CHAPTER 7
INTEREST RATES AND BOND VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

3. No. If the bid price were higher than the ask price, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

6. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells exactly at par.

8. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

Solutions to Questions and Problems

2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.

3. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:

$P = €58(\{ 1 – [1 / (1 + .0475^{23})] \} / .047) + €1,000[1 / (1 + .047)^{23}]$
$P = €1,152.66$

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:

$PVIF_{R,t} = 1 / (1 + R)^{t}$

which stands for <u>P</u>resent <u>V</u>alue <u>I</u>nterest <u>F</u>actor

$PVIFA_{R,t} = (\{ 1 – [1/(1 + R)^{t}] \} / R)$

which stands for <u>P</u>resent <u>V</u>alue <u>I</u>nterest <u>F</u>actor of an <u>A</u>nnuity

These abbreviations are shorthand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in the remainder of the solutions key.

4. Here we need to find the YTM of a bond. The equation for the bond price is:

$P = ¥91,530 = ¥3,400(PVIFA_{R\%,16}) + ¥100,000(PVIF_{R\%,16})$

Notice the equation cannot be solved directly for R. Using a spreadsheet, a financial calculator, or trial and error, we find:

$R = YTM = 4.13\%$

If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be higher than the coupon rate since the bond is a discount bond. That still leaves a lot of interest rates to check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate YTM = [Annual interest payment + (Price difference from par / Years to maturity)] /
[(Price + Par value) / 2]

Solving for this problem, we get:

Approximate YTM = [¥3,400 + (¥8,470 / 16] / [(¥91,530 + 100,000) / 2] = 4.10%

This is not the exact YTM, but it is close, and it will give you a place to start.

5. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$$P = \$948 = C(\text{PVIFA}_{5.90\%,8}) + \$1,000(\text{PVIF}_{5.90\%,8})$$

Solving for the coupon payment, we get:

$$C = \$50.66$$

The coupon payment is the coupon rate times par value. Using this relationship, we get:

Coupon rate = $50.66 / $1,000
Coupon rate = .0507, or 5.07%

6. To find the price of this bond, we need to realize that the maturity of the bond is 14 years. The bond was issued 1 year ago, with 15 years to maturity, so there are 14 years left on the bond. Also, the coupons are semiannual, so we need to use the semiannual interest rate and the number of semiannual periods. The price of the bond is:

$$P = \$20.50(\text{PVIFA}_{2.25\%,28}) + \$1,000(\text{PVIF}_{2.25\%,28})$$
$$P = \$958.78$$

18. Here we are finding the YTM of annual coupon bonds for various maturity lengths. The bond price equation is:

$$P = C(\text{PVIFA}_{R\%,t}) + \$1,000(\text{PVIF}_{R\%,t})$$

X: P_0 = $42.50(PVIFA$_{3.5\%,26}$) + $1,000(PVIF$_{3.5\%,26}$) = $1,126.68
P_1 = $42.50(PVIFA$_{3.5\%,24}$) + $1,000(PVIF$_{3.5\%,24}$) = $1,120.44
P_3 = $42.50(PVIFA$_{3.5\%,20}$) + $1,000(PVIF$_{3.5\%,20}$) = $1,106.59
P_8 = $42.50(PVIFA$_{3.5\%,10}$) + $1,000(PVIF$_{3.5\%,10}$) = $1,062.37
P_{12} = $42.50(PVIFA$_{3.5\%,2}$) + $1,000(PVIF$_{3.5\%,2}$) = $1,014.25
P_{13} = $1,000
Y: P_0 = $35(PVIFA$_{4.25\%,26}$) + $1,000(PVIF$_{4.25\%,26}$) = $883.33
P_1 = $35(PVIFA$_{4.25\%,24}$) + $1,000(PVIF$_{4.25\%,24}$) = $888.52
P_3 = $35(PVIFA$_{4.25\%,20}$) + $1,000(PVIF$_{4.25\%,20}$) = $900.29
P_8 = $35(PVIFA$_{4.25\%,10}$) + $1,000(PVIF$_{4.25\%,10}$) = $939.92
P_{12} = $35(PVIFA$_{4.25\%,2}$) + $1,000(PVIF$_{4.25\%,2}$) = $985.90
P_{13} = $1,000

