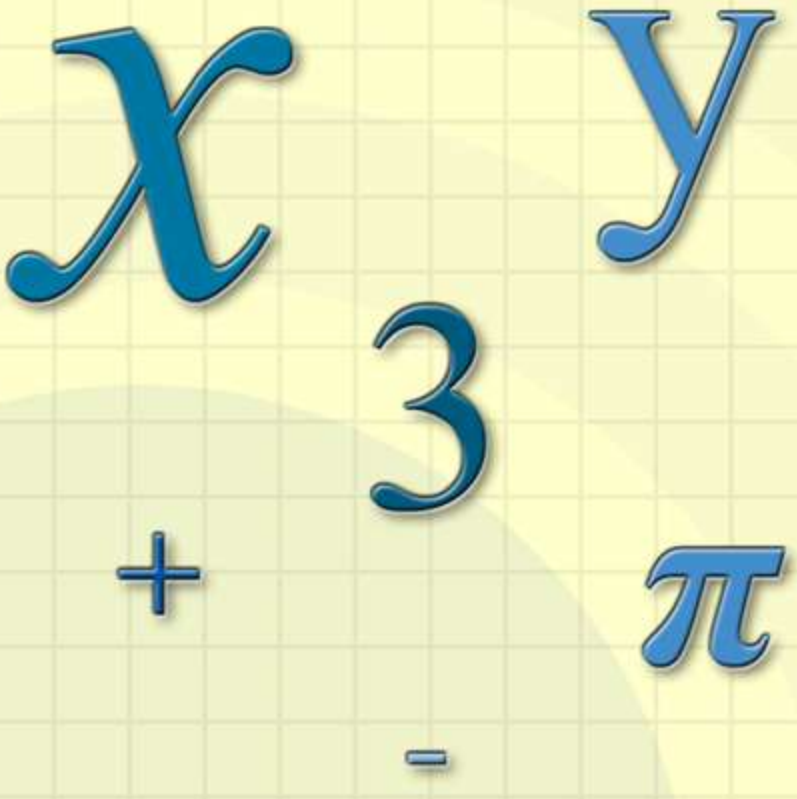
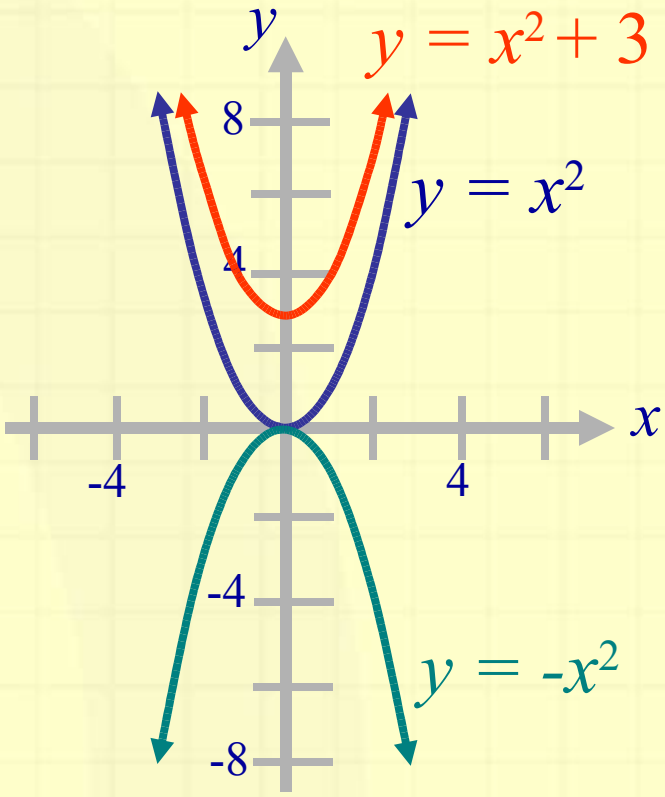


Digital Lesson

Shifting Graphs

The graphs of many functions are transformations of the graphs of very basic functions.

Example:
The graph of $y = x^2 + 3$ is the graph of $y = x^2$ shifted *upward three units*. This is a vertical **shift**.

