[Warm Up]

[Lesson Presentation]

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## Warm Up

**Determine if each statement is true or false.**

**1.** The measure of an obtuse angle is less than 90°. F

**2.** All perfect-square numbers are positive. T

**3.** Every prime number is odd. F

**4.** Any three points are coplanar. T

## Objectives

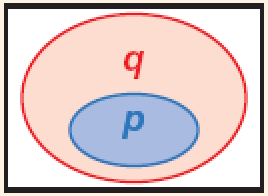
Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.

# Vocabulary

conditional statement
hypothesis
conclusion
truth value
negation
converse
inverse
contrapostive
logically equivalent statements

**Conditional Statements**

| DEFINITION | SYMBOLS | VENN DIAGRAM |
|---|---|---|
| A **conditional statement** is a statement that can be written in the form "if *p*, then *q*." | $p \rightarrow q$ |  |
| The **hypothesis** is the part *p* of a conditional statement following the word *if*. | | |
| The **conclusion** is the part *q* of a conditional statement following the word *then*. | | |

By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.

## Example 1: Identifying the Parts of a Conditional Statement

**Identify the hypothesis and conclusion of each conditional.**

**A. If today is Thanksgiving Day, then today is Thursday.**

Hypothesis: Today is Thanksgiving Day.

Conclusion: Today is Thursday.

**B. A number is a rational number if it is an integer.**

Hypothesis: A number is an integer.

Conclusion: The number is a rational number.

## Check It Out! Example 1

**Identify the hypothesis and conclusion of the statement.**

**"A number is divisible by 3 if it is divisible by 6."**

Hypothesis: A number is divisible by 6.

Conclusion: A number is divisible by 3.

**Writing Math**

"If *p*, then *q*" can also be written as "if *p*, *q*," "*q*, if *p*," "*p* implies *q*," and "*p* only if *q*."

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence's hypothesis and conclusion by figuring out which part of the statement depends on the other.

## Example 2A: Writing a Conditional Statement

**Write a conditional statement from the following.**

**An obtuse triangle has exactly one obtuse angle.**

**An obtuse triangle**

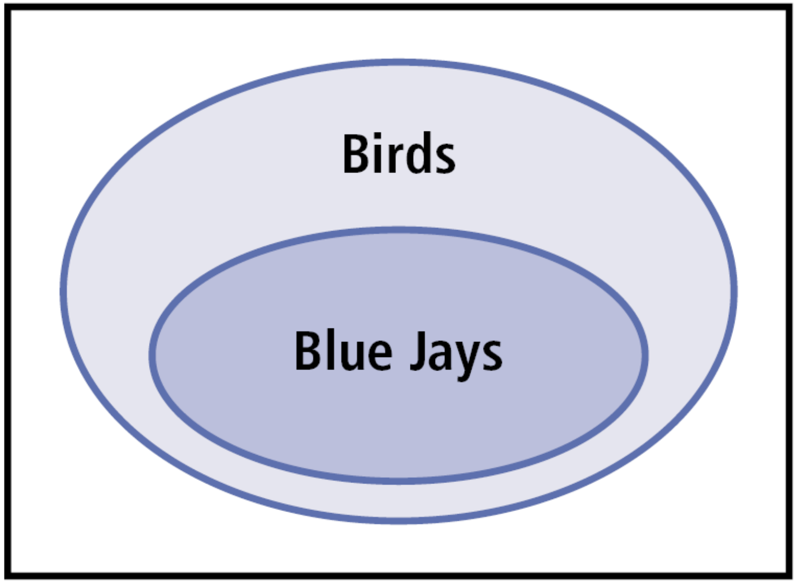
**has exactly one obtuse angle.**

*Identify the hypothesis and the conclusion.*

If a triangle is obtuse, then it has exactly one obtuse angle.

## Example 2B: Writing a Conditional Statement

**Write a conditional statement from the following.**




If an animal is a blue jay, then it is a bird.

The inner oval represents the hypothesis, and the outer oval represents the conclusion.

## Check It Out! Example 2

**Write a conditional statement from the sentence “Two angles that are complementary are acute.”**

**Two angles that are complementary**

**are acute.**

*Identify the hypothesis and the conclusion.*

If two angles are complementary, then they are acute.

