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Chapter 7 Resource Masters

Algebra 1

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CONSUMABLE WORKBOOKS Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

	MHID	ISBN
Study Guide and Intervention Workbook	0-07-660292-3	978-0-07-660292-6
Homework Practice Workbook	0-07-660291-5	978-0-07-660291-9
Spanish Version		
Homework Practice Workbook	0-07-660294-X	978-0-07-660294-0

Answers For Workbooks The answers for Chapter 7 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

ConnectED All of the materials found in this booklet are included for viewing, printing, and editing at connected.mcgraw-hill.com.

Spanish Assessment Masters (MHID: 0-07-660289-3, ISBN: 978-0-07-660289-6) These masters contain a Spanish version of Chapter 7 Test Form 2A and Form 2C.

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Contents

Teacher's Guide to Using the *Chapter 7 Resource Masters*

The *Chapter 7 Resource Masters* includes the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing, printing, and editing at connectED.mcgraw-hill.com.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 7-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, TI-Nspire, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the ***Chapter 7 Resource Masters*** offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 11 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- ***Form 1*** contains multiple-choice questions and is intended for use with below grade level students.
- ***Forms 2A and 2B*** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Forms 2C and 2D*** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Form 3*** is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
- Full-size answer keys are provided for the assessment masters.

NAME _____ DATE _____ PERIOD _____

7 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
binomial by·NOH·mee·uhl		
constant		
common ratio		
compound interest		
cube root		
exponential decay function		
exponential equation		
exponential function		
exponential growth function		

(continued on the next page)

NAME _____ DATE _____ PERIOD _____

7 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
geometric sequence		
monomial mah·NOH·mee·uhl		
negative exponent		
*n*th root		
order of magnitude		
rational exponent		
recursive formula		
scientific notation		
zero exponent		

NAME _____ DATE _____ PERIOD _____

7 Anticipation Guide

Exponents and Exponential Functions

Step 1 ***Before you begin Chapter 7***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. When multiplying two powers that have the same base, multiply the exponents.	
	2. $(k^3)^4$ is equivalent to k^{12}.	
	3. To divide two powers that have the same base, subtract the exponents.	
	4. $\left(\frac{2}{5}\right)^3$ is the same as $\frac{2^3}{5}$.	
	5. A polynomial may contain one or more monomials.	
	6. The degree of the polynomial $3x^2y^3 - 5y^2 + 8x^3$ is 3 because the greatest exponent is 3.	
	7. A function containing powers is called an exponential function.	
	8. Receiving compound interest on a bank account is one example of exponential growth.	

Step 2 ***After you complete Chapter 7***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE _____ FECHA _____ PERÍODO _____

7 Ejercicios preparatorios

Exponentes y Funciones Exponenciales

Paso 1 ***Antes de comenzar el Capítulo 7***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. Al multiplicar dos potencias con la misma base, multiplica los exponentes.	
	2. $(k^3)^4$ es equivalente a k^{12}.	
	3. Para dividir dos potencias que tienen la misma base, resta los exponentes.	
	4. $\left(\frac{2}{5}\right)^3$ es lo mismo que $\frac{2^3}{5}$.	
	5. Un polinomio puede contener uno o más monomios.	
	6. El grado del polinomio $3x^2y^3 - 5y^2 + 8x^3$ es 3 porque el exponente más grande es 3.	
	7. Una función que contiene potencias se denomina función exponencial.	
	8. El recibir interés compuesto en una cuenta bancaria es un ejemplo de crecimiento exponencial.	

Paso 2 ***Después de completar el Capítulo 7***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME _____ DATE _____ PERIOD _____

7-1 Study Guide and Intervention

Multiplication Properties of Exponents

Multiply Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Group the coefficients and the variables

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Lesson 7-1

Exercises

Simplify each expression.

1. $y(y^5)$

2. $n^2 \cdot n^7$

3. $(-7x^2)(x^4)$

4. $x(x^2)(x^4)$

5. $m \cdot m^5$

6. $(-x^3)(-x^4)$

7. $(2a^2)(8a)$

8. $(rn)(rn^3)(n^2)$

9. $(x^2y)(4xy^3)$

10. $\frac{1}{3}(2a^3b)(6b^3)$

11. $(-4x^3)(-5x^7)$

12. $(-3j^2k^4)(2jk^6)$

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$

14. $(-5xy)(4x^2)(y^4)$

15. $(10x^3yz^2)(-2xy^5z)$

NAME _____ DATE _____ PERIOD _____

7-1 Study Guide and Intervention *(continued)*

Multiplication Properties of Exponents

Simplify Expressions An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and any integers m and p, $(a^m)^p = a^{mp}$.
Power of a Product	For any numbers a and b and any integer m, $(ab)^m = a^m b^m$.

We can combine and use these properties to simplify expressions involving monomials.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power

$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product

$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables

$= (-2)^3(a^{11})(b^2)^3$ Product of Powers

$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify each expression.

1. $(y^5)^2$

2. $(n^7)^4$

3. $(x^2)^5(x^3)$

4. $-3(ab^4)^3$

5. $(-3ab^4)^3$

6. $(4x^2b)^3$

7. $(4a^2)^2(b^3)$

8. $(4x)^2(b^3)$

9. $(x^2y^4)^5$

10. $(2a^3b^2)(b^3)^2$

11. $(-4xy)^3(-2x^2)^3$

12. $(-3j^2k^3)^2(2j^2k)^3$

13. $(25a^2b)^3\left(\frac{1}{5}abf\right)^2$

14. $(2xy)^2(-3x^2)(4y^4)$

15. $(2x^3y^2z^2)^3(x^2z)^4$

16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$

17. $(-3a^3n^4)(-3a^3n)^4$

18. $-3(2x)^4(4x^5y)^2$

NAME _____ DATE _____ PERIOD _____

7-1 Skills Practice

Multiplication Properties of Exponents

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11

2. $a - b$

3. $\frac{p^2}{r^2}$

4. y

5. j^3k

6. $2a + 3b$

Simplify.

7. $a^2(a^3)(a^6)$

8. $x(x^2)(x^7)$

9. $(y^2z)(yz^2)$

10. $(\ell^2k^2)(\ell^3k)$

11. $(a^2b^4)(a^2b^2)$

12. $(cd^2)(c^3d^2)$

13. $(2x^2)(3x^5)$

14. $(5a^7)(4a^2)$

15. $(4xy^3)(3x^3y^5)$

16. $(7a^5b^2)(a^2b^3)$

17. $(-5m^3)(3m^8)$

18. $(-2c^4d)(-4cd)$

19. $(10^2)^3$

20. $(p^3)^{12}$

21. $(-6p)^2$

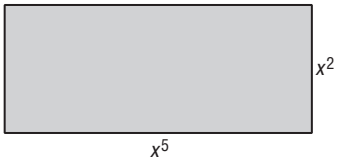
22. $(-3y)^3$

23. $(3pr^2)^2$

24. $(2b^3c^4)^2$

GEOMETRY Express the area of each figure as a monomial.

25.



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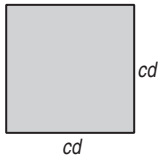

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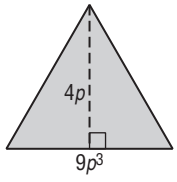


Lesson 7-1

NAME _____ DATE _____ PERIOD _____

7-1 Practice

Multiplication Properties of Exponents

Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

1. $\frac{21a^2}{7b}$

2. $\frac{b^3c^2}{2}$

Simplify each expression.

3. $(-5x^2y)(3x^4)$

4. $(2ab^2f^2)(4a^3b^2f^2)$

5. $(3ad^4)(-2a^2)$

6. $(4g^3h)(-2g^5)$

7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$

8. $(-xy)^3(xz)$

9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$

10. $(0.2a^2b^3)^2$

11. $\left(\frac{2}{3}p\right)^2$

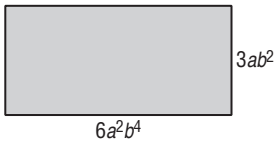
12. $\left(\frac{1}{4}ad^3\right)^2$

13. $(0.4k^3)^3$

14. $[(4^2)^2]^2$

GEOMETRY Express the area of each figure as a monomial.

15.



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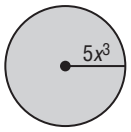

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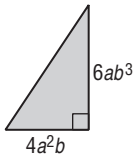
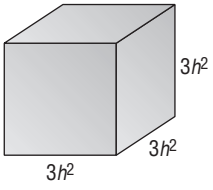


GEOMETRY Express the volume of each solid as a monomial.

18.



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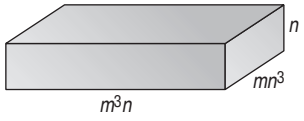

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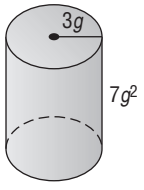


21. COUNTING A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set?

22. HOBBIES Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year?

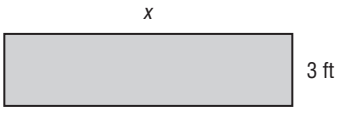
NAME _____ DATE _____ PERIOD _____

7-1 Word Problem Practice

Multiplication Properties of Exponents

1. GRAVITY An egg that has been falling for x seconds has dropped at an average speed of $16x$ feet per second. If the egg is dropped from the top of a building, its total distance traveled is the product of the average rate times the time. Write a simplified expression to show the distance the egg has traveled after x seconds.

2. CIVIL ENGINEERING A developer is planning a sidewalk for a new development. The sidewalk can be installed in rectangular sections that have a fixed width of 3 feet and a length that can vary. Assuming that each section is the same length, express the area of a 4-section sidewalk as a monomial.



3. PROBABILITY If you flip a coin 3 times in a row, there are 2^3 outcomes that can occur.

Outcomes	
HHH	TTT
HTT	THH
HTH	TTH
HHT	THT

If you then flip the coin two more times, there are $2^3 \times 2^2$ outcomes that can occur. How many outcomes can occur if you flip the coin as mentioned above four more times? Write your answer in the form 2^x.

4. SPORTS The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Find the volume of air in three different basketballs. Use $\pi = 3.14$. Round your answers to the nearest whole number.

Ball	Radius (in.)	Volume (in^3)
Child's	4	
Women's	4.5	
Men's	4.8	

5. ELECTRICITY An electrician uses the formula $W = I^2R$, where W is the power in watts, I is the current in amperes, and R is the resistance in ohms.

a. Find the power in a household circuit that has 20 amperes of current and 5 ohms of resistance.

b. If the current is reduced by one half, what happens to the power?

Lesson 7-1

NAME _____ DATE _____ PERIOD _____

7-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned} 1001101_2 &= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 64 + 0 \times 32 + 0 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1 \\ &= 77 \end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 **2.** 10000_2 **3.** 11000011_2 **4.** 10111001_2

Write each decimal number as a binary number.

5. 8 **6.** 11 **7.** 29 **8.** 117

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers.

The American Standard Guide for Information Interchange (ASCII)							
A	65	**N**	78	**a**	97	**n**	110
B	66	**O**	79	**b**	98	**o**	111
C	67	**P**	80	**c**	99	**p**	112
D	68	**Q**	81	**d**	100	**q**	113
E	69	**R**	82	**e**	101	**r**	114
F	70	**S**	83	**f**	102	**s**	115
G	71	**T**	84	**g**	103	**t**	116
H	72	**U**	85	**h**	104	**u**	117
I	73	**V**	86	**i**	105	**v**	118
J	74	**W**	87	**j**	106	**w**	119
K	75	**X**	88	**k**	107	**x**	120
L	76	**Y**	89	**l**	108	**y**	121
M	77	**Z**	90	**m**	109	**z**	122

NAME _____ DATE _____ PERIOD _____

7-2 Study Guide and Intervention

Division Properties of Exponents

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Lesson 7-2

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{5^5}{5^2}$

2. $\frac{m^6}{m^4}$

3. $\frac{p^5n^4}{p^2n}$

4. $\frac{a^2}{a}$

5. $\frac{x^5y^3}{x^5y^2}$

6. $\frac{-2y^7}{14y^5}$

7. $\frac{xy^6}{y^4x}$

8. $\left(\frac{2a^2b}{a}\right)^3$

9. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^3$

10. $\left(\frac{2r^5w^3}{r^4w^3}\right)^4$

11. $\left(\frac{3r^6n^3}{2r^5n}\right)^4$

12. $\frac{r^7n^7t^2}{n^3r^3t^2}$

NAME _____ DATE _____ PERIOD _____

7-2 Study Guide and Intervention *(continued)*

Division Properties of Exponents

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.**

$$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$$ Group powers with the same base.

$$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$$ Quotient of Powers and Negative Exponent Properties

$$= \frac{1}{4}a^{-5}b^0c^5$$ Simplify.

$$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$$ Negative Exponent and Zero Exponent Properties

$$= \frac{c^5}{4a^5}$$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{2^2}{2^{-3}}$
2. $\frac{m}{m^{-4}}$
3. $\frac{p^{-8}}{p^3}$
4. $\frac{b^{-4}}{b^{-5}}$
5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$
6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$
7. $\frac{x^4y^0}{x^{-2}}$
8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$
9. $\frac{(3rt)^2u^{-4}}{r^{-1}t^2u^7}$
10. $\frac{m^{-3}t^{-5}}{(m^2t^3)^{-1}}$
11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$
12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

NAME _____ DATE _____ PERIOD _____

7-2 Skills Practice

Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{6^5}{6^4}$

2. $\frac{9^{12}}{9^8}$

3. $\frac{x^4}{x^2}$

4. $\frac{r^3t^2}{r^3t^4}$

5. $\frac{m}{m^3}$

6. $\frac{9d^7}{3d^6}$

7. $\frac{12n^5}{36n}$

8. $\frac{w^4x^3}{w^4x}$

9. $\frac{a^3b^5}{ab^2}$

10. $\frac{m^7p^2}{m^3p^2}$

11. $\frac{-21w^5x^2}{7w^4x^5}$

12. $\frac{32x^3y^2z^5}{-8xyz^2}$

13. $\left(\frac{4p^7}{7r^2}\right)^2$

14. 4^{-4}

15. 8^{-2}

16. $\left(\frac{5}{3}\right)^{-2}$

17. $\left(\frac{9}{11}\right)^{-1}$

18. $\frac{h^3}{h^{-6}}$

19. $k^0(k^4)(k^{-6})$

20. $k^{-1}(\ell^{-6})(m^3)$

21. $\frac{f^{-7}}{f^4}$

22. $\left(\frac{16p^5w^2}{2p^3w^3}\right)^0$

23. $\frac{f^{-5}g^4}{h^{-2}}$

24. $\frac{15x^6y^{-9}}{5xy^{-11}}$

25. $\frac{-15t^0u^{-1}}{5u^3}$

26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$

Lesson 7-2

NAME _____ DATE _____ PERIOD _____

7-2 Practice

Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{8^8}{8^4}$

2. $\frac{a^4b^6}{ab^3}$

3. $\frac{xy^2}{xy}$

4. $\frac{m^5np}{m^4p}$

5. $\frac{5c^2d^3}{-4c^2d}$

6. $\frac{8y^7z^6}{4y^6z^5}$

7. $\left(\frac{4f^3g}{3h^6}\right)^3$

8. $\left(\frac{6w^5}{7p^6r^3}\right)^2$

9. $\frac{-4x^2}{24x^5}$

10. $x^3(y^{-5})(x^{-8})$

11. $p(q^{-2})(r^{-3})$

12. 12^{-2}

13. $\left(\frac{3}{7}\right)^{-2}$

14. $\left(\frac{4}{3}\right)^{-4}$

15. $\frac{22r^3s^2}{11r^2s^{-3}}$

16. $\frac{-15w^0u^{-1}}{5u^3}$

17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$

18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$

19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$

20. $\frac{-12t^{-1}u^5x^{-4}}{2t^{-3}ux^5}$

21. $\frac{r^4}{(3r)^3}$

22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$

23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$

24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$

25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$

26. $\left(\frac{7c^{-3}d^3}{c^5dh^{-4}}\right)^{-1}$

27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$

28. BIOLOGY A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells?

29. COUNTING The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"?

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7-2 Word Problem Practice

Division Properties of Exponents

1. CHEMISTRY The nucleus of a certain atom is 10^{-13} centimeters across. If the nucleus of a different atom is 10^{-11} centimeters across, how many times as large is it as the first atom?

2. SPACE The Moon is approximately 25^4 kilometers away from Earth on average. The Olympus Mons volcano on Mars stands 25 kilometers high. How many Olympus Mons volcanoes, stacked on top of one another, would fit between the surface of the Earth and the Moon?

3. E-MAIL Spam (also known as junk e-mail) consists of identical messages sent to thousands of e-mail users. People often obtain anti-spam software to filter out the junk e-mail messages they receive. Suppose Yvonne's anti-spam software filtered out 10^2 e-mails, and she received 10^4 e-mails last year. What fraction of her e-mails were filtered out? Write your answer as a monomial.

4. METRIC MEASUREMENT Consider a dust mite that measures 10^{-3} millimeters in length and a caterpillar that measures 10 centimeters long. How many times as long as the mite is the caterpillar?

5. COMPUTERS In 1995, standard capacity for a personal computer hard drive was 40 megabytes (MB). In 2010, a standard hard drive capacity was 500 gigabytes (GB or Gig). Refer to the table below.

Memory Capacity Approximate Conversions
8 bits = 1 byte
10^3 bytes = 1 kilobyte
10^3 kilobytes = 1 megabyte (meg)
10^3 megabytes = 1 gigabyte (gig)
10^3 gigabytes = 1 terabyte
10^3 terabytes = 1 petabyte

a. The newer hard drives have about how many times the capacity of the 1995 drives?

b. Predict the hard drive capacity in the year 2025 if this rate of growth continues.

c. One kilobyte of memory is what fraction of one terabyte?

Lesson 7-2

NAME _____ DATE _____ PERIOD _____

7-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a.	**b.**	**c.**
$2^{10} =$ _____	$5^{10} =$ _____	$4^{10} =$ _____
$2^9 =$ _____	$5^9 =$ _____	$4^9 =$ _____
$2^8 =$ _____	$5^8 =$ _____	$4^8 =$ _____
$2^7 =$ _____	$5^7 =$ _____	$4^7 =$ _____
$2^6 =$ _____	$5^6 =$ _____	$4^6 =$ _____
$2^5 =$ _____	$5^5 =$ _____	$4^5 =$ _____
$2^4 =$ _____	$5^4 =$ _____	$4^4 =$ _____
$2^3 =$ _____	$5^3 =$ _____	$4^3 =$ _____
$2^2 =$ _____	$5^2 =$ _____	$4^2 =$ _____
$2^1 =$ _____	$5^1 =$ _____	$4^1 =$ _____

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

2. Describe the pattern of the powers from the top of the column to the bottom.

3. What would you expect the following powers to be?

2^0 5^0 4^0

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases.

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$ _?_ 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?

6. Based upon the pattern, can you determine whether 0^0 exists?

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?

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7-3 Study Guide and Intervention

Rational Exponents

Rational Exponents For any real numbers a and b and any positive integer n, if $a^n = b$, then a is an nth root of b. Rational exponents can be used to represent nth roots.

Square Root	$b^{\frac{1}{2}} = \sqrt{b}$
Cube Root	$b^{\frac{1}{3}} = \sqrt[3]{b}$
nth Root	$b^{\frac{1}{n}} = \sqrt[n]{b}$

Example 1 **Write $(6xy)^{\frac{1}{2}}$ in radical form.**

$(6xy)^{\frac{1}{2}} = \sqrt{6xy}$ Definition of $b^{\frac{1}{2}}$

Example 2 **Simplify $625^{\frac{1}{4}}$.**

$625^{\frac{1}{4}} = \sqrt[4]{625}$ $b^{\frac{1}{n}} = \sqrt[n]{b}$

$= \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5}$ $625 = 5^4$

$= 5$ Simplify.

Exercises

Write each expression in radical form, or write each radical in exponential form.

1. $14^{\frac{1}{2}}$ **2.** $5x^{\frac{1}{2}}$ **3.** $17y^{\frac{1}{2}}$

4. $12^{\frac{1}{2}}$ **5.** $19ab^{\frac{1}{2}}$ **6.** $\sqrt{17}$

7. $\sqrt{12n}$ **8.** $\sqrt{18b}$ **9.** $\sqrt{37}$

Simplify.

10. $\sqrt[3]{343}$ **11.** $\sqrt[5]{1024}$ **12.** $512^{\frac{1}{3}}$

13. $\sqrt[4]{2401}$ **14.** $\sqrt[6]{64}$ **15.** $243^{\frac{1}{5}}$

16. $\sqrt[3]{1331}$ **17.** $\sqrt[4]{6561}$ **18.** $4096^{\frac{1}{4}}$

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7-3 Study Guide and Intervention *(continued)*

Rational Exponents

Solve Exponential Equations In an **exponential equation**, variables occur as exponents. Use the Power Property of Equality and the other properties of exponents to solve exponential equations.

Example **Solve $1024^{x-1} = 4$.**

$1024^{x-1} = 4$	Original equation
$(4^5)^{x-1} = 4$	Rewrite 1024 as 4^5.
$4^{5x-5} = 4^1$	Power of a Power, Distributive Property
$5x - 5 = 1$	Power Property of Equality
$5x = 6$	Add 5 to each side.
$x = \frac{6}{5}$	Divide each side by 5.

Exercises

Solve each equation.

1. $2^x = 128$

2. $3^{3x+1} = 81$

3. $4^{x-3} = 32$

4. $5^x = 15{,}625$

5. $6^{3x+2} = 216$

6. $4^{5x-3} = 16$

7. $8^x = 4096$

8. $9^{3x+3} = 6561$

9. $11^{x-1} = 1331$

10. $3^x = 6561$

11. $2^{5x+4} = 512$

12. $7^{x-2} = 343$

13. $8^x = 262{,}144$

14. $5^{5x} = 3125$

15. $9^{2x-6} = 6561$

16. $7^x = 2401$

17. $7^{3x} = 117{,}649$

18. $6^{2x-7} = 7776$

19. $9^x = 729$

20. $8^{3x+1} = 4096$

21. $13^{3x-8} = 28{,}561$

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7-3 Skills Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $(8x)^{\frac{1}{2}}$

2. $6z^{\frac{1}{2}}$

3. $\sqrt{19}$

4. $\sqrt{11}$

5. $19x^{\frac{1}{2}}$

6. $\sqrt{34}$

7. $\sqrt{27g}$

8. $33gh^{\frac{1}{2}}$

9. $\sqrt{13abc}$

Simplify.

10. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

11. $\sqrt[5]{3125}$

12. $729^{\frac{1}{3}}$

13. $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

14. $\sqrt[6]{4096}$

15. $1024^{\frac{1}{5}}$

16. $\left(\frac{16}{625}\right)^{\frac{1}{4}}$

17. $\sqrt[6]{15{,}625}$

18. $117{,}649^{\frac{1}{6}}$

Solve each equation.

19. $2^x = 512$

20. $3^x = 6561$

21. $6^x = 46{,}656$

22. $5^x = 125$

23. $3^{x-3} = 243$

24. $4^{x-1} = 1024$

25. $6^{x-1} = 1296$

26. $2^{4x+3} = 2048$

27. $3^{3x+3} = 6561$

NAME _____ DATE _____ PERIOD _____

7-3 Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $\sqrt{13}$

2. $\sqrt{37}$

3. $\sqrt{17x}$

4. $(7ab)^{\frac{1}{2}}$

5. $21z^{\frac{1}{2}}$

6. $13(ab)^{\frac{1}{2}}$

Simplify.

7. $\left(\frac{1}{81}\right)^{\frac{1}{4}}$

8. $\sqrt[5]{1024}$

9. $512^{\frac{1}{3}}$

10. $\left(\frac{32}{1024}\right)^{\frac{1}{5}}$

11. $\sqrt[4]{1296}$

12. $3125^{\frac{1}{5}}$

Solve each equation.

13. $3^x = 729$

14. $4^x = 4096$

15. $5^x = 15{,}625$

16. $6^{x+3} = 7776$

17. $3^{x-3} = 2187$

18. $4^{3x+4} = 16{,}384$

19. WATER The flow of water F in cubic feet per second over a wier, a small overflow dam, can be represented by $F = 1.26H^{\frac{3}{2}}$, where H is the height of the water in meters above the crest of the wier. Find the height of the water if the flow of the water is 10.08 cubic feet per second.

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7-3 Word Problem Practice

Rational Exponents

1. VELOCITY The velocity v in feet per second of a freely falling object that has fallen h feet can be represented by $v = 8h^{\frac{1}{2}}$. Find the distance that an object has fallen if its velocity is 96 feet per second.

2. ELECTRICITY The relationship of the current, the power, and the resistance in an appliance can be modeled by $IR^{\frac{1}{2}} = \sqrt{P}$, where I is the current in amperes, P is the power in watts, and R is the resistance in ohms. Find the power that an appliance is using if the current is 2.5 amps and the resistance is 16 ohms.

3. GEOMETRY The surface area T of a cube in square inches can be determined by $T = 6V^{\frac{2}{3}}$, where V is the volume of the cube in cubic inches.

a. Find the surface area of a cube that has a volume of 4096 cubic inches.

b. Find the volume of a cube that has a surface area of 96 square inches.

4. PLANETS The average distance d in astronomical units that a planet is from the Sun can be modeled by $d = t^{\frac{2}{3}}$, where t is the number of Earth years that it takes for the planet to orbit the Sun.

a. Find the average distance a planet is from the Sun if the planet has an orbit of 27 Earth years.

b. Find the time that it would take a planet to orbit the Sun if its average distance from the Sun is 4 astronomical units.

5. BIOLOGY The relationship between the mass m in kilograms of an organism and its metabolism P in Calories per day can be represented by $P = 73.3\sqrt[4]{m^3}$. Find the mass of an organism that has a metabolism of 586.4 Calories per day.