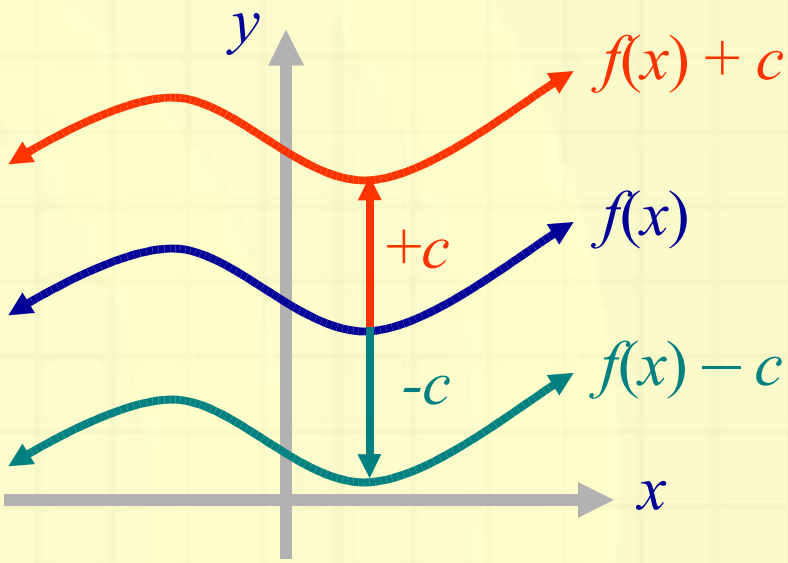


The graph of $y = -x^2$ is the **reflection** of the graph of $y = x^2$ in the x-axis.

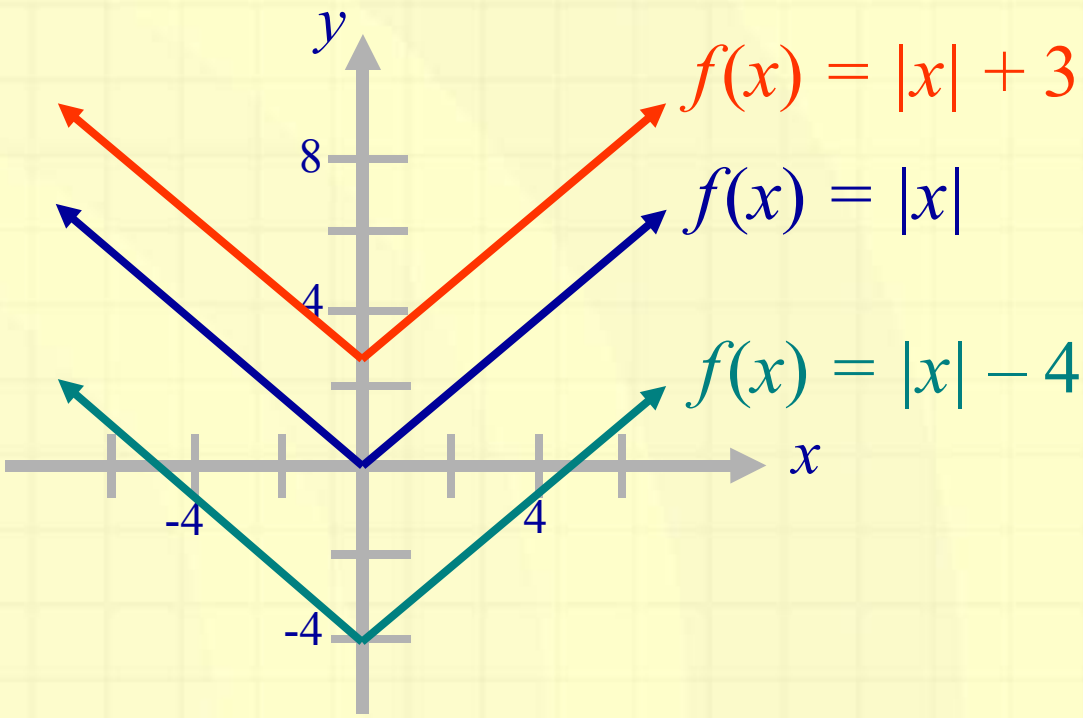
Vertical Shifts

If c is a positive real number, the graph of $f(x) + c$ is the graph of $y = f(x)$ **shifted upward** c units.

If c is a positive real number, the graph of $f(x) - c$ is the graph of $y = f(x)$ **shifted downward** c units.