Maturity and Bond Price
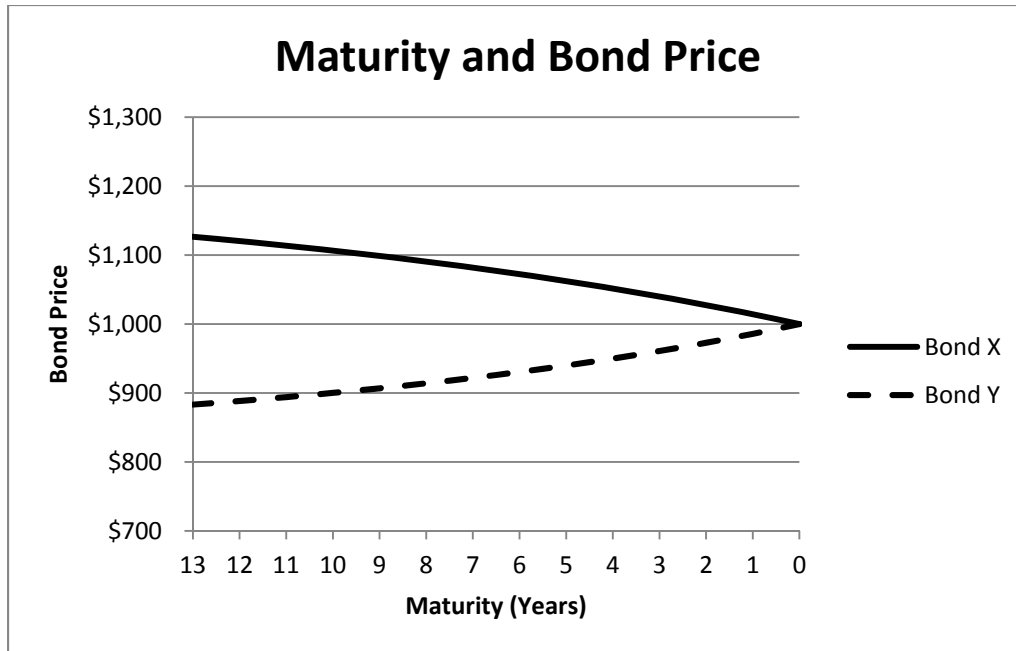
All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

20. Initially, at a YTM of 6 percent, the prices of the two bonds are:

$$P_J = \$15(PVIFA_{3\%,38}) + \$1,000(PVIF_{3\%,38}) = \$662.61$$

$$P_K = \$45(PVIFA_{3\%,38}) + \$1,000(PVIF_{3\%,38}) = \$1,337.39$$

If the YTM rises from 6 percent to 8 percent:

$$P_J = \$15(PVIFA_{4\%,38}) + \$1,000(PVIF_{4\%,38}) = \$515.80$$

$$P_K = \$45(PVIFA_{4\%,38}) + \$1,000(PVIF_{4\%,38}) = \$1,096.84$$

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$$\Delta P_J\% = (\$515.80 - 662.61) / \$662.61 = -.2216, \text{ or } -22.16\%$$
$$\Delta P_K\% = (\$1,096.84 - 1,337.39) / \$1,337.39 = -.1799, \text{ or } -17.99\%$$