6. MANUFACTURING The profit P of a company in thousands of dollars can be modeled by $P = 12.75\sqrt[5]{c^2}$, where c is the number of customers in hundreds. If the profit of the company is \$51,000, how many customers do they have?

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7-3 Enrichment

Counterexamples

Some statements in mathematics can be proven false by **counterexamples**.

Consider the following statement.

For any numbers a and b, $a - b = b - a$.

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let $a = 7$ and $b = 3$. Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$

$$4 \neq -4$$

In general, for any numbers a and b, the statement $a - b = b - a$ is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

In each of the following exercises *a*, *b*, and *c* are any numbers. Prove that the statement is false by counterexample.

1. $a - (b - c) \stackrel{?}{=} (a - b) - c$

2. $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$

3. $a \div b \stackrel{?}{=} b \div a$

4. $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$

5. $a^{\frac{1}{2}} + a^{\frac{1}{2}} \stackrel{?}{=} a^1$

6. $a^c \cdot b^d \stackrel{?}{=} (ab)^{c+d}$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

8. For the Distributive Property $a(b + c) = ab + ac$, it is said that multiplication distributes over addition. Exercise 4 proves that some operations do not distribute. Write a statement for the exercise that indicates this.

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7-4 Study Guide and Intervention

Scientific Notation

Scientific Notation Very large and very small numbers are often best represented using a method known as **scientific notation**. Numbers written in scientific notation take the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. Any number can be written in scientific notation.

Example 1 **Express 34,020,000,000 in scientific notation.**

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number a. Here, $a = 3.402$.

Step 2 Note the number of places n and the direction that you moved the decimal point. The decimal point moved 10 places to the left, so $n = 10$.

Step 3 Because the decimal moved to the left, write the number as $a \times 10^n$.

$34{,}020{,}000{,}000 = 3.4020000000 \times 10^{10}$

Step 4 Remove the extra zeros. 3.402×10^{10}

Example 2 **Express 4.11×10^{-6} in standard notation.**

Step 1 The exponent is –6, so $n = -6$.

Step 2 Because $n < 0$, move the decimal point 6 places to the left.

$4.11 \times 10^{-6} \Rightarrow .00000411$

Step 3 $4.11 \times 10^{-6} \Rightarrow 0.00000411$

Rewrite; insert a 0 before the decimal point.

Exercises

Express each number in scientific notation.

1. 5,100,000 **2.** 80,300,000,000 **3.** 14,250,000

4. 68,070,000,000,000 **5.** 14,000 **6.** 901,050,000,000

7. 0.0049 **8.** 0.000301 **9.** 0.0000000519

10. 0.000000185 **11.** 0.002002 **12.** 0.00000771

Express each number in standard form.

13. 4.91×10^4 **14.** 3.2×10^{-5} **15.** 6.03×10^8

16. 2.001×10^{-6} **17.** 1.00024×10^{10} **18.** 5×10^5

19. 9.09×10^{-5} **20.** 3.5×10^{-2} **21.** 1.7087×10^7

NAME _____ DATE _____ PERIOD _____

7-4 Study Guide and Intervention *(continued)*

Scientific Notation

Products and Quotients in Scientific Notation You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

Example 1 **Evaluate $(9.2 \times 10^{-3}) \times (4 \times 10^{8})$. Express the result in both scientific notation and standard form.**

$(9.2 \times 10^{-3})(4 \times 10^{8})$	Original expression
$= (9.2 \times 4)(10^{-3} \times 10^{8})$	Commutative and Associative Properties
$= 36.8 \times 10^{5}$	Product of Powers
$= (3.68 \times 10^{1}) \times 10^{5}$	$36.8 = 3.68 \times 10$
$= 3.68 \times 10^{6}$	Product of Powers
$= 3{,}680{,}000$	Standard Form

Example 2 **Evaluate $\frac{(2.76 \times 10^{7})}{(6.9 \times 10^{5})}$. Express the result in both scientific notation and standard form.**

$\frac{(2.76 \times 10^{7})}{(6.9 \times 10^{5})} = \left(\frac{2.76}{6.9}\right)\left(\frac{10^{7}}{10^{5}}\right)$	Product rule for fractions
$= 0.4 \times 10^{2}$	Quotient of Powers
$= 4.0 \times 10^{-1} \times 10^{2}$	$0.4 = 4.0 \times 10^{-1}$
$= 4.0 \times 10^{1}$	Product of Powers
$= 40$	Standard form

Exercises

Evaluate each product. Express the results in both scientific notation and standard form.

1. $(3.4 \times 10^{3})(5 \times 10^{4})$

2. $(2.8 \times 10^{-4})(1.9 \times 10^{7})$

3. $(6.7 \times 10^{-7})(3 \times 10^{3})$

4. $(8.1 \times 10^{5})(2.3 \times 10^{-3})$

5. $(1.2 \times 10^{-4})^{2}$

6. $(5.9 \times 10^{5})^{2}$

Evaluate each quotient. Express the results in both scientific notation and standard form.

7. $\frac{(4.9 \times 10^{-3})}{(2.5 \times 10^{-4})}$

8. $\frac{5.8 \times 10^{4}}{5 \times 10^{-2}}$

9. $\frac{(1.6 \times 10^{5})}{(4 \times 10^{-4})}$

10. $\frac{8.6 \times 10^{6}}{1.6 \times 10^{-3}}$

11. $\frac{(4.2 \times 10^{-2})}{(6 \times 10^{-7})}$

12. $\frac{8.1 \times 10^{5}}{2.7 \times 10^{4}}$

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7-4 Skills Practice

Scientific Notation

Express each number in scientific notation.

1. 3,400,000,000

2. 0.000000312

3. 2,091,000

4. 980,200,000,000,000

5. 0.00000000008

6. 0.00142

Express each number in standard form.

7. 2.1×10^{5}

8. 8.023×10^{-7}

9. 3.63×10^{-6}

10. 7.15×10^{8}

11. 1.86×10^{-4}

12. 4.9×10^{5}

Evaluate each product. Express the results in both scientific notation and standard form.

13. $(6.1 \times 10^{5})(2 \times 10^{5})$

14. $(4.4 \times 10^{6})(1.6 \times 10^{-9})$

15. $(8.8 \times 10^{8})(3.5 \times 10^{-13})$

16. $(1.35 \times 10^{8})(7.2 \times 10^{-4})$

17. $(2.2 \times 10^{-3})^{2}$

18. $(3.4 \times 10^{2})^{2}$

Evaluate each quotient. Express the results in both scientific notation and standard form.

19. $\dfrac{(9.2 \times 10^{-8})}{(2 \times 10^{-6})}$

20. $\dfrac{(4.8 \times 10^{4})}{(3 \times 10^{-5})}$

21. $\dfrac{(1.161 \times 10^{-9})}{(4.3 \times 10^{-6})}$

22. $\dfrac{(4.625 \times 10^{10})}{(1.25 \times 10^{4})}$

23. $\dfrac{(2.376 \times 10^{-4})}{(7.2 \times 10^{-8})}$

24. $\dfrac{(8.74 \times 10^{-3})}{(1.9 \times 10^{5})}$

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7-4 Practice

Scientific Notation

Express each number in scientific notation.

1. 1,900,000

2. 0.000704

3. 50,040,000,000

4. 0.0000000661

Express each number in standard form.

5. 5.3×10^{7}

6. 1.09×10^{-4}

7. 9.13×10^{3}

8. 7.902×10^{-6}

Evaluate each product. Express the results in both scientific notation and standard form.

9. $(4.8 \times 10^{4})(6 \times 10^{6})$

10. $(7.5 \times 10^{-5})(3.2 \times 10^{7})$

11. $(2.06 \times 10^{4})(5.5 \times 10^{-9})$

12. $(8.1 \times 10^{-6})(1.96 \times 10^{11})$

13. $(7.2 \times 10^{-5})^{2}$

14. $(5.29 \times 10^{6})^{2}$

Evaluate each quotient. Express the results in both scientific notation and standard form.

15. $\dfrac{(4.2 \times 10^{5})}{(3 \times 10^{-3})}$

16. $\dfrac{(1.76 \times 10^{-11})}{(2.2 \times 10^{-5})}$

17. $\dfrac{(7.05 \times 10^{12})}{(9.4 \times 10^{7})}$

18. $\dfrac{(2.04 \times 10^{-4})}{(3.4 \times 10^{5})}$

19. GRAVITATION Issac Newton's theory of universal gravitation states that the equation $F = G\dfrac{m_1 m_2}{r^2}$ can be used to calculate the amount of gravitational force in newtons between two point masses m_1 and m_2 separated by a distance r. G is a constant equal to 6.67×10^{-11} Nm^2kg^{-2}. The mass of Earth m_1 is equal to 5.97×10^{24} kg, the mass of the Moon m_2 is equal to 7.36×10^{22} kg, and the distance r between the two is 384,000,000 m.

a. Express the distance r in scientific notation.

b. Compute the amount of gravitational force between Earth and the Moon. Express your answer in scientific notation.

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7-4 Word Problem Practice

Scientific Notation

1. PLANETS Neptune's mean distance from the Sun is 4,500,000,000 kilometers. Uranus' mean distance from the Sun is 2,870,000,000 kilometers. Express these distances in scientific notation.

2. PATHOLOGY The common cold is caused by the rhinovirus, which commonly measures 2×10^{-8} m in diameter. The E. coli bacterium, which causes food poisoning, commonly measures 3×10^{-6} m in length. Express these measurements in standard form.

3. COMMERCIALS A 30-second commercial aired during the 2007 Super Bowl cost \$2,600,000. A 30-second commercial aired during the 1967 Super Bowl cost \$40,000. Express these values in scientific notation. How many times more expensive was it to air an advertisement during the 2007 Super Bowl than the 1967 Super Bowl?

4. AVOGADRO'S NUMBER Avogadro's number is an important concept in chemistry. It states that the number 6.022×10^{23} is approximately equal to the number of molecules in 12 grams of carbon 12. Use Avogadro's number to determine the number of molecules in 5×10^{-7} grams of carbon 12.

5. COAL RESERVES The table below shows the number of kilograms of coal select countries had in proven reserve at the end of a recent year.

Coal Reserves	
Country	**Coal (kg)**
United States	2.46×10^{14}
Russia	1.57×10^{14}
India	9.24×10^{13}
Romania	4.94×10^{11}

Source: British Petroleum

a. Express each country's coal reserves in standard form.

b. How many times more coal does the United States have than Romania?

c. One kilogram of coal has an energy density of 2.4×10^{7} joules. What is the total energy density of the United States' coal reserve? Express your answer in scientific notation.

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7-4 Enrichment

Engineering Notation

Engineering notation is a variation on scientific notation where numbers are expressed as powers of 1000 rather than as powers of 10. Engineering notation takes the familiar form of $a \times 10^n$, but n is restricted to multiples of three and $1 \leq |a| < 1000$.

One advantage to engineering notation is that numbers can be neatly expressed using SI prefixes. These prefixes are typically used for scientific measurements.

1000^n	10^n	SI Prefix	Symbol
1000^5	10^{15}	peta	P
1000^4	10^{12}	tera	T
1000^3	10^9	giga	G
1000^2	10^6	mega	M
1000^1	10^3	kilo	k
1000^{-1}	10^{-3}	milli	m
1000^{-2}	10^{-6}	micro	μ
1000^{-3}	10^{-9}	nano	n
1000^{-4}	10^{-12}	pico	p
1000^{-5}	10^{-15}	femto	f

Example **NUCLEAR POWER** **The output of a nuclear power plant is measured to be 620,000,000 watts. Express this number in engineering notation and using SI prefixes.**

To express a number in engineering notation, first convert the number to scientific notation.

Step 1 $620{,}000{,}000 \Rightarrow 6.20000000$ $a = 6.20000000$

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $620{,}000{,}000 = 6.20000000 \times 10^8 = 6.2 \times 10^8$

Because 8 is not a multiple of 3, we need to round down n to the next multiple of 3.

Step 4 $6.2 \times 10^8 = (6.2 \times 10^2) \times 10^6$ $620 = 6.2 \times 10^2$

Step 5 $= 620 \times 10^6$ Product of Powers

The output of the power plant is 620×10^6 watts. Using the chart above, the prefix for 10^6 is found to be mega, or M. The output of the power plant is 620 megawatts, or 620MW.

Exercises

Express each number in engineering notation.

1. 40,000,000,000

2. 180,000,000,000,000

3. 0.00006

4. 0.00000000000039

Express each measurement using SI prefixes.

5. 0.00000000014 gram

6. 40,000,000,000 watts

7. 63,100,000,000,000 bytes

8. 0.0000002 meter

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7-5 Study Guide and Intervention

Exponential Functions

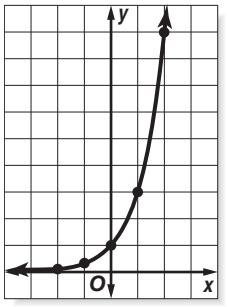
Graph Exponential Functions

Exponential Function	a function defined by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$

You can use values of x to find ordered pairs that satisfy an exponential function. Then you can use the ordered pairs to graph the function.

Example 1 **Graph $y = 3^x$. Find the y-intercept and state the domain and range.**

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

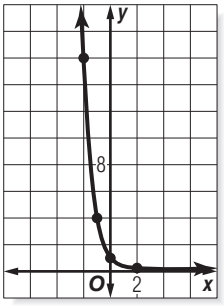


The y-intercept is 1.

The domain is all real numbers, and the range is all positive numbers.

Example 2 **Graph $y = \left(\frac{1}{4}\right)^x$. Find the y-intercept and state the domain and range.**

x	y
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



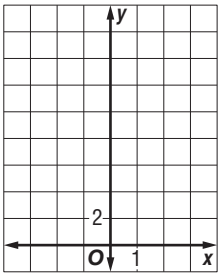
The y-intercept is 1.

The domain is all real numbers, and the range is all positive numbers.

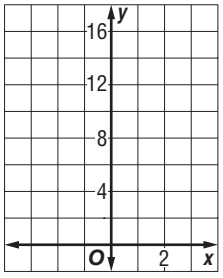
Exercises

Graph each function. Find the y-intercept and state the domain and range.

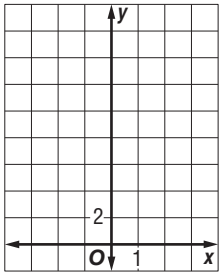
1. $y = 0.3^x$



2. $y = 3x + 1$



3. $y = \left(\frac{1}{3}\right)^x + 1$



Lesson 7-5

NAME _____ DATE _____ PERIOD _____

7-5 Study Guide and Intervention *(continued)*

Exponential Functions

Identify Exponential Behavior It is sometimes useful to know if a set of data is exponential. One way to tell is to observe the shape of the graph. Another way is to observe the pattern in the set of data.

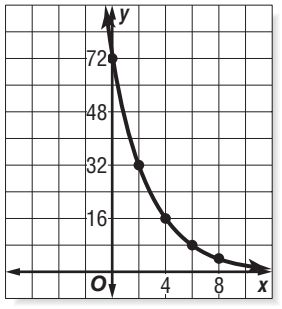
Example **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

x	0	2	4	6	8	10
y	64	32	16	8	4	2

Method 1: Look for a Pattern

The domain values increase by regular intervals of 2, while the range values have a common factor of $\frac{1}{2}$. Since the domain values increase by regular intervals and the range values have a common factor, the data are probably exponential.

Method 2: Graph the Data



The graph shows rapidly decreasing values of y as x increases. This is characteristic of exponential behavior.

Exercises

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

1.

x	0	1	2	3
y	5	10	15	20

2.

x	0	1	2	3
y	3	9	27	81

3.

x	−1	1	3	5
y	32	16	8	4

4.

x	−1	0	1	2	3
y	3	3	3	3	3

5.

x	−5	0	5	10
y	1	0.5	0.25	0.125

6.

x	0	1	2	3	4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

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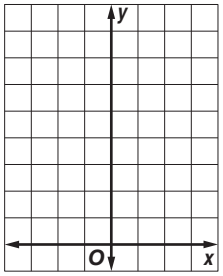
7-5 Skills Practice

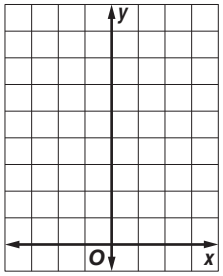
Exponential Functions

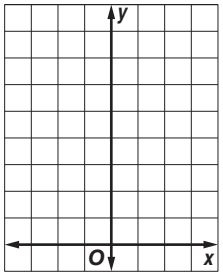
Graph each function. Find the *y*-intercept, and state the domain and range.

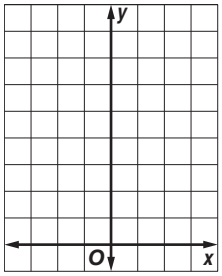
1. $y = 2^x$



2. $y = \left(\frac{1}{3}\right)^x$







3. $y = 3(2^x)$

4. $y = 3^x + 2$

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

5.

x	−3	−2	−1	0
y	9	12	15	18

6.

x	0	5	10	15
y	20	10	5	2.5

7.

x	4	8	12	16
y	20	40	80	160

8.

x	50	30	10	−10
y	90	70	50	30

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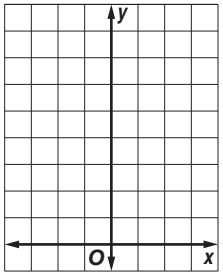
7-5 Practice

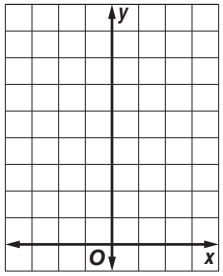
Exponential Functions

Graph each function. Find the *y*-intercept and state the domain and range.

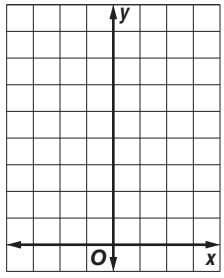
1. $y = \left(\frac{1}{10}\right)^x$



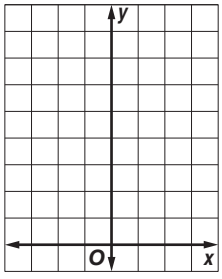
2. $y = 3^x$



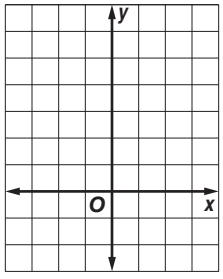
3. $y = \left(\frac{1}{4}\right)^x$



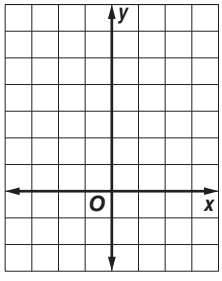
4. $y = 4(2^x) + 1$



5. $y = 2(2^x - 1)$



6. $y = 0.5(3^x - 3)$



Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

7.

x	2	5	8	11
y	480	120	30	7.5

8.

x	21	18	15	12
y	30	23	16	9

9. LEARNING Ms. Klemperer told her English class that each week students tend to forget one sixth of the vocabulary words they learned the previous week. Suppose a student learns 60 words. The number of words remembered can be described by the function $W(x) = 60\left(\frac{5}{6}\right)^x$, where x is the number of weeks that pass. How many words will the student remember after 3 weeks?

10. BIOLOGY Suppose a certain cell reproduces itself in four hours. If a lab researcher begins with 50 cells, how many cells will there be after one day, two days, and three days? (*Hint:* Use the exponential function $y = 50(2^x)$.)

NAME _____ DATE _____ PERIOD _____

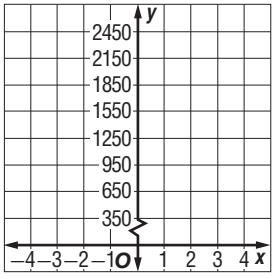
7-5 Word Problem Practice

Exponential Functions

1. WASTE Suppose the waste generated by nonrecycled paper and cardboard products is approximated by the following function.

$$y = 1000(2)^{0.3x}$$

Sketch the exponential function on the coordinate grid below.



2. MONEY Tatyana's grandfather gave her one penny on the day she was born. He plans to double the amount he gives her every day. Estimate how much she will receive from her grandfather on the 12th day of her life.

3. PICTURE FRAMES Since a picture frame includes a border, the picture must be smaller in area than the entire frame. The table shows the relationship between picture area and frame length for a particular line of frames. Is this an exponential relationship? Explain.

Side Length (in.)	Picture Area (in^2)
5	6
6	12
7	20
8	30
9	42

4. DEPRECIATION The value of Royce Company's computer equipment is decreasing in value according to the following function.

$$y = 4000(0.87)^x$$

In the equation, x is the number of years that have elapsed since the equipment was purchased and y is the value in dollars. What was the value 5 years after it was purchased? Round your answer to the nearest dollar.

5. METEOROLOGY The atmospheric pressure in millibars at altitude x meters can be approximated by the following function. The function is valid for values of x between 0 and 10,000.

$$f(x) = 1038(1.000134)^{-x}$$

a. What is the pressure at sea level?

b. The McDonald Observatory in Texas is at an altitude of 2000 meters. What is the approximate atmospheric pressure there?

c. As altitude increases, what happens to atmospheric pressure?

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7-5 Enrichment

Logarithmic Functions

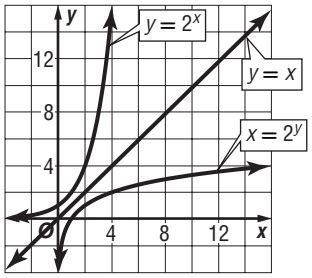
You have found the inverse of a linear function by interchanging x and y in the equation of the function. The inverse of an exponential function is called a **logarithmic function**.

Example **Find the inverse function of the exponential function $y = 2^x$. Make a table of values for each function. Then graph both functions.**

To find the inverse function, interchange x and y. The inverse of $y = 2^x$ is $x = 2^y$.

$y = 2^x$	
x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x = 2^y$	
x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



In general the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, the variable y is called the **logarithm** of x. This is usually written as $y = \log_b x$.

Exercises

1. What are the domain and range of $y = \log_2 x$? How are the domain and range related to the domain and range of $y = 2^x$?

2. How are the graphs of $y = \log_2 x$ and $y = 2^x$ related?

3. Through which quadrants do the graphs of $y = \log_2 x$ and $y = 2^x$ pass?

4. If an exponential function finds population as a function of time, what can you conclude about its inverse logarithmic function?

5. RESEARCH Investigate real-world uses of logarithmic functions. How are logarithmic functions used to model real-world situations?

NAME _____ DATE _____ PERIOD _____

7-6 Study Guide and Intervention

Growth and Decay

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	The general equation for exponential growth is $y = a(1 + r)^t$. • y represents the final amount. • a represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.

Example 1 **POPULATION** **The population of Johnson City in 2005 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.**

a. Write an equation to represent the population of Johnson City since 2005.

The rate 3.2% can be written as 0.032.

$y = a(1 + r)^t$

$y = 25{,}000(1 + 0.032)^t$

$y = 25{,}000(1.032)^t$

b. According to the equation, what will the population of Johnson City be in 2015?

In 2015 t will equal 2015 − 2005 or 10.

Substitute 10 for t in the equation from part **a**.

$y = 25{,}000(1.032)^{10}$ $t = 10$

$\approx 34{,}256$

In 2015 the population of Johnson City will be about 34,256.

Example 2 **INVESTMENT** **The Garcias have $12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.**

The rate 3.5% can be written as 0.035. The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 12{,}000\left(1 + \frac{0.035}{12}\right)^{3(12)}$$

$$\approx 13{,}326.49$$

In three years, the balance of the account will be $13,326.49.

Exercises

1. **POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population was about 303,146,000 in 2008, predict the population in 2012.

2. **INVESTMENT** Determine the value of an investment of $2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years.

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the 2008 population was about 6,641,000,000, predict the 2015 population.

4. **INVESTMENT** Determine the value of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

NAME _____ DATE _____ PERIOD _____

7-6 Study Guide and Intervention *(continued)*

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of **exponential decay**. This means that an initial amount decreases at a steady rate over a period of time.

Exponential Decay	The general equation for exponential decay is $y = a(1 - r)^t$. • y represents the final amount. • a represents the initial amount. • r represents the rate of decay expressed as a decimal. • t represents time.

Example **DEPRECIATION** **The original price of a tractor was $45,000. The value of the tractor decreases at a steady rate of 12% per year.**

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

$y = a(1 - r)^t$ General equation for exponential decay

$y = 45{,}000(1 - 0.12)^t$ $a = 45{,}000$ and $r = 0.12$

$y = 45{,}000(0.88)^t$ Simplify.

b. What is the value of the tractor in 5 years?

$y = 45{,}000(0.88)^t$ Equation for decay from part a

$y = 45{,}000(0.88)^5$ $t = 5$

$y \approx 23{,}747.94$ Use a calculator.

In 5 years, the tractor will be worth about $23,747.94.

Exercises

1. **POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015.

2. **DEPRECIATION** Mr. Gossell is a machinist. He bought some new machinery for about $125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest $100) at the end of 10 years?

3. **ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of carbon-14 of 100 grams.
   a. If the organism lived 5730 years ago, what is the concentration of carbon-14 today?
   b. If the organism lived 11,460 years ago, determine the concentration of carbon-14 today.

4. **DEPRECIATION** A new car costs $32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years.

NAME _____ DATE _____ PERIOD _____

7-6 Skills Practice

Growth and Decay

1. **POPULATION** The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%.

   a. Write an equation for the population t years after 2000.

   b. Use the equation to predict the population of New York City in 2015.

2. **SAVINGS** The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.

   a. If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account?

   b. If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account?

3. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 7%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2009?

4. **MANUFACTURING** Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.

   a. Write an equation for the value of the piece of equipment after t years.

   b. Find the value of the piece of equipment after 6 years.

5. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks?

6. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car?

NAME _____ DATE _____ PERIOD _____

7-6 Practice

Growth and Decay

1. **COMMUNICATIONS** Sports radio stations numbered 220 in 1996. The number of sports radio stations has since increased by approximately 14.3% per year.

   a. Write an equation for the number of sports radio stations for t years after 1996.

   b. If the trend continues, predict the number of sports radio stations in 2015.