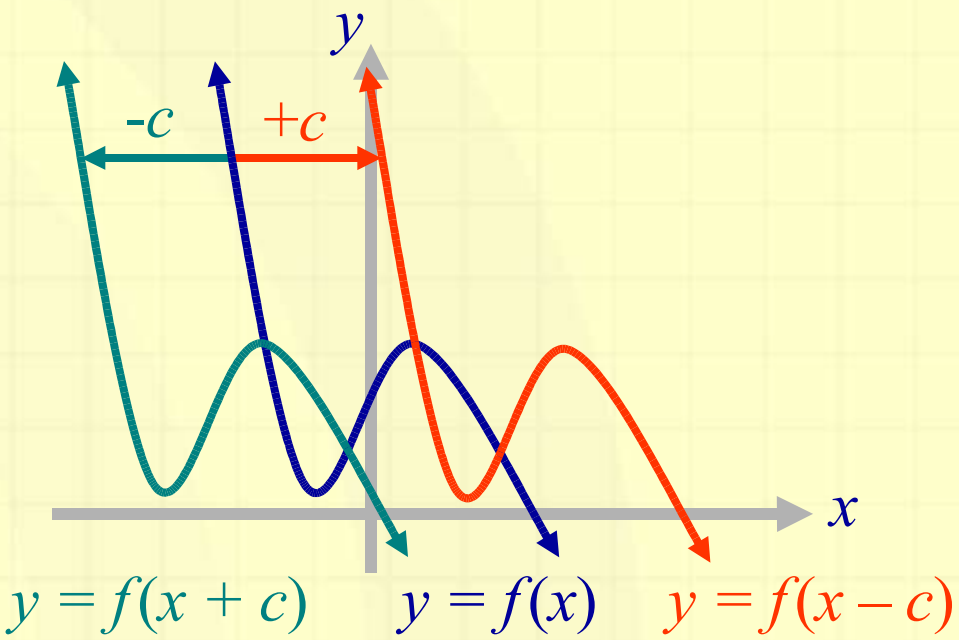
Example: Use the graph of $f(x) = |x|$ to graph the functions $f(x) = |x| + 3$ and $f(x) = |x| - 4$.



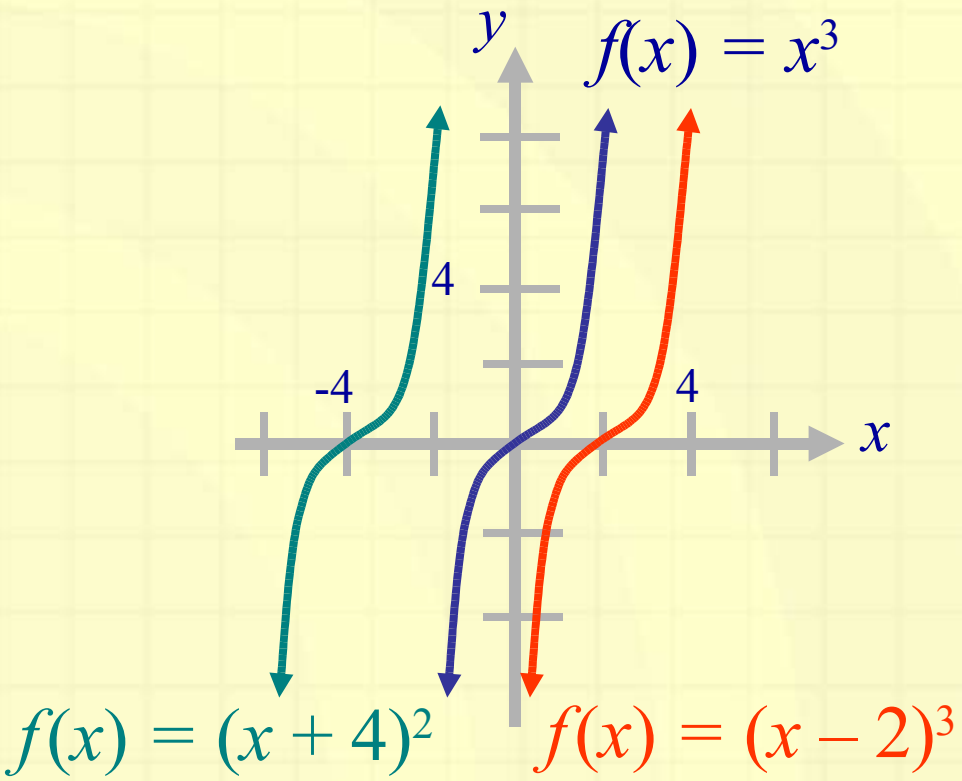
Horizontal Shifts

If c is a positive real number, then the graph of $f(x - c)$ is the graph of $y = f(x)$ **shifted to the right** c units.