If the YTM declines from 6 percent to 4 percent:

$$P_J \quad = \$15(\text{PVIFA}_{2\%,38}) + \$1,000(\text{PVIF}_{2\%,38}) \quad = \$867.80$$

$$P_K \quad = \$45(\text{PVIFA}_{2\%,38}) + \$1,000(\text{PVIF}_{2\%,38}) \quad = \$1,661.02$$

$$\Delta P_J\% \quad = (\$867.80 - 662.61) / \$662.61 \quad = .3097, \text{ or } 30.97\%$$

$$\Delta P_K\% \quad = (\$1,661.02 - 1,337.39) / \$1,337.39 \quad = .2420, \text{ or } 24.20\%$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

21. The current yield is:

Current yield = Annual coupon payment / Price
Current yield = \$64 / \$1,068
Current yield = .0599, or 5.99%

The bond price equation for this bond is:

$$P_0 = \$1,068 = \$32(\text{PVIFA}_{R\%,36}) + \$1,000(\text{PVIF}_{R\%,36})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 2.893\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 2.893%
YTM = 5.79%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = $(1 + .02893)^2 - 1$
Effective annual yield = .0587, or 5.87%

22. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on the outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$$P = \$1,083 = \$35(\text{PVIFA}_{R\%,40}) + \$1,000(\text{PVIF}_{R\%,40})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.133\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.133%
YTM = 6.27%

26. The bond has 13 years to maturity, so the bond price equation is:

$P = \$1,089.60 = \$28.50(\text{PVIFA}_{R\%,26}) + \$1,000(\text{PVIF}_{R\%,26})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 2.384\%$

This is the semiannual interest rate, so the YTM is:

$\text{YTM} = 2 \times 2.384\%$
$\text{YTM} = 4.77\%$

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = $57 / $1,089.60
Current yield = .0523, or 5.23%

29. *a.*   The coupon bonds have a 6 percent coupon which matches the 6 percent required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = $47,000,000 / $1,000
Number of coupon bonds to sell = 47,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $1,000 / 1.03^{40}
Price of zero coupon bonds = $306.56

Number of zero coupon bonds to sell = $47,000,000 / $306.56
Number of zero coupon bonds to sell = 153,316

b.   The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 47,000($1,030)
Coupon bonds repayment = $48,410,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes repayment = 153,316($1,000)
Zeroes repayment = $153,315,776

32. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year are:

P: $P_0 = \$100(\text{PVIFA}_{7\%, 10}) + \$1,000(\text{PVIF}_{7\%, 10}) = \$1,210.71$

$P_1 = \$100(\text{PVIFA}_{7\%, 9}) + \$1,000(\text{PVIF}_{7\%, 9}) = \$1,195.46$

So, the capital gains yield is:

Capital gains yield = (New price – Original price) / Original price
Capital gains yield = ($1,195.46 – 1,210.71) / $1,210.71
Capital gains yield = –.0126, or –1.26%

And the current yield is:

Current yield = $100 / $1,210.71
Current yield = .0826, or 8.26%

The current price of Bond D and the price of Bond D in one year is:

D: $P_0 = \$40(\text{PVIFA}_{7\%, 10}) + \$1,000(\text{PVIF}_{7\%, 10}) = \$789.29$

$P_1 = \$40(\text{PVIFA}_{7\%, 9}) + \$1,000(\text{PVIF}_{7\%, 9}) = \$804.54$

So, the capital gains yield is:

Capital gains yield = ($804.54 – 789.29) / $789.29
Capital gains yield = .0193, or 1.93%

And the current yield is:

Current yield = $40 / $789.29
Current yield = .0507, or 5.07%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 7 percent, but this return is distributed differently between current income and capital gains.

Calculator Solutions

3.

Enter	23	4.7%		€58	€1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			€1,152.66		

4.

Enter	18		±¥91,530	¥3,400	¥100,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		4.13%			

5.

Enter	8	5.9%	±$948		$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$50.66	

Coupon rate = $50.66 / $1,000 = 5.07%

6.