2. **INVESTMENTS** Determine the amount of an investment if \$500 is invested at an interest rate of 4.25% compounded quarterly for 12 years.

3. **INVESTMENTS** Determine the amount of an investment if \$300 is invested at an interest rate of 6.75% compounded semiannually for 20 years.

4. **HOUSING** The Greens bought a condominium for \$110,000 in 2010. If its value appreciates at an average rate of 6% per year, what will the value be in 2015?

5. **DEFORESTATION** During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.

   a. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for t years after 1990.

   b. If this trend continues, predict the forested area in 2015.

6. **BUSINESS** A piece of machinery valued at \$25,000 depreciates at a steady rate of 10% yearly. What will the value of the piece of machinery be after 7 years?

7. **TRANSPORTATION** A new car costs \$18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car in 8 years.

8. **POPULATION** The population of Osaka, Japan, declined at an average annual rate of 0.05% for the five years between 1995 and 2000. If the population of Osaka was 11,013,000 in 2000 and it continues to decline at the same rate, predict the population in 2050.

NAME _____ DATE _____ PERIOD _____

7-6 Word Problem Practice

Growth and Decay

1. DEPRECIATION The value of a new plasma television depreciates by about 7% each year. Aeryn purchases a 50-inch plasma television for $3000. What is its value after 4 years? Round your answer to the nearest hundred.

2. MONEY Hans opens a savings account by depositing $1200 in an account that earns 3 percent interest compounded weekly. How much will his investment be worth in 10 years? Assume that there are exactly 52 weeks in a year and round your answer to the nearest cent.

3. HIGHER EDUCATION The table lists the average costs of attending a four-year college in the United States during a recent year.

College Sector	Tuition and Fees	Room and Board
Four-year Public	$5941	$6636
Four-year Private	$21,235	$7,791

Source: College Board

Russ's parents invested money in a savings account earning an average of 4.5 percent interest, compounded monthly. After 15 years, they have exactly the right amount to cover the tuition, fees, room and board for Russ's first year at a public college. What was their initial investment? Round your answer to the nearest dollar.

4. POPULATION In 2007 the U.S. Census Bureau estimated the population of the United States estimated at 301 million. The annual rate of growth is about 0.89%. At this rate, what is the expected population at the time of the 2020 census? Round your answer to the nearest ten million.

5. MEDICINE When doctors prescribe medication, they have to consider the rate at which the body filters a drug from the bloodstream. Suppose it takes the human body 6 days to filter out half of the Flu-B-Gone vaccine. The amount of Flu-B-Gone vaccine remaining in the bloodstream x days after an injection is given by the equation $y = y_0(0.5)^{\frac{x}{6}}$, where y_0 is the initial amount. Suppose a doctor injects a patient with 20 μg (micrograms) of Flu-B-Gone.

a. How much of the vaccine will remain after 1 day? Round your answer to the nearest tenth.

b. How much of the vaccine will remain after 12 days? Round your answer to the nearest tenth.

c. After how many days will the amount of vaccine be less than 1 μg?

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7-6 Enrichment

Continuously Compounding Interest

You can use the formula for compound interest $A = P\left(1 + \frac{r}{n}\right)^{nt}$ when n, the number of times a year interest is compounded, is known. A special type of compound interest calculation regularly used in finance is **continuously compounded interest,** where n approaches infinity.

General formula: $A = Pe^{rt}$
A = current amount of investment
P = initial amount of investment
e = natural logarithm, a constant approximately equal to 2.71828
r = annual rate of interest, expressed as a decimal
t = number of years the money is invested

Example **INVESTING Mr. Rivera placed $5000 in an investment account that has an interest rate of 9.5% per year.**

a. How much money will be in the account after 5 years if interest is compounded monthly?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 5000\left(1 + \frac{0.095}{12}\right)^{12(5)}$ $P = 5000$, $r = 0.095$, $n = 12$, and $t = 5$

$= 5000(1.0079)^{60}$ Simplify.

$= 8025.05$ Use a calculator.

There will be about $8025.05 in the account if interest is compounded monthly.

b. How much money will be in the account after 5 years if interest is compounded continuously?

$A = Pe^{rt}$ Continuous interest equation

$= 5000(2.71828)^{0.095(5)}$ $P = 5000$, $e = 2.71828$, $r = 0.095$, and $t = 5$

$= 5000(2.71828)^{0.475}$ Simplify.

$= 8040.07$ Use a calculator.

There will be about $8040.07 in the account if interest is compounded continuously.

Exercises

1. **SAVINGS** Mr. Harris saves $20,000 in a money-market account at a rate of 5.2%.
   - **a.** Determine the value of his investment after 10 years if interest is compounded quarterly.
   - **b.** Determine the value of his investment after 10 years if interest is compounded continuously.

2. **COLLEGE SAVINGS** Shannon is choosing between two different savings accounts for her college fund. The first account compounds interest semiannually at a rate of 11.0%. The second account compounds interest continuously at a rate of 10.8%. If Shannon plans to keep her money in the account for 5 years, which account should she choose? Explain.

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7-6 Spreadsheet Activity

Compound Interest

Banks often use spreadsheets to calculate and store financial data. One application is to calculate compound interest on an account.

Example **Use a spreadsheet to find the time it will take an investment of $1000 to double. Suppose you can choose from investments that have annual interest rates of 5%, 8%, or 10% compounded monthly.**

The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is paid, or compounded, each year, and t is the number of years. In this case, $P = 1000$ and $n = 12$.

Step 1 Use Column A of the spreadsheet for the years.

Step 2 Columns B, C, and D contain the formulas for the final amounts of the investments. Format the cells in these columns as currency so that the amounts are shown in dollars and cents. Each formula will use the values in Column A as the value of t. For example, the formula that is in cell C3 is =1000*(1 + (0.08/12))^(12 * A3).

	A	B	C	D
1	Years	5%	8%	10%
2	1	$1,051.16	$1,083.00	$1,104.71
3	2	$1,104.94	$1,172.89	$1,220.39
4	3	$1,161.47	$1,270.24	$1,348.18
5	4	$1,220.90	$1,375.67	$1,489.35
6	5	$1,283.36	$1,489.85	$1,645.30
7	6	$1,349.02	$1,613.50	$1,817.59
8	7	$1,418.04	$1,747.42	$2,007.92
9	8	$1,490.59	$1,892.46	$2,218.18
10	9	$1,566.85	$2,049.53	$2,450.45
11	10	$1,647.01	$2,219.64	$2,707.04
12	11	$1,731.27	$2,403.87	$2,990.50
13	12	$1,819.85	$2,603.39	$3,303.65
14	13	$1,912.96	$2,819.47	$3,649.58
15	14	$2,010.83	$3,053.48	$4,031.74
16				

Sheet 1 Sheet 2 Sheet 3

Study the spreadsheet for the times when each investment exceeds $2000. At 5%, the $1000 will double in 14 years, at 8% it will double in 9 years, and at 10% it will double in 7 years.

Exercises

Use the spreadsheet of accounts involving compound interest.

1. How are the doubling times affected if the accounts compound interest quarterly instead of monthly?

2. How long will it take each account to reach $4000 if the interest is compounded monthly? quarterly?

3. How do the interest rate and the number of times the interest is compounded affect the growth of an investment?

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7-7 Study Guide and Intervention

Geometric Sequences as Exponential Functions

Recognize Geometric Sequences A geometric sequence is a sequence in which each term after the first is found by multiplying the previous term by a nonzero constant r called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

Example 1 **Determine whether the sequence is *arithmetic, geometric,* or *neither:* 21, 63, 189, 567, . . .**

Find the ratios of the consecutive terms. If the ratios are constant, the sequence is geometric.

$$21 \quad 63 \quad 189 \quad 567$$

$$\frac{63}{21} = \frac{189}{63} = \frac{567}{189} = 3$$

Because the ratios are constant, the sequence is geometric. The common ratio is 3.

Example 2 **Find the next three terms in this geometric sequence: −1215, 405, −135, 45, . . .**

Step 1 Find the common ratio.

$$-1215 \quad 405 \quad -135 \quad 45$$

$$\frac{405}{-1215} = \frac{-135}{405} = \frac{45}{-135} = \frac{-1}{3}$$

The value of r is $-\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$45 \quad -15 \quad 5 \quad -\frac{5}{3}$$

$$\times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right)$$

The next three terms of the sequence are –15, 5, and $-\frac{5}{3}$.

Exercises

Determine whether each sequence is *arithmetic, geometric,* or *neither.* Explain.

1. 1, 2, 4, 8, . . .

2. 9, 14, 6, 11, . . .

3. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$

4. –2, 5, 12, 19, . . .

Find the next three terms in each geometric sequence.

5. 648, –216, 72, . . .

6. 25, –5, 1, . . .

7. $\frac{1}{16}, \frac{1}{2}$, 4, . . .

8. 72, 36, 18, . . .

NAME _____ DATE _____ PERIOD _____

7-7 Study Guide and Intervention *(continued)*

Geometric Sequences as Exponential Functions

Geometric Sequences and Functions The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer: $a_n = a_1 \cdot r^{n-1}$.

Example **a. Write an equation for the nth term of the geometric sequence 5, 20, 80, 320, . . .**

The first term of the sequence is 320. So, $a_1 = 320$. Now find the common ratio.

5 → 20 → 80 → 320

$$\frac{20}{5} = \frac{80}{20} = \frac{320}{80} = 4$$

The common ratio is 4. So, $r = 4$.

$a_n = a_1 \cdot r^{n-1}$ Formula for nth term

$a_n = 5 \cdot 4^{n-1}$ $a_1 = 5$ and $r = 4$

b. Find the seventh term of this sequence.

Because we are looking for the seventh term, $n = 7$.

$a_n = a_1 \cdot r^{n-1}$ Formula for nth term

$a_7 = 5 \cdot 4^{7-1}$ $n = 7$

$= 5 \cdot 4^6$ Simplify.

$= 5 \cdot 4096$ $4^6 = 4096$

$= 20{,}480$ Multiply.

The seventh term of the sequence is 20,480.

Lesson 7-7

Exercises

1. Write an equation for the nth term of the geometric sequence –2, 10, –50, ... Find the eleventh term of this sequence.

2. Write an equation for the nth term of the geometric sequence 512, 128, 32, ... Find the sixth term of this sequence.

3. Write an equation for the nth term of the geometric sequence $\frac{4}{9}$, 4, 36, ... Find the eighth term of this sequence.

4. Write an equation for the nth term of the geometric sequence 6, –54, 486, ... Find the ninth term of this sequence.

5. Write an equation for the nth term of the geometric sequence 100, 80, 64, ... Find the seventh term of this sequence.

6. Write an equation for the nth term of the geometric sequence $\frac{2}{5}, \frac{1}{10}, \frac{1}{40}, \ldots$. Find the sixth term of this sequence.

7. Write an equation for the nth term of the geometric sequence $\frac{3}{8}, -\frac{3}{2}, 6, \ldots$. Find the tenth term of this sequence.

8. Write an equation for the nth term of the geometric sequence –3, –21, –147, ... Find the fifth term of this sequence.

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7-7 Skills Practice

Geometric Sequences as Exponential Functions

Determine whether each sequence is *arithmetic, geometric,* or *neither*. Explain.

1. 7, 13, 19, 25, …

2. –96, –48, –24, –12, …

3. 108, 66, 141, 99, …

4. 3, 9, 81, 6561, …

5. $\frac{7}{3}$, 14, 84, 504, …

6. $\frac{3}{8}$, $-\frac{1}{8}$, $-\frac{5}{8}$, $-\frac{9}{8}$, …

Find the next three terms in each geometric sequence.

7. 2500, 500, 100, …

8. 2, 6, 18, …

9. –4, 24, –144, …

10. $\frac{4}{5}$, $\frac{2}{5}$, $\frac{1}{5}$, …

11. –3, –12, –48, …

12. 72, 12, 2, …

13. Write an equation for the nth term of the geometric sequence 3, – 24, 192, ….
Find the ninth term of this sequence.

14. Write an equation for the nth term of the geometric sequence $\frac{9}{16}$, $\frac{3}{8}$, $\frac{1}{4}$, ….
Find the seventh term of this sequence.

15. Write an equation for the nth term of the geometric sequence 1000, 200, 40, ….
Find the fifth term of this sequence.

16. Write an equation for the nth term of the geometric sequence – 8, – 2, $-\frac{1}{2}$, ….
Find the eighth term of this sequence.

17. Write an equation for the nth term of the geometric sequence 32, 48, 72, ….
Find the sixth term of this sequence.

18. Write an equation for the nth term of the geometric sequence $\frac{3}{100}$, $\frac{3}{10}$, 3, ….
Find the ninth term of this sequence.

NAME _____ DATE _____ PERIOD _____

7-7 Practice

Geometric Sequences as Exponential Functions

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 1, −5, −11, −17, …

2. 3, $\frac{3}{2}$, 1, $\frac{3}{4}$, …

3. 108, 36, 12, 4, …

4. −2, 4, −6, 8, …

Find the next three terms in each geometric sequence.

5. 64, 16, 4, …

6. 2, −12, 72, …

7. 3750, 750, 150, …

8. 4, 28, 196, …

9. Write an equation for the nth term of the geometric sequence 896, −448, 224, … . Find the eighth term of this sequence.

10. Write an equation for the nth term of the geometric sequence 3584, 896, 224, … . Find the sixth term of this sequence.

11. Find the sixth term of a geometric sequence for which $a_2 = 288$ and $r = \frac{1}{4}$.

12. Find the eighth term of a geometric sequence for which $a_3 = 35$ and $r = 7$.

13. PENNIES Thomas is saving pennies in a jar. The first day he saves 3 pennies, the second day 12 pennies, the third day 48 pennies, and so on. How many pennies does Thomas save on the eighth day?

Lesson 7-7

NAME _____ DATE _____ PERIOD _____

7-7 Word Problem Practice

Geometric Sequences as Exponential Functions

1. WORLD POPULATION The CIA estimates the world population is growing at a rate of 1.167% each year. The world population for 2007 was about 6.6 billion.

a. Write an equation for the world population after n years. (*Hint:* The common ratio is not 0.01167.)

b. What will the estimated world population be in 2017?

2. MUSEUMS The table shows the annual visitors to a museum in millions. Write an equation for the projected number of visitors after n years.

Year	Visitors (millions)
1	4
2	6
3	9
4	$13\frac{1}{2}$
n	?

3. BANKING Arnold has a bank account with a beginning balance of $5000. He spends one-fifth of the balance each month. How much money will be in the account after 6 months?

4. POPULATION The table shows the projected population of the United States through 2050. Does this table show an *arithmetic sequence*, a *geometric sequence* or neither? Explain.

Year	Projected Population
2000	282,125,000
2010	308,936,000
2020	335,805,000
2030	363,584,000
2040	391,946,000
2050	419,854,000

Source: U.S. Census Bureau

5. SAVINGS ACCOUNTS A bank offers a savings account with a 0.5% return each month.

a. Write an equation for the balance of a savings account after n months. (*Hint*: The common ratio is not 0.005.)

b. Given an initial deposit of $500, what will the account balance be after 15 months?

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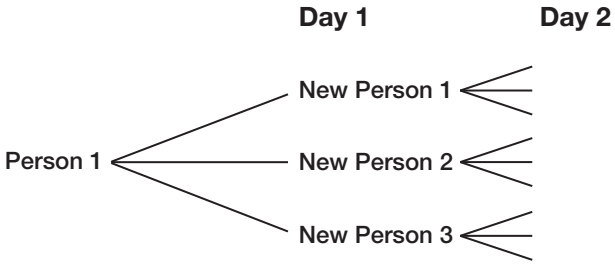
7-7 Enrichment

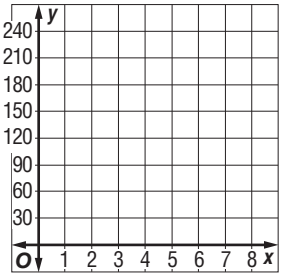
Pay it Forward

The idea behind "pay it forward" is that on the first day, one person does a good deed for three different people. Then, on the second day, those three people will each perform good deeds for 3 more people, so that on Day 2, there are 3×3 or 9 good deeds being done. Continue this process to fill in the chart. A tree diagram will help you fill in the chart.

Day	# of Deeds
0	1
1	3
2	9
3	
4	
5	



1. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.



2. What type of function is your graph from Exercise 1? Write an equation that can be used to determine the number of good deeds on any given day, x.

3. How many good deeds will be performed on Day 21?

The formula $T = \frac{3^{n+1} - 3}{2}$ can be used to determine the **total** number of good deeds that have been performed, where n represents the day. For example, on Day 2, there have been $3 + 9$ or 12 good deeds performed. Using the formula, you get, $\frac{3^{2+1} - 3}{2}$ or $\frac{3^3 - 3}{2} + \frac{27 - 3}{2}$ or a total of 12 good deeds performed.

4. Use the formula to determine the approximate number of good deeds that have been performed through Day 21.

5. Look up the world population. How does your number from Exercise 4 compare to the world population?

NAME _____ DATE _____ PERIOD _____

7-8 Study Guide and Intervention

Recursive Formulas

Using Recursive Formulas A **recursive formula** allows you to find the nth term of a sequence by performing operations on one or more of the terms that precede it.

Example **Find the first five terms of the sequence in which $a_1 = 5$ and $a_n = -2a_{n-1} + 14$, if $n \geq 2$.**

The given first term is $a_1 = 5$. Use this term and the recursive formula to find the next four terms.

$a_2 = -2a_{2-1} + 14$	$n = 2$	$a_4 = -2a_{4-1} + 14$	$n = 4$
$= -2a_1 + 14$	Simplify.	$= -2a_3 + 14$	Simplify.
$= -2(5) + 14$ or 4	$a_1 = 5$	$= -2(6) + 14$ or 2	$a_3 = 6$
$a_3 = -2a_{3-1} + 14$	$n = 3$	$a_5 = -2a_{5-1} + 14$	$n = 5$
$= -2a_2 + 14$	Simplify.	$= -2a_4 + 14$	Simplify.
$= -2(4) + 14$ or 6	$a_2 = 4$	$= -2(2) + 14$ or 10	$a_4 = 2$

The first five terms are 5, 4, 6, 2, and 10.

Exercises

Find the first five terms of each sequence.

1. $a_1 = -4, a_n = 3a_{n-1}, n \geq 2$

2. $a_1 = 5, a_n = 2a_{n-1}, n \geq 2$

3. $a_1 = 8, a_n = a_{n-1} - 6, n \geq 2$

4. $a_1 = -32, a_n = a_{n-1} + 13, n \geq 2$

5. $a_1 = 6, a_n = -3a_{n-1} + 20, n \geq 2$

6. $a_1 = -9, a_n = 2a_{n-1} + 11, n \geq 2$

7. $a_1 = 12, a_n = 2a_{n-1} - 10, n \geq 2$

8. $a_1 = -1, a_n = 4a_{n-1} + 3, n \geq 2$

9. $a_1 = 64, a_n = 0.5a_{n-1} + 8, n \geq 2$

10. $a_1 = 8, a_n = 1.5a_{n-1}, n \geq 2$

11. $a_1 = 400, a_n = \frac{1}{2}a_{n-1}, n \geq 2$

12. $a_1 = \frac{1}{4}, a_n = a_{n-1} + \frac{3}{4}, n \geq 2$

NAME _____ DATE _____ PERIOD _____

7-8 Study Guide and Intervention *(continued)*

Recursive Formulas

Writing Recursive Formulas Complete the following steps to write a recursive formula for an arithmetic or geometric sequence.

Step 1	Determine if the sequence is arithmetic or geometric by finding a common difference or a common ratio.
Step 2	Write a recursive formula. **Arithmetic Sequences** $a_n = a_{n-1} + d$, where d is the common difference **Geometric Sequences** $a_n = r \cdot a_{n-1}$, where r is the common ratio
Step 3	State the first term and the domain for n.

Example **Write a recursive formula for 216, 36, 6, 1, … .**

Step 1 First subtract each term from the term that follows it.

$216 - 36 = 180$ $36 - 6 = 30$ $6 - 1 = 5$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$\frac{36}{216} = \frac{1}{6}$ $\frac{6}{36} = \frac{1}{6}$ $\frac{1}{6} = \frac{1}{6}$

There is a common ratio of $\frac{1}{6}$. The sequence is geometric.

Step 2 Use the formula for a geometric sequence.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence

$a_n = \frac{1}{6}a_{n-1}$ $r = \frac{1}{6}$

Step 3 The first term a_1 is 216 and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 216$, $a_n = \frac{1}{6}a_{n-1}$, $n \geq 2$.

Exercises

Write a recursive formula for each sequence.

1. 22, 16, 10, 4, …

2. −8, −3, 2, 7, …

3. 5, 15, 45, 135, …

4. 243, 81, 27, 9, …

5. −3, 14, 31, 48, …

6. 8, −20, 50, −125, …

Lesson 7-8

NAME _____ DATE _____ PERIOD _____

7-8 Skills Practice

Recursive Formulas

Find the first five terms of each sequence.

1. $a_1 = -11, a_n = a_{n-1} - 7, n \geq 2$

2. $a_1 = 14, a_n = a_{n-1} + 6.5, n \geq 2$

3. $a_1 = -3, a_n = 3a_{n-1} + 10, n \geq 2$

4. $a_1 = 187.5, a_n = -0.8a_{n-1}, n \geq 2$

5. $a_1 = \frac{1}{2}, a_n = \frac{3}{2}a_{n-1} + \frac{1}{4}, n \geq 2$

6. $a_1 = \frac{1}{5}, a_n = \frac{5}{2}a_{n-1} - \frac{1}{2}, n \geq 2$

Write a recursive formula for each sequence.

7. 7, 28, 112, 448, …

8. 73, 52, 31, 10, …

9. 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, …

10. −37, −15, 7, 29, …

11. 0.25, 0.37, 0.49, 0.61, …

12. 64, −80, 100, −125, 156.25, …

For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.

13. $a_1 = 38, a_n = a_{n-1} - 17, n \geq 2$

14. $a_n = 5n - 16$

15. $a_n = 50(0.75)^{n-1}$

16. $a_1 = 16, a_n = 4a_{n-1}, n \geq 2$

NAME _____ DATE _____ PERIOD _____

7-8 Practice

Recursive Formulas

Find the first five terms of each sequence.

1. $a_1 = 25, a_n = a_{n-1} - 12, n \geq 2$

2. $a_1 = -101, a_n = a_{n-1} + 38, n \geq 2$

3. $a_1 = 3.3, a_n = a_{n-1} + 2.7, n \geq 2$

4. $a_1 = 7, a_n = -3a_{n-1} + 20, n \geq 2$

5. $a_1 = 20, a_n = \frac{1}{5}a_{n-1}, n \geq 2$

6. $a_1 = \frac{2}{3}, a_n = \frac{1}{3}a_{n-1} - \frac{2}{9}, n \geq 2$

Write a recursive formula for each sequence.

7. 80, −40, 20, −10, …

8. 87, 52, 17, −18, …

9. $\frac{1}{3}, \frac{4}{15}, \frac{16}{75}, \frac{64}{375}, \ldots$

10. $\frac{4}{5}, \frac{3}{10}, -\frac{1}{5}, -\frac{7}{10}, \ldots$

11. 2.6, 5.2, 7.8, 10.4, …

12. 100, 120, 144, 172.8, …

13. PIZZA The total costs for ordering one to five cheese pizzas from Luigi's Pizza Palace are shown.

Total Number of Pizzas Ordered	Cost
1	$7.00
2	$12.50
3	$18.00
4	$23.50
5	$29.00

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.

Lesson 7-8

NAME _____ DATE _____ PERIOD _____

7-8 Word Problem Practice

Recursive Formulas

1. ASSEMBLY An assembly line can create 175 widgets in one hour, 350 widgets in two hours, 525 widgets in three hours, 700 widgets in four hours, and so on.

a. Find the next 5 terms in the sequence.

b. Write a recursive formula for the sequence.

c. Write an explicit formula for the sequence.

2. PAPER A piece of paper is folded several times. The number of sections into which the piece of paper is divided after each fold is shown below.

Number of Folds	Sections
1	2
2	4
3	8
4	16
5	32

a. Find the next 5 terms in the sequence.

b. Write a recursive formula for the sequence.

c. Write an explicit formula for the sequence.

3. CLEANING An equation for the cost a_n in dollars that a carpet cleaning company charges for cleaning n rooms is $a_n = 50 + 25(n - 1)$. Write a recursive formula to represent the cost a_n.

4. SAVINGS A recursive formula for the balance of a savings account a_n in dollars at the beginning of year n is $a_1 = 500$, $a_n = 1.05a_{n-1}$, $n \geq 2$. Write an explicit formula to represent the balance of the savings account a_n.

5. SNOW A snowman begins to melt as the temperature rises. The height of the snowman in feet after each hour is shown.

Hour	Height (ft)
1	6.0
2	5.4
3	4.86
4	4.374

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.

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7-8 Enrichment

Arithmetic Series

Consider a situation in which an uncle gives his niece a dollar amount equal to her age each year on her birthday. An arithmetic sequence that represents this situation is 1, 2, 3, … . How much total money will the niece have received by her 30th birthday?

When the terms of an arithmetic sequence are added, an *arithmetic series* is formed. The solution of the problem above is the sum of the arithmetic series $1 + 2 + 3 + \ldots + 30$.

If we write the series twice, as shown below, and add the terms in each column, we can see a pattern.

$$\begin{array}{r} 1 + 2 + 3 + 4 + \ldots + 27 + 28 + 29 + 30 \\ 30 + 29 + 28 + 27 + \ldots + 4 + 3 + 2 + 1 \\ \hline 31 + 31 + 31 + 31 + \ldots + 31 + 31 + 31 + 31 \end{array}$$

The sum of each pair of terms is 31. Multiplying 31 by the number of terms in the bottom row, 30, gives us the sum of the bottom row, or 930. Since the series was written twice, and resultantly, each term in the series was accounted for twice, we can divide 930 by 2 and find that the sum of the series $1 + 2 + 3 + \ldots + 30$ is 465.