If c is a positive real number, then the graph of $f(x + c)$ is the graph of $y = f(x)$ **shifted to the left** c units.

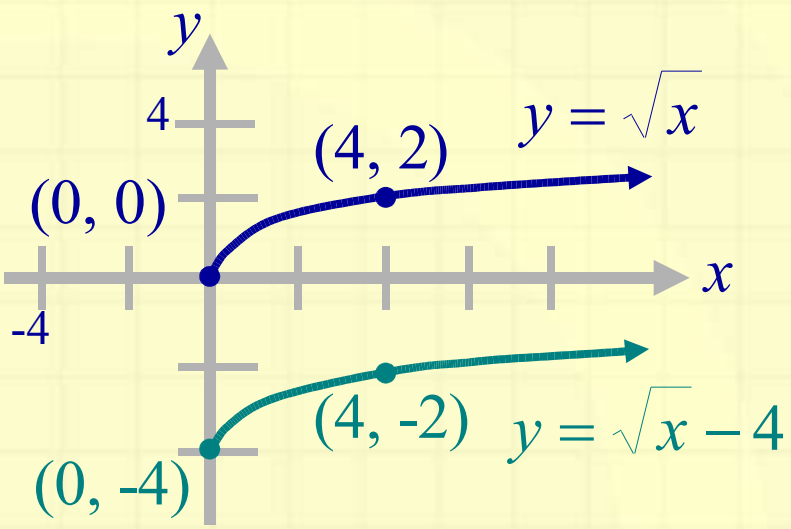


Example: Use the graph of $f(x) = x^3$ to graph $f(x) = (x - 2)^3$ and $f(x) = (x + 4)^2$.

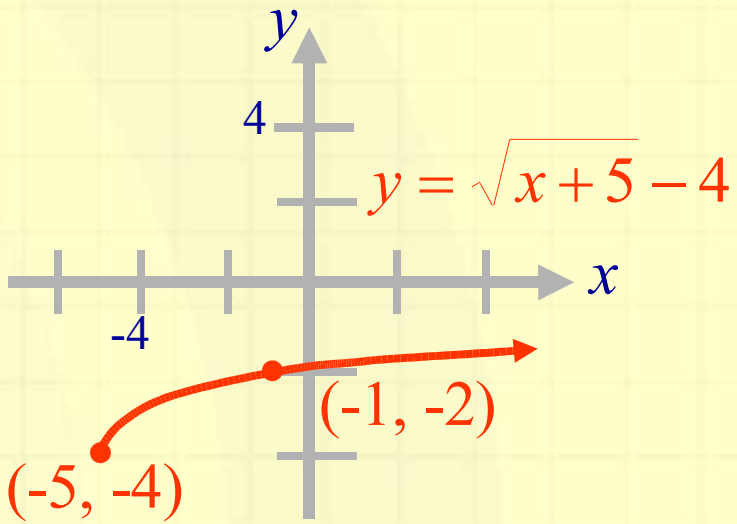


Example: Graph the function $y = \sqrt{x+5} - 4$ using the graph of $y = \sqrt{x}$.

