Significant Figure Rules for Logarithms

- Things to remember: significant figures include all certain digits and the first uncertain digit. There is always some uncertainty in the last digit.
- Regular sig fig rules are guidelines, and they don't always predict the correct number of significant figures. We use the sig fig rules as a shortcut so that we don't have to do a detailed error analysis on everything we calculate.
- Logs are exponents.

Playing with some numbers:

log 0.00722 = -2.14146
log 0.00723 = -2.14086
log 0.00724 = -2.14026

The numbers we took the log of had three significant figures. The results of the calculations differed in the third decimal place.

Here are some more. In this case, the numbers we are taking the log of have two significant figures.

log 0.0056 = -2.2518
log 0.0057 = -2.2441
log 0.0058 = -2.2366

The results above start to differ in the second decimal place.

Here is another set:

log 0.00056 = -3.2518
log 0.00057 = -3.2441
log 0.00058 = -3.2366

Again, the numbers we took the log of have two significant figures, and the results differed in the second decimal place.

Conclusion: When you take the log of a number with **N significant figures**, the result should have **N decimal places**. The number in front of the decimal place indicates only the order of magnitude. It is not a significant figure.
The rule for natural logs (ln) is similar, but not quite as clear-cut. For simplicity, we will use the above rules for natural logs too.

Using natural logs:

ln 0.0056 = -5.1850
ln 0.0057 = -5.1673
ln 0.0058 = -5.1499

Note that the numbers each had two significant figures, and the results started to differ in the second decimal place.

Going the other way:

The opposite of taking the log of a number is to raise 10 to the power of that number. This corresponds to the 10^x button on your calculator. The sig fig rule for this function is the opposite of the sig fig rule for logs.

Let's try some numbers:

$10^{2.890} = 776.25$
$10^{2.891} = 778.04$
$10^{2.892} = 779.83$

Notice that the original numbers had 3 digits behind the decimal place. The results differ in the third place.

$10^{1.2} = 15.8$
$10^{1.3} = 19.9$
$10^{1.4} = 25.1$

In this case, the original numbers had 1 digit behind the decimal place. The results differ in the first place.

Conclusion: When raising 10 to a power: If the power of 10 has **N decimal places**, the result should have **N significant figures**. The rule for raising e to a power is similar. For simplicity, we will use this rule for both situations.

For example:
$e^{-2.55} = 0.078082$
$e^{-2.56} = 0.077305$
$e^{-2.57} = 0.076535$
The numbers used in the power of e each had two decimal places. The results started to differ in the second place. The result should have 2 significant figures.