1. Write an expression to represent the sum of the series above using the first term 1, the last term 30, and the number of terms in the series 30.

2. Write an expression to represent the sum of an arithmetic series with n terms, first term a_1, and last term a_n.

Use the expression that you found in Exercise 2 to find the sum of each arithmetic series.

3. $1 + 2 + 3 + \ldots + 100$

4. $5 + 10 + 15 + \ldots + 100$

5. $1 + 3 + 5 + \ldots + 29$

6. $2 + 4 + 6 + \ldots + 60$

7. $6 + 13 + 20 + \ldots + 125$

8. $8 + 12 + 16 + \ldots + 84$

Lesson 7-8

NAME _____ DATE _____ PERIOD _____

7 Student Recording Sheet

SCORE _____

Use this recording sheet with pages 458–459 of the Student Edition.

Multiple Choice

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ	**3.** Ⓐ Ⓑ Ⓒ Ⓓ	**5.** Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓕ Ⓖ Ⓗ Ⓙ	**4.** Ⓕ Ⓖ Ⓗ Ⓙ	**6.** Ⓕ Ⓖ Ⓗ Ⓙ

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

7a. _____

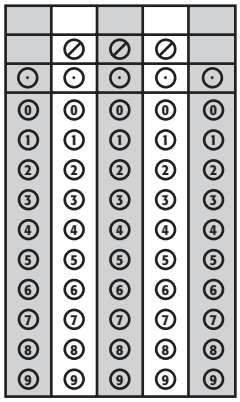
7b. _____

8. _____

9. _____

10. _____ *(grid-in)*

10.



Extended Response

Record your answers for Question 11 on the back of this paper.

NAME _____ DATE _____ PERIOD _____

7 Rubric for Scoring Extended Response

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 11 Rubric

Score	Specific Criteria
4	Part **a** identifies Mercury as the planet closest to the Sun, because it has the smallest exponent. In part **b,** students set up and solve the expression $\frac{2.28 \times 10^8}{1.50 \times 10^8}$ to find that Mars is about 1.52 times further from the Sun than Earth. In part **c,** students set up and solve the expression $\frac{1.43 \times 10^9}{3 \times 10^5}$ to find that it takes approximately 4767 seconds for light from the Sun to reach Saturn.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Quiz 1

SCORE _____

(Lessons 7-1 and 7-2)

Simplify.

1. $(r^3)(2r^5)$

2. $(x^5)^4$

3. $(-4t^2n^3)(3tn^4)$

4. $(-5x^4y^2)^3$

5. $(2cd)^2(-4c^3)^2$

6. $(5y^2w^4)^2 + 2(yw^2)^4$

Simplify. Assume that no denominator is equal to zero.

7. $\frac{6^{15}}{6^9}$

8. $\frac{y^8}{y^3}$

9. $\frac{r^6n^{-7}}{r^4n^2}$

10. MULTIPLE CHOICE Write the ratio of the area of a circle with radius r to the circumference of the same circle.

A $\frac{2}{r}$ **B** 2 **C** $\frac{r}{2}$ **D** $\frac{1}{2r}$

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Quiz 2

SCORE _____

(Lessons 7-3 and 7-4)

Simplify.

1. $\sqrt[4]{81}$

2. $4^{\frac{3}{2}}$

Solve each equation.

3. $5^x = 125$

4. $2^{5x-4} = 64$

5. Write $2\sqrt{7xy}$ in exponential form.

Express each number in scientific notation.

6. 14,000,000

7. 0.0000308

8. MULTIPLE CHOICE Evaluate $(4 \times 10^7)(3.6 \times 10^{-4})$.

A 1.11×10^3 **C** 1.44×10^2

B 1.44×10^4 **D** 1.44×10^{-27}

Evalute each quotient. Express the results in both scientific notation and standard form.

9. $\frac{7.2 \times 10^4}{8 \times 10^{-3}}$

10. $\frac{-3.6 \times 10^{-5}}{6 \times 10^3}$

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Quiz 3

SCORE _____

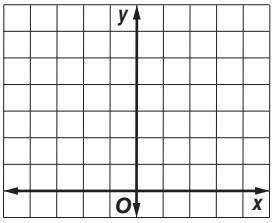
(Lessons 7-5 and 7-6)

Graph each function. Find the *y*-intercept and state the domain and range.

1. $y = \left(\frac{1}{3}\right)^x$

2. $y = 4(2)^x - 1$

1.



Robin invests $1500 at 4.85% compounded quarterly.

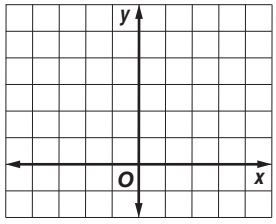
3. Write an equation to represent the amount of money he will have in n years.

4. How much money will Robin have in 12 years?

5. MULTIPLE CHOICE The population of Camden, New Jersey, has been decreasing by 0.12% a year on average. If this trend continues, and the population was 79,318 in 2006, estimate Camden's population in 2015.

A 71,152 **C** 78,465
B 78,461 **D** 80,179

2.



3. _____

4. _____

5. _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Quiz 4

SCORE _____

(Lessons 7-7 and 7-8)

1. Determine whether the sequence 32, 16, 8, 4, . . . is *arithmetic*, *geometric*, or *neither*. Explain.

1. _____

2. Find the next three terms of the geometric sequence −3, −12, −48, ...

2. _____

3. MULTIPLE CHOICE What is the eighth term of the geometric series −2, 10, −50, ...?

A −781,250 **C** 156,250
B −156,250 **D** 781,250

3. _____

Write a recursive formula for each sequence.

4. 3, 20, 37, 54, …

4. _____

5. 125, −25, 5, $-\frac{1}{5}$

5. _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Mid-Chapter Test

SCORE _____

(Lessons 7-1 through 7-4)

Assessment

Part I **Write the letter for the correct answer in the blank at the right of each question.**

1. Simplify $(n^5)(n^2)(r^3)(r^4)$.

A $n^{10}r^{12}$ **B** n^7r^7 **C** nr^7 **D** nr^{14} **1.** _____

2. Simplify $(3w^2r)^2(-2w^5r^2)^3$.

F $-72w^{19}r^8$ **G** $72w^{12}r^7$ **H** $-36w^{32}r^{10}$ **J** $36w^{19}r^6$ **2.** _____

Simplify. Assume the denominator is not equal to zero.

3. $\frac{m^6n^3}{m^2n^6}$

A $\frac{m^4}{n^3}$ **B** $-\frac{m^4}{n^3}$ **C** $-\frac{m^8}{n^3}$ **D** $\frac{m^8}{n^3}$ **3.** _____

4. $\frac{(z^2w^{-1})^3}{(z^3w^2)^2}$

F $\frac{1}{w^7}$ **G** $\frac{z^{12}}{w^7}$ **H** w **J** $\frac{1}{w}$ **4.** _____

5. Solve $2^{3x+10} = 128$.

A 3 **B** 2 **C** 1 **D** -1 **5.** _____

6. Express 0.000702 in scientific notation.

F 7.02×10^{-3} **H** 7.02×10^{-4} **G** 7.02^{-4} **J** 7.02×10^{-5} **6.** _____

7. Evaluate $\frac{6.08 \times 10^7}{3.8 \times 10^{-2}}$.

A 1.6×10^5 **B** 1.6×10^9 **C** 2.3104×10^6 **D** 2.3104×10^{10} **7.** _____

Part II **Evalute each product. Express the results in both scientific notation and standard form.**

8. $(4.2 \times 10^9)(1.6 \times 10^{-5})$ **8.** _____

9. $(7.8 \times 10^{-3})(4 \times 10^{-4})$ **9.** _____

Simplify. Assume that no denominator is equal to zero.

10. $\frac{2x^3y^5}{5(x^4y^2)^3}$ **10.** _____

11. $\frac{-10m^{-1}y^0r}{-14m^{-7}y^{-3}r^{-4}}$ **11.** _____

Solve each equation.

12. $8^x = 16$ **12.** _____

13. $216^{x+1} = 6$ **13.** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Vocabulary Test

SCORE _____

common ratio	exponential growth	order of magnitude
compound interest	geometric sequence	rational exponent
constant	monomial	recursive formula
exponential decay	negative exponent	scientific notation
exponential function	*n*th root	zero exponent

State whether each sentence is *true* or *false*. If false, replace the underlined word or phrase to make a true sentence.

1. The zero exponent property tells us that any nonzero number raised to the zero power is 1. **1.** _____

2. A very large or very small number written in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer, is said to be in scientific notation. **2.** _____

3. The scientific notation of a quantity is the number rounded to the nearest power of 10. **3.** _____

4. The negative exponent of a geometric sequence can be found by dividing any term by its previous term. **4.** _____

5. Constants are monomials that are real numbers. **5.** _____

6. The equation $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is the general equation for compound interest. **6.** _____

7. A recursive formula allows you to find the nth term of a sequence by performing operations to one or more of the terms that precede it. **7.** _____

8. Exponential decay is when an initial amount decreases by the same percent over a given period of time. **8.** _____

9. A number, a variable, or a product of a number and one or more variables is a geometric sequence. **9.** _____

Define each term in your own words.

10. exponential function **10.** _____

11. geometric sequence **11.** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 1

SCORE _____

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $y^5 \cdot y^3$.

A y^2 **B** y^8 **C** y^{15} **D** $2y^8$ **1.** _____

2. Simplify $(b^4)^3$.

F b^7 **G** $3b^4$ **H** b^{12} **J** $3b^7$ **2.** _____

3. Simplify $\frac{a^7}{a^4}$. Assume the denominator is not equal to zero.

A a^{11} **B** a^{28} **C** a^3 **D** 1 **3.** _____

4. A rectangle has a length of $25x^3$ and a width of $5x^2$. Find the area in square units.

F $25x^6$ **G** $25x^5$ **H** $125x^6$ **J** $125x^5$ **4.** _____

5. Simplify $\frac{m^5r^2}{m^2r^3}$. Assume the denominator is not equal to zero.

A m^7r^5 **B** $\frac{m^3}{r}$ **C** m^3r **D** $\frac{r}{m^3}$ **5.** _____

6. Express 0.000024 in scientific notation.

F 2.4×10^5 **G** 0.24×10^{-4} **H** 2.4×10^{-4} **J** 2.4×10^{-5} **6.** _____

7. Evaluate $(7 \times 10^8)(2.4 \times 10^{-4})$.

A 1.68×10^4 **B** 2.92×10^4 **C** 1.68×10^5 **D** 1.68×10^{13} **7.** _____

8. Evaluate $16^{\frac{3}{4}}$.

F 2 **G** 4 **H** 8 **J** 32 **8.** _____

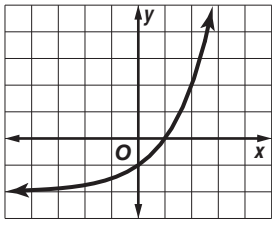
9. Solve $3^{x+2} = 81$.

A 0 **B** 1 **C** 2 **D** 3 **9.** _____

10. Which equation corresponds to the graph shown?

F $y = 2^x + 2$ **H** $y = 2^x - 2$

G $y = \left(\frac{1}{2}\right)^x - 2$ **J** $y = \left(\frac{1}{2}\right)^x + 2$



10. _____

11. What is the y-intercept on the graph shown?

A $\frac{1}{2}$ **B** 1 **C** 0 **D** -1 **11.** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 1 *(continued)*

12. TOURNAMENTS A chess tournament starts with 16 people competing. The exponential function $y = 16\left(\frac{1}{2}\right)^x$ describes how many people are remaining in the tournament after x rounds. How many people are left in the tournament after 2 rounds?

F 4 **G** 2 **H** 8 **J** 1 **12.** _____

13. INVESTMENTS Determine the amount of an investment if \$1000 is invested at an interest rate of 8% compounded quarterly for 2 years.

A \$1160.00 **B** \$1171.66 **C** \$1040.40 **D** \$1166.40 **13.** _____

14. BIOLOGY If $y = 10(2.5)^t$ represents the number of bacteria in a culture at time t, how many will there be at time $t = 6$?

F 2441 **G** 244 **H** 24 **J** none **14.** _____

15. DEPRECIATION A \$60,000 piece of machinery depreciates in value at a rate of 11% per year. About what will its value be in 5 years?

A \$47,526 **B** \$42,298 **C** \$33,504 **D** \$37,645 **15.** _____

16. Which is the equation for the nth term of the geometric sequence $-2, 8, -32, \ldots$?

F $a_n = -2 \cdot 4^n$ **H** $a_n = -2 \cdot 4^{n-1}$

G $a_n = 4 \cdot (-2)^n$ **J** $a_n = -2 \cdot (-4)^{n-1}$ **16.** _____

17. What is the ninth term of the geometric sequence 3, 9, 27, …?

A 2187 **B** 6561 **C** 19,683 **D** 59,049 **17.** _____

18. Find the third term of the sequence in which $a_1 = 12$ and $a_n = 5a_{n-1} - 14$, if $n \geq 2$.

F 1 **G** 46 **H** 216 **J** 1066 **18.** _____

19. Find an explicit formula for $a_1 = 17$, $a_n = a_{n-1} + 4$, $n \geq 2$.

A $a_n = 4n + 13$ **C** $a_n = 4n + 17$

B $a_n = n + 4$ **D** $a_n = 17n + 4$ **19.** _____

20. Find a recursive formula for the arithmetic sequence 18, 12, 6, 2, ...

F $a_1 = 18, a_n = -6a_{n-1}, n \geq 2$ **H** $a_1 = 18, a_n = \frac{2}{3}a_{n-1}, n \geq 2$

G $a_1 = 18, a_n = a_{n-1} - 6, n \geq 2$ **J** $a_1 = 18, a_n = \frac{1}{2}a_{n-1} + 9, n \geq 2$ **20.** _____

Bonus Simplify $(3^{n+1})(3^{2n})^4$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 2A

SCORE _____

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $(x^3)^8$.

A x^{24} **B** x^{11} **C** $8x^{24}$ **D** $8x^{11}$ **1.** _____

2. Simplify $(-2hk)^4(4h^3k^5)^2$.

F $2h^{24}k^{40}$ **G** $-64h^9k^{11}$ **H** $-256h^{10}k^{14}$ **J** $256h^{10}k^{14}$ **2.** _____

3. Simplify $\frac{36b^4c^2}{9b^{-1}c^5}$. Assume the denominator is not equal to zero.

A $\frac{27b^4}{c^3}$ **B** $\frac{4b^4}{c^3}$ **C** $\frac{27b^3}{c^3}$ **D** $\frac{4b^5}{c^3}$ **3.** _____

4. Simplify $\frac{(3y^4n^6)^2}{(y^2n^{-3})^4}$. Assume the denominator is not equal to zero.

F $\frac{9}{y^{16}}$ **G** $\frac{9}{n^{24}}$ **H** $9y^{16}$ **J** $9n^{24}$ **4.** _____

5. Which monomial represents the number of square units in the area of a circle with radius $4x^3$ units?

A $16\pi x^6$ **B** $8\pi x^6$ **C** $16\pi x^9$ **D** $8\pi x^5$ **5.** _____

6. Express 46,100,000 in scientific notation.

F 4.61×10^7 **G** 4.61×10^6 **H** 4.61×10^5 **J** 4.61×10^8 **6.** _____

7. Evaluate $\frac{7 \times 10^4}{1.4 \times 10^{-5}}$.

A 5×10^9 **B** 5×10^{-20} **C** 5×10^{-1} **D** 5×10^1 **7.** _____

8. ATTENDANCE The total attendance for a professional baseball team this season was 3.24×10^6 and two years ago was 2.43×10^6. About how many times as large was this season's attendance as attendance two years ago?

F 0.8 **G** 0.9 **H** 1.1 **J** 1.3 **8.** _____

9. Write $10y^{\frac{1}{2}}$ in radical form.

A $\sqrt{10y}$ **B** $10\sqrt{y}$ **C** $10\sqrt{10y}$ **D** $y\sqrt{10}$ **9.** _____

10. Evaluate $81^{\frac{3}{4}}$.

F 3 **G** 9 **H** 27 **J** 243 **10.** _____

11. Solve $5^{x-2} = 125$.

A 2 **B** 3 **C** 4 **D** 5 **11.** _____

7 Chapter 7 Test, Form 2A *(continued)*

12. Which is the equation for the nth term of the geometric sequence $-4, 8, -16, \ldots$?

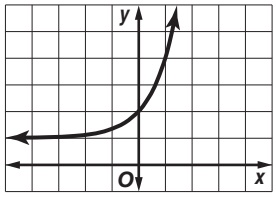
F $a_n = -4 \cdot 2^{n-1}$ **H** $a_n = -4 \cdot (-2)^{n-1}$

G $a_n = -2 \cdot (-4)^{n-1}$ **J** $a_n = -2 \cdot 4^n$ **12.** _____

13. Which equation represents exponential growth?

A $y = 5(0.84)^x$ **B** $y = 5x$ **C** $y = 0.3x^3$ **D** $y = 5(1.06)^x$ **13.** _____

14. Which equation corresponds to the graph shown?



F $y = (3)^x + 1$ **H** $y = 2(3^x)$

G $y = 2(3^x + 1)$ **J** $y = (2 \cdot 3)^x + 1$ **14.** _____

15. A weightlifter can increase the weight $W(x)$ that she can lift according to $W(x) = 315(1.05)^x$, where x represents the number of training cycles completed. How much will she lift after 4 training cycles?

A 365 lb **B** 383 lb **C** 378 lb **D** 402 lb **15.** _____

16. BIOLOGY A certain fast-growing bacteria increases 6% per minute. If there are 100 bacteria now, about how many will there be 12 minutes later?

F 172 **G** 201 **H** 48 **J** 190 **16.** _____

17. POPULATION A city's population is about 954,000 and is decreasing at an annual rate of 0.1%. Predict the population in 50 years.

A 577,176 **B** 906,300 **C** 1,002,888 **D** 907,450 **17.** _____

18. Find the third term of the sequence in which $a_1 = 7$ and $a_n = -2a_{n-1} + 11$, if $n \geq 2$.

F -23 **G** -3 **H** 5 **J** 17 **18.** _____

19. Find an explicit formula for $a_1 = -4$, $a_n = a_{n-1} + 9$, $n \geq 2$.

A $a_n = 9n - 13$ **C** $a_n = 9n - 4$

B $a_n = n + 9$ **D** $a_n = -4n + 9$ **19.** _____

20. Find a recursive formula for the arithmetic sequence $24, 32, 40, 48, \ldots$.

F $a_1 = 24, a_n = 8a_{n-1}, n \geq 2$ **H** $a_1 = 24, a_n = \frac{4}{3}a_{n-1}, n \geq 2$

G $a_1 = 24, a_n = \frac{1}{2}a_{n-1} + 20, n \geq 2$ **J** $a_1 = 24, a_n = a_{n-1} + 8, n \geq 2$ **20.** _____

Bonus Simplify $\frac{7^{x-3}}{7^{3x-1}}$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 2B

SCORE _____

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $(m^4)^2$.

A $6m$ **B** m^8 **C** m^6 **D** $2m^4$ **1.** _____

2. Simplify $(-2xy^2)^4(2x^3y^4)^2$.

F $4x^{24}y^{32}$ **G** $-8x^9y^6$ **H** $64x^{10}y^{16}$ **J** $-4x^{10}y^{16}$ **2.** _____

3. Simplify $\frac{6n^{-3}y}{2n^{-1}y^{-3}}$. Assume the denominator is not equal to zero.

A $\frac{4y^3}{n^2}$ **B** $\frac{3y^4}{n^2}$ **C** $\frac{3}{n^4y^2}$ **D** $\frac{3n^2}{y^4}$ **3.** _____

4. Simplify $\frac{(a^{-2}b^4)^{-6}}{(a^4b^{-8})^3}$. Assume the denominator is not equal to zero.

F ab^3 **G** 1 **H** $\frac{a^{24}}{b^{48}}$ **J** $\frac{b^{48}}{a^{24}}$ **4.** _____

5. Which monomial represents the number of square units in the area of a circle with radius $3x^3$ units?

A $9\pi x^6$ **B** $6\pi x^6$ **C.** $9\pi x^9$ **D** $6\pi x^5$ **5.** _____

6. Express 8,450,000 in scientific notation.

F 8.45×10^4 **G** 8.45×10^7 **H** 8.45×10^5 **J** 8.45×10^6 **6.** _____

7. Evaluate $\frac{4.65 \times 10^{-4}}{5 \times 10^{-6}}$.

A 9.3×10^{11} **B** 9.3×10^1 **C** 9.3×10^2 **D** 9.3×10^0 **7.** _____

8. SOLAR SYSTEM The average distance Earth is from the Sun is about 9.296×10^7 miles, and the average distance Mars is from the Sun is about 1.4162×10^8. About how many times as far is Mars from the Sun as Earth is from the Sun?

F 0.7 **G** 0.9 **H** 1.3 **J** 1.5 **8.** _____

9. Write $(8x)^{\frac{1}{2}}$ in radical form.

A $8\sqrt{x}$ **B** $\sqrt{8x}$ **C** $8\sqrt{8x}$ **D** $x\sqrt{8}$ **9.** _____

10. Evaluate $125^{\frac{2}{3}}$.

F 5 **G** 25 **H** 625 **J** 3125 **10.** _____

11. Solve $6^{x+1} = 1296$.

A 1 **B** 2 **C** 3 **D** 4 **11.** _____

7 Chapter 7 Test, Form 2B *(continued)*

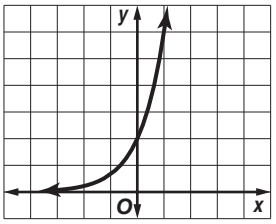
12. Which is the equation for the nth term of the geometric sequence 6, 12, 24, ...?

F $a_n = 6 \cdot 2^n$ **G** $a_n = 2 \cdot 3^n$ **H** $a_n = 3 \cdot 2^n$ **J** $a_n = 3 \cdot 2^{n-1}$ **12.** _____

13. Which equation represents exponential decay?

A $y = 0.5x^3$ **B** $y = 0.5x^2 - x$ **C** $y = 0.5(1.07)^x$ **D** $y = 0.5(.87)^x$ **13.** _____

14. Which equation corresponds to the graph shown?



F $y = 3^x + 2$ **H** $y = 2(3^x)$

G $y = 2(3^x + 1)$ **J** $y = (2 \cdot 3)^x + 1$ **14.** _____

15. A weight lifter can deadlift 275 pounds. She can increase the weight $W(x)$ that she can lift according to the function $W(x) = 275(1.05)^x$, where x represents the number of training cycles completed. How much will she deadlift after 5 training cycles?

A 334 lb **B** 369 lb **C** 344 lb **D** 351 lb **15.** _____

16. POPULATION A city's population is about 763,000 and is increasing at an annual rate of 1.5%. Predict the population of the city in 50 years.

F 1,335,250 **G** 826,830,628 **H** 358,374 **J** 1,606,300 **16.** _____

17. BUSINESS A printing press valued at $120,000 depreciates 12% per year. What will be the approximate value of the printing press in 7 years?

A $19,200 **B** $265,282 **C** $49,041 **D** $55,728 **17.** _____

18. Find the third term of the sequence in which $a_1 = -1$ and $a_n = 5a_{n-1} - 3$, if $n \geq 2$.

F −218 **G** −43 **H** −8 **J** 12 **18.** _____

19. Find an explicit formula for $a_1 = 10$, $a_n = a_{n-1} - 3$, $n \geq 2$.

A $a_n = n - 3$ **C** $a_n = -3n + 10$

B $a_n = 10n - 3$ **D** $a_n = -3n + 13$ **19.** _____

20. Find a recursive formula for the arithmetic sequence 8, −2, −12, −22, ...

F $a_1 = 8, a_n = -10a_{n-1}, n \geq 2$ **H** $a_1 = 8, a_n = -\frac{1}{2}a_{n-1} + 2, n \geq 2$

G $a_1 = 8, a_n = a_{n-1} - 10, n \geq 2$ **J** $a_1 = 8, a_n = \frac{1}{2}a_{n-1} - 6, n \geq 2$ **20.** _____

Bonus Simplify $3^{2n-1} \cdot (3^{5n})$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 2C

SCORE _____

Simplify.

1. $(9t^3n^5)(-2tn^2)$ **1.** _____

2. $(w^5y^4)^3$ **2.** _____

3. $4a^3n^6 + 4(a^3n)^6 + 4(an^2)^3$ **3.** _____

Simplify. Assume that no denominator is equal to zero.

4. $\frac{16r^3t^{-5}}{4r^{-1}t^2}$ **4.** _____

5. $\frac{(-8x^2y^2)^2}{(4x^3y)^3}$ **5.** _____

6. Express 0.000000607 in scientific notation. **6.** _____

Evalute each product or quotient. Express the results in both scientific notation and standard form.

7. $(2.8 \times 10^{-4})(7.7 \times 10^9)$ **7.** _____

8. $\frac{8.1 \times 10^4}{1.8 \times 10^{-4}}$ **8.** _____

9. ATTENDANCE The total home attendance for a professional basketball team in 2010 was about 8.2×10^5, and in 2008 was about 7.175×10^5. About how many times as large was the attendance in 2010 as the attendance in 2008? **9.** _____

Solve each equation.

10. $36^{x-2} = 6$ **10.** _____

11. $4^{3x-2} = 256$ **11.** _____

12. $32^{x+3} = 2$ **12.** _____

7 Chapter 7 Test, Form 2C *(continued)*

Simplify.

