the mortgage contract rate (j_2 = 7%) was less than the market rate (j_2 = 16%) which meant the market value of the offer ($167,614.01) was less than the stated offer price ($245,000).

Note that this analysis did not consider the impact of upward rounding of the regular stream of payments on the final mortgage payment. In fact, this loan would require 299 payments of $1,155.69 and a slightly lower final, 300th payment of $1,152.50 because the payments have been rounded up to the next higher cent. Since this smaller final payment does not affect the analysis significantly it has been ignored here.[11]

[11] The techniques for calculating the final payment in a fully amortized loan are described in BUSI 121 (AIC 2200).

Illustration 2

The agent suggested that the vendor counter the offer with a similar arrangement except that the mortgage contain a three year term. The market value of the proposed counter offer is determined in a similar manner to the previous example; however, there is also the outstanding balance to consider at the end of the loan term.

Analysis of Market Value of the Proposed Counter Offer (Partially Amortized Loan)

***Proposed Counter Offer*:**

	Amount Offered:	$ 245,000.00
−	Cash Down payment:	−$ 80,000.00
	Vendor Mortgage:	$ 165,000.00

***Terms of Proposed Vendor Mortgage*:**

Face Value:	$165,000.00
Interest Rate:	7% per annum compounded semi-annually
Amortization Period:	25 years
Contractual Term:	3 years
Payments:	Monthly

Solution:

1. Calculate the Loan Information with the Contract Rate

(i) Calculate the Equivalent Nominal Contract Rate with Monthly Compounding

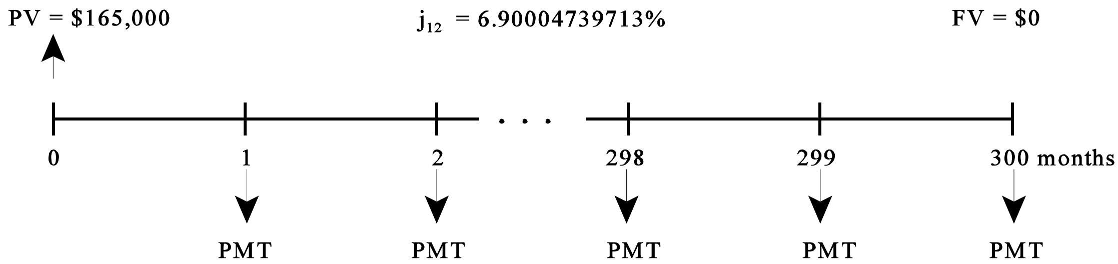
Calculation

Press	**Display**	**Comments**
7 ■ [NOM%]	7	Entered stated nominal rate
2 ■ [P/YR]	2	Entered stated compounded frequency
■ [EFF%]	7.1225	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	6.90004739713	Compute nominal rate with monthly compounding

The equivalent nominal contract rate with monthly compounding is j_{12} = 6.90004739713%

(ii) Calculate the Monthly Payment and the Outstanding Balance due at the end of the term

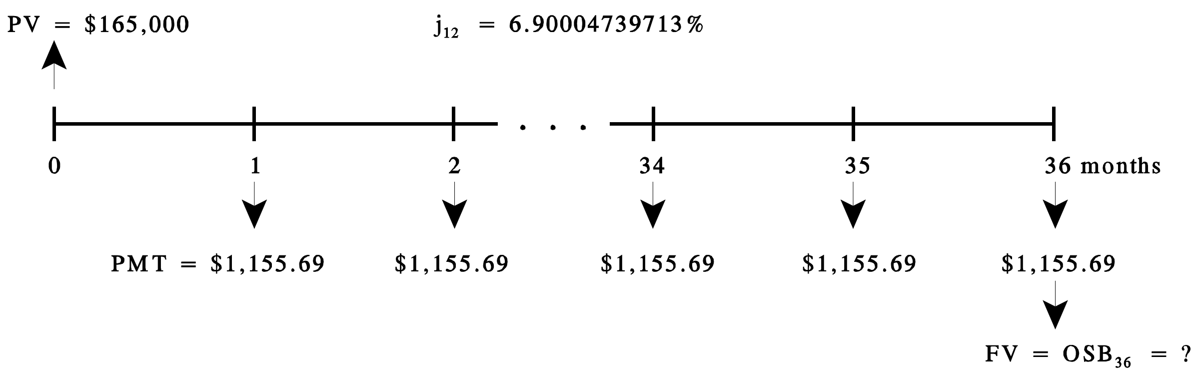
(a) Payment:



Calculation (continued)

Press	**Display**	**Comments**
	6.90004739713	j_{12} already stored
165000 PV	165,000	Actual loan amount
300 N	300	Enter amortization period in months
0 FV	0	Payments should fully amortize loan over 300 months
PMT	−1,155.68599428	Calculated payments
1155.69 +/− PMT	−1,155.69	Actual (rounded) payments

(b) Outstanding Balance:



Calculation (continued)

Press	**Display**	**Comments**
36 INPUT ■ AMORT	PEr 36-36	
= = =	156,749.516995	Outstanding balance after 36th payment

2. Calculate the Market Value of Proposed Vendor Mortgage with the Market Rate

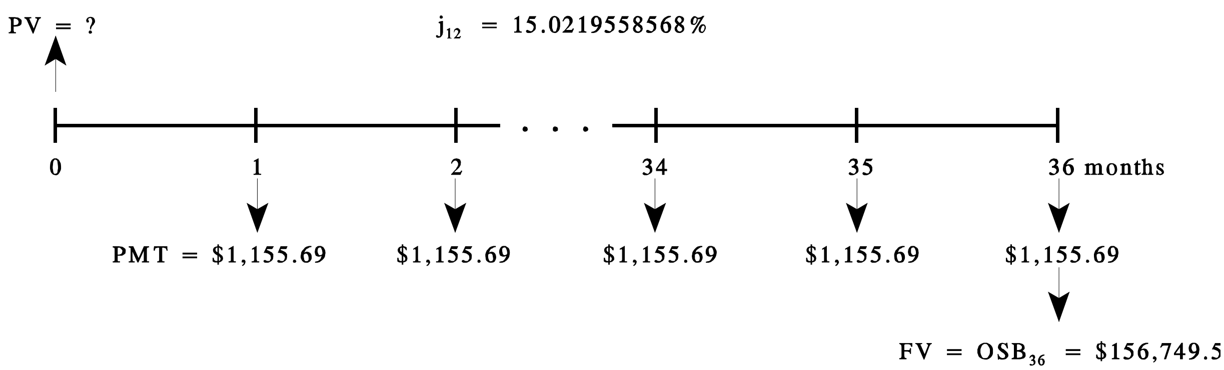
Under the terms of the proposed vendor mortgage, the vendor would have the contractual right to receive 36 monthly payments of $1,155.69 as well as the outstanding balance payment of $156,749.52 at the end of the loan term. In valuing this mortgage proposal, the agent used an interest rate of j_2 = 15.5% because it is the market rate for three year term mortgages.