A conditional statement has a **truth value** of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

## Example 3A: Analyzing the Truth Value of a Conditional Statement

**Determine if the conditional is true. If false, give a counterexample.**

**If this month is August, then next month is September.**

When the hypothesis is true, the conclusion is also true because September follows August. So the conditional is true.

## Example 3B: Analyzing the Truth Value of a Conditional Statement

**Determine if the conditional is true. If false, give a counterexample.**

**If two angles are acute, then they are congruent.**

You can have acute angles with measures of 80° and 30°. In this case, the hypothesis is true, but the conclusion is false.

Since you can find a counterexample, the conditional is false.

## Example 3C: Analyzing the Truth Value of a Conditional Statement

**Determine if the conditional is true. If false, give a counterexample.**

**If an even number greater than 2 is prime, then 5 + 4 = 8.**

An even number greater than 2 will never be prime, so the hypothesis is false. 5 + 4 is not equal to 8, so the conclusion is false. However, the conditional is true because the hypothesis is false.

## Check It Out! Example 3

**Determine if the conditional "If a number is odd, then it is divisible by 3" is true. If false, give a counterexample.**

An example of an odd number is 7. It is not divisible by 3. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.

**Remember!**

If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.

The **negation** of statement *p* is "not *p*," written as $\sim p$. The negation of a true statement is false, and the negation of a false statement is true.

**Related Conditionals**

| Definition | Symbols |
| --- | --- |
| A conditional is a statement that can be written in the form "If *p*, then *q*." | $p \to q$ |

**Related Conditionals**

| Definition | Symbols |
| --- | --- |
| The **converse** is the statement formed by exchanging the hypothesis and conclusion. | $q \to p$ |

**Related Conditionals**

| Definition | Symbols |
|---|---|
| The **inverse** is the statement formed by negating the hypothesis and conclusion. | $\sim p \rightarrow \sim q$ |

**Related Conditionals**

| Definition | Symbols |
|---|---|
| The **contrapositive** is the statement formed by both exchanging and negating the hypothesis and conclusion. | $\sim q \rightarrow \sim p$ |

## Example 4: Biology Application

**Write the converse, inverse, and contrapositive of the conditional statement. Use the Science Fact to find the truth value of each.**

***If an animal is an adult insect, then it has six legs.***

**Science Fact**

Adult insects have six legs.
No other animals have six legs.

## Example 4: Biology Application

*If an animal is an adult insect, then it has six legs.*

Converse: If an animal has six legs, then it is an adult insect.

No other animals have six legs so the converse is true.

Inverse: If an animal is not an adult insect, then it does not have six legs.

No other animals have six legs so the converse is true.

Contrapositive: If an animal does not have six legs, then it is not an adult insect.

Adult insects must have six legs. So the contrapositive is true.

## Check It Out! Example 4

**Write the converse, inverse, and contrapostive of the conditional statement "If an animal is a cat, then it has four paws." Find the truth value of each.**

*If an animal is a cat, then it has four paws.*

## Check It Out! Example 4

*If an animal is a cat, then it has four paws.*

Converse: If an animal has 4 paws, then it is a cat.

There are other animals that have 4 paws that are not cats, so the converse is false.

Inverse: If an animal is not a cat, then it does not have 4 paws.

There are animals that are not cats that have 4 paws, so the inverse is false.

Contrapositive: If an animal does not have 4 paws, then it is not a cat; True.

Cats have 4 paws, so the contrapositive is true.

Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

**Helpful Hint**

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.

## Lesson Quiz: Part I

**Identify the hypothesis and conclusion of each conditional.**

**1.** A triangle with one right angle is a right triangle.
H: A triangle has one right angle.
C: The triangle is a right triangle.

**2.** All even numbers are divisible by 2.
H: A number is even.
C: The number is divisible by 2.

**3.** Determine if the statement "If $n^2 = 144$, then $n = 12$" is true. If false, give a counterexample.
False; $n = -12$.

## Lesson Quiz: Part II

**Identify the hypothesis and conclusion of each conditional.**

**4.** Write the converse, inverse, and contrapositive of the conditional statement “If Maria’s birthday is February 29, then she was born in a leap year.” Find the truth value of each.

**Converse**: If Maria was born in a leap year, then her birthday is February 29; False.
**Inverse**: If Maria’s birthday is not February 29, then she was not born in a leap year; False.
**Contrapositive**: If Maria was not born in a leap year, then her birthday is not February 29; True.