13. $64^{\frac{5}{6}}$ **13.** _____

14. $81^{\frac{5}{4}}$ **14.** _____

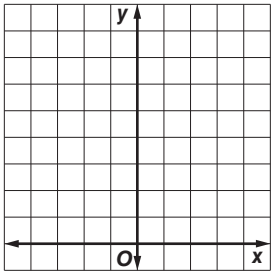
Write each expression in radical form.

15. $22^{\frac{1}{3}}$ **15.** _____

16. $3x^{\frac{1}{4}}$ **16.** _____

17. A weight lifter can bench-press 165 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 165(1.05)^x$, where x represents the number of training cycles she completes. How much will she bench-press after 4 training cycles? **17.** _____

18. Graph $y = 7^x$. Find the y-intercept and state the domain and range.



18. _____

19. INVESTMENTS Determine the amount of an investment if \$700 is invested at an interest rate of 8% compounded monthly for 9 years. **19.** _____

20. BUSINESS A new welding machine valued at \$38,000 depreciates at a steady rate of 9% per year. What is the value of the welding machine in 10 years? **20.** _____

21. Write an equation for the nth term of the geometric sequence $-3, 9, -27, \ldots$. **21.** _____

Find the first three terms of each sequence.

22. $a_1 = 5, a_n = a_{n-1} - 3, n \geq 2$ **22.** _____

23. $a_1 = 3, a_n = 2a_{n-1} + 4, n \geq 2$ **23.** _____

Write a recursive formula for each sequence.

24. 18, 29, 40, 51, … **24.** _____

25. 2, –12, 72, –432, … **25.** _____

Bonus Find the first term of the geometric sequence with $a_6 = 256$ and $a_7 = 1024$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 2D

SCORE _____

Simplify.

1. $(3a^2b^5)(-2ab^3)$ **1.** _____

2. $(w^3z^7)^3$ **2.** _____

3. $4a^4b^8 + 2(ab^2)^4 + 4(a^2b^4)^2$ **3.** _____

Simplify. Assume that no denominator is equal to zero.

4. $\frac{4a^{-3}d^2}{8a^2d^{-5}}$ **4.** _____

5. $\frac{(3r^3t^5)^3}{(-3r^2t^7)^2}$ **5.** _____

6. Express 0.00000402 in scientific notation. **6.** _____

Evaluate each product or quotient. Express the results in both scientific notation and standard form.

7. $(4.6 \times 10^3)(9.12 \times 10^{-7})$ **7.** _____

8. $\frac{1.6 \times 10^{-3}}{8 \times 10^{-7}}$ **8.** _____

9. ATTENDANCE The total home attendance for a professional football team in 2010 was about 5.44×10^5, and in 2008 was about 4.32×10^5. About how many times as large was the attendance in 2010 as the attendance in 2008? **9.** _____

Solve each equation.

10. $125^{x-1} = 5$ **10.** _____

11. $3^{3x+1} = 81$ **11.** _____

12. $64^{2x+3} = 2$ **12.** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 2D *(continued)*

Simplify.

13. $1000^{\frac{2}{3}}$ **13.** _____

14. $4^{\frac{5}{2}}$ **14.** _____

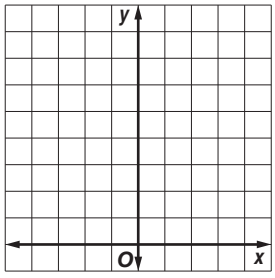
Write each expression in radical form.

15. $57^{\frac{1}{4}}$ **15.** _____

16. $10x^{\frac{1}{3}}$ **16.** _____

17. A weight lifter can bench-press 145 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 145(1.05)^x$, where x represents the number of training cycles she completes. How much will she bench-press after 5 training cycles? **17.** _____

18. Graph $y = \left(\frac{1}{6}\right)^x$. Find the y-intercept and state the domain and range.



19. ART An oil painting originally cost \$2500 and increases in value at a rate of 6% per year. Find the value of the painting after 12 years. **18.** _____

19. _____

20. CARS A new car valued at \$16,500 depreciates at a steady rate of 12% per year. What is the value of the car in 10 years? **20.** _____

21. Write an equation for the nth term of the geometric sequence 4, 8, 16,… . **21.** _____

Find the first three terms of each sequence.

22. $a_1 = -4, a_n = a_{n-1} + 7, n \geq 2$ **22.** _____

23. $a_1 = 1, a_n = 4a_{n-1} - 2, n \geq 2$ **23.** _____

Write a recursive formula for each sequence.

24. 27, 19, 11, 3, … **24.** _____

25. 1296, 216, 36, 6, … **25.** _____

Bonus Find the first term of the geometric sequence with $a_5 = 625$ and $a_6 = 3125$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 3

SCORE _____

Simplify.

1. $\left(\frac{2}{3}h^3\right)^4$ **1.** _____

2. $(4^{3x+7})(4^{2x-9})$ **2.** _____

Simplify. Assume that no denominator is equal to zero.

3. $\frac{(-2mx^{-3})^{-4}}{8m^{-5}x^0}$ **3.** _____

4. $\left(\frac{-3a^2b^{-3}}{6a^3b^{-4}}\right)^2\left(\frac{-5b}{4a^{-3}}\right)^{-3}$ **4.** _____

5. Express 473 in scientific notation. **5.** _____

Evaluate each product or quotient. Express the results in both scientific notation and standard form.

6. $(5.2 \times 10^4)(5.9 \times 10^6)$ **6.** _____

7. $\frac{2.485 \times 10^3}{7.1 \times 10^9}$ **7.** _____

8. ATTENDANCE The total home attendance for a professional football team was about 5.2×10^5. In the same year, the total home attendance for a professional baseball team was about 2.673×10^6. About how many times as large was the attendance for the baseball team as the attendance for the football team? **8.** _____

Solve each equation.

9. $4^{3x-2} = 256$ **9.** _____

10. $625^{x-1} = 5$ **10.** _____

11. $3^{3x+2} = 729$ **11.** _____

12. $256^{4x-5} = 2$ **12.** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Test, Form 3 *(continued)*

Simplify.

13. $1296^{\frac{3}{4}}$ **13.** _____

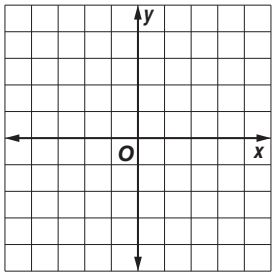
14. $8^{\frac{7}{3}}$ **14.** _____

Write each expression in radical form.

15. $32x^{\frac{1}{6}}$ **15.** _____

16. $(4yz)^{\frac{1}{8}}$ **16.** _____

17. WEIGHT LIFTING A weight lifter can squat 475 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 475(1.05)^x$, where x represents the number of training cycles she completes. How much will she squat after 4 training sessions? **17.** _____



18. Graph $y = 4(2^x - 1)$. State the y-intercept. **18.** _____

19. INVESTMENTS Determine the amount of an investment if \$96,000 is invested at an interest rate of 5.2% compounded daily for 3 years. **19.** _____

20. Suppose a car that sells for \$40,000 depreciates 10% per year. How many years would it take for the car to have a value less than \$25,000? **20.** _____

21. Write an equation for the nth term of the geometric sequence $-7, 1.75, -0.4375, \ldots$. **21.** _____

Find the first three terms of each sequence.

22. $a_1 = 2.25, a_n = a_{n-1} + 0.5, n \geq 2$ **22.** _____

23. $a_1 = 3.5, a_n = -0.5a_{n-1} + 0.75, n \geq 2$ **23.** _____

Write a recursive formula for each sequence.

24. 50, 3, −44, −91 … **24.** _____

25. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots$ **25.** _____

Bonus Find the first term of the geometric sequence with $a_6 = \frac{1}{4}$ and $a_7 = \frac{1}{8}$. **B:** _____

NAME _____ DATE _____ PERIOD _____

7 Chapter 7 Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Simplify $\left(\frac{4a^3}{2a^{-2}}\right)^4$ by using the Quotient of Powers property first. Then use the Power of a Power property.

b. Simplify $\left(\frac{4a^3}{2a^{-2}}\right)^4$ by using the Power of a Quotient property first. Then use the Quotient of Powers property.

c. Write a statement that generalizes the results of part **a** and part **b**.

2. Give a counterexample for each statement.

a. As n increases in a geometric sequence, the value of a_n will move farther away from zero.

b. In a recursive sequence, if $a_1 = a_2$, then $a_2 = a_3$, and so on.

3. Honovi purchased a new car for $25,000 and has $5000 left to invest.

a. Choose an interest rate between 4% and 7% for Honovi's investment, and find the length of time it would take for the investment to double.

b. Choose an annual depreciation rate from 8% to 10% for the new car that Honovi purchased, and find the length of time it would take for the car's value to be equal to one-half of the purchase price.

c. Using the rates from part **a** and part **b**, find the length of time it would take for the investment to be equal to the value of the car. What is the value at that time?

4. The mass of Jupiter is 1.8987×10^{27} kilograms and is roughly 317.8 times the mass of Earth. The mass of the solar system is about 1.992×10^{30} kilograms. The Sun accounts for about 99.8% of this mass.

a. Find the mass of Earth in kilograms.

b. What is the approximate mass of all of the other objects in the solar system, not counting the Sun?

c. What percentage of the mass obtained in part **b** is represented by Jupiter?

NAME _____ DATE _____ PERIOD _____

7 Standardized Test Practice

SCORE _____

(Chapters 1–7)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Write $y - 11 = \frac{1}{2}(x - 12)$ in standard form. (Lesson 4-2)

A $y = \frac{1}{2}x + 5$ **B** $y = \frac{1}{2}x - 17$ **C** $x - 2y = -10$ **D** $x - 2y = 17$ **1.** Ⓐ Ⓑ Ⓒ Ⓓ

2. Ashanti wants to collect more than 50 food items for the local homeless shelter. If he has already collected 13, how many more items must he collect? (Lesson 5-1)

F 37 **G** at least 38 **H** at least 37 **J** more than 36 **2.** Ⓕ Ⓖ Ⓗ Ⓙ

3. Solve $-5n + 22 \geq -73$. (Lesson 5-3)

A $\{n \mid n \leq 19\}$ **B** $\{n \mid n \geq 19\}$ **C** $\left\{n \mid n \leq -\frac{51}{5}\right\}$ **D** $\{n \mid n \geq 90\}$ **3.** Ⓐ Ⓑ Ⓒ Ⓓ

4. Solve $|5k + 2| > 1$. (Lesson 5-5)

F $\left\{k \mid k < -\frac{3}{5} \text{ or } k > -\frac{1}{5}\right\}$ **H** $\left\{k \mid k - \frac{3}{5} < k < \frac{1}{5}\right\}$

G $\varnothing$ **J** $\{k \mid k \text{ is a real number.}\}$ **4.** Ⓕ Ⓖ Ⓗ Ⓙ

5. Use substitution to solve the system of equations. (Lesson 6-2)

$$y = -2x$$
$$5x + 3y = 4$$

A $(-4, 8)$ **B** $(8, -4)$ **C** $(-8, 4)$ **D** $(4, -8)$ **5.** Ⓐ Ⓑ Ⓒ Ⓓ

6. Use elimination to solve the system of equations. (Lesson 6-3)

$$x + 3y = -6$$
$$2x + 3y = -9$$

F $(-2, -4)$ **G** $(-6, 1)$ **H** $(-3, -1)$ **J** $(3, -5)$ **6.** Ⓕ Ⓖ Ⓗ Ⓙ

7. Simplify $\frac{(3^2)(3^3)}{(3^{-2})(3^{-3})}$. (Lesson 7-2)

A 3^{10} **B** 3^{12} **C** -1 **D** $\frac{1}{3}$ **7.** Ⓐ Ⓑ Ⓒ Ⓓ

8. Simplify $\sqrt[4]{1296}$. (Lesson 7-3)

F 36 **G** 9 **H** 6 **J** 3 **8.** Ⓕ Ⓖ Ⓗ Ⓙ

9. Evaluate $(3.2 \times 10^6)(4.6 \times 10^{-2})$ (Lesson 7-4)

A 1.472×10^4 **C** 1.472×10^3

B 1.472×10^5 **D** 14.72×10^4 **9.** Ⓐ Ⓑ Ⓒ Ⓓ

10. Find the sixth term of the geometric sequence $-16, 40, -100, \ldots$. (Lesson 7-7)

F $-15{,}625$ **H** 6250

G -6250 **J** 1562.5 **10.** Ⓕ Ⓖ Ⓗ Ⓙ

11. Simplify $24 + 6 - \{2^3 + 7(5 - 2)\}$. (Lesson 1-2)

A 43 **B** 3 **C** 1 **D** 45 **11.** Ⓐ Ⓑ Ⓒ Ⓓ

NAME _____ DATE _____ PERIOD _____

7 Standardized Test Practice *(continued)*

12. Find the percent of change. (Lesson 2-7)

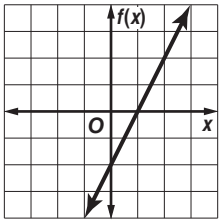
Original: \$28 New: \$21

F 25% **G** 7% **H** 33% **J** 14% **12.** Ⓕ Ⓖ Ⓗ Ⓙ

13. Solve $a(b - c) = \frac{a}{b}$ for c. (Lesson 2-8)

A $c = \frac{b^2 - 1}{b}$ **B** $c = \frac{-1}{b - b}$ **C** $c = \frac{1}{b + ab}$ **D** $c = \frac{a}{b - ab}$ **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. Write an equation in function notation for the relation. (Lesson 3-6)



F $f(x) = -2x - 2$ **H** $f(x) = -2x + 2$

G $f(x) = 2x - 2$ **J** $f(x) = \frac{1}{2}x - 2$ **14.** Ⓕ Ⓖ Ⓗ Ⓙ

15. Find the slope of the line that passes through the points $(-4, 4)$ and $(6, -8)$. (Lesson 3-3)

A $\frac{6}{5}$ **B** $-\frac{6}{5}$ **C** $-\frac{5}{6}$ **D** $\frac{5}{6}$ **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Write $y + 1 = 4(x - 2)$ in slope-intercept form. (Lesson 4-3)

F $y = 4x - 3$ **G** $y = 4x + 7$ **H** $y = 4x - 9$ **J** $y = x - 9$ **16.** Ⓕ Ⓖ Ⓗ Ⓙ

17. Solve $14u + 9 - 10u > 21$. (Lesson 5-3)

A $u > 3$ **B** $u > -3$ **C** $u < 3$ **D** $u < -3$ **17.** Ⓐ Ⓑ Ⓒ Ⓓ

18. Simplify $(4x^2)^2(-2x^4)^3$. (Lesson 7-1)

F $-48x^{16}$ **G** $-48x^{11}$ **H** $128x^{11}$ **J** $-128x^{16}$ **18.** Ⓕ Ⓖ Ⓗ Ⓙ

Part 2: Gridded Response

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

19. Evaluate $4x(y^2 - z^2)$ if $x = 3$, $y = 5$ and $z = 4$. (Lesson 1-2)

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

20. After receiving his allowance, Spencer spent half of it on a Mother's Day card. He bought 2 toy cars for \$0.49 each to give to his brothers, and a pack of gum for \$0.35. How much did Spencer receive for his allowance if \$0.42 is left over? (Lesson 2-3)

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

NAME _____ DATE _____ PERIOD _____

7 Standardized Test Practice *(continued)*

Part 3: Short Response

Instructions: Write your answers in the space provided.

21. Translate the sentence into an equation.
The quotient of 35 and 6 plus a number is equal to twice the sum of that number and 3. (Lesson 2-1)

21. _____

22. TUITION The average cost of tuition and fees at Trenton College was $33,560 for a recent school year. Two school years before, the average cost was $31,110. Find the rate of change between the two school years. (Lesson 3-3)

22. _____

23. Solve $3y - 4 \leq 7$. (Lesson 5-3)

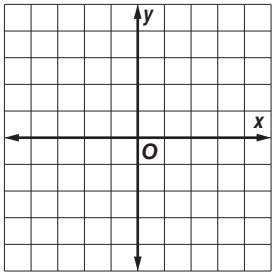
23. _____

24. RECREATION Barrington needs to buy snorkels and fins for his family for their annual beach vacation. Snorkels cost $8 a set and fins cost $12 a pair. Write an inequality that represents this situation if Barrington has $62 to spend. (Lesson 5-6)

24. _____

25. Graph the system of equations and determine the number of solutions it has. If it has one solution, name it. (Lesson 6-1)
$x + y = 4$
$y = x$

25.



26. Determine the best method to solve the system of equations. Then solve the system. (Lesson 6-5)
$5x - 2y = 7$
$2x + 5y = -3$

_____

26. _____

27. Simplify $\frac{51x^{-1}y^3}{17x^2y}$ Assume the denominator is not equal to zero. (Lesson 7-2)

27. _____

28. Solve $9^{4n-3} = 3^6$. (Lesson 7-3)

28. _____

29. Write a recursive formula for 39, 32, 25, 18, … . (Lesson 7-8)

29. _____

30. As of the end of the 2010 football season, the Dallas Cowboys and the Denver Broncos had won a total of 7 Super Bowls. The Cowboys had won 2.5 times as many Super Bowls as the Broncos. (Lesson 6-2)

a. Define variables and write a system of linear equations to find each team's wins.

30a. _____

b. How many Super Bowls had the Dallas Cowboys won?

30b. _____

NAME _____ DATE _____ PERIOD _____

7 Anticipation Guide

Exponents and Exponential Functions

Step 1 ***Before you begin Chapter 7***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. When multiplying two powers that have the same base, multiply the exponents.	D
	2. $(k^3)^4$ is equivalent to k^{12}.	A
	3. To divide two powers that have the same base, subtract the exponents.	A
	4. $\left(\frac{2}{5}\right)^3$ is the same as $\frac{2^3}{5}$.	D
	5. A polynomial may contain one or more monomials.	A
	6. The degree of the polynomial $3x^2y^3 - 5y^2 + 8x^3$ is 3 because the greatest exponent is 3.	D
	7. A function containing powers is called an exponential function.	D
	8. Receiving compound interest on a bank account is one example of exponential growth.	A

Step 2 ***After you complete Chapter 7***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME _____ DATE _____ PERIOD _____

7-1 Study Guide and Intervention

Multiplication Properties of Exponents

Multiply Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Group the coefficients and the variables

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Exercises

Simplify each expression.

1. $y(y^5)$ — y^6

2. $n^2 \cdot n^7$ — n^9

3. $(-7x^2)(x^4)$ — $-7x^6$

4. $x(x^2)(x^4)$ — x^7

5. $m \cdot m^5$ — m^6

6. $(-x^3)(-x^4)$ — x^7

7. $(2a^2)(8a)$ — $16a^3$

8. $(rn)(rn^3)(n^2)$ — r^2n^6

9. $(x^2y)(4xy^3)$ — $4x^3y^4$

10. $\frac{1}{3}(2a^3b)(6b^3)$ — $4a^3b^4$

11. $(-4x^3)(-5x^7)$ — $20x^{10}$

12. $(-3j^2k^4)(2jk^6)$ — $-6j^3k^{10}$

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$ — $a^3b^2c^7$

14. $(-5xy)(4x^2)(y^4)$ — $-20x^3y^5$

15. $(10x^3yz^2)(-2xy^5z)$ — $-20x^4y^6z^3$

Answers

NAME _____ DATE _____ PERIOD _____

7-1 Study Guide and Intervention *(continued)*

Multiplication Properties of Exponents

Simplify Expressions An expression of the form $(x^m)^p$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and any integers m and p, $(a^m)^p = a^{mp}$.
Power of a Product	For any numbers a and b and any integer m, $(ab)^m = a^m b^m$.

We can combine and use these properties to simplify expressions involving monomials.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power

$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product

$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables

$= (-2)^3(a^{11})(b^2)^3$ Product of Powers

$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify each expression.

1. $(y^5)^2$ — y^{10}
2. $(n^7)^4$ — n^{28}
3. $(x^2)^5(x^3)$ — x^{13}
4. $-3(ab^4)^3$ — $-3a^3b^{12}$
5. $(-3ab^4)^3$ — $-27a^3b^{12}$
6. $(4x^2b)^3$ — $64x^6b^3$
7. $(4a^2)^2(b^3)$ — $16a^4b^3$
8. $(4x)^2(b^3)$ — $16x^2b^3$
9. $(x^2y^4)^5$ — $x^{10}y^{20}$
10. $(2a^3b^2)(b^3)^2$ — $2a^3b^8$
11. $(-4xy)^3(-2x^2)^3$ — $512x^9y^3$
12. $(-3j^2k^3)^2(2j^2k)^3$ — $72j^{10}k^9$
13. $(25a^2b)^3\left(\frac{1}{5}abf\right)^2$ — $625a^8b^5f^2$
14. $(2xy)^2(-3x^2)(4y^4)$ — $-48x^4y^6$
15. $(2x^3y^2z^2)^3(x^2z)^4$ — $8x^{17}y^6z^{10}$
16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$ — $12n^{12}y^{10}$
17. $(-3a^3n^4)(-3a^3n)^4$ — $-243a^{15}n^8$
18. $-3(2x)^4(4x^5y)^2$ — $-768x^{14}y^2$

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7-1 Skills Practice

Multiplication Properties of Exponents

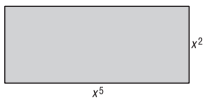
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11 **Yes; 11 is a real number and an example of a constant.**
2. $a - b$ **No; this is the difference, not the product, of two variables.**
3. $\frac{p^2}{r^2}$ **No; this is the quotient, not the product, of two variables.**
4. y **Yes; single variables are monomials.**
5. j^3k **Yes; this is the product of two variables.**
6. $2a + 3b$ **No; this is the sum of two monomials.**

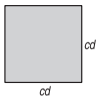
Simplify.

7. $a^2(a^3)(a^6)$ — a^{11}
8. $x(x^2)(x^7)$ — x^{10}
9. $(y^2z)(yz^2)$ — y^3z^3
10. $(\ell^2k^2)(\ell^3k)$ — ℓ^5k^3
11. $(a^2b^4)(a^2b^2)$ — a^4b^6
12. $(cd^2)(c^3d^2)$ — c^4d^4
13. $(2x^2)(3x^5)$ — $6x^7$
14. $(5a^7)(4a^2)$ — $20a^9$
15. $(4xy^3)(3x^3y^5)$ — $12x^4y^8$
16. $(7a^5b^2)(a^2b^3)$ — $7a^7b^5$
17. $(-5m^3)(3m^8)$ — $-15m^{11}$
18. $(-2c^4d)(-4cd)$ — $8c^5d^2$
19. $(10^2)^3$ — 10^6 or 1,000,000
20. $(p^3)^{12}$ — p^{36}
21. $(-6p)^2$ — $36p^2$
22. $(-3y)^3$ — $-27y^3$
23. $(3pr^2)^2$ — $9p^2r^4$
24. $(2b^3c^4)^2$ — $4b^6c^8$

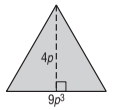
GEOMETRY Express the area of each figure as a monomial.

25. 

x^7

26. 

c^2d^2

27. 

$18p^4$

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7-1 Practice

Multiplication Properties of Exponents

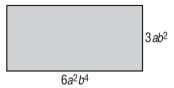
Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

1. $\frac{21a^2}{7b}$ **No; this involves the quotient, not the product, of variables.**
2. $\frac{b^3c^2}{2}$ **Yes; this is the product of a number, $\frac{1}{2}$, and two variables.**

Simplify each expression.

3. $(-5x^2y)(3x^4)$ $-15x^6y$
4. $(2ab^2f^2)(4a^3b^2f^2)$ $8a^4b^4f^4$
5. $(3ad^4)(-2a^2)$ $-6a^3d^4$
6. $(4g^3h)(-2g^5)$ $-8g^8h$
7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$ $5x^2y^7$
8. $(-xy)^3(xz)$ $-x^4y^3z$
9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$ $-54m^5n^4$
10. $(0.2a^2b^3)^2$ $0.04a^4b^6$
11. $\left(\frac{2}{3}p\right)^2$ $\frac{4}{9}p^2$
12. $\left(\frac{1}{4}ad^3\right)^2$ $\frac{1}{16}a^2d^6$
13. $(0.4k^3)^3$ $0.064k^9$
14. $[(4^2)^2]^2$ 4^8 **or** $65{,}536$

GEOMETRY Express the area of each figure as a monomial.

15. 

$18a^3b^6$

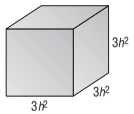
16. 

$(25x^6)\pi$

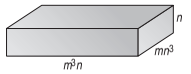
17. 

$12a^3b^4$

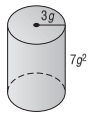
GEOMETRY Express the volume of each solid as a monomial.

18. 

$27h^6$

19. 

m^4n^5

20. 

$(63g^4)\pi$

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? 2^5 **or** 32

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? 2^6 **or** 64

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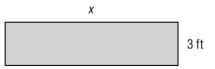
7-1 Word Problem Practice

Multiplication Properties of Exponents

1. **GRAVITY** An egg that has been falling for x seconds has dropped at an average speed of $16x$ feet per second. If the egg is dropped from the top of a building, its total distance traveled is the product of the average rate times the time. Write a simplified expression to show the distance the egg has traveled after x seconds. $16x^2$

2. **CIVIL ENGINEERING** A developer is planning a sidewalk for a new development. The sidewalk can be installed in rectangular sections that have a fixed width of 3 feet and a length that can vary. Assuming that each section is the same length, express the area of a 4-section sidewalk as a monomial. $12x$



3. **PROBABILITY** If you flip a coin 3 times in a row, there are 2^3 outcomes that can occur.

Outcomes	
HHH	TTT
HTT	THH
HTH	TTH
HHT	THT

If you then flip the coin two more times, there are $2^3 \times 2^2$ outcomes that can occur. How many outcomes can occur if you flip the coin as mentioned above four more times? Write your answer in the form 2^x. 2^9

4. **SPORTS** The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Find the volume of air in three different basketballs. Use $\pi = 3.14$. Round your answers to the nearest whole number.

