**Teaching trigonometry**

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A summary of responses to a questionnaire: 29 responses received. These are in no way representative of practice in Oxfordshire, because the replies came in clusters of between 1 and 5 from a wide range of individual schools, all within the Oxford Partnership. I have summarised their contents to give me a starting point for further research. Where something was only mentioned by one person I have said so; in all other cases 2 or more teachers offered similar ideas**.**

**1. Where you start**

Many teachers start with collections of triangles, measurement of sides and calculation of ratios. Results are then tabulated on the board. Within this general approach there are significant differences:

**Similarity approach (the most common):** students fix the angle, or are given a fixed angle and look at right-angled triangles of different sizes to see that the ratio (or ratios) is invariant for a fixed angle. They can see that the ratios define the angle and *vice versa.* Some teachers start with tangent and talk about gradient; others start with all three ratios.

**Functional approach (a close second)**: students are given or asked to construct right-angled triangles with angles of 10, 20, 30 … degrees and then to see how the ratios of sides varies. Most who use this method use either a fixed radius (e.g. a metre stick on the board, or motion of a point round a circle) and focus on height from the horizontal (sine) or a fixed ‘base’ of 10 cm. and varying height (tan). This approach emphasises that the angle is the ‘driver’ or the ‘parameter’, and also avoids students assuming a linear relationship. They can see from the beginning that the values of sine, tan etc. do not double when the angle doubles.

**Multiplier approach:** a few teachers use the functional approach as above but focus on sin and cos as the *multiplier* for a unit radius circle (related to radian measure), so that you have to scale it by the length of any non-unit hypotenuse.

**Combination:** students are given, or construct, a range of right-angled triangles; measure sides and calculate ratios; compare results and, with a large enough set of learner-generated examples and a nimble teacher organising particular students to show each other their results, conjectures about equality emerge.

**Problems** and how you deal with them:

i) measurement is inaccurate and students often do not recall how to use a protractor. One teacher starts with a discussion of measurement as never being exact. Another uses dynamic geometry software and lets that do the measurements, but the students just sit and watch. The multiplier approach does not need accurate measurement.

ii) this may be the first time students have had to express a ratio as one number. Previously they may only have met this notation for ratios 3:4. Some teachers talk about ‘division’ or ‘proportion’ instead. Some teachers ensure that trig follows work on similar triangles so that students are already thinking about ratios of sides, scaling, multipliers and so on before they relate these to angles in right-angled triangles. Some emphasise that sine, cosine, tan etc. are multipliers by using fixed radii or bases of 10, 20, 30 cm. etc.

**Purposeful tools/more about triangles**

This usually follows work on Pythagoras’ theorem, discussing what this can and cannot do in the context of a general reminder about what we do and do not know about triangles so far, e.g. ‘Here is a triangle for which we know one side and one angle – Pythagoras is no help’. Introduce trig ratios as the formulae for finding out everything about a right-angled triangle, i.e. as tools.

Some teachers use contexts, such as trees and tall buildings and the problem of finding their heights – either following this with given formulae of with one of the exploratory approaches above.

One teacher tells students the three ratios and showing them how to use them with worked examples.

**2. What is on the board?**

I was interested in what is on the board since for people whose mind wanders (i.e. most adolescents) the board can tell the story of the lesson, and for me it indicates what the most important points might be.

For many the board has triangles, either similar triangles nested with varying bases or non-similar triangles with many orientations. Some have a standard diagram of a triangle with its usual labels, hypotenuse, opposite, adjacent and θ (one teacher says she avoids using θ in the first lesson).

One has the three ratios written up: sine θ = O/H and so on

Several have tables recording the findings of the measurement and calculation tasks

Many have key words written up

One teacher says that what is written varies as he responds to what students notice from their explorations

One has written SOHCAHTOA and another has this in triangular formats

One teacher said ‘TREES!!’

One teacher has “what patterns can you find?” and “ is there a relationship between the same sides in different triangles?”

**3. Language**

Nearly everyone talked of emphasising ‘hypotenuse, opposite and adjacent’ (apart from one who emphasises multiplication and scaling of a unit circle) and relating these to the orientation of the triangle and the angle under consideration. Labelling the givens was emphasised.