First make a *vertical shift 4 units downward.*



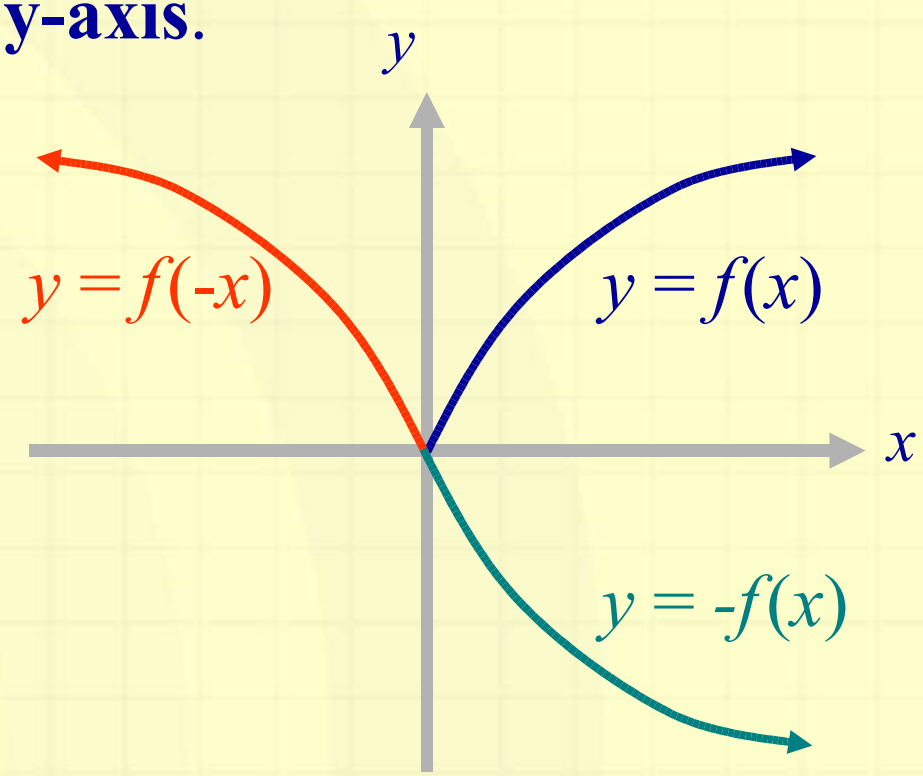
Then a *horizontal shift 5 units left.*



The graph of a function may be a **reflection** of the graph of a basic function.

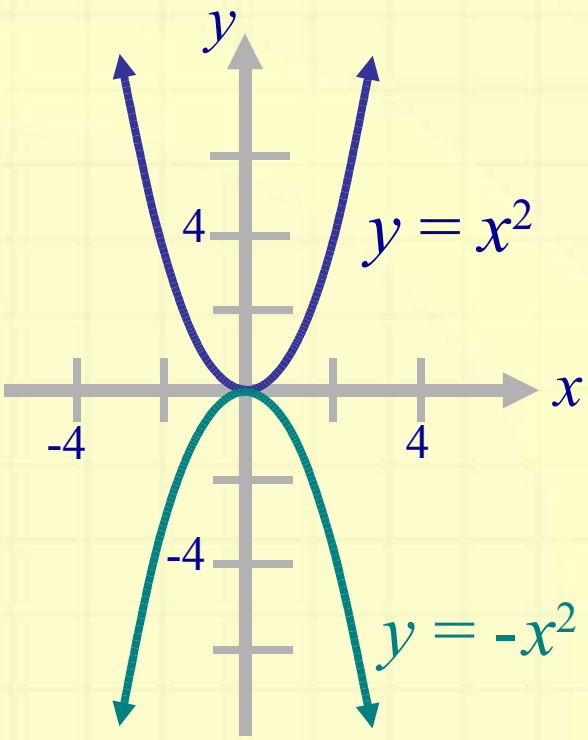
The graph of the function $y = f(-x)$ is the graph of $y = f(x)$ **reflected in the y-axis**.

The graph of the function $y = -f(x)$ is the graph of $y = f(x)$ **reflected in the x-axis**.