Ball	Radius (in.)	Volume (in³)
Child's	4	268
Women's	4.5	382
Men's	4.8	463

5. **ELECTRICITY** An electrician uses the formula $W = I^2R$, where W is the power in watts, I is the current in amperes, and R is the resistance in ohms.

   a. Find the power in a household circuit that has 20 amperes of current and 5 ohms of resistance. **2000 watts**

   b. If the current is reduced by one half, what happens to the power?

   The power is one-fourth the previous amount.

Answers

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7-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned}1001101_2 &= 1 \times \underline{2}^6 + 0 \times \underline{2}^5 + 0 \times \underline{2}^4 + 1 \times \underline{2}^3 + 1 \times \underline{2}^2 + 0 \times \underline{2}^1 + 1 \times \underline{2}^0 \\ &= 1 \times \underline{64} + 0 \times \underline{32} + 0 \times \underline{16} + 1 \times \underline{8} + 1 \times \underline{4} + 0 \times \underline{2} + 1 \times \underline{1} \\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1 \\ &= 77\end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 **15**
2. 10000_2 **16**
3. 11000011_2 **195**
4. 10111001_2 **185**

Write each decimal number as a binary number.

5. 8 **1000_2**
6. 11 **1011_2**
7. 29 **11101_2**
8. 117 **1110101_2**

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers. **Answers will vary.**

The American Standard Guide for Information Interchange (ASCII)							
A	65	N	78	a	97	n	110
B	66	O	79	b	98	o	111
C	67	P	80	c	99	p	112
D	68	Q	81	d	100	q	113
E	69	R	82	e	101	r	114
F	70	S	83	f	102	s	115
G	71	T	84	g	103	t	116
H	72	U	85	h	104	u	117
I	73	V	86	i	105	v	118
J	74	W	87	j	106	w	119
K	75	X	88	k	107	x	120
L	76	Y	89	l	108	y	121
M	77	Z	90	m	109	z	122

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7-2 Study Guide and Intervention

Division Properties of Exponents

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{5^5}{5^2}$ **5^3 or 125**
2. $\frac{m^6}{m^4}$ **m^2**
3. $\frac{p^5n^4}{p^2n}$ **p^3n^3**
4. $\frac{a^2}{a}$ **a**
5. $\frac{x^5y^3}{x^5y^2}$ **y**
6. $\frac{-2y^7}{14y^5}$ **$-\frac{1}{7}y^2$**
7. $\frac{xy^6}{y^4x}$ **y^2**
8. $\left(\frac{2a^2b}{a}\right)^3$ **$8a^3b^3$**
9. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^3$ **$\frac{64}{27}p^6r^6$**
10. $\left(\frac{2r^5w^3}{r^4w^3}\right)^4$ **$16r^4$**
11. $\left(\frac{3r^6n^3}{2r^5n}\right)^4$ **$\frac{81}{16}r^4n^8$**
12. $\frac{r^7n^7t^2}{n^3r^3t^2}$ **r^4n^4**

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7-2 Study Guide and Intervention *(continued)*

Division Properties of Exponents

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{2^2}{2^{-3}}$ **2^5 or 32**
2. $\frac{m}{m^{-4}}$ **m^5**
3. $\frac{p^{-8}}{p^3}$ **$\frac{1}{p^{11}}$**
4. $\frac{b^{-4}}{b^{-5}}$ **b**
5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ **$\frac{w}{4y^2}$**
6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$ **a^6b^8**
7. $\frac{x^4y^0}{x^{-2}}$ **x^6**
8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$ **$\frac{36}{a^2b^6}$**
9. $\frac{(3rt)^2u^{-4}}{r^{-1}t^2u^7}$ **$\frac{9r^3}{u^{11}}$**
10. $\frac{m^{-3}t^{-5}}{(m^2t^3)^{-1}}$ **$\frac{1}{mt^2}$**
11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$ **1**
12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$ **$-\frac{m^3}{32n^{10}}$**

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7-2 Skills Practice

Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{6^5}{6^4}$ **6^1 or 6**
2. $\frac{9^{12}}{9^8}$ **9^4 or 6561**
3. $\frac{x^4}{x^2}$ **x^2**
4. $\frac{r^3t^2}{r^3t^4}$ **$\frac{1}{t^2}$**
5. $\frac{m}{m^3}$ **$\frac{1}{m^2}$**
6. $\frac{9d^7}{3d^6}$ **$3d$**
7. $\frac{12n^5}{36n}$ **$\frac{n^4}{3}$**
8. $\frac{w^4x^3}{w^4x}$ **x^2**
9. $\frac{a^3b^5}{ab^2}$ **a^2b^3**
10. $\frac{m^7p^2}{m^3p^2}$ **m^4**
11. $\frac{-21w^5x^2}{7w^4x^5}$ **$-\frac{3w}{x^3}$**
12. $\frac{32x^3y^2z^5}{-8xyz^2}$ **$-4x^2yz^3$**
13. $\left(\frac{4p^7}{7r^2}\right)^2$ **$\frac{16p^{14}}{49r^4}$**
14. 4^{-4} **$\frac{1}{4^4}$ or $\frac{1}{256}$**
15. 8^{-2} **$\frac{1}{8^2}$ or $\frac{1}{64}$**
16. $\left(\frac{5}{3}\right)^{-2}$ **$\frac{9}{25}$**
17. $\left(\frac{9}{11}\right)^{-1}$ **$\frac{11}{9}$**
18. $\frac{h^3}{h^{-6}}$ **h^9**
19. $k^0(k^4)(k^{-6})$ **$\frac{1}{k^2}$**
20. $k^{-1}(\ell^{-6})(m^3)$ **$\frac{m^3}{k\ell^6}$**
21. $\frac{f^{-7}}{f^4}$ **$\frac{1}{f^{11}}$**
22. $\left(\frac{16p^5w^2}{2p^3w^3}\right)^0$ **1**
23. $\frac{f^{-5}g^4}{h^{-2}}$ **$\frac{g^4h^2}{f^5}$**
24. $\frac{15x^6y^{-9}}{5xy^{-11}}$ **$3x^5y^2$**
25. $\frac{-15t^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$ **$-\frac{8x^5y^2}{z}$**

Answers

NAME _____ DATE _____ PERIOD _____

7-2 Practice

Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{8^8}{8^4}$ **8^4 or 4096**
2. $\frac{a^4b^6}{ab^3}$ **a^3b^3**
3. $\frac{xy^2}{xy}$ **y**
4. $\frac{m^5np}{m^4p}$ **mn**
5. $\frac{5c^2d^3}{-4c^2d}$ **$-\frac{5d^2}{4}$**
6. $\frac{8y^7z^6}{4y^6z^5}$ **$2yz$**
7. $\left(\frac{4f^3g}{3h^6}\right)^3$ **$\frac{64f^9g^3}{27h^{18}}$**
8. $\left(\frac{6w^5}{7p^6r^3}\right)^2$ **$\frac{36w^{10}}{49p^{12}r^6}$**
9. $\frac{-4x^2}{24x^5}$ **$-\frac{1}{6x^3}$**
10. $x^3(y^{-5})(x^{-8})$ **$\frac{1}{x^5y^5}$**
11. $p(q^{-2})(r^{-3})$ **$\frac{p}{q^2r^3}$**
12. 12^{-2} **$\frac{1}{144}$**
13. $\left(\frac{3}{7}\right)^{-2}$ **$\frac{49}{9}$**
14. $\left(\frac{4}{3}\right)^{-4}$ **$\frac{81}{256}$**
15. $\frac{22r^3s^2}{11r^2s^{-3}}$ **$2rs^5$**
16. $\frac{-15w^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$ **$2c^4f^7$**
18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$ **1**
19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$ **$\frac{g^8h^2}{9}$**
20. $\frac{-12t^{-1}u^5x^{-4}}{2t^{-3}ux^5}$ **$-\frac{6t^2u^4}{x^9}$**
21. $\frac{r^4}{(3r)^3}$ **$\frac{r}{27}$**
22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$ **$\frac{m^2}{n^2}$**
23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$ **$\frac{j}{k^{15}}$**
24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$ **$\frac{a^4}{40b^7}$**
25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$ **$\frac{q^{10}}{r^{25}}$**
26. $\left(\frac{7c^{-3}d^3}{c^5dh^{-4}}\right)^{-1}$ **$\frac{c^8}{7d^2h^4}$**
27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$ **$\frac{9x^2}{4y^2z^6}$**

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells? **$\frac{1}{484}$**

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"? **676**

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7-2 Word Problem Practice

Division Properties of Exponents

1. **CHEMISTRY** The nucleus of a certain atom is 10^{-13} centimeters across. If the nucleus of a different atom is 10^{-11} centimeters across, how many times as large is it as the first atom? **100**

2. **SPACE** The Moon is approximately 25^4 kilometers away from Earth on average. The Olympus Mons volcano on Mars stands 25 kilometers high. How many Olympus Mons volcanoes, stacked on top of one another, would fit between the surface of the Earth and the Moon?
$25^3 = 15{,}625$

3. **E-MAIL** Spam (also known as junk e-mail) consists of identical messages sent to thousands of e-mail users. People often obtain anti-spam software to filter out the junk e-mail messages they receive. Suppose Yvonne's anti-spam software filtered out 10^2 e-mails, and she received 10^4 e-mails last year. What fraction of her e-mails were filtered out? Write your answer as a monomial.
10^{-2}

4. **METRIC MEASUREMENT** Consider a dust mite that measures 10^{-3} millimeters in length and a caterpillar that measures 10 centimeters long. How many times as long as the mite is the caterpillar?
$10^4 = 10{,}000$

5. **COMPUTERS** In 1995, standard capacity for a personal computer hard drive was 40 megabytes (MB). In 2010, a standard hard drive capacity was 500 gigabytes (GB or Gig). Refer to the table below.

Memory Capacity Approximate Conversions
8 bits = 1 byte
10^3 bytes = 1 kilobyte
10^3 kilobytes = 1 megabyte (meg)
10^3 megabytes = 1 gigabyte (gig)
10^3 gigabytes = 1 terabyte
10^3 terabytes = 1 petabyte

a. The newer hard drives have about how many times the capacity of the 1995 drives? **12,500**

b. Predict the hard drive capacity in the year 2025 if this rate of growth continues. **6.25 petabytes**

c. One kilobyte of memory is what fraction of one terabyte? **$\frac{1}{10^9} = 10^{-9}$**

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7-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a. 2^{10} = 1024	b. 5^{10} = 9,765,625	c. 4^{10} = 1,048,576
2^9 = 512	5^9 = 1,953,125	4^9 = 262,144
2^8 = 256	5^8 = 390,625	4^8 = 65,536
2^7 = 128	5^7 = 78,125	4^7 = 16,384
2^6 = 64	5^6 = 15,625	4^6 = 4096
2^5 = 32	5^5 = 3125	4^5 = 1024
2^4 = 16	5^4 = 625	4^4 = 256
2^3 = 8	5^3 = 125	4^3 = 64
2^2 = 4	5^2 = 25	4^2 = 16
2^1 = 2	5^1 = 5	4^1 = 4

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.
   The exponents decrease by one from each row to the one below.

2. Describe the pattern of the powers from the top of the column to the bottom. **To get each power, divide the power on the row above by the base (2, 5, or 4).**

3. What would you expect the following powers to be?
   2^0 **1** 5^0 **1** 4^0 **1**

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases. **Any nonzero number to the zero power equals one.**

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$? 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?
   Negative exponents are not defined unless the base is nonzero.

6. Based upon the pattern, can you determine whether 0^0 exists?
   No, since the pattern $0^n = 0$ breaks down for $n < 1$.

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?
   Answers will vary. One answer is that if $0^0 = 1$, then $1 = \frac{1}{1} = \frac{1^0}{0^0} = \left(\frac{1}{0}\right)^0$, which is a false result, since division by zero is not allowed. Thus, 0^0 cannot equal 1.

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7-3 Study Guide and Intervention

Rational Exponents

Rational Exponents For any real numbers a and b and any positive integer n, if $a^n = b$, then a is an nth root of b. Rational exponents can be used to represent nth roots.

Square Root	$b^{\frac{1}{2}} = \sqrt{b}$
Cube Root	$b^{\frac{1}{3}} = \sqrt[3]{b}$
nth Root	$b^{\frac{1}{n}} = \sqrt[n]{b}$

Example 1 **Write $(6xy)^{\frac{1}{2}}$ in radical form.**

$(6xy)^{\frac{1}{2}} = \sqrt{6xy}$ Definition of $b^{\frac{1}{2}}$

Example 2 **Simplify $625^{\frac{1}{4}}$.**

$625^{\frac{1}{4}} = \sqrt[4]{625}$ $b^{\frac{1}{n}} = \sqrt[n]{b}$

$= \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5}$ $625 = 5^4$

$= 5$ Simplify.

Exercises

Write each expression in radical form, or write each radical in exponential form.

1. $14^{\frac{1}{2}}$ $\sqrt{14}$
2. $5x^{\frac{1}{2}}$ $5\sqrt{x}$
3. $17y^{\frac{1}{2}}$ $17\sqrt{y}$
4. $12^{\frac{1}{2}}$ $\sqrt{12}$
5. $19ab^{\frac{1}{2}}$ $19a\sqrt{b}$
6. $\sqrt{17}$ $17^{\frac{1}{2}}$
7. $\sqrt{12n}$ $(12n)^{\frac{1}{2}}$
8. $\sqrt{18b}$ $(18b)^{\frac{1}{2}}$
9. $\sqrt{37}$ $37^{\frac{1}{2}}$

Simplify.

10. $\sqrt[3]{343}$ 7
11. $\sqrt[5]{1024}$ 4
12. $512^{\frac{1}{3}}$ 8
13. $\sqrt[4]{2401}$ 7
14. $\sqrt[6]{64}$ 2
15. $243^{\frac{1}{5}}$ 3
16. $\sqrt[3]{1331}$ 11
17. $\sqrt[4]{6561}$ 9
18. $4096^{\frac{1}{4}}$ 8

Answers

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7-3 Study Guide and Intervention *(continued)*

Rational Exponents

Solve Exponential Equations In an **exponential equation**, variables occur as exponents. Use the Power Property of Equality and the other properties of exponents to solve exponential equations.

Example **Solve $1024^{x-1} = 4$.**

$1024^{x-1} = 4$	Original equation
$(4^5)^{x-1} = 4$	Rewrite 1024 as 4^5.
$4^{5x-5} = 4^1$	Power of a Power, Distributive Property
$5x - 5 = 1$	Power Property of Equality
$5x = 6$	Add 5 to each side.
$x = \frac{6}{5}$	Divide each side by 5.

Exercises

Solve each equation.

1. $2^x = 128$ **7**
2. $3^{3x+1} = 81$ **1**
3. $4^{x-3} = 32$ $\frac{11}{2}$
4. $5^x = 15{,}625$ **6**
5. $6^{3x+2} = 216$ $\frac{1}{3}$
6. $4^{5x-3} = 16$ **1**
7. $8^x = 4096$ **4**
8. $9^{3x+3} = 6561$ $\frac{1}{3}$
9. $11^{x-1} = 1331$ **4**
10. $3^x = 6561$ **8**
11. $2^{5x+4} = 512$ **1**
12. $7^{x-2} = 343$ **5**
13. $8^x = 262{,}144$ **6**
14. $5^{5x} = 3125$ **1**
15. $9^{2x-6} = 6561$ **5**
16. $7^x = 2401$ **4**
17. $7^{3x} = 117{,}649$ **2**
18. $6^{2x-7} = 7776$ **6**
19. $9^x = 729$ **3**
20. $8^{3x+1} = 4096$ **1**
21. $13^{3x-8} = 28{,}561$ **4**

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7-3 Skills Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $(8x)^{\frac{1}{2}}$ $\sqrt{8x}$
2. $6z^{\frac{1}{2}}$ $6\sqrt{z}$
3. $\sqrt{19}$ $19^{\frac{1}{2}}$
4. $\sqrt{11}$ $11^{\frac{1}{2}}$
5. $19x^{\frac{1}{2}}$ $19\sqrt{x}$
6. $\sqrt{34}$ $34^{\frac{1}{2}}$
7. $\sqrt{27g}$ $(27g)^{\frac{1}{2}}$
8. $33gh^{\frac{1}{2}}$ $33g\sqrt{h}$
9. $\sqrt{13abc}$ $(13abc)^{\frac{1}{2}}$

Simplify.

10. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$ $\frac{1}{2}$
11. $\sqrt[5]{3125}$ **5**
12. $729^{\frac{1}{3}}$ **9**
13. $\left(\frac{1}{32}\right)^{\frac{1}{5}}$ $\frac{1}{2}$
14. $\sqrt[6]{4096}$ **4**
15. $1024^{\frac{1}{5}}$ **4**
16. $\left(\frac{16}{625}\right)^{\frac{1}{4}}$ $\frac{2}{5}$
17. $\sqrt[6]{15{,}625}$ **5**
18. $117{,}649^{\frac{1}{6}}$ **7**

Solve each equation.

19. $2^x = 512$ **9**
20. $3^x = 6561$ **8**
21. $6^x = 46{,}656$ **6**
22. $5^x = 125$ **3**
23. $3^{x-3} = 243$ **8**
24. $4^{x-1} = 1024$ **6**
25. $6^{x-1} = 1296$ **5**
26. $2^{4x+3} = 2048$ **2**
27. $3^{3x+3} = 6561$ $\frac{5}{3}$

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7-3 Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $\sqrt{13}$ **$13^{\frac{1}{2}}$**
2. $\sqrt{37}$ **$37^{\frac{1}{2}}$**
3. $\sqrt{17x}$ **$(17x)^{\frac{1}{2}}$**
4. $(7ab)^{\frac{1}{2}}$ **$\sqrt{7ab}$**
5. $21z^{\frac{1}{2}}$ **$21\sqrt{z}$**
6. $13(ab)^{\frac{1}{2}}$ **$13\sqrt{ab}$**

Simplify.

7. $\left(\frac{1}{81}\right)^{\frac{1}{4}}$ **$\frac{1}{3}$**
8. $\sqrt[5]{1024}$ **4**
9. $512^{\frac{1}{3}}$ **8**
10. $\left(\frac{32}{1024}\right)^{\frac{1}{5}}$ **$\frac{1}{2}$**
11. $\sqrt[4]{1296}$ **6**
12. $3125^{\frac{1}{5}}$ **5**

Solve each equation.

13. $3^x = 729$ **6**
14. $4^x = 4096$ **6**
15. $5^x = 15{,}625$ **6**
16. $6^{x+3} = 7776$ **2**
17. $3^{x-3} = 2187$ **10**
18. $4^{3x+4} = 16{,}384$ **1**

19. **WATER** The flow of water F in cubic feet per second over a wier, a small overflow dam, can be represented by $F = 1.26H^{\frac{3}{2}}$, where H is the height of the water in meters above the crest of the wier. Find the height of the water if the flow of the water is 10.08 cubic feet per second. **4 ft**

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7-3 Word Problem Practice

Rational Exponents

1. **VELOCITY** The velocity v in feet per second of a freely falling object that has fallen h feet can be represented by $v = 8h^{\frac{1}{2}}$. Find the distance that an object has fallen if its velocity is 96 feet per second. **144 ft**

2. **ELECTRICITY** The relationship of the current, the power, and the resistance in an appliance can be modeled by $IR^{\frac{1}{2}} = \sqrt{P}$, where I is the current in amperes, P is the power in watts, and R is the resistance in ohms. Find the power that an appliance is using if the current is 2.5 amps and the resistance is 16 ohms. **100 watts**

3. **GEOMETRY** The surface area T of a cube in square inches can be determined by $T = 6V^{\frac{2}{3}}$, where V is the volume of the cube in cubic inches.

   a. Find the surface area of a cube that has a volume of 4096 cubic inches.
   1536 in²

   b. Find the volume of a cube that has a surface area of 96 square inches.
   64 in³

4. **PLANETS** The average distance d in astronomical units that a planet is from the Sun can be modeled by $d = t^{\frac{2}{3}}$, where t is the number of Earth years that it takes for the planet to orbit the Sun.

   a. Find the average distance a planet is from the Sun if the planet has an orbit of 27 Earth years.
   9 astronomical units

   b. Find the time that it would take a planet to orbit the Sun if its average distance from the Sun is 4 astronomical units.
   8 Earth years

5. **BIOLOGY** The relationship between the mass m in kilograms of an organism and its metabolism P in Calories per day can be represented by $P = 73.3\sqrt[4]{m^3}$. Find the mass of an organism that has a metabolism of 586.4 Calories per day.
   16 kg

6. **MANUFACTURING** The profit P of a company in thousands of dollars can be modeled by $P = 12.75\sqrt[4]{c^2}$, where c is the number of customers in hundreds. If the profit of the company is $51,000, how many customers do they have?
   3200 customers

Answers

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7-3 Enrichment

Counterexamples

Some statements in mathematics can be proven false by **counterexamples**.

Consider the following statement.

For any numbers a and b, $a - b = b - a$.

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let $a = 7$ and $b = 3$. Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$
$$4 \neq -4$$

In general, for any numbers a and b, the statement $a - b = b - a$ is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

In each of the following exercises a, b, and c are any numbers. Prove that the statement is false by counterexample. **Sample answers are given.**

1. $a - (b - c) \stackrel{?}{=} (a - b) - c$

$6 - (4 - 2) \stackrel{?}{=} (6 - 4) - 2$

$6 - 2 \stackrel{?}{=} 2 - 2$

$4 \neq 0$

2. $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$

$6 \div (4 \div 2) \stackrel{?}{=} (6 \div 4) \div 2$

$\frac{6}{2} \stackrel{?}{=} \frac{1.5}{2}$

$3 \neq 0.75$

3. $a \div b \stackrel{?}{=} b \div a$

$6 \div 4 \stackrel{?}{=} 4 \div 6$

$\frac{3}{2} \neq \frac{2}{3}$

4. $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$

$6 \div (4 + 2) \stackrel{?}{=} (6 \div 4) + (6 \div 2)$

$6 \div 6 \stackrel{?}{=} 1.5 + 3$

$1 \neq 4.5$

5. $a^{\frac{1}{2}} + a^{\frac{1}{2}} \stackrel{?}{=} a^1$

$9^{\frac{1}{2}} + 9^{\frac{1}{2}} \stackrel{?}{=} 9^1$

$3 + 3 \stackrel{?}{=} 9$

$6 \neq 9$

6. $a^c \cdot b^d \stackrel{?}{=} (ab)^{c+d}$

$2^3 \cdot 5^2 \stackrel{?}{=} [(2)(5)]^{(3+2)}$

$8 \cdot 25 \stackrel{?}{=} 10^5$

$200 \neq 100{,}000$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

1. Subtraction is not an associative operation.

2. Division is not an associative operation.

3. Division is not a commutative operation.

8. For the Distributive Property $a(b + c) = ab + ac$, it is said that multiplication distributes over addition. Exercise 4 proves that some operations do not distribute. Write a statement for the exercise that indicates this.

4. Division does not distribute over addition.

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7-4 Study Guide and Intervention

Scientific Notation

Scientific Notation Very large and very small numbers are often best represented using a method known as **scientific notation**. Numbers written in scientific notation take the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. Any number can be written in scientific notation.

Example 1 **Express 34,020,000,000 in scientific notation.**

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number a. Here, $a = 3.402$.

Step 2 Note the number of places n and the direction that you moved the decimal point. The decimal point moved 10 places to the left, so $n = 10$.

Step 3 Because the decimal moved to the left, write the number as $a \times 10^n$.

$34{,}020{,}000{,}000 = 3.4020000000 \times 10^{10}$

Step 4 Remove the extra zeros. 3.402×10^{10}

Example 2 **Express 4.11×10^{-6} in standard notation.**

Step 1 The exponent is –6, so $n = -6$.

Step 2 Because $n < 0$, move the decimal point 6 places to the left.

$4.11 \times 10^{-6} \Rightarrow .00000411$

Step 3 $4.11 \times 10^{-6} \Rightarrow 0.00000411$

Rewrite; insert a 0 before the decimal point.

Exercises

Express each number in scientific notation.

1. 5,100,000
5.1 × 10⁶

2. 80,300,000,000
8.03 × 10¹⁰

3. 14,250,000
1.425 × 10⁷

4. 68,070,000,000,000
6.807 × 10¹³

5. 14,000
1.4 × 10⁴

6. 901,050,000,000
9.0105 × 10¹¹

7. 0.0049
4.9 × 10⁻³

8. 0.000301
3.01 × 10⁻⁴

9. 0.0000000519
5.19 × 10⁻⁸

10. 0.000000185
1.85 × 10⁻⁷

11. 0.002002
2.002 × 10⁻³

12. 0.00000771
7.71 × 10⁻⁶

Express each number in standard form.

13. 4.91×10^4
49,100

14. 3.2×10^{-5}
0.000032

15. 6.03×10^8
603,000,000

16. 2.001×10^{-6}
0.000002001

17. 1.00024×10^{10}
10,002,400,000

18. 5×10^5
500,000

19. 9.09×10^{-5}
0.0000909

20. 3.5×10^{-2}
0.035

21. 1.7087×10^7
17,087,000

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7-4 Study Guide and Intervention *(continued)*

Scientific Notation

Products and Quotients in Scientific Notation You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

Example 1 **Evaluate $(9.2 \times 10^{-3}) \times (4 \times 10^{8})$. Express the result in both scientific notation and standard form.**

$(9.2 \times 10^{-3})(4 \times 10^{8})$	Original expression
$= (9.2 \times 4)(10^{-3} \times 10^{8})$	Commutative and Associative Properties
$= 36.8 \times 10^{5}$	Product of Powers
$= (3.68 \times 10^{1}) \times 10^{5}$	$36.8 = 3.68 \times 10$
$= 3.68 \times 10^{6}$	Product of Powers
$= 3{,}680{,}000$	Standard Form

Example 2 **Evaluate $\frac{(2.76 \times 10^{7})}{(6.9 \times 10^{5})}$. Express the result in both scientific notation and standard form.**

$\frac{(2.76 \times 10^{7})}{(6.9 \times 10^{5})} = \left(\frac{2.76}{6.9}\right)\left(\frac{10^{7}}{10^{5}}\right)$	Product rule for fractions
$= 0.4 \times 10^{2}$	Quotient of Powers
$= 4.0 \times 10^{-1} \times 10^{2}$	$0.4 = 4.0 \times 10^{-1}$
$= 4.0 \times 10^{1}$	Product of Powers
$= 40$	Standard form

Exercises

Evaluate each product. Express the results in both scientific notation and standard form.