(i) Calculate the Equivalent Nominal Market Rate with Monthly Compounding

j_2 = 15.5%
j_{12} = 15.0219558568%

(ii) Calculate Market Value of the Mortgage (Present Value of Payments and Outstanding Balance at Market Interest Rate)

N = 36 j_{12} = 15.0219558568% PMT = $1,155.69 FV = $156,749.52 PV = ?



Calculation

Press	**Display**	**Comments**
	15.021955857	j_{12} already stored from previous calculation
1155.69 [+/−] [PMT]	−1,155.69	Monthly payments
36 [N]	36	36 regular payments to be received
156749.52 [+/−] [FV]	−156,749.52	OSB to be received at the end of the term
[PV]	133,490.058616	Market value of payment stream over the loan term

The market value of the vendor mortgage as proposed in the counter offer is $133,490.06.

3. Calculate the Market Value of the Counter Offer

Thus, the counter offer has a market value to the vendor which is much higher than the purchaser's original offer:

	Cash	$ 80,000.00
+	Market Value of Vendor Mortgage	+ $ 133,490.06
	Market Value of Offer	$ 213,490.06

The counter offer increases the cash value of the transaction from $167,614.01 to $213,490.06 which may be within the range of the property's true market value. In this way, the agent has both protected her client's interests and found a method of providing low rate financing to the purchaser.

Illustration 3

Assume that fully amortized mortgages are currently available at 16% per annum, compounded semi-annually, and that five year term first mortgages are offered at 15% per annum, compounded semi-annually. Consider the case of a property listed for sale at $75,000. A potential purchaser makes an offer of $73,000, subject to the vendor taking back a $50,000 mortgage at 10% per annum, compounded semi-annually, amortized with monthly payments (round payments to the next higher cent) over 25 years. Calculate the market value of the offer assuming:

(a) the loan is to be fully amortized, and
(b) the loan is partially amortized over a five year term.

Abbreviated Solution:

(a) Calculate the Market Value of the Offer, Subject to Fully Amortized Mortgage

1. Calculate the Loan Information with the Contract Rate

j_2 = 10%
j_{12} = 9.79781526228%
PMT = $447.25

2. Calculate the Market Value of Mortgage with the Market Rate

j_2 = 16%
j_{12} = 15.4913483562%
PV = $33,906.47 (Market Value of the Mortgage)

3. Calculate the Market Value of the Offer

	Market Value of the Mortgage	$ 33,906.47
+	Downpayment	+ $ 23,000.00
	Market Value of Offer	$ 56,906.47

(b) Calculate the Market Value of the Offer, Subject to Partially Amortized Mortgage

1. Calculate the Loan Information with the Contract Rate

j_2 = 10%
j_{12} = 9.79781526228%
PMT = $447.25
OSB_{60} = $46,995.44

2. Calculate the Market Value of the Mortgage with the Market Rate

j_2 = 15%
j_{12} = 14.5516549135%
PV of 60 payments and OSB_{60} = $41,789.19 (Market Value of the Mortgage)

3. Calculate the Market Value of the Offer

	Market Value of the Mortgage	$ 41,789.19
+	Downpayment	+ $ 23,000.00
	Market Value of the Offer	$ 64,789.19

If the offer were accepted subject to a fully amortized mortgage, the vendor would receive $23,000 cash plus a contract with a face value of $50,000, but with a market value of only $33,906.47. Thus, the offer to the vendor has a cash value of $56,906.47, rather than the stated $73,000. If the vendor accepts the offer subject to the partially amortized loan, the market value of the mortgage would increase to $41,789.19 and the cash value of the offer would increase to $64,789.19, which is still much lower than the $73,000 indicated offer price.

By accepting a mortgage at a rate less than the prevailing market rate, the vendor is, in effect, accepting less for the property than the stated value of the offer.

In summary, a mortgage with a contract interest rate less than the prevailing market rate will have a market value lower than the face value of the loan. Mortgage investors will only buy the mortgage for the amount that the required payments will repay at the current market rate; the contract rate is only of use in determining the payments required under the mortgage.

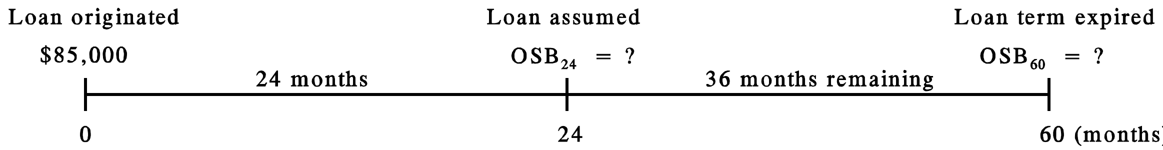
B. Mortgage Assumption

A vendor may also receive an offer to purchase which specifies that the purchaser will assume (take over the payment of monies owing under) an existing mortgage loan. If the existing loan has a rate of interest which is different from the market rate, a similar type of analysis as that done for a vendor take-back mortgage is required.

Illustration 4

Assume that John Smith bought his house two years ago at which time he arranged an $85,000 mortgage. This loan was written at a rate of 10.25% compounded semi-annually, with a 5 year term, 25 year amortization period and monthly payments. Today John has received an offer from Mary Jones to buy his house for $40,000 cash, plus assume the existing mortgage on the property which has 3 years remaining in the term. If current mortgage rates today are 13% per annum, compounded semi-annually, for three year loans, calculate Mary's indicated or stated offer price.

The events involving this loan can be illustrated on a simple time line:



Solution:

1. Calculate the Loan Information with the Contract Rate (Calculate the Outstanding Balance when the loan is assumed, as well as the Outstanding Balance at the end of the term)

j_2 = 10.25%
j_{12} = 10.0377355308%
PV = $85,000

Calculation

Press	**Display**	**Comments**
	10.0377355308	j_{12} already stored from previous calculation
85000 [PV]	85,000	Actual loan amount
300 [N]	300	Amortization period in months
0 [FV]	0	Payments should fully amortize loan over 300 months
[PMT]	−774.657893966	Calculated payments
774.66 [+/−] [PMT]	−774.66	Actual (rounded) payments
24 [INPUT] ■ [AMORT]	PEr 24 − 24	
= = =	83,315.9303395	Outstanding balance after 24th payment
60 [INPUT] ■ [AMORT]	PEr 60 − 60	
= = =	80,065.922325	Outstanding balance at end of loan term

Therefore, the loan has monthly payments of $774.66 over the 5-year term; it has an outstanding balance of $83,315.93 when Mary assumes it; and she will be required to repay the outstanding balance of $80,065.92 at the end of another 3 years when the loan term expires.

Mary has offered a downpayment of $40,000, plus she will take over the remaining payments on John's mortgage. Thus, her offer has a stated price of $40,000 + $83,315.93, or $123,315.93.

If Mary offered $40,000 cash, plus the proceeds from a new mortgage at the market rate of interest, which required exactly the same payments as she is offering to assume, she would be able to borrow less money and, in turn, be able to pass less money to the vendor.

Given that she has agreed she can make 36 payments of $774.66 and a balloon payment of $80,065.92 at the end of three years, how much could she borrow at the market rate of interest?

2. Calculate the Market Value of the Assumed Loan using the Market Rate

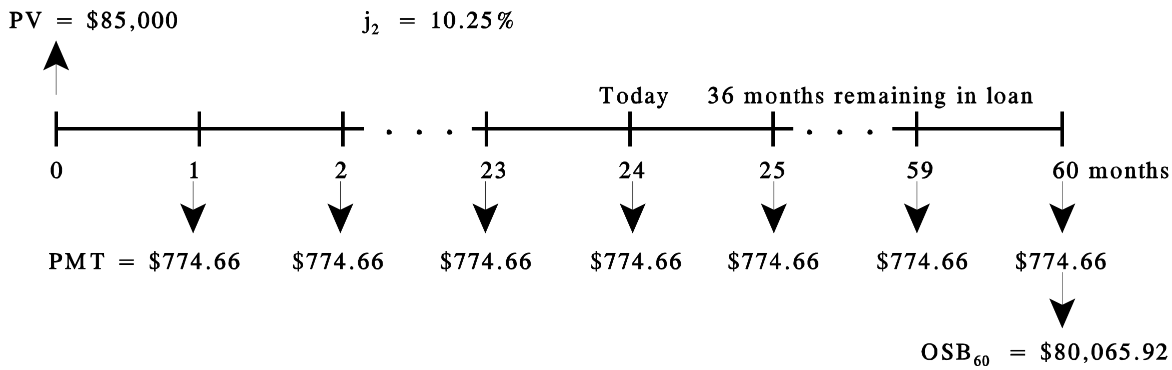
(i) Calculate the Equivalent Nominal Market Rate with Monthly Compounding

j_2 = 13%
j_{12} = 12.6612887764%

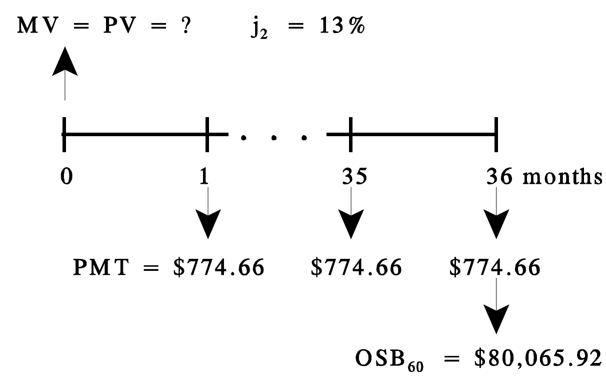
(ii) Calculate the Market Value of the Assumed Loan

N = 36 j_{12} = 12.6612887764% PMT = $774.66 FV = 80,065.92 PV = ?

John Smith's Original Loan: Payments and Outstanding Balance set out in loan contract.



Mary Jones' Assumed Loan: Find the market value of the remaining 36 payments and outstanding balance, given the current market interest rate.



Calculation

Press	Display	Comments
	12.6612887764%	j_{12} already stored from previous calculation
774.66 [+/-] [PMT]	−774.66	Monthly payments
36 [N]	36	36 regular payments remaining
80065.92 [+/-] [FV]	−80,065.92	OSB to be paid at the end of term
[PV]	77,974.6801515	Market value of remaining payments and OSB

This analysis shows that the payments Mary agreed to assume would only purchase a mortgage loan of $77,974.68 at the market interest rate.

3. Calculate the Market Value of the Offer

Therefore, the cash-equivalent price of Mary's offer (the amount of cash she could raise if the assumable mortgage did not exist) is actually:

	Cash	$ 40,000.00
+	Market Value of Assumed Loan	+ $ 77,974.68
	Market Value of the Offer	$ 117,974.68

If John Smith feels that another buyer might value his property at more than $117,974.68, he should consider rejecting Mary's offer.

Illustration 5

Three years ago Jim bought a house at which time he arranged a mortgage in the amount of $120,000. The loan was written at a rate of 9.75% per annum compounded semi-annually, with a 5 year term, 25 year amortization period and monthly payments.

Jim has just received an offer from Alice to buy his house. Alice offers to provide $25,000 cash and to assume the existing financing for the remainder of the term. If current lending rates for 2 year term mortgages are 11.75% per annum, compounded semi-annually, what is the market value of Alice's offer?

Abbreviated Solution:

1. Calculate the Loan Information Using the Contract Rate

Nominal contract rate with monthly compounding = 9.55765685944%
Monthly Payment = $1,053.25 (calculated using the full 25 year amortization period)
Outstanding Balance after 5 year term = $112,538.71

2. Calculate the Market Value of the Assumed Mortgage

Nominal market rate with monthly compounding = 11.4722852558%
Market value of the 2 years of payments remaining and the outstanding balance owing at the end of the loan term = $112,055.09

3. Calculate the Market Value of the Offer

	Cash	$ 25,000.00
+	Market Value of Assumed Loan	+ $ 112,055.09
	Market Value of the Offer	$ 137,055.09

II. Interest Rate Buydowns

An interest rate is "bought down" in a mortgage when a lump sum is paid to a lender by a developer in exchange for the lender making mortgage loans available to potential purchasers which are below current market interest rate levels. Interest rate buydowns are used as an incentive for purchasers and to improve affordability.

A. Buydowns as Purchaser Incentives

Interest rate buydown are commonly used by developers to help sell a property. By offering a below-market interest rate, the developer can create an immediate benefit to purchasers which does not diminish the face value of the selling price. For example, say a developer has sold 27 of 30 units in a townhouse development. The developer wants to sell the remaining 3 units, but does not want to advertise a lower price, diminishing the prestige of the overall development with a "fire sale" as well as possibly angering the other 27 owners. At this stage, the developer may choose other non-price incentives, i.e., throwing in optional features such as higher quality fixtures, free appliances, entertainment units, club memberships, mountain bikes, etc. Alternatively, the developer may offer the benefit of a pre-arranged mortgage with an interest rate bought down to substantially below current market levels.

By using any of these incentives, the developer sells a property at a price which on its face appears higher than its market value. For example, if the property is sold for $200,000, but the purchase price includes a $5,000 entertainment unit, the value of the property without the entertainment unit should be $195,000. The same will be true for the sale with the interest rate buydown – the face value of the offer will be greater than its market value, or cash equivalent price. If you needed to find the value of the property alone, you would have to determine how much value was attached to the beneficial financing; this is similar to analyzing offers with vendor takeback or assumed loans.

Illustration 6

Marina Development Corp. (MDC) is holding a pre-construction open house for its proposed waterfront condominium high-rise in Sooke, and hopes to sell out the whole building in this one weekend. MDC is asking $149,000 for its three-bedroom condos. For offers signed this weekend, MDC will give the purchaser either: 1) an appliance set valued at $3,000, or 2) a pre-arranged below-market interest rate loan. In the below-market interest rate loan arrangement, qualified purchasers can borrow up to 75% of the purchase price at an interest rate of 5%. Current market rates for similar loans are 7% (all rates are compounded semi-annually, j_2). The loan would be for a three year term, with a 25 year amortization period and monthly payments.

Your client Grania is considering purchasing one of these condos in order to take advantage of one of these incentives. However, she also suspects that if she chooses to forego these incentives, she may be able to negotiate a lower price for the condominium. Advise Grania on the costs and benefits of each of the alternatives.

Analysis of the Purchase Incentives (Purchaser)

If Grania chooses to purchase a condo for $149,000 and receives the appliance set as a gift, the purchase price paid for the condo is effectively $146,000 ($149,000 − $3,000). If she accepts the below-market interest rate loan instead, this also represents a benefit to her, and the amount effectively paid for the condo will also be less than its $149,000 face value. The question is, how much less? To determine this, it is necessary to calculate the cash equivalent price of this offer, in terms of the present value of the financing advantage.

***Terms of Proposed Bought Down Mortgage*:**

Face Value of Mortgage:	$149,000 × 0.75 = $111,750
Cash Down Payment:	$37,250
Interest Rate:	j_2 = 5%
Amortization:	25 years
Contractual Term:	3 years
Payments:	Monthly

Solution:

1. Calculate the Loan Information with the Contract Rate

(i) Calculate the Equivalent Nominal Rate with Monthly Compounding

Calculation

Press	**Display**	**Comments**
5 ■ [NOM%]	5	Entered stated nominal rate
2 ■ [P/YR]	2	Entered stated compounded frequency
■ [EFF%]	5.0625	Compute equivalent effective annual rate
12 ■ [P/YR]	12	Enter desired compounding frequency
■ [NOM%]	4.94869855817	Compute nominal rate with monthly compounding

The equivalent nominal contract rate with monthly compounding is j_{12} = 4.94869855817%

(ii) Calculate the Monthly Payment and the Outstanding Balance due at the end of the term

(a) Payment:

Calculation (continued)

Press	**Display**	**Comments**
	4.94869855817	j_{12} already stored
111750 [PV]	111,750	Actual loan amount
300 [N]	300	Enter amortization period in months
0 [FV]	0	Payments should fully amortize loan over 300 months
[PMT]	−649.943570779	Calculated payments
649.95 [+/−] [PMT]	−649.95	Actual (rounded) payments

(b) Outstanding Balance:

Calculation (continued)

Press	**Display**	**Comments**
36 [INPUT] ■ [AMORT]	PEr 36-36	
= = =	104,427.24657	Outstanding balance after 36th payment

2. Calculate the Market Value of Bought Down Mortgage using the Market Rate

Under the terms of the bought down mortgage, the lender will receive 36 monthly payments of $649.95 as well as the outstanding balance payment of $104,427.25 at the end of the loan term. However, the market interest rate for similar mortgages is $j_2 = 7\%$ -- what would this payment stream be worth at the market rate? If an investor was offered these payments and this outstanding balance, how much would they pay for this investment if they required a return on their investment of $j_2 = 7\%$? Or alternatively, if a borrower offered a lender these payments and outstanding balance, and the lender required an interest rate of $j_2 = 7\%$, how much would the lender be willing to lend?

(i) Calculate the Equivalent Nominal Market Rate with Monthly Compounding

$j_2 = 7\%$
$j_{12} = 6.90004739713\%$

(ii) Calculate Market Value of the Mortgage (Present Value of Payments and Outstanding Balance at Market Interest Rate)

N = 36 $j_{12} = 6.90004739713\%$ PMT = $649.95 FV = $104,427.25 PV = ?

Calculation

Press	Display	Comments
	6.90004739713	j_{12} already stored from previous calculation
649.95 [+/−] [PMT]	−649.95	Monthly payments
36 [N]	36	36 regular payments to be received
104427.25 [+/−] [FV]	−104,427.25	OSB to be received at the end of the term
[PV]	106,032.404479	Market value of payment stream over the loan term

The face value of the loan is $111,750, but the market value when considering the buydown is $106,032.40.

3. Calculate the Market Value of the Offer

The cash equivalent price of purchasing this property and taking advantage of the interest rate buydown is:

	Cash		$ 37,250.00
+	Market Value of Bought Down Mortgage	+	$ 106,032.40
	Market Value of Offer		$ 143,282.40

If Grania uses this pre-arranged financing, she will effectively purchase the property for a cash-equivalent price (or market value) of $143,282.40 rather than the $149,000 stated offer price. This $5,717 difference is due to having the benefit of lower payments over the three year mortgage term (without this interest rate buydown, the monthly payments would have been $132.76 higher at $782.72). Assuming she requires financing and faces the market interest rate as her best alternative, she may wish to consider pursuing this option instead of the free appliances. On the other hand, if cash flow is tight and she needs to buy appliances in any case, the $3,000 in appliances upfront may be appealing as well. The third option she should consider is to see how much of a price reduction she can negotiate given the cash equivalent value of the interest rate buydown. Knowing that the present value of this benefit is calculated to be $5,717 may be helpful in these negotiations -- another way of looking at this is that MDC would have to reduce the price of the condominium by $5,717 in order to match the increased affordability of this 2% buydown.

A third party analyzing this transaction, such as an appraiser, who is looking to use this sale as evidence of market value for similar condominiums would have to determine how much any of these incentives affected the purchase price. For example, the value of the property after adjusting for the buydown could be $143,282 or it could be higher, depending on how much value it appears that purchasers are attaching to this beneficial financing. Purchasers may value the benefit of this interest rate buydown at the $5,717 present value calculated above, or they may attach less or more value to it depending on their circumstances and the market situation. However, in the absence of further information, the cash equivalent price of $143,282 can likely serve as a good indication of the effect of this buydown.

Analysis of the Purchase Incentives (Developer)

The other side of this situation is to analyze the developer's position – Marina Development Corp. needs to sell these condominiums as fast as possible, but with the lowest possible marketing expense. To offer this form of low interest financing, they will have had to make prior arrangements with a lender to buy down the interest rate. Typically this would involve paying a lump sum that represents the present value of the difference between the loan payments at the bought down rate with what would have been required at the market rate. In this case, the difference in payments is $132.76 per month and the present value of this difference is $5,717. This would likely form the upper-end of the negotiated payment to buy down the interest rate, since the developer may be able to negotiate a lower fee with the lender given their mutually beneficial relationship.

Note also that developers often structure the interest rate buydown to minimize the fee required. For example, if this loan was written for a one year term, the present value of the buydown would only be $2,081; a six month term would only require $1,064. This is because the lender will only be obligated to allow the lower payments for six months or a year, as opposed to three years. However, the developer would have to determine if the savings in the buydown fee would be worth reducing this benefit to purchasers such that they may not be enticed into completing the sale. Market evidence indicates that these shorter-term arrangements are quite common, suggesting that affordability is not a key concern in many of these transactions and that purchasers either do not understand or do not care about the implications of the shorter term (or perhaps they do not attach significant value to the interest rate buydown in these cases).

B. The Effect of Buydowns on Affordability

A second benefit of interest rate buydowns is their potential to improve affordability. Borrowers qualify for loans based on the ratio of mortgage payments to income; e.g., most lenders limit the portion of income which can be spent on mortgage payments to 30-35%. If the interest rate drops on a mortgage due to a buydown, monthly payments decline, and smaller monthly payments mean less income is needed for potential purchasers to qualify for financing. For example, in Illustration 6, a purchaser applying for this loan at the 7% market interest rate might need a minimum income of $32,500 to qualify, whereas at the 5% bought down rate, the required income would only be $27,500.[12]

Clearly there is potential for abuse, if a developer offered extremely low interest rate loans for very short terms in order to qualify more purchasers (i.e., a 1% interest rate for only 2 months). Lenders are somewhat wary of considering buydowns when evaluating loan applications because when the term of the bought down loan expires, the borrower must be able to afford to pay the full market rate. Canada Mortgage and Housing Corporation (CMHC) does allow buydowns to be considered in qualifying borrowers for insured loans, but has set limits for the length of term required for a buydown (e.g., 3 years) and the maximum number of percentage points which can be bought down (e.g., if market rates are 14% or greater, the maximum buydown is 3%).

☞ CanadaMortgage.com is an excellent web site, with more information on buydowns and an on-line calculator that will find the present value of a buydown. http://www.canadamortgage.com/

[12] Mortgage underwriting calculations are explained in other courses In this example, it is assumed that the lender's gross debt service ratio is 32% and the annual property taxes are $1,000.

Illustration 7

Kai is shopping for a house for his new family. He has talked to the lending officer at his bank, and she has calculated that he can afford up to $1,200 per month in mortgage payments. The bank has offered him a rate of j_2 = 10% on a 4 year term loan, with monthly payments and a 25 year amortization period. He is looking at houses in Ottawa in the price range of $200,000. However, he is also interested in a house in a new subdivision in Kanata because the developer is offering a pre-arranged loan with an interest rate buydown to j_2 = 6% for a 4 year term loan (25 year amortization, monthly payments).

(a) If Kai purchases the house in Kanata, how much more can he afford to pay than if he used the bank's loan?

(b) If Kai purchases one of the developer's houses for $250,000, with a $175,000 loan at the bought down rate, what would be the cash equivalent value of this purchase?

Abbreviated Solution:

(a) Maximum mortgage with the bank:
j_2 = 10% → j_{12} = 9.79781526228%
Pmt = $1,200 → PV = $134,155.07

Maximum mortgage with the developer:
j_2 = 6% → j_{12} = 5.92634643744%
Pmt = $1,200 → PV = $187,556.67

Difference = $53,401.60

Because of the lower interest rate, Kai could afford to pay $53,401.60 more for the house in Kanata.

(b) **Cash Equivalent Value of this Purchase**

1. Calculate the Loan Information Using the Bought Down Rate (j_2 = 6%)

Nominal contract rate with monthly compounding = 5.92634643744%
Monthly Payment = $1,119.67 (calculated using the full 25 year amortization period)
Outstanding Balance after 4 year term = $161,203.43

2. Calculate the Market Value of Bought Down Mortgage with the Market Rate (j_2 = 10%)

Nominal market rate with monthly compounding = 9.79781526228%
Market value of the 4 years of payments and the outstanding balance owing at the end of the loan term = $153,424.82

3. Calculate the Market Value of the Offer

	Cash Downpayment		$75,000.00
+	Market Value of Bought Down Loan	+	$153,424.82
	Cash Equivalent Value of Property		$228,424.82

REVIEW PROBLEMS

1. A prospective purchaser offers to purchase a property for $195,000 providing the vendor takes back a mortgage in the amount of $150,000. This vendor financing is to be written at j_2 = 12% per annum, and is to be fully amortized over a 25 year period with constant monthly payments (rounded up to the next higher cent). If the market rate of interest on a fully amortized mortgage is j_2 = 15%, what is the market value of this offer?

   (a) Convert the contract rate of interest (which is stated as a nominal rate of 12% per annum, compounded semi-annually), to an equivalent nominal rate with monthly compounding.
   (b) Calculate the monthly payment (rounded up to the next higher cent) required under the $150,000 mortgage.
   (c) Calculate the equivalent nominal rate with monthly compounding for the market rate of j_2 = 15%.
   (d) Calculate the present value of the vendor mortgage at the market rate of interest.
   (e) What is the market value of the offer?

2. A prospective purchaser has made an offer to purchase a property for $115,000, given that the vendor will provide financing at the rate of 1.2% per month. The vendor take-back is to be in the amount of $75,000, providing for constant monthly payments (rounded up to the next higher cent). The mortgage is to be written over a 25-year period, but will have a term of only five years. What is the market value of this offer if the market rate of interest on a partially amortized mortgage is 18% per annum compounded monthly?

   (a) Calculate the monthly payment (rounded up to the next higher cent) which is required under the $75,000 mortgage.
   (b) Calculate the outstanding balance at the end of the term.
   (c) Using the market rate of interest, determine the total present value of the vendor financing arrangements.
   (d) What is the market value of the offer?

3. A proposed vendor take-back mortgage has a face value of $114,000 with an interest rate of 12.34% per annum, compounded semi-annually; monthly payments rounded to the next higher dollar; an amortization period of 15 years; and a term of 2 years. Current market mortgage rates are 14.75% per annum, compounded semi-annually. Calculate the market value of the vendor mortgage.

4. A vendor has agreed to grant a purchaser a vendor mortgage in order to facilitate the sale of his house. The mortgage is to be in the amount of $75,000 and is to be fully amortized over a period of 20 years, at an interest rate of 8.5% per annum, compounded semi-annually. Payments are to be made monthly, rounded to the next higher cent. If the purchaser has offered to purchase the house for a total of $115,000, calculate the market value of the offer. Assume market interest rates are currently 10.75% per annum, compounded semi-annually.

5. A vendor bought a property 2 years ago and arranged the following mortgage:

   Face Value = \$85,000

   Contract Rate = j_{12} = 12%

   25 year amortization period; 5 year term; and monthly payments.

   Today, Lucy wants to buy the property and offers \$40,000 cash and assumption of the mortgage. Current market rates for similar mortgages are j_{12} = 15%. Find the market value of the mortgage and the market value of the offer.

6. A vendor wishes to sell his property, but is facing a problem: the property was bought four years ago when market interest rates were j_2 = 13% and financed with a closed five year mortgage (i.e., no prepayments at all for five years). Current market interest rates for 1 year mortgages are j_2 = 11.75%. To sell the property, a purchaser will have to assume this high interest loan, which is not an attractive selling feature. In order to make the sale of this property more attractive, the vendor wishes to buy down the interest rate on the first mortgage from j_2 = 13% to the current market rate of j_2 = 11.75%. The mortgage was originally written for \$150,000 and called for monthly payments, a five year term, and a 20 year amortization period.

   (a) What fee will the vendor have to pay for this buydown? Assume that the vendor will pay 80% of the buydown fee and the lender will assume 20%. Also, remember that there is only one year remaining in the loan term.

   (b) If the property is sold for \$200,000 with this bought down loan, what would be the cash equivalent value of this property?

7. A prospective purchaser has requested vendor financing in the amount of \$50,000. This financing arrangement is to be fully amortized over a 20 year period by monthly payments (rounded up to the next higher cent). The interest rate to be paid is 0.85% per month. What is the market value of this "vendor take-back" if the market interest rate on a fully amortized mortgage is 1.15% per month?

   (a) Calculate the monthly payment (rounded up to the next higher cent).
   (b) Calculate the present value of the vendor financing arrangement.

   Assume instead that the vendor financing arrangement above had a five year term. What would be the market value of the "vendor take-back"?

   (c) Calculate the outstanding balance at the end of the term.
   (d) What is the market value of the "vendor take-back"?

8. A vendor has accepted a $137,000 first mortgage "take back" to facilitate the sale of a commercial property. The loan calls for annual payments to amortize the loan over 20 years at an interest rate of eight per cent per annum, compounded annually.

   (a) Calculate the required payment.
   (b) If interest rates at the time the vendor sold the property were 12% per annum compounded semi-annually for financial arrangements of this type, what was the cash value (or market value) of the mortgage?

9. A vendor agrees to "take back" a mortgage of $93,500 at a rate of 7% per annum compounded semi-annually, amortized over 15 years, but with a two year term. Payments are to be rounded to the next higher $100 and made monthly.

   (a) Calculate the required payment.
   (b) Calculate the outstanding balance at the end of the term.
   (c) Calculate the market value of the mortgage if similar mortgages are currently available at 9% per annum, compounded semi-annually.

10. Jerzy Kozyniak has had his house appraised and been told by several experts that its value is in the $1 to $1.2 million dollar range. Dolly has offered to purchase the mini-palace by giving Jerzy a down payment of $700,000 and assuming Jerzy's existing mortgage. Jerzy has consulted his mortgagee and found that on the day Dolly wishes to take possession of the property, the outstanding balance on the mortgage would be $280,580.00 and there would be 27 monthly payments of $4,352.92 remaining in the mortgage term. The mortgagee tells Jerzy that he should probably accept Dolly's offer because current mortgage rates are 15% per annum compounded semi-annually and the contract rate on the existing mortgage is 19% per annum, compounded semi-annually. Give Jerzy the benefit of your financial expertise.

SOLUTIONS

1. (a) 11.710553015%
   (b) $1,547.85
   (c) 14.5516549135%
   (d) $124,211.00
   (e) $169,211.00

2. (a) $925.85
   (b) $72,747.50
   (c) $66,235.48
   (d) $106,235.48

3. Market Value of the Mortgage = $109,604.62

4. Market Value of the Offer = $104,423.38

5. Market Value of the Mortgage= $77,812.33
   Market Value of the Offer = $117,812.33

6. (a) First calculate the payments and outstanding balance for the loan using the contract interest rate of j_2 = 13%:

   j_2 = 13% → j_{12} = 12.6612887764%
   Pmt = $1,721.30; OSB_{60} = $138,474.60
   OSB_{48} = $141,393.49

   Then calculate the present value of the remaining 12 payments and the OSB at the end of the loan term using the market interest rate of j_2 = 11.75%:

   j_2 = 11.75% → j_{12} = 11.4722852558%
   PV = $142,960.38

   Value of Buydown = Market Value of the loan − Current Face Value of the loan
   = $142,960.38 − 141,393.49 = $1,566.89

   Vendor's Share of Buydown Fee = $1,566.89 × 0.8 = $1,253.51

   (b) Cash equivalent value = $200,000 − 1,253.51 = $198,746.49

7. (a) $489.16
   (b) $39,800.76
   (c) $45,005.29
   (d) $43,779.25

8. (a) $13,953.76
   (b) $101,918.65

9. (a) $900.00
   (b) $84,203.00
   (c) $90,342.15

10. Dolly's stated offer is:

Down Payment	$ 700,000
Assumed Loan	280,580
Stated Value	$ 980,580

However, on loan assumptions, the market value of the mortgage must be calculated. Dolly is offering to assume an above market rate of interest; therefore her offer is actually higher than the stated price.

Step 1 - Calculate the OSB at the end of the term. There are 27 payments remaining in the term. Payments = $4,352.92; loan amount = $280,580 on the day Dolly assumes the mortgage loan; contract interest rate: $j_2 = 19\%$; $j_{12} = 18.2888399689\%$.

$OSB_{27} = 278{,}042.08$

Step 2 - Use the market rate of interest to find the market value of 27 payments of $4,352.92 plus an outstanding balance payment of $278,042.08 paid with the 27th payment.
Market rate: $j_2 = 15\%$; $j_{12} = 14.5516549135\%$.

PV = $300,521.28

Step 3 - Calculate the market value of the offer.

	Down Payment	$ 700,000.00
+	Market Value of Assumed Mortgage	$300,521.28
=	Total value of offer	$1,000,521.28

Jerzy must now ask himself if he is willing to accept an offer which has a cash equivalent price of approximately $1,000,500.

APPENDIX A
HP 10BII and Six Functions of One Dollar Tables

In previous courses, students may have solved financial calculations using the publication *Compound Interest and Discount Factor Tables for Appraisers - The Six Functions of One Dollar*. This book lists tables of compounding and discounting factors for common interest rates which, before the existence of preprogrammed financial calculators, allowed real estate practitioners to avoid having to solve problems using algebraic formulae. However, the continued improvements in computing power and miniaturization over the past twenty years have made these tables obsolete. A knowledgeable user with a handheld financial calculators can do everything that is in the "Six Functions of a Dollar" tables, plus many more functions and with much more flexibility. Real Estate Division courses do not support the use of these tables. It is imperative that every student owns a financial calculator and learns how to operate it - please note that students will **NOT** be allowed to bring these tables with them to the final examination for their course.

For those students who wish to continue using the same methodology as they use for the tables, this appendix contains a review of how to use the HP 10BII calculator to calculate the factors which would otherwise be provided in the Six Functions of a Dollar tables.

The *Compound Interest and Discount Factor Tables for Appraisers - The Six Functions of One Dollar* publication consists of three compounding functions and three discounting functions. These six functions for the basis for all financing mathematics. The functions are arranged into six columns:

- Amount of One Dollar (Column 1);
- Accumulation of One Dollar (Column 2);
- Sinking Fund Factor (Column 3);
- Reversion Factor (Column 4);
- Annuity Factor (Column 5);
- Partial Payment Factor (Column 6).

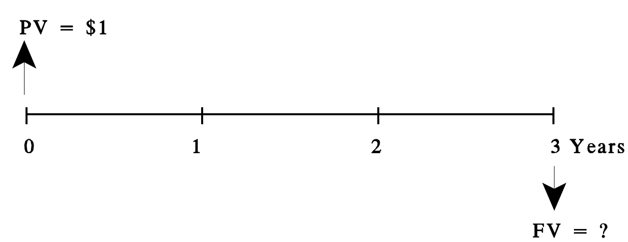
The factors in these columns are based on a value of one dollar. This creates a coefficient or multiplier which can then be applied to any dollar amount.

The following section will examine each of these six functions, explaining the purpose of each, illustrating it with a cash flow diagram, and listing the HP 10BII calculator steps which can be used to solve for the factor.

1. Amount of One Dollar

Column 1 shows the value at some future point in time of $1 invested today at a given compound interest rate.

Example: Deposit $1 for 3 years at 10% per annum, compounded annually.