Enter	28	4.5% / 2		$41 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$958.78		

18. Bond X

P_0

Enter	26	7% / 2		$85 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,126.68		

P_1

Enter	24	7% / 2		$85 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,120.44		

P_3

Enter	20	7% / 2		$85 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,106.59		

P_8

Enter	10	7% / 2		$85 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,062.37		

P_{12}

Enter	2	7% / 2		$85 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,014.25		

Bond Y

P_0

Enter	26	8.5% / 2		$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$883.33		

P_1

Enter	24	8.5% / 2		$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$888.52		

P_3

Enter	20	8.5% / 2		$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$900.29		

P_8

Enter	10	8.5% / 2		$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$939.92		

P_{12}

Enter	2	8.5% / 2		$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$985.90		

20. Initially, at a YTM of 6 percent, the prices of the two bonds are:

P_J

Enter	38	6% / 2		$30 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$662.61		

P_K

Enter	38	6% / 2		$90 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,337.39		

If the YTM rises from 6 percent to 8 percent:

P_J

Enter	38	8%2		$30 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$515.80		

$\Delta P_J\% = (\$515.80 - 662.61) / \$662.61 = -22.16\%$

P_K

Enter	38	8% / 2		$90 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,096.84		

$\Delta P_K\% = (\$1,096.84 - 1,337.39) / \$1,337.39 = -17.99\%$

If the YTM declines from 6 percent to 4 percent:

P_J

Enter	38	4% / 2		$30 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$867.80		

$\Delta P_J\% = (\$867.80 - 662.61) / \$662.61 = +30.97\%$

P_K

Enter	38	4% / 2		$90 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,661.02		

$\Delta P_K\% = (\$1,661.02 - 1,337.39) / \$1,337.39 = +24.20\%$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

21.

Enter	36		±$1,068	$64 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		2.893%			

$2.893\% \times 2 = 5.79\%$

Enter	5.79%		2		
	NOM	**EFF**	**C/Y**		
Solve for		5.87%			

22. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter	40		±$1,083	$70 / 2	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		3.133%			

$3.133\% \times 2 = 6.27\%$

26.

	N	I/Y	PV	PMT	FV
Enter	26		±$1,089.60	$57 / 2	$1,000
Solve for		2.384%			

2.384% × 2 = 4.77%

29. *a.* The coupon bonds have a 6% coupon rate, which matches the 6% required return, so they will sell at par; number of bonds = $47,000,000 / $1,000 = 47,000.

For the zeroes:

	N	I/Y	PV	PMT	FV
Enter	40	6% / 2			$1,000
Solve for			$306.56		

$47,000,000 / $306.56 = 153,316 will be issued.

b. Coupon bonds: repayment = 47,000($1,030) = $48,410,000
Zeroes: repayment = 153,316($1,000) = $153,315,776

c. Coupon bonds: (47,000)($60)(1 – .35) = $1,833,000 cash outflow
Zeroes:

	N	I/Y	PV	PMT	FV
Enter	38	6% / 2			$1,000
Solve for			$325.23		

Year 1 interest deduction = $325.23 – 306.56 = $18.67
(153,316)($18.67)(.35) = $1,001,805 cash inflow

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

32.
Bond P
P_0

	N	I/Y	PV	PMT	FV
Enter	10	7%		$100	$1,000
Solve for			$1,210.71		

P_1

	N	I/Y	PV	PMT	FV
Enter	9	7%		$100	$1,000
Solve for			$1,195.46		

Current yield = $100 / $1,210.71 = .0826, or 8.26%
Capital gains yield = ($1,195.46 – 1,210.71) / $1,210.71 = –.0126, or –1.26%

Bond D
P_0

	N	I/Y	PV	PMT	FV
Enter	10	7%		$40	$1,000
Solve for			$789.29		

P_1

Enter	9	7%		$40	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$804.54		

Current yield = $40 / $789.29 = .0507, or 5.07%

Capital gains yield = ($804.54 – 789.29) / $789.29 = .0193, or 1.93%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 7 percent, but this return is distributed differently between current income and capital gains.