1. $(3.4 \times 10^{3})(5 \times 10^{4})$
   1.7×10^{8}; 170,000,000
2. $(2.8 \times 10^{-4})(1.9 \times 10^{7})$
   5.32×10^{3}; 5320
3. $(6.7 \times 10^{-7})(3 \times 10^{3})$
   2.01×10^{-3}; 0.00201
4. $(8.1 \times 10^{5})(2.3 \times 10^{-3})$
   1.863×10^{3}; 1863
5. $(1.2 \times 10^{-4})^{2}$
   1.44×10^{-8}; 0.0000000144
6. $(5.9 \times 10^{5})^{2}$
   3.481×10^{11}; 348,100,000,000

Evaluate each quotient. Express the results in both scientific notation and standard form.

7. $\frac{(4.9 \times 10^{-3})}{(2.5 \times 10^{-4})}$
   1.96×10^{1}; 19.6
8. $\frac{5.8 \times 10^{4}}{5 \times 10^{-2}}$
   1.16×10^{6}; 1,160,000
9. $\frac{(1.6 \times 10^{5})}{(4 \times 10^{-4})}$
   4.0×10^{8}; 400,000,000
10. $\frac{8.6 \times 10^{6}}{1.6 \times 10^{-3}}$
    5.375×10^{9}; 5,375,000,000
11. $\frac{(4.2 \times 10^{-2})}{(6 \times 10^{-7})}$
    7×10^{4}; 70,000
12. $\frac{8.1 \times 10^{5}}{2.7 \times 10^{4}}$
    3×10^{1}; 30

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7-4 Skills Practice

Scientific Notation

Express each number in scientific notation.

1. 3,400,000,000
   3.4×10^{9}
2. 0.000000312
   3.12×10^{-7}
3. 2,091,000
   2.091×10^{6}
4. 980,200,000,000,000
   9.802×10^{14}
5. 0.00000000008
   8×10^{-11}
6. 0.00142
   1.42×10^{-3}

Express each number in standard form.

7. 2.1×10^{5}
   210,000
8. 8.023×10^{-7}
   0.0000008023
9. 3.63×10^{-6}
   0.00000363
10. 7.15×10^{8}
    715,000,000
11. 1.86×10^{-4}
    0.000186
12. 4.9×10^{5}
    490,000

Evaluate each product. Express the results in both scientific notation and standard form.

13. $(6.1 \times 10^{5})(2 \times 10^{5})$
    1.22×10^{11}; 122,000,000,000
14. $(4.4 \times 10^{6})(1.6 \times 10^{-9})$
    7.04×10^{-3}; 0.00704
15. $(8.8 \times 10^{8})(3.5 \times 10^{-13})$
    3.08×10^{-4}; 0.000308
16. $(1.35 \times 10^{8})(7.2 \times 10^{-4})$
    9.72×10^{4}; 97,200
17. $(2.2 \times 10^{-3})^{2}$
    4.84×10^{-6}; 0.00000484
18. $(3.4 \times 10^{2})^{2}$
    1.156×10^{5}; 115,600

Evaluate each quotient. Express the results in both scientific notation and standard form.

19. $\frac{(9.2 \times 10^{-8})}{(2 \times 10^{-6})}$
    4.6×10^{-2}; 0.046
20. $\frac{(4.8 \times 10^{4})}{(3 \times 10^{-5})}$
    1.6×10^{9}; 1,600,000,000
21. $\frac{(1.161 \times 10^{-9})}{(4.3 \times 10^{-6})}$
    2.7×10^{-4}; 0.00027
22. $\frac{(4.625 \times 10^{10})}{(1.25 \times 10^{4})}$
    3.7×10^{6}; 3,700,000
23. $\frac{(2.376 \times 10^{-4})}{(7.2 \times 10^{-8})}$
    3.3×10^{3}; 3,300
24. $\frac{(8.74 \times 10^{-3})}{(1.9 \times 10^{5})}$
    4.6×10^{-8}; 0.000000046

Answers

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7-4 Practice

Scientific Notation

Express each number in scientific notation.

1. 1,900,000
   1.9×10^6
2. 0.000704
   7.04×10^{-4}
3. 50,040,000,000
   5.004×10^{10}
4. 0.0000000661
   6.61×10^{-8}

Express each number in standard form.

5. 5.3×10^7
   53,000,000
6. 1.09×10^{-4}
   0.000109
7. 9.13×10^3
   9130
8. 7.902×10^{-6}
   0.000007902

Evaluate each product. Express the results in both scientific notation and standard form.

9. $(4.8 \times 10^4)(6 \times 10^6)$
   2.88×10^{11}; 288,000,000,000
10. $(7.5 \times 10^{-5})(3.2 \times 10^7)$
    2.4×10^3; 2400
11. $(2.06 \times 10^4)(5.5 \times 10^{-9})$
    1.133×10^{-4}; 0.0001133
12. $(8.1 \times 10^{-6})(1.96 \times 10^{11})$
    1.5876×10^6; 1,587,600
13. $(7.2 \times 10^{-5})^2$
    5.184×10^{-9}; 0.000000005184
14. $(5.29 \times 10^6)^2$
    2.79841×10^{13}; 27,984,100,000,000

Evaluate each quotient. Express the results in both scientific notation and standard form.

15. $\frac{(4.2 \times 10^5)}{(3 \times 10^{-3})}$
    1.4×10^8; 140,000,000
16. $\frac{(1.76 \times 10^{-11})}{(2.2 \times 10^{-5})}$
    8×10^{-7}; 0.0000008
17. $\frac{(7.05 \times 10^{12})}{(9.4 \times 10^7)}$
    7.5×10^4; 75,000
18. $\frac{(2.04 \times 10^{-4})}{(3.4 \times 10^5)}$
    6×10^{-10}; 0.0000000006

19. **GRAVITATION** Issac Newton's theory of universal gravitation states that the equation $F = G\frac{m_1 m_2}{r^2}$ can be used to calculate the amount of gravitational force in newtons between two point masses m_1 and m_2 separated by a distance r. G is a constant equal to 6.67×10^{-11} Nm^2kg^{-2}. The mass of Earth m_1 is equal to 5.97×10^{24} kg, the mass of the Moon m_2 is equal to 7.36×10^{22} kg, and the distance r between the two is 384,000,000 m.

   a. Express the distance r in scientific notation. **3.84×10^8 m**

   b. Compute the amount of gravitational force between Earth and the Moon. Express your answer in scientific notation. **1.99×10^{20} Newtons**

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7-4 Word Problem Practice

Scientific Notation

1. **PLANETS** Neptune's mean distance from the Sun is 4,500,000,000 kilometers. Uranus' mean distance from the Sun is 2,870,000,000 kilometers. Express these distances in scientific notation.
   Neptune: 4.5×10^9 km; Uranus: 2.87×10^9 km

2. **PATHOLOGY** The common cold is caused by the rhinovirus, which commonly measures 2×10^{-8} m in diameter. The E. coli bacterium, which causes food poisoning, commonly measures 3×10^{-6} m in length. Express these measurements in standard form.
   Rhinovirus: 0.00000002 m; E. coli: 0.000003 m

3. **COMMERCIALS** A 30-second commercial aired during the 2007 Super Bowl cost $2,600,000. A 30-second commercial aired during the 1967 Super Bowl cost $40,000. Express these values in scientific notation. How many times more expensive was it to air an advertisement during the 2007 Super Bowl than the 1967 Super Bowl?
   2007 Super Bowl: $\$2.6 \times 10^6$; 1967 Super Bowl: $\$4 \times 10^4$; 6.5×10^1 or 65 times more expensive

4. **AVOGADRO'S NUMBER** Avogadro's number is an important concept in chemistry. It states that the number 6.022×10^{23} is approximately equal to the number of molecules in 12 grams of carbon 12. Use Avogadro's number to determine the number of molecules in 5×10^{-7} grams of carbon 12.
   2.509×10^{16} molecules

5. **COAL RESERVES** The table below shows the number of kilograms of coal select countries had in proven reserve at the end of a recent year.

Coal Reserves	
Country	**Coal (kg)**
United States	2.46×10^{14}
Russia	1.57×10^{14}
India	9.24×10^{13}
Romania	4.94×10^{11}

Source: British Petroleum

   a. Express each country's coal reserves in standard form. **United States: 246,000,000,000,000 kg; Russia: 157,000,000,000,000 kg; India: 92,400,000,000,000 kg; Romania: 494,000,000,000 kg**

   b. How many times more coal does the United States have than Romania?
   4.98×10^2 or 498 times

   c. One kilogram of coal has an energy density of 2.4×10^7 joules. What is the total energy density of the United States' coal reserve? Express your answer in scientific notation.
   5.90×10^{21} joules

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7-4 Enrichment

Engineering Notation

Engineering notation is a variation on scientific notation where numbers are expressed as powers of 1000 rather than as powers of 10. Engineering notation takes the familiar form of $a \times 10^n$, but n is restricted to multiples of three and $1 \leq |a| < 1000$.

One advantage to engineering notation is that numbers can be neatly expressed using SI prefixes. These prefixes are typically used for scientific measurements.

1000^n	10^n	SI Prefix	Symbol
1000^5	10^{15}	peta	P
1000^4	10^{12}	tera	T
1000^3	10^9	giga	G
1000^2	10^6	mega	M
1000^1	10^3	kilo	k
1000^{-1}	10^{-3}	milli	m
1000^{-2}	10^{-6}	micro	μ
1000^{-3}	10^{-9}	nano	n
1000^{-4}	10^{-12}	pico	p
1000^{-5}	10^{-15}	femto	f

Example **NUCLEAR POWER** **The output of a nuclear power plant is measured to be 620,000,000 watts. Express this number in engineering notation and using SI prefixes.**

To express a number in engineering notation, first convert the number to scientific notation.

Step 1 $620{,}000{,}000 \Rightarrow 6.20000000$ $a = 6.20000000$

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $620{,}000{,}000 = 6.20000000 \times 10^8 = 6.2 \times 10^8$

Because 8 is not a multiple of 3, we need to round down n to the next multiple of 3.

Step 4 $6.2 \times 10^8 = (6.2 \times 10^2) \times 10^6$ $620 = 6.2 \times 10^2$

Step 5 $= 620 \times 10^6$ Product of Powers

The output of the power plant is 620×10^6 watts. Using the chart above, the prefix for 10^6 is found to be mega, or M. The output of the power plant is 620 megawatts, or 620MW.

Exercises

Express each number in engineering notation.

1. 40,000,000,000 **40×10^9**
2. 180,000,000,000,000 **180×10^{12}**
3. 0.00006 **60×10^{-6}**
4. 0.00000000000039 **390×10^{-15}**

Express each measurement using SI prefixes.

5. 0.00000000014 gram
   140 picograms (pg)
6. 40,000,000,000 watts
   40 gigawatts (GW)
7. 63,100,000,000,000 bytes
   63.1 terabytes (TB)
8. 0.0000002 meter
   200 nanometers (nm)

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7-5 Study Guide and Intervention

Exponential Functions

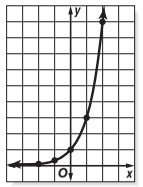
Graph Exponential Functions

Exponential Function	a function defined by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$

You can use values of x to find ordered pairs that satisfy an exponential function. Then you can use the ordered pairs to graph the function.

Example 1 **Graph $y = 3^x$. Find the y-intercept and state the domain and range.**

x	y
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9

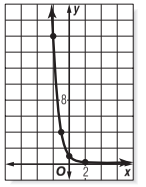


The y-intercept is 1.

The domain is all real numbers, and the range is all positive numbers.

Example 2 **Graph $y = \left(\frac{1}{4}\right)^x$. Find the y-intercept and state the domain and range.**

x	y
−2	16
−1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



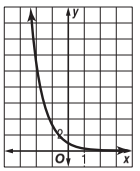
The y-intercept is 1.

The domain is all real numbers, and the range is all positive numbers.

Exercises

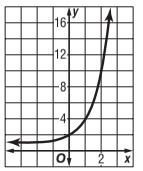
Graph each function. Find the y-intercept and state the domain and range.

1. $y = 0.3^x$



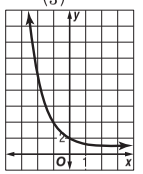
1; D = {all real numbers}, R = {$y|y > 0$}

2. $y = 3x + 1$



2; D = {all real numbers}, R = {$y|y > 1$}

3. $y = \left(\frac{1}{3}\right)^x + 1$



2; D = {all real numbers}, R = {$y|y > 1$}

Answers

NAME _____ DATE _____ PERIOD _____

7-5 Study Guide and Intervention *(continued)*

Exponential Functions

Identify Exponential Behavior It is sometimes useful to know if a set of data is exponential. One way to tell is to observe the shape of the graph. Another way is to observe the pattern in the set of data.

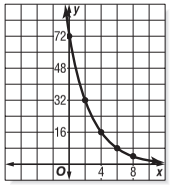
Example **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

x	0	2	4	6	8	10
y	64	32	16	8	4	2

Method 1: Look for a Pattern

The domain values increase by regular intervals of 2, while the range values have a common factor of $\frac{1}{2}$. Since the domain values increase by regular intervals and the range values have a common factor, the data are probably exponential.

Method 2: Graph the Data



The graph shows rapidly decreasing values of y as x increases. This is characteristic of exponential behavior.

Exercises

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

1.

x	0	1	2	3
y	5	10	15	20

No; the domain values are at regular intervals, and the range values have a common difference 5.

2.

x	0	1	2	3
y	3	9	27	81

Yes; the domain values are at regular intervals, and the range values have a common factor 3.

3.

x	−1	1	3	5
y	32	16	8	4

Yes; the domain values are at regular intervals, and the range values have a common factor $\frac{1}{2}$.

4.

x	−1	0	1	2	3
y	3	3	3	3	3

No; the domain values are at regular intervals, but the range values do not change.

5.

x	−5	0	5	10
y	1	0.5	0.25	0.125

Yes; the domain values are at regular intervals, and the range values have a common factor 0.5.

6.

x	0	1	2	3	4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

Yes; the domain values are at regular intervals, and the range values have a common factor $\frac{1}{3}$.

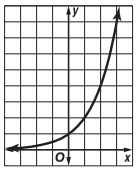
NAME _____ DATE _____ PERIOD _____

7-5 Skills Practice

Exponential Functions

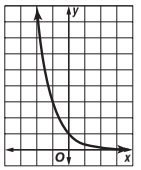
Graph each function. Find the *y*-intercept, and state the domain and range.

1. $y = 2^x$



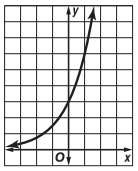
1; D = {all real numbers}, R = {y | y > 0}

2. $y = \left(\frac{1}{3}\right)^x$



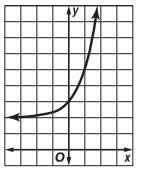
1; D = {all real numbers}, R = {y | y > 0}

3. $y = 3(2^x)$



3; D = {all real numbers}, R = {y | y > 0}

4. $y = 3^x + 2$



3; D = {all real numbers}, R = {y | y > 2}

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

5.

x	−3	−2	−1	0
y	9	12	15	18

No; the domain values are at regular intervals and the range values have a common difference 3.

6.

x	0	5	10	15
y	20	10	5	2.5

Yes; the domain values are at regular intervals and the range values have a common factor 0.5.

7.

x	4	8	12	16
y	20	40	80	160

Yes; the domain values are at regular intervals and the range values have a common factor 2.

8.

x	50	30	10	−10
y	90	70	50	30

No; the domain values are at regular intervals and the range values have a common difference 20.

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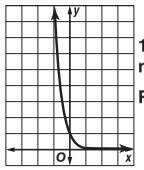
7-5 Practice

Exponential Functions

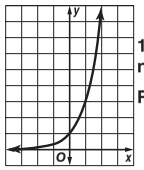
Graph each function. Find the *y*-intercept and state the domain and range.

1. $y = \left(\frac{1}{10}\right)^x$



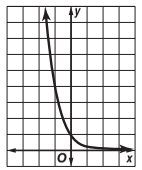
1; D = {all real numbers}; R = { y | y > 0}

2. $y = 3^x$



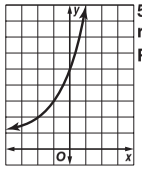
1; D = {all real numbers}; R = { y | y > 0}

3. $y = \left(\frac{1}{4}\right)^x$



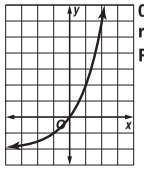
1; D = {all real numbers}; R = { y | y > 0}

4. $y = 4(2^x) + 1$



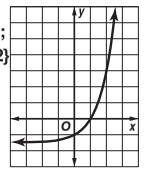
5; D = {all real numbers}; R = { y | y > 1}

5. $y = 2(2^x - 1)$



0; D = {all real numbers}; R = { y | y > −2}

6. $y = 0.5(3^x - 3)$



−1; D = {all real numbers}; R = { y | y > −1.5}

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

7.

x	2	5	8	11
y	480	120	30	7.5

Yes; the domain values are at regular intervals and the range values have a common factor 0.25.

8.

x	21	18	15	12
y	30	23	16	9

No; the domain values are at regular intervals and the range values have a common difference 7.

9. LEARNING Ms. Klemperer told her English class that each week students tend to forget one sixth of the vocabulary words they learned the previous week. Suppose a student learns 60 words. The number of words remembered can be described by the function $W(x) = 60\left(\frac{5}{6}\right)^x$, where x is the number of weeks that pass. How many words will the student remember after 3 weeks? **about 35**

10. BIOLOGY Suppose a certain cell reproduces itself in four hours. If a lab researcher begins with 50 cells, how many cells will there be after one day, two days, and three days? (*Hint:* Use the exponential function $y = 50(2^x)$.)
3200 cells; 204,800 cells; 13,107,200 cells

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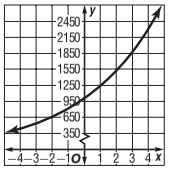
7-5 Word Problem Practice

Exponential Functions

1. WASTE Suppose the waste generated by nonrecycled paper and cardboard products is approximated by the following function.

$$y = 1000(2)^{0.3x}$$

Sketch the exponential function on the coordinate grid below.



2. MONEY Tatyana's grandfather gave her one penny on the day she was born. He plans to double the amount he gives her every day. Estimate how much she will receive from her grandfather on the 12th day of her life.
about $20

3. PICTURE FRAMES Since a picture frame includes a border, the picture must be smaller in area than the entire frame. The table shows the relationship between picture area and frame length for a particular line of frames. Is this an exponential relationship? Explain.

Side Length (in.)	Picture Area (in²)
5	6
6	12
7	20
8	30
9	42

No; there is no common factor between the picture areas.

4. DEPRECIATION The value of Royce Company's computer equipment is decreasing in value according to the following function.

$$y = 4000(0.87)^x$$

In the equation, x is the number of years that have elapsed since the equipment was purchased and y is the value in dollars. What was the value 5 years after it was purchased? Round your answer to the nearest dollar.
$1994

5. METEOROLOGY The atmospheric pressure in millibars at altitude x meters can be approximated by the following function. The function is valid for values of x between 0 and 10,000.

$$f(x) = 1038(1.000134)^{-x}$$

a. What is the pressure at sea level?
1038 millibars

b. The McDonald Observatory in Texas is at an altitude of 2000 meters. What is the approximate atmospheric pressure there?
794 millibars

c. As altitude increases, what happens to atmospheric pressure?
It decreases.

Answers

NAME _____ DATE _____ PERIOD _____

7-5 Enrichment

Logarithmic Functions

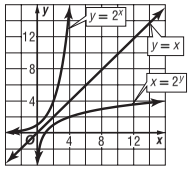
You have found the inverse of a linear function by interchanging x and y in the equation of the function. The inverse of an exponential function is called a **logarithmic function**.

Example **Find the inverse function of the exponential function $y = 2^x$. Make a table of values for each function. Then graph both functions.**

To find the inverse function, interchange x and y. The inverse of $y = 2^x$ is $x = 2^y$.

$y = 2^x$	
x	y
−3	$\frac{1}{8}$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x = 2^y$	
x	y
$\frac{1}{8}$	−3
$\frac{1}{4}$	−2
$\frac{1}{2}$	−1
1	0
2	1
4	2
8	3



In general the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, the variable y is called the **logarithm** of x. This is usually written as $y = \log_b x$.

Exercises

1. What are the domain and range of $y = \log_2 x$? How are the domain and range related to the domain and range of $y = 2^x$?
D = {x|x > 0}, R = {all real numbers}; The domain of $y = \log_2 x$ is the range of $y = 2^x$ and the range of $y = \log_2 x$ is the domain of $y = 2^x$.

2. How are the graphs of $y = \log_2 x$ and $y = 2^x$ related?
They are reflections in the line y = x.

3. Through which quadrants do the graphs of $y = \log_2 x$ and $y = 2^x$ pass?
$y = \log_2 x$ passes through I and II; $y = 2^x$ passes through I and IV.

4. If an exponential function finds population as a function of time, what can you conclude about its inverse logarithmic function?
The logarithmic function finds time as a function of population.

5. **RESEARCH** Investigate real-world uses of logarithmic functions. How are logarithmic functions used to model real-world situations?
Sample answer: The Richter scale is a logarithmic function used to measure earthquake intensity.

NAME _____ DATE _____ PERIOD _____

7-6 Study Guide and Intervention

Growth and Decay

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	The general equation for exponential growth is $y = a(1 + r)^t$. • y represents the final amount. • a represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.

Example 1 **POPULATION** **The population of Johnson City in 2005 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.**

a. Write an equation to represent the population of Johnson City since 2005.

The rate 3.2% can be written as 0.032.

$y = a(1 + r)^t$

$y = 25{,}000(1 + 0.032)^t$

$y = 25{,}000(1.032)^t$

b. According to the equation, what will the population of Johnson City be in 2015?

In 2015 t will equal 2015 − 2005 or 10.

Substitute 10 for t in the equation from part **a**.

$y = 25{,}000(1.032)^{10}$ $t = 10$

$\approx 34{,}256$

In 2015 the population of Johnson City will be about 34,256.

Example 2 **INVESTMENT** **The Garcias have $12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.**

The rate 3.5% can be written as 0.035. The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 12{,}000\left(1 + \frac{0.035}{12}\right)^{3(12)}$$
$$\approx 13{,}326.49$$

In three years, the balance of the account will be $13,326.49.

Exercises

1. **POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population was about 303,146,000 in 2008, predict the population in 2012.
about 314,332,051

2. **INVESTMENT** Determine the value of an investment of $2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years. **$3082.78**

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the 2008 population was about 6,641,000,000, predict the 2015 population.
about 7,269,417,259

4. **INVESTMENT** Determine the value of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.
$185,888.87

NAME _____ DATE _____ PERIOD _____

7-6 Study Guide and Intervention *(continued)*

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of **exponential decay**. This means that an initial amount decreases at a steady rate over a period of time.

Exponential Decay	The general equation for exponential decay is $y = a(1 - r)^t$. • y represents the final amount. • a represents the initial amount. • r represents the rate of decay expressed as a decimal. • t represents time.

Example **DEPRECIATION** **The original price of a tractor was $45,000. The value of the tractor decreases at a steady rate of 12% per year.**

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

$y = a(1 - r)^t$	General equation for exponential decay
$y = 45{,}000(1 - 0.12)^t$	$a = 45{,}000$ and $r = 0.12$
$y = 45{,}000(0.88)^t$	Simplify.

b. What is the value of the tractor in 5 years?

$y = 45{,}000(0.88)^t$	Equation for decay from part a
$y = 45{,}000(0.88)^5$	$t = 5$
$y \approx 23{,}747.94$	Use a calculator.

In 5 years, the tractor will be worth about $23,747.94.