<u>Formula</u>

$$
\begin{aligned}
S^n &= (1 + i)^n \\
&= (1 + .10)^3 \\
&= 1.331
\end{aligned}
$$

Calculation

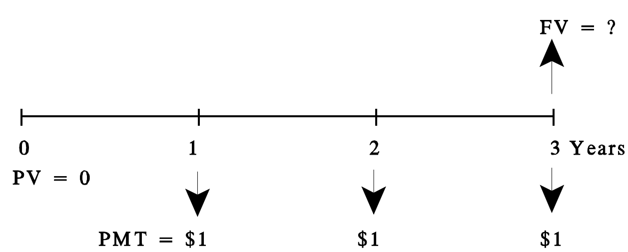
Press	**Display**	**Comments**
10 I/YR	10	Enter nominal interest rate
1 ■ P/YR	1	Enter compounding frequency
3 N	3	Enter the number of compounding periods
1 PV	1	Enter present value
0 PMT	0	No payments during term
FV	-1.331	Compute future value

The future value of $1 deposited for 3 years at an interest rate of 10% per annum, compounded annually is $1.331.

2. Accumulation of One Dollar

Column 2 shows the accumulated value at some future point in time of $1 invested per period at a given compound interest rate.

Example: Deposit $1 per year for 3 years at 10% per annum, compounded annually.



<u>Formula</u>

$$S\llbracket n,i\% \rrbracket = \frac{S^n - 1}{i} \qquad \text{where } S^n = (1+i)^n$$

$$\frac{(1 + i)^3 - 1}{.10} = \frac{.331}{.10} = 3.31$$

Calculation

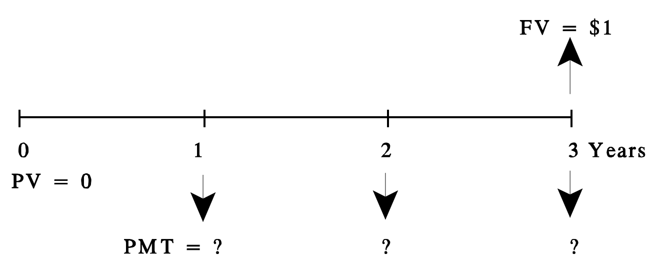
Press	Display	Comments
10 [I/YR]	10	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
3 [N]	3	Enter the number of payments
0 [PV]	0	Enter present value
1 [+/−] [PMT]	-1	Enter payments during term
[FV]	3.31	Computed future value

The future value at the end of 3 years of $1 deposited each year into an account earning interest at 10% per annum, compounded annually, is $3.31.

3. Sinking Fund Factor

Column 3 shows the amount which would have to be invested per period at a given compound interest rate in order to accumulate $1 at some future point in time.

Example: Want to accumulate $1 at the end of 3 years with an interest rate of 10% per annum, compounded annually.



Formula

$$\frac{1}{S[\![n,i\%]\!]} = \frac{i}{S^{n} - 1} \text{ where } S^{n} = (1 + i)^{n}$$

$$\frac{.10}{(1 + .10)^{3} - 1} = \frac{.10}{1.331 - 1} = .302115$$

Calculation

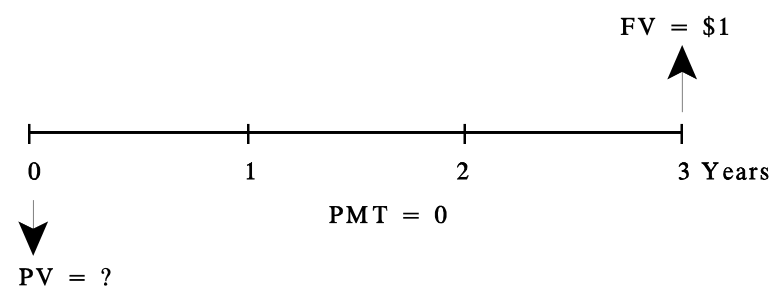
Press	Display	Comments
10 [I/YR]	10	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
3 [N]	3	Enter the number of payments
0 [PV]	0	Enter present value
1 [FV]	1	Enter future value
[PMT]	-3.02114804E-1	Compute necessary payments per year.[13]

In order to accumulate $1 at the end of 3 years in an account earning interest at 10% per annum, compounded annually, annual payments of $0.302 would be required.

4. Reversion Factor

Column 4 shows the value today of an investment of $1 at some future point in time discounted back to present value terms using a given compound interest rate.

Example: What is the value today of the promise of $1 in three years time with an interest rate of 10% per annum, compounded annually?



Formula

$$\frac{1}{S^n} = \frac{1}{(1 + i)^n} = \frac{1}{(1 + .10)^3} = \frac{1}{1.331} = .7513$$

[13] Note that if your calculator is set to a "floating decimal", this answer is shown in scientific notation with E-1 at the end. This tells you to move the decimal one place to the left, giving -0.302114804. See Section J of the Introduction to the HP 10BII calculator for more information.

Calculation

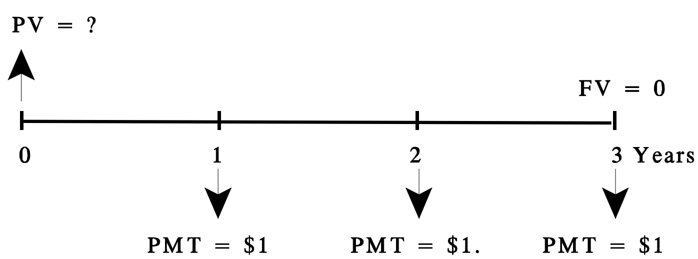
Press	Display	Comments
10 [I/YR]	10	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
0 [PMT]	0	Enter payments during term
3 [N]	3	Enter the number of payments
1 [FV]	1	Enter future value
[PV]	-7.51314801E-1	Compute present value

The present value of $1 to be received in 3 years is $0.7513 assuming a discount rate of 10% per annum, compounded annually.

5. Annuity Factor

Column 5 shows the value today of an investment of $1 per period in the future discounted back to present value terms using a given compound interest rate.

Example: What is the value today of $1 per year for 3 years discounted with an interest rate of 10% per annum, compounded annually.



Formula

$$a[\![n, i\%]\!] = \frac{1 - 1/s^n}{i}, \text{ where } 1/s^n = \frac{1}{(1 + i)^n}$$

$$= \frac{1 - \frac{1}{(1 + .10)^3}}{i} = \frac{1 - .7513}{.10} = 2.4868$$

Calculation

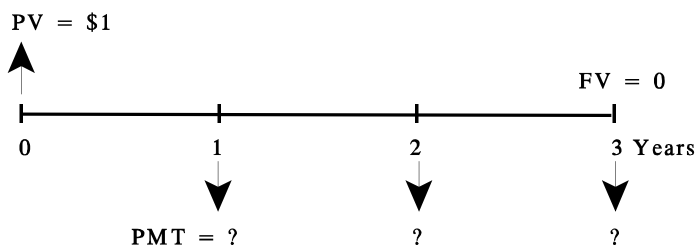
Press	Display	Comments
10 [I/YR]	10	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
3 [N]	3	Enter the number of payments
1 [+/−] [PMT]	-1	Enter payments during term
0 [FV]	0	Enter future value
[PV]	2.48685199098	Compute present value

The present value of $1 per year to be paid for 3 years is $2.4869, assuming a discount rate of 10% per annum, compounded annually.

6. Partial Payment Factor (also known as Mortgage Constant)

Column 6 shows the amount which would have to be paid per period in order to amortize a loan of $1 (i.e., pay off the loan amount plus all interest owing), given an amortization period and a compound interest rate.

Example: What would be required in order to repay all principal plus the interest owing on a loan of $1 over 3 years at an interest rate of 10%, compounded annually?



Formula

$$\frac{1}{a\rrbracket n,i\%\rrbracket} = \frac{i}{1 - 1/s^{n}} = \frac{.10}{1 - .7513} = .402115$$

Calculation

Press	Display	Comments
10 [I/YR]	10	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
3 [N]	3	Enter the number of payments
1 [PV]	1	Enter present value
0 [FV]	0	Zero future value (loan fully repaid)
[PMT]	-4.02114804E-1	Compute required annual payment

In order to repay all principal and interest owing for a loan of $1, assuming an interest rate of 10% per annum, compounded annually, the required payment would be $0.4021 per year.

APPENDIX B
ADDITIONAL CALCULATIONS

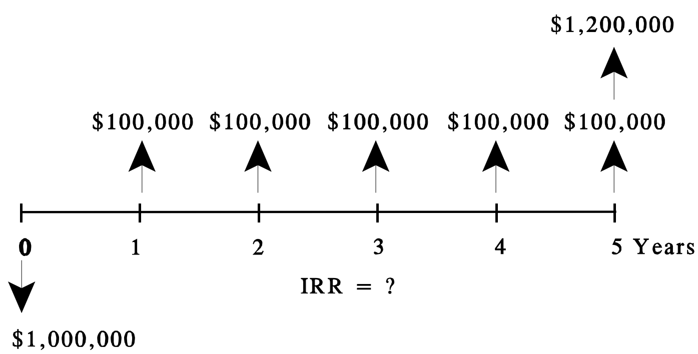
Below are additional calculations that may be required in more advanced courses.

INTERNAL RATE OF RETURN

The Internal Rate of Return (IRR), sometimes called the Free and Clear Rate of Return (FCRR), is the return on capital based on the present value of the net operating income and the present value of appreciation/depreciation over the investment cycle. Before the advent of financial calculators, the only possible method for calculating these rate of return problems was by the interpolation method, or in other words, by trial and error. Most financial calculators have this formula programmed into them and can quickly compute this return. The steps for calculating an IRR problem are shown below.

Example of IRR: What is the Internal Rate of Return on the following investment?

Annual Net Operating Income:	$100,000
Holding Period	5 years
Purchase Price	$1,000,000
Sale Price	$1,200,000



Calculation

Press	Display	Comments
■ [C ALL]	0	Clears memory registers
1 ■ [P/YR]	1	Enter compounding frequency
1000000 [+/−] [CFj]	-1,000,000	Enter initial cash flow
100000 [CFj]	100,000	Enter first cash flow
4 ■ [Nj]	4	Repeated 4 times (Years 1 - 4)
1300000 [CFj]	1,300,000	Enter total cash flow for Year 5
■ [IRR/YR]	13.081313987	Compute annual nominal yield rate

The internal rate of return on this investment is approximately 13.1%.

Note that the calculator took several seconds to calculate this answer, as compared to other calculations which are more-or-less instantaneous. This pause is because the calculator must also go through the very time-consuming discounting process in estimating this rate of return.

SIMPLIFIED EXPLANATION OF THE HOSKOLD AND INWOOD PREMISES

Both the Inwood Premise and the Hoskold Sinking Fund Premise are discounting factors which are used to discount future periodic income to their present value. They are used to appraise interests in Real Estate that have limited terms, like leasehold interests (leasehold interests generally last 1 to 999 years, as compared to fee simple interests, which last in perpetuity).

Both premises are based on the notion that investors in real estate require both a return *on* their invested capital and a return *of* this capital over the investment period. For example, an investor purchasing an apartment building expects to earn income while holding the investment and to receive some or all of their initial investment back when property is eventually re-sold. However, some interests in real estate, like leasehold interests, do not provide the investor an opportunity to receive their capital back at the end of the lease term. In these situations, it is critical that the investor receives a return *of* capital during the lease term. Therefore, when evaluating this type of investment opportunity, the return to the investor will implicitly include both a return *on* and the return *of* capital. Because these are the returns an investor uses to evaluate investment opportunities, these types of interests also must be used.

The Inwood Premise assumes that the income received during the investment period can be reinvested at the same rate as that of the original investment.

The Hoskold Premise assumes that the income received during the investment period will be reinvested at a different rate from that of the original investment (i.e., typically a lower rate, reflecting the fact that the investor cannot earn the same rate on the small amounts of returned capital as is possible on the entire investment amount).

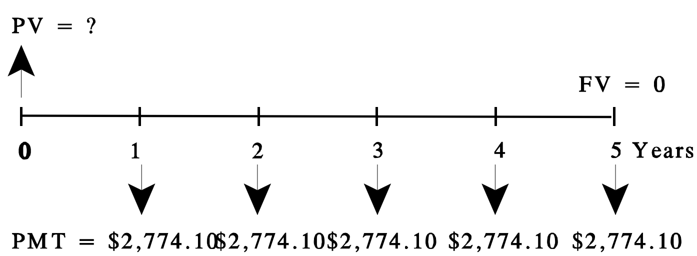
1. Inwood Example

Inwood problems can be solved using Column 5 of the Discount Factor T
ables or with a financial calculator.

The following example is from "Fundamentals of Real Estate Investment Analysis and Feasibility Studies", by F.S.C. Young, and published by the Appraisal Institute of Canada. (Page 32)

Example: Mr. Able intends to purchase a leasehold interest which will produce a net profit rent of $2,774.10 per annum for 5 years. If he requires an annual return of 12% for his capital, how much can he afford to pay for the leasehold interest?

This is solved below using the calculator:



Calculation

Press	Display	Comments
12 [I/YR]	12	Enter nominal interest rate
1 ■ [P/YR]	1	Enter compounding frequency
5 [N]	5	Enter the number of payments
2774.1 [+/−] [PMT]	-2,774.1	Enter payments during term
0 [FV]	0	Enter future value
[PV]	10,000.0096629	Compute present value

In other words, the present value of a 5 year leasehold interest paying a profit rent of $2,774.10 per year is $10,000, given a discount rate of 12%.

2. Hoskold Example

Hoskold sinking fund problems must be solved algebraically, as there are no tables available and the calculator cannot solve them directly since there are two rates of return to deal with.

The example above is solved below using the Hoskold Sinking Fund Factor. Assume that the reinvestment rate "r" is 6%.

$$\text{Hoskold Sinking Fund Factor} = \frac{1}{i + \dfrac{r}{s^n - 1}} = \frac{1}{.12 + \dfrac{.06}{(1.06)^5 - 1}} = 3.3625$$

Therefore the Present Value of the leasehold interest is:

$$\$2{,}774.10 \text{ per year} \times 3.3625 = \$9{,}327.95$$

Note that this value is less than the value of $10,000 calculated by the Inwood method. The difference in value is due entirely to the difference in reinvestment assumptions. The Inwood method has implicitly assumed that both the return on and return of capital occurred at the investment rate of return. The Hoskold method adds the assumption that reinvested capital will earn a lower rate of return than the return of capital (6% as opposed to 12%).