Other words teachers found important are: similar, ratio, function, angle, length, sine, cosine, tangent, inverse sine etc., multiplier, scale; accuracy, equality, horizontal, vertical, compare, trigonometric, elevation, depression. One teacher said they would not use names for ratios in the first lesson but would explore their meaning and use and name them later. Two teachers emphasised layout of answers as providing a language frame.

4. **What examples do you use?**

Some teachers interpreted this to mean ‘real life contexts’ and talked about heights of buildings, ladders, trees, pushing (?) in sports, surveying, clinometers, bearings, old books of tables, and so on.

One teacher talked about diagonals of rectangles as providing the initial stimulus. Another enters trig by talking about similar shapes more generally and then narrowing the field to right-angled triangles.

Specific examples teachers use with students include: 30, 60 triangles; angles varying by 10 degrees; triangles with deliberately obscure angles, such as 37 degrees; triangles with fixed angles. One teacher said she would always calculate ratios for BOTH angles in a triangle to show their relationships and also that labelling depended on the angle.

5. **Prior knowledge**

Teachers differed a lot about what they would assume students already know and what they would think would ‘come up’ while studying trig:

Language and properties of triangles

Pythagoras

Multiplicative relationships

Ratio as a/b rather than as a:b

Similar shapes and their properties

Calculator use for division

Scale drawing

Enlargement

Gradients

Functions and inverses

Scaling shapes

Protractor use

6. **What sort of questions do you set?**

As well as the approaches described above, teachers would set the following kinds of work early on:

Labelling triangles

Which ratio would you use to ….?

Find sin/cos/tan of some angles

Use your own results so far with a fixed radius to find ‘opposite’ length for the same angles and different radii

Build up to opp = sinA x hyp as ‘scaling’

Use a given ratio to find some unknown lengths

Draw three right-angled triangles with YOUR choice of angle and repeat measure and calculate task

One teacher said: the first lesson is discussion only

Others offered no content information, saying:’ tenticks’, ‘AQA’, ‘worksheet’, with no further information

7. **Homework**

Find contexts for trig

Research trig on the net

Label these triangles

What about angles of (10n + 5) degrees (for students who have done multiples of 10 degrees in class

Plot your ratios for various angles

Calculator familiarisation

Questions using tan (for those who have only met tan so far)

Use certain calculator buttons and compare to your lesson results

Find same ratio from several similar triangles

‘mymaths’

‘online homework wherever they are up to’

8 & 9 **What next and future work: a range of comments**

*General comments:*

The first lesson is crucial for what follows, no point in rushing on

I would emphasise that this is just the very beginning of something big

Get them comfortable, show them the wonder and power.

*Follow on from homework*

Discuss what they learnt from their homework (researching trig on net)

Use own constructed graph to find angles and side lengths of various triangles, then use calculator

*Mathematical contexts*

Discuss what fixes a triangle (conditions for congruence)

Standard triangles

3D trig

*Practical contexts*

Go outside and use trundle wheels

Pitch of roof on new garage extension must not collide with landing window

Ladders up walls

Diagrams from word problems

Get good at drawing diagrams

*Graduated difficulty of using formulae*

Formalise after they understand

Descriptions of how use of formulae gets gradually more difficult. Although most who did this showed that finding the angle might be done last, no one write specifically about how students might understand the inverse process – this was implicit in the graph-use and possibly in the calculator use approaches.

Practice on basic triangles

One teacher uses tables rather than calculator at first

SOHCAHTOA was mentioned by two more teachers

What the denominator in the formula means

How to apply to isosceles triangles

Find sides first, then angles

Layout of answers

*Beyond right-angled triangles*

How can we use this for non-right angled triangles?

Beyond acute angles v. stick with acute angles; e.g. on teacher would ask them to fill in a table for hyp = 1 for angles up to 360

*Where else does this go?*

Several wanted to end the lesson sequence with practical applications; others with exam questions.

**Different groups**

In all questions, only three teachers mentioned that they might do things differently with different groups:

(i) *with lower sets I would do more drawing and contexts, with higher sets just give the formulae*

(ii) *depends on syllabus for that set what I would do next*

(iii) *depends on how students respond what order we do things in.*