Exercises

1. **POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015. **about 6,813,204**

2. **DEPRECIATION** Mr. Gossell is a machinist. He bought some new machinery for about $125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest $100) at the end of 10 years? **about $24,600**

3. **ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of carbon-14 of 100 grams.
   - **a.** If the organism lived 5730 years ago, what is the concentration of carbon-14 today? **50 g**
   - **b.** If the organism lived 11,460 years ago, determine the concentration of carbon-14 today. **25 g**

4. **DEPRECIATION** A new car costs $32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years. **about $16,242.63**

NAME _____ DATE _____ PERIOD _____

7-6 Skills Practice

Growth and Decay

1. **POPULATION** The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%.
   - **a.** Write an equation for the population t years after 2000. **$P = 8{,}008{,}278(1.004)^t$**
   - **b.** Use the equation to predict the population of New York City in 2015. **about 8,502,465**

2. **SAVINGS** The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.
   - **a.** If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account? **$1485.95**
   - **b.** If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account? **$15,996.47**

3. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 7%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2009? **about $247,539**

4. **MANUFACTURING** Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.
   - **a.** Write an equation for the value of the piece of equipment after t years. **$E = 60{,}000(0.90)^t$**
   - **b.** Find the value of the piece of equipment after 6 years. **about $31,886**

5. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks? **$102.95**

6. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car? **about $3993**

Answers

NAME _____ DATE _____ PERIOD _____

7-6 Practice

Growth and Decay

1. **COMMUNICATIONS** Sports radio stations numbered 220 in 1996. The number of sports radio stations has since increased by approximately 14.3% per year.

   a. Write an equation for the number of sports radio stations for t years after 1996.
   $R = 220(1.143)^t$

   b. If the trend continues, predict the number of sports radio stations in 2015.
   about 2788 stations

2. **INVESTMENTS** Determine the amount of an investment if $500 is invested at an interest rate of 4.25% compounded quarterly for 12 years.
   $830.41

3. **INVESTMENTS** Determine the amount of an investment if $300 is invested at an interest rate of 6.75% compounded semiannually for 20 years.
   $1131.73

4. **HOUSING** The Greens bought a condominium for $110,000 in 2010. If its value appreciates at an average rate of 6% per year, what will the value be in 2015?
   about $147,205

5. **DEFORESTATION** During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.

   a. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for t years after 1990.
   $C = 34{,}400(0.983)^t$

   b. If this trend continues, predict the forested area in 2015.
   about 22,407.65 km²

6. **BUSINESS** A piece of machinery valued at $25,000 depreciates at a steady rate of 10% yearly. What will the value of the piece of machinery be after 7 years?
   about $11,957

7. **TRANSPORTATION** A new car costs $18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car in 8 years.
   about $6473

8. **POPULATION** The population of Osaka, Japan, declined at an average annual rate of 0.05% for the five years between 1995 and 2000. If the population of Osaka was 11,013,000 in 2000 and it continues to decline at the same rate, predict the population in 2050.
   about 10,741,021

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7-6 Word Problem Practice

Growth and Decay

1. **DEPRECIATION** The value of a new plasma television depreciates by about 7% each year. Aeryn purchases a 50-inch plasma television for $3000. What is its value after 4 years? Round your answer to the nearest hundred. **about $2200**

2. **MONEY** Hans opens a savings account by depositing $1200 in an account that earns 3 percent interest compounded weekly. How much will his investment be worth in 10 years? Assume that there are exactly 52 weeks in a year and round your answer to the nearest cent.
   $1619.69

3. **HIGHER EDUCATION** The table lists the average costs of attending a four-year college in the United States during a recent year.

College Sector	Tuition and Fees	Room and Board
Four-year Public	$5941	$6636
Four-year Private	$21,235	$7,791

Source: College Board

Russ's parents invested money in a savings account earning an average of 4.5 percent interest, compounded monthly. After 15 years, they have exactly the right amount to cover the tuition, fees, room and board for Russ's first year at a public college. What was their initial investment? Round your answer to the nearest dollar. **$6182**

4. **POPULATION** In 2007 the U.S. Census Bureau estimated the population of the United States estimated at 301 million. The annual rate of growth is about 0.89%. At this rate, what is the expected population at the time of the 2020 census? Round your answer to the nearest ten million.
   340 million

5. **MEDICINE** When doctors prescribe medication, they have to consider the rate at which the body filters a drug from the bloodstream. Suppose it takes the human body 6 days to filter out half of the Flu-B-Gone vaccine. The amount of Flu-B-Gone vaccine remaining in the bloodstream x days after an injection is given by the equation $y = y_0(0.5)^{\frac{x}{6}}$, where y_0 is the initial amount. Suppose a doctor injects a patient with 20 μg (micrograms) of Flu-B-Gone.

   a. How much of the vaccine will remain after 1 day? Round your answer to the nearest tenth.
   17.8 μg

   b. How much of the vaccine will remain after 12 days? Round your answer to the nearest tenth.
   5 μg

   c. After how many days will the amount of vaccine be less than 1 μg?
   after 26 days

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7-6 Enrichment

Continuously Compounding Interest

You can use the formula for compound interest $A = P\left(1 + \frac{r}{n}\right)^{nt}$ when n, the number of times a year interest is compounded, is known. A special type of compound interest calculation regularly used in finance is **continuously compounded interest,** where n approaches infinity.

General formula: $A = Pe^{rt}$
A = current amount of investment
P = initial amount of investment
e = natural logarithm, a constant approximately equal to 2.71828
r = annual rate of interest, expressed as a decimal
t = number of years the money is invested

Example **INVESTING Mr. Rivera placed $5000 in an investment account that has an interest rate of 9.5% per year.**

a. How much money will be in the account after 5 years if interest is compounded monthly?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 5000\left(1 + \frac{0.095}{12}\right)^{12(5)}$ $P = 5000$, $r = 0.095$, $n = 12$, and $t = 5$

$= 5000(1.0079)^{60}$ Simplify.

$= 8025.05$ Use a calculator.

There will be about $8025.05 in the account if interest is compounded monthly.

b. How much money will be in the account after 5 years if interest is compounded continuously?

$A = Pe^{rt}$ Continuous interest equation

$= 5000(2.71828)^{0.095(5)}$ $P = 5000$, $e = 2.71828$, $r = 0.095$, and $t = 5$

$= 5000(2.71828)^{0.475}$ Simplify.

$= 8040.07$ Use a calculator.

There will be about $8040.07 in the account if interest is compounded continuously.

Exercises

1. **SAVINGS** Mr. Harris saves $20,000 in a money-market account at a rate of 5.2%.
   a. Determine the value of his investment after 10 years if interest is compounded quarterly. **$33,528.01**
   b. Determine the value of his investment after 10 years if interest is compounded continuously. **$33,640.54**

2. **COLLEGE SAVINGS** Shannon is choosing between two different savings accounts for her college fund. The first account compounds interest semiannually at a rate of 11.0%. The second account compounds interest continuously at a rate of 10.8%. If Shannon plans to keep her money in the account for 5 years, which account should she choose? Explain. **She should choose the continuous compounding account at 10.8%. After 5 years, the value of the continuously compounding account will be 1.716*P*, whereas the quarterly compounding account will only have a value of 1.708*P*.**

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7-6 Spreadsheet Activity

Compound Interest

Banks often use spreadsheets to calculate and store financial data. One application is to calculate compound interest on an account.

Example **Use a spreadsheet to find the time it will take an investment of $1000 to double. Suppose you can choose from investments that have annual interest rates of 5%, 8%, or 10% compounded monthly.**

The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is paid, or compounded, each year, and t is the number of years. In this case, $P = 1000$ and $n = 12$.

Step 1 Use Column A of the spreadsheet for the years.

Step 2 Columns B, C, and D contain the formulas for the final amounts of the investments. Format the cells in these columns as currency so that the amounts are shown in dollars and cents. Each formula will use the values in Column A as the value of t. For example, the formula that is in cell C3 is =1000*(1 + (0.08/12))^(12 * A3).

	A	B	C	D
1	Years	5%	8%	10%
2	1	$1,051.16	$1,083.00	$1,104.71
3	2	$1,104.94	$1,172.89	$1,220.39
4	3	$1,161.47	$1,270.24	$1,348.18
5	4	$1,220.90	$1,375.67	$1,489.35
6	5	$1,283.36	$1,489.85	$1,645.30
7	6	$1,349.02	$1,613.50	$1,817.59
8	7	$1,418.04	$1,747.42	$2,007.92
9	8	$1,490.59	$1,892.46	$2,218.18
10	9	$1,566.85	$2,049.53	$2,450.45
11	10	$1,647.01	$2,219.64	$2,707.04
12	11	$1,731.27	$2,403.87	$2,990.50
13	12	$1,819.85	$2,603.39	$3,303.65
14	13	$1,912.96	$2,819.47	$3,649.58
15	14	$2,010.83	$3,053.48	$4,031.74
16				

Sheet 1 Sheet 2 Sheet 3

Study the spreadsheet for the times when each investment exceeds $2000. At 5%, the $1000 will double in 14 years, at 8% it will double in 9 years, and at 10% it will double in 7 years.

Exercises

Use the spreadsheet of accounts involving compound interest.

1. How are the doubling times affected if the accounts compound interest quarterly instead of monthly? **The 5% and 8% accounts double in the same number of years as before, but the 10% account will double in 8 years.**

2. How long will it take each account to reach $4000 if the interest is compounded monthly? quarterly? **monthly: 5% in 28 yr, 8% in 18 yr, 10% in 14 yr; quarterly: 5% in 28 yr, 8% in 18 yr, 10% in 15 yr**

3. How do the interest rate and the number of times the interest is compounded affect the growth of an investment? **The accounts with higher interest rate and interest compounded more often grow more quickly.**

Answers

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7-7 Study Guide and Intervention

Geometric Sequences as Exponential Functions

Recognize Geometric Sequences A geometric sequence is a sequence in which each term after the first is found by multiplying the previous term by a nonzero constant r called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

Example 1 **Determine whether the sequence is *arithmetic, geometric,* or *neither*: 21, 63, 189, 567, . . .**

Find the ratios of the consecutive terms. If the ratios are constant, the sequence is geometric.

$$21 \xrightarrow{\times} 63 \xrightarrow{\times} 189 \xrightarrow{\times} 567$$

$$\frac{63}{21} = \frac{189}{63} = \frac{567}{189} = 3$$

Because the ratios are constant, the sequence is geometric. The common ratio is 3.

Example 2 **Find the next three terms in this geometric sequence: −1215, 405, −135, 45, . . .**

Step 1 Find the common ratio.

$$-1215 \xrightarrow{\times} 405 \xrightarrow{\times} -135 \xrightarrow{\times} 45$$

$$\frac{405}{-1215} = \frac{-135}{405} = \frac{45}{-135} = \frac{-1}{3}$$

The value of r is $-\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$45 \xrightarrow{\times\left(-\frac{1}{3}\right)} -15 \xrightarrow{\times\left(-\frac{1}{3}\right)} 5 \xrightarrow{\times\left(-\frac{1}{3}\right)} -\frac{5}{3}$$

The next three terms of the sequence are −15, 5, and $-\frac{5}{3}$.

Exercises

Determine whether each sequence is *arithmetic, geometric,* or *neither*. Explain.

1. 1, 2, 4, 8, . . .
   Geometric; common ratio is 2.
2. 9, 14, 6, 11, . . .
   Neither; there is no common difference or ratio.
3. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$
   Geometric; common ratio is $\frac{1}{2}$.
4. −2, 5, 12, 19, . . .
   Arithmetic; common difference is 7.

Find the next three terms in each geometric sequence.

5. 648, −216, 72, . . .
   $-24, 8, -2\frac{2}{3}$
6. 25, −5, 1, . . .
   $-\frac{1}{5}, \frac{1}{25}, -\frac{1}{125}$
7. $\frac{1}{16}, \frac{1}{2}, 4, \ldots$
   32, 256, 2048
8. 72, 36, 18, . . .
   $9, 4\frac{1}{2}, 2\frac{1}{4}$

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7-7 Study Guide and Intervention *(continued)*

Geometric Sequences as Exponential Functions

Geometric Sequences and Functions The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer: $a_n = a_1 \cdot r^{n-1}$.

Example **a. Write an equation for the nth term of the geometric sequence 5, 20, 80, 320, . . .**

The first term of the sequence is 320. So, $a_1 = 320$. Now find the common ratio.

$$5 \xrightarrow{\times} 20 \xrightarrow{\times} 80 \xrightarrow{\times} 320$$

$$\frac{20}{5} = \frac{80}{20} = \frac{320}{80} = 4$$

The common ratio is 4. So, $r = 4$.

$a_n = a_1 \cdot r^{n-1}$	Formula for nth term
$a_n = 5 \cdot 4^{n-1}$	$a_1 = 5$ and $r = 4$

b. Find the seventh term of this sequence.

Because we are looking for the seventh term, $n = 7$.

$a_n = a_1 \cdot r^{n-1}$	Formula for nth term
$a_7 = 5 \cdot 4^{7-1}$	$n = 7$
$= 5 \cdot 4^6$	Simplify.
$= 5 \cdot 4096$	$4^6 = 4096$
$= 20{,}480$	Multiply.

The seventh term of the sequence is 20,480.

Exercises

1. Write an equation for the nth term of the geometric sequence −2, 10, −50, ... Find the eleventh term of this sequence. **$a_n = -2 \cdot (-5)^{n-1}$; −19,531,250**
2. Write an equation for the nth term of the geometric sequence 512, 128, 32, ... Find the sixth term of this sequence. **$a_n = 512 \cdot \left(\frac{1}{4}\right)^{n-1}$; $\frac{1}{2}$**
3. Write an equation for the nth term of the geometric sequence $\frac{4}{9}$, 4, 36, ... Find the eighth term of this sequence. **$a_n = \frac{4}{9} \cdot 9^{n-1}$; 2,125,764**
4. Write an equation for the nth term of the geometric sequence 6, −54, 486, ... Find the ninth term of this sequence. **$a_n = 6 \cdot (-9)^{n-1}$; 258,280,326**
5. Write an equation for the nth term of the geometric sequence 100, 80, 64, ... Find the seventh term of this sequence. **$a_n = 100 \cdot \left(\frac{4}{5}\right)^{n-1}$; $26\frac{134}{625}$**
6. Write an equation for the nth term of the geometric sequence $\frac{2}{5}, \frac{1}{10}, \frac{1}{40}, \ldots$. Find the sixth term of this sequence. **$a_n = \frac{2}{5} \cdot \left(\frac{1}{4}\right)^{n-1}$; $\frac{1}{2560}$**
7. Write an equation for the nth term of the geometric sequence $\frac{3}{8}, -\frac{3}{2}, 6, \ldots$. Find the tenth term of this sequence. **$a_n = \frac{3}{8} \cdot (-4)^{n-1}$; −98,304**
8. Write an equation for the nth term of the geometric sequence −3, −21, −147, ... Find the fifth term of this sequence. **$a_n = -3 \cdot 7^{n-1}$; −7203**

NAME _____ DATE _____ PERIOD _____

7-7 Skills Practice

Geometric Sequences as Exponential Functions

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 7, 13, 19, 25, …
   Arithmetic; common difference is 6.
2. –96, –48, –24, –12, …
   Geometric; the common ratio is $\frac{1}{2}$.
3. 108, 66, 141, 99, …
   Neither; there is no common difference or ratio.
4. 3, 9, 81, 6561, …
   Neither; there is no common difference or ratio.
5. $\frac{7}{3}$, 14, 84, 504, …
   Geometric; common ratio is 6.
6. $\frac{3}{8}$, $-\frac{1}{8}$, $-\frac{5}{8}$, $-\frac{9}{8}$, …
   Arithmetic; common difference is $-\frac{1}{2}$.

Find the next three terms in each geometric sequence.

7. 2500, 500, 100, … **20, 4, $\frac{4}{5}$**
8. 2, 6, 18, … **54, 162, 486**
9. –4, 24, –144, … **864; −5184; 31,104**
10. $\frac{4}{5}, \frac{2}{5}, \frac{1}{5}$, … **$\frac{1}{10}, \frac{1}{20}, \frac{1}{40}$**
11. –3, –12, –48, … **−192, −768, −3072**
12. 72, 12, 2, … **$\frac{1}{3}, \frac{1}{18}, \frac{1}{108}$**
13. Write an equation for the nth term of the geometric sequence 3, – 24, 192, ….
    Find the ninth term of this sequence.
    $3(-8)^{n-1}$; 50,331,648
14. Write an equation for the nth term of the geometric sequence $\frac{9}{16}, \frac{3}{8}, \frac{1}{4}$, ….
    Find the seventh term of this sequence.
    $\frac{9}{16}\left(\frac{2}{3}\right)^{n-1}$; $\frac{4}{81}$
15. Write an equation for the nth term of the geometric sequence 1000, 200, 40, ….
    Find the fifth term of this sequence.
    $1000\left(\frac{1}{5}\right)^{n-1}$; $\frac{8}{5}$
16. Write an equation for the nth term of the geometric sequence – 8, – 2, $-\frac{1}{2}$, ….
    Find the eighth term of this sequence.
    $-8\left(\frac{1}{4}\right)^{n-1}$; $-\frac{1}{2048}$
17. Write an equation for the nth term of the geometric sequence 32, 48, 72, ….
    Find the sixth term of this sequence.
    $32\left(\frac{3}{2}\right)^{n-1}$; 243
18. Write an equation for the nth term of the geometric sequence $\frac{3}{100}$, $\frac{3}{10}$, 3, ….
    Find the ninth term of this sequence.
    $\frac{3}{100} \cdot 10^{n-1}$; 3,000,000

NAME _____ DATE _____ PERIOD _____

7-7 Practice

Geometric Sequences as Exponential Functions

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

1. 1, –5, –11, –17, …
   Arithmetic; the common difference is −6.
2. 3, $\frac{3}{2}$, 1, $\frac{3}{4}$, …
   Neither; there is no common difference or ratio.
3. 108, 36, 12, 4, …
   Geometric; the common ratio is $\frac{1}{3}$.
4. –2, 4, –6, 8, …
   Neither; there is no common difference or ratio.

Find the next three terms in each geometric sequence.

5. 64, 16, 4, … **1; $\frac{1}{4}$; $\frac{1}{16}$**
6. 2, –12, 72, … **−432; 2592; −15,552**
7. 3750, 750, 150, … **30; 6; $\frac{6}{5}$**
8. 4, 28, 196, … **1372; 9604; 67,228**
9. Write an equation for the nth term of the geometric sequence 896, −448, 224, … .
   Find the eighth term of this sequence.
   $a_n = 896\left(-\frac{1}{2}\right)^{n-1}$; –7
10. Write an equation for the nth term of the geometric sequence 3584, 896, 224, … .
    Find the sixth term of this sequence.
    $a_n = 3584\left(\frac{1}{4}\right)^{n-1}$; 3.5
11. Find the sixth term of a geometric sequence for which $a_2 = 288$ and $r = \frac{1}{4}$.
    1.125
12. Find the eighth term of a geometric sequence for which $a_3 = 35$ and $r = 7$.
    588,245
13. **PENNIES** Thomas is saving pennies in a jar. The first day he saves 3 pennies, the second day 12 pennies, the third day 48 pennies, and so on. How many pennies does Thomas save on the eighth day?
    49,152

Answers

NAME _____ DATE _____ PERIOD _____

7-7 Word Problem Practice

Geometric Sequences as Exponential Functions

1. WORLD POPULATION The CIA estimates the world population is growing at a rate of 1.167% each year. The world population for 2007 was about 6.6 billion.

a. Write an equation for the world population after n years. (*Hint:* The common ratio is not 0.01167.)

$a_n = 6{,}600{,}000{,}000 \cdot 1.01167^{n-1}$

b. What will the estimated world population be in 2017?

about 7.4 billion

2. MUSEUMS The table shows the annual visitors to a museum in millions. Write an equation for the projected number of visitors after n years.

Year	Visitors (millions)
1	4
2	6
3	9
4	$13\frac{1}{2}$
n	?

$a_n = 4 \cdot \left(\frac{3}{2}\right)^{n-1}$

3. BANKING Arnold has a bank account with a beginning balance of $5000. He spends one-fifth of the balance each month. How much money will be in the account after 6 months?

$1310.72

4. POPULATION The table shows the projected population of the United States through 2050. Does this table show an *arithmetic sequence*, a *geometric sequence* or neither? Explain.

Year	Projected Population
2000	282,125,000
2010	308,936,000
2020	335,805,000
2030	363,584,000
2040	391,946,000
2050	419,854,000

Source: U.S. Census Bureau

Neither, there is no common ratio or difference.

5. SAVINGS ACCOUNTS A bank offers a savings account with a 0.5% return each month.

a. Write an equation for the balance of a savings account after n months. (*Hint*: The common ratio is not 0.005.)

$a_n = p \cdot 1.005^n$

b. Given an initial deposit of $500, what will the account balance be after 15 months?

$538.84

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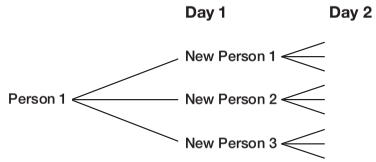
7-7 Enrichment

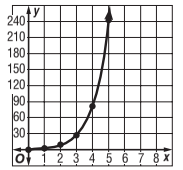
Pay it Forward

The idea behind "pay it forward" is that on the first day, one person does a good deed for three different people. Then, on the second day, those three people will each perform good deeds for 3 more people, so that on Day 2, there are 3×3 or 9 good deeds being done. Continue this process to fill in the chart. A tree diagram will help you fill in the chart.

Day	# of Deeds
0	1
1	3
2	9
3	**27**
4	**81**
5	**243**



1. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.



2. What type of function is your graph from Exercise 1? Write an equation that can be used to determine the number of good deeds on any given day, x.

exponential; $y = 3^x$

3. How many good deeds will be performed on Day 21? **10,460,353,203**

The formula $T = \frac{3^{n+1} - 3}{2}$ can be used to determine the **total** number of good deeds that have been performed, where n represents the day. For example, on Day 2, there have been $3 + 9$ or 12 good deeds performed. Using the formula, you get, $\frac{3^{2+1} - 3}{2}$ or $\frac{3^3 - 3}{2} + \frac{27 - 3}{2}$ or a total of 12 good deeds performed.

4. Use the formula to determine the approximate number of good deeds that have been performed through Day 21.

15,690,529,800

5. Look up the world population. How does your number from Exercise 4 compare to the world population?

The world population is approximately 6,560,526,046. The number of good deeds performed is greater than the entire world population.

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7-8 Study Guide and Intervention

Recursive Formulas

Using Recursive Formulas A **recursive formula** allows you to find the nth term of a sequence by performing operations on one or more of the terms that precede it.

Example **Find the first five terms of the sequence in which $a_1 = 5$ and $a_n = -2a_{n-1} + 14$, if $n \geq 2$.**

The given first term is $a_1 = 5$. Use this term and the recursive formula to find the next four terms.

$a_2 = -2a_{2-1} + 14$ $n = 2$
$= -2a_1 + 14$ Simplify.
$= -2(5) + 14$ or 4 $a_1 = 5$

$a_3 = -2a_{3-1} + 14$ $n = 3$
$= -2a_2 + 14$ Simplify.
$= -2(4) + 14$ or 6 $a_2 = 4$

$a_4 = -2a_{4-1} + 14$ $n = 4$
$= -2a_3 + 14$ Simplify.
$= -2(6) + 14$ or 2 $a_3 = 6$

$a_5 = -2a_{5-1} + 14$ $n = 5$
$= -2a_4 + 14$ Simplify.
$= -2(2) + 14$ or 10 $a_4 = 2$

The first five terms are 5, 4, 6, 2, and 10.

Exercises

Find the first five terms of each sequence.

1. $a_1 = -4, a_n = 3a_{n-1}, n \geq 2$
   –4, –12, –36, –108, –324
2. $a_1 = 5, a_n = 2a_{n-1}, n \geq 2$
   5, 10, 20, 40, 80
3. $a_1 = 8, a_n = a_{n-1} - 6, n \geq 2$
   8, 2, –4, –10, –16
4. $a_1 = -32, a_n = a_{n-1} + 13, n \geq 2$
   –32, –19, –6, 7, 20
5. $a_1 = 6, a_n = -3a_{n-1} + 20, n \geq 2$
   6, 2, 14, –22, 86
6. $a_1 = -9, a_n = 2a_{n-1} + 11, n \geq 2$
   –9, –7, –3, 5, 21
7. $a_1 = 12, a_n = 2a_{n-1} - 10, n \geq 2$
   12, 14, 18, 26, 42
8. $a_1 = -1, a_n = 4a_{n-1} + 3, n \geq 2$
   –1, –1, –1, –1, –1
9. $a_1 = 64, a_n = 0.5a_{n-1} + 8, n \geq 2$
   64, 40, 28, 22, 19
10. $a_1 = 8, a_n = 1.5a_{n-1}, n \geq 2$
    8, 12, 18, 27, 40.5
11. $a_1 = 400, a_n = \frac{1}{2}a_{n-1}, n \geq 2$
    400, 200, 100, 50, 25
12. $a_1 = \frac{1}{4}, a_n = a_{n-1} + \frac{3}{4}, n \geq 2$
    $\frac{1}{4}, 1, \frac{7}{4}, \frac{5}{2}, \frac{13}{4}$

NAME _____ DATE _____ PERIOD _____

7-8 Study Guide and Intervention *(continued)*

Recursive Formulas

Writing Recursive Formulas Complete the following steps to write a recursive formula for an arithmetic or geometric sequence.

Step 1	Determine if the sequence is arithmetic or geometric by finding a common difference or a common ratio.
Step 2	Write a recursive formula. **Arithmetic Sequences** $a_n = a_{n-1} + d$, where d is the common difference **Geometric Sequences** $a_n = r \cdot a_{n-1}$, where r is the common ratio
Step 3	State the first term and the domain for n.

Example **Write a recursive formula for 216, 36, 6, 1, … .**

Step 1 First subtract each term from the term that follows it.

$216 - 36 = 180$ $36 - 6 = 30$ $6 - 1 = 5$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$\frac{36}{216} = \frac{1}{6}$ $\frac{6}{36} = \frac{1}{6}$ $\frac{1}{6} = \frac{1}{6}$

There is a common ratio of $\frac{1}{6}$. The sequence is geometric.

Step 2 Use the formula for a geometric sequence.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence
$a_n = \frac{1}{6}a_{n-1}$ $r = \frac{1}{6}$

Step 3 The first term a_1 is 216 and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 216, a_n = \frac{1}{6}a_{n-1}, n \geq 2$.

Exercises

Write a recursive formula for each sequence.

1. 22, 16, 10, 4, …
   $a_1 = 22, a_n = a_{n-1} - 6, n \geq 2$
2. –8, –3, 2, 7, …
   $a_1 = -8, a_n = a_{n-1} + 5, n \geq 2$
3. 5, 15, 45, 135, …
   $a_1 = 5, a_n = 3a_{n-1}, n \geq 2$
4. 243, 81, 27, 9, …
   $a_1 = 243, a_n = \frac{1}{3}a_{n-1}, n \geq 2$
5. –3, 14, 31, 48, …
   $a_1 = -3, a_n = a_{n-1} + 17, n \geq 2$
6. 8, –20, 50, –125, …
   $a_1 = 8, a_n = -2.5a_{n-1}, n \geq 2$

Answers

NAME _____ DATE _____ PERIOD _____

7-8 Skills Practice

Recursive Formulