Uncertainties and Significant Figures

All measurements always have some uncertainty. We refer to the uncertainty as the error in the measurement. Errors fall into two categories:

1. **Systematic Error** - errors resulting from measuring devices being out of calibration. Such measurements will be consistently too small or too large. These errors can be eliminated by pre-calibrating against a known, trusted standard.

2. **Random Errors** - errors resulting in the fluctuation of measurements of the same quantity about the average. The measurements are equally probable of being too large or too small. These errors generally result from the fineness of scale division of a measuring device.

Physics is a quantitative science and that means a lot of measurements and calculations. These calculations involve measurements with uncertainties and thus it is essential for the physics student to learn how to analyze these uncertainties (errors) in any calculation.

Systematic errors are generally "simple" to analyze but random errors require a more careful analysis and thus it will be our focus. There is a statistical method for calculating random uncertainties in measurements. This requires taking at least 10 measurements of a quantity. We will consider such method later on in the lab. For now we will consider the uncertainty associated with a single measurement.

The following general rules of thumb are often used to determine the uncertainty in a single measurement when using a scale or digital measuring device.

1. **Uncertainty in a Scale Measuring Device is equal to the smallest increment divided by 2.**

$$\sigma_x = \frac{\text{smallest increment}}{2}$$

2. **Uncertainty in a Digital Measuring Device is equal to the smallest increment.**

$$\sigma_x = \text{smallest increment}$$

Ex. Meter Stick (scale device)

$$\sigma_x = \frac{1\,\text{mm}}{2} = 0.5mm = 0.05cm$$

Ex. Digital Balance (digital device)

$$\boxed{5 \mid . \mid 7 \mid 5 \mid 1 \mid 3} \;\; kg$$

$$\sigma_x = 0.0001 kg$$

When stating a measurement the uncertainty should be stated explicitly so that there is no question about the uncertainty in the measurement. However, if the is not stated explicitly, an uncertainty is still implied.

For example, if we measure a length of 5.7 cm with a meter stick, this implies that the length can be anywhere in the range 5.65 cm $\leq$ L $\leq$ 5.75 cm. Thus, L =5 .7 cm measured with a meter stick implies an uncertainty of 0.05 cm. A common rule of thumb is to take one-half the unit of the last decimal place in a measurement to obtain the uncertainty.

In general, any measurement can be stated in the following preferred form:

$$measurement = x_{best} \pm \sigma_x$$
$$x_{best} = \text{best estimate of measurement}$$
$$\sigma_x = \text{uncertainty (error) in measurement}$$

Rule For Stating Uncertainties - Experimental uncertainties should be stated to 1-significant figure.

      Ex. v = 31.25 $\pm$ 0.034953 m/s
         v = 31.25 $\pm$ 0.03 m/s (correct)

The uncertainty is just an estimate and thus it cannot be more precise (more significant figures) than the best estimate of the measured value.

Rule For Stating Answers – The last significant figure in any answer should be in the same place as the uncertainty.

Ex. a = 1261.29 $\pm$ 200 cm/s^2
   a = 1300 $\pm$ 200 cm/s^2 (correct)

Since the uncertainly is stated to the hundreds place, we also state the answer to the hundreds place. Note that the uncertainty determines the number of significant figures in the answer.

Significant Figures

Calculating uncertainties in calculations involving measurements (error propagation) can sometimes be time consuming. A quicker and approximate method that is used to determine the number of significant figures in a calculation is to use a couple rules.

DEF: A significant figure is a reliably known digit.

- Because zeros serve as counters and to set the decimal point, they present a problem when determining significant figures in a number.

A. Rules for Determining Significant Figures in a Number

1. All non-zero numbers are significant.
2. Zeros within a number are always significant.
3. Zeros that do nothing but set the decimal point are not significant. Both 0.000098 and 0.98 contain two significant figures.
4. Zeros that aren't needed to hold the decimal point are significant. For example, 4.00 has three significant figures.
5. Zeros that follow a number *may* be significant.

B. Rule for Adding and Subtracting Significant Figures

When measurements are added or subtracted, the number of decimal places in the final answer should equal the smallest number of decimal places of any term.

Ex. 256.5895 g
  + 8.1 g
M = 264.6895 g
M = 264.7 g (answer)

C. Rule for Multiplying/Dividing Significant Figures
When measurements are multiplied or divided, the number of   significant figures in the final answer should be the same as the term with the lowest number of significant figures.

Ex. L_1=2.2 cm
  L_2=38.2935 cm
   $A=L_1L_2$=84.126900000 cm^2
   A=84 cm^2 (answer)

D. Stating a number in scientific notation removes all ambiguities with regard to how many significant figures a number has.
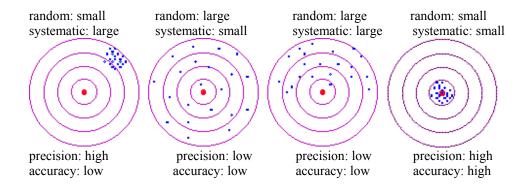
Accuracy and Precision

The terms accuracy and precision are often mistakenly used interchangeably. In error analysis there is a clear distinction between the two.

Accuracy – an indication of how close a set of measurements is to the exact (true) value.

Precision – a measure of the closeness of a set of measurements. (sometimes it is used to specify the exactness of a measurement)

To get a better feeling for the difference between accuracy & precision and random & systematic errors, let's consider the following shooting-target analogy. The experiment is to shoot a set of rounds at a stationary target and analyze the results. The results are summarized below.

random: small   random: large   random: large   random: small
systematic: large   systematic: small   systematic: large   systematic: small

precision: high   precision: low   precision: low   precision: high
accuracy: low   accuracy: low   accuracy: low   accuracy: high

Fractional Uncertainty

The uncertainty σ_x in the *measurement* $= x_{best} \pm \sigma_x$ indicates the precision of a measurement. However, σ_x by itself does not.

Ex.   Highly Precise Measurement
$$x_{best} = 1005 \text{ m}$$
$$\sigma_x = 1 \text{ m}$$

Ex.   Poorly Precise Measurement
$$x_{best} = 3 \text{ m}$$
$$\sigma_x = 1 \text{ m}$$

Therefore, the quality of a measurement is indicated not just by σ_x , but also by the ratio of σ_x to x_{best}.

We define the fractional uncertainty of a measurement, also called the precision of the measurement, by the following:

$$\text{fractional uncertainty} = \frac{\sigma_x}{|x_{best}|} = \text{relative uncertainty} = \text{precision}$$

To avoid confusion with fractional uncertainty, the uncertainty is sometimes called the absolute uncertainty. The fractional uncertainty (precision) of a measurement is often expressed a percentage.

Ex. $x = 47 \pm 2$ cm

$\sigma_x = 2$ cm

$x_{best} = 47$ cm

$\dfrac{\sigma_x}{|x_{best}|} = \dfrac{2}{47} = 0.043 \text{ or } 4.3\%$

$x = 47 \text{ cm} \pm 4.3 \%$