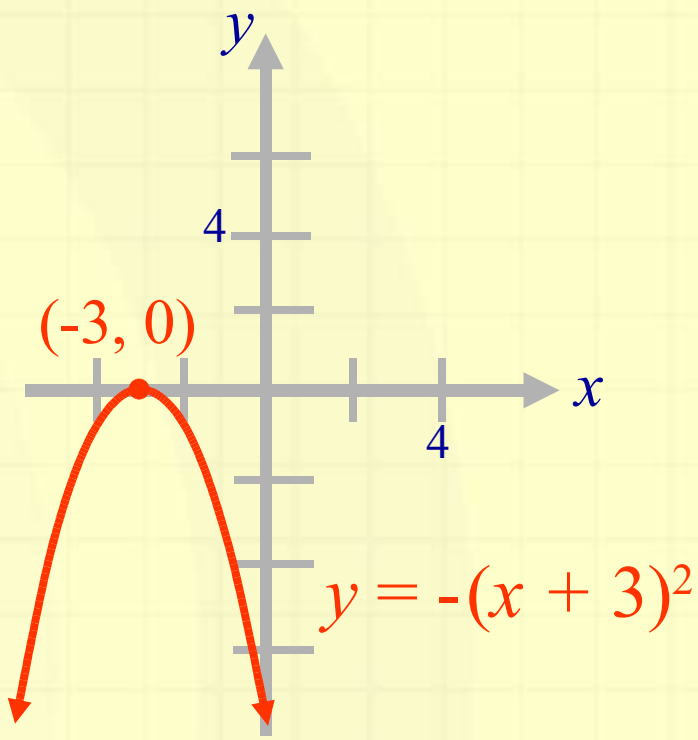


Example: Graph $y = -(x + 3)^2$ using the graph of $y = x^2$.

First *reflect the graph* in the x-axis.



Then *shift the graph* three units to the left.



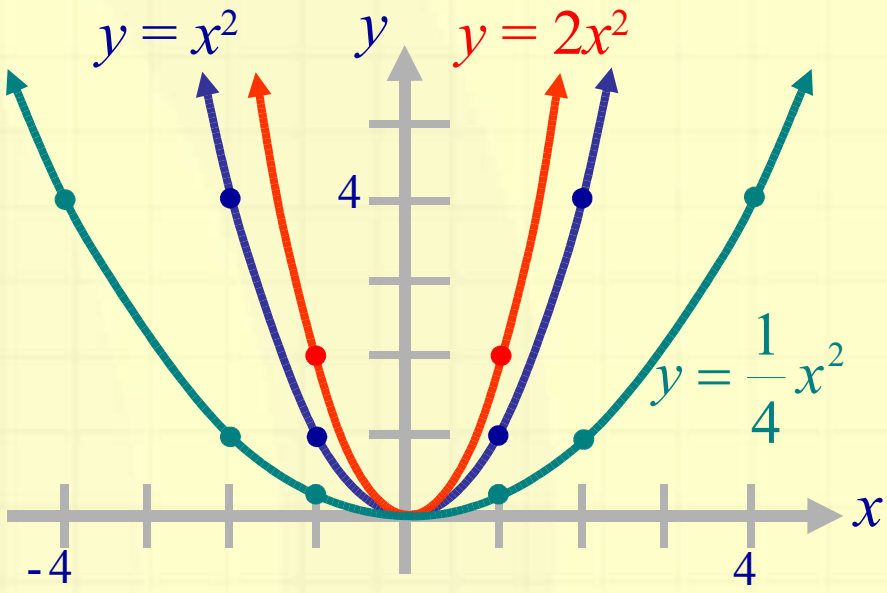
Vertical Stretching and Shrinking

If $c > 1$ then the graph of $y = cf(x)$ is the graph of $y = f(x)$ **stretched vertically by *c*.**

If $0 < c < 1$ then the graph of $y = cf(x)$ is the graph of $y = f(x)$ **shrunk vertically by *c*.**

Example: $y = 2x^2$ is the graph of $y = x^2$ *stretched vertically by 2.*

$y = \frac{1}{4}x^2$ is the graph of $y = x^2$ *shrunk vertically by* $\frac{1}{4}$.



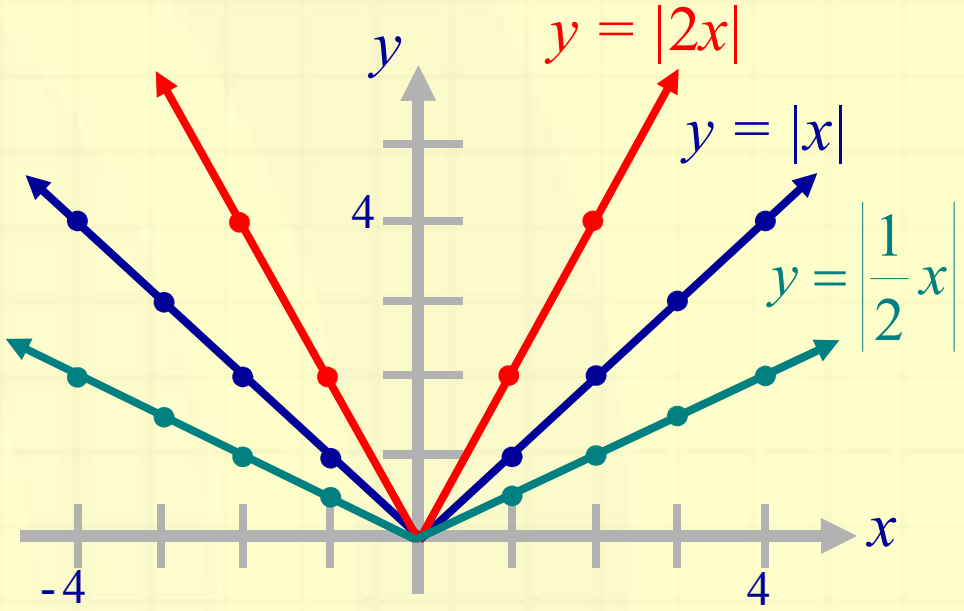
Horizontal Stretching and Shrinking

If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ **shrunk horizontally by *c*.**

If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ **stretched horizontally by *c*.**

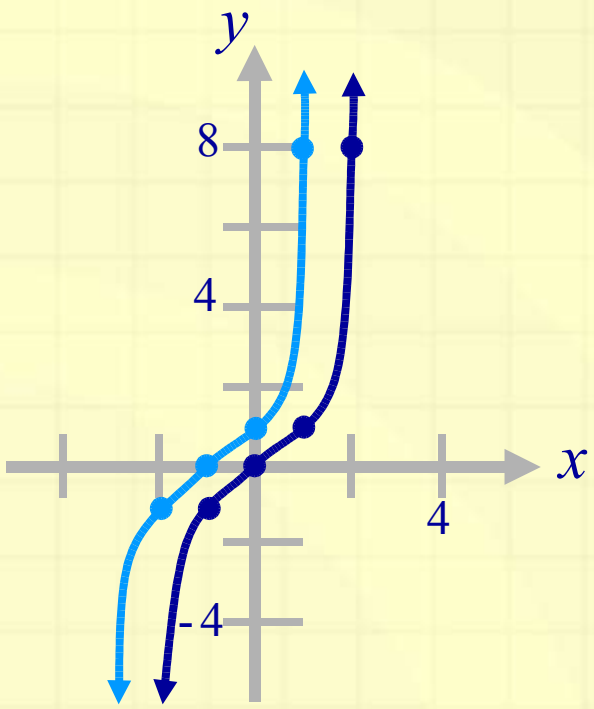
Example: $y = |2x|$ is the graph of $y = |x|$ *shrunk horizontally by 2.*

$y = \left|\frac{1}{2}x\right|$ is the graph of $y = |x|$ *stretched horizontally by* $\frac{1}{2}$.



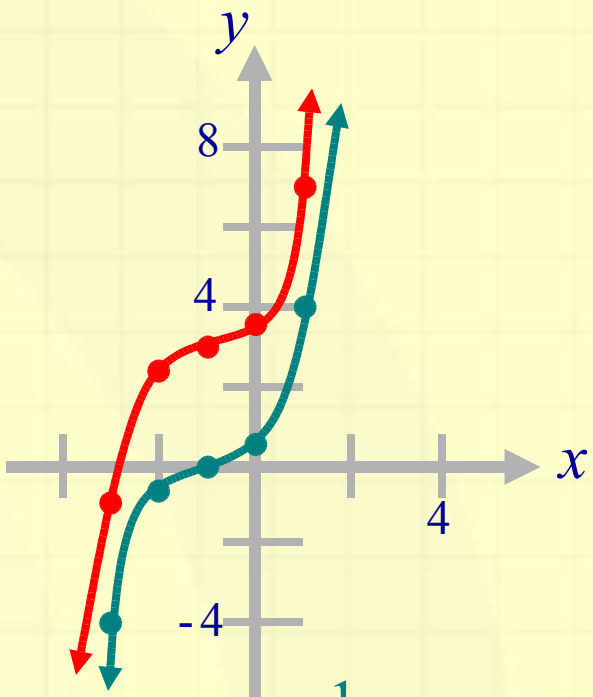
Example: Graph $y = \frac{1}{2}(x+1)^3 + 3$ given the graph $y = x^3$.

Graph $y = x^3$ and do one transformation at a time.



Step 1: $y = x^3$

Step 2: $y = (x + 1)^3$



Step 3: $y = \frac{1}{2}(x+1)^3$

Step 4: $y = \frac{1}{2}(x+1)^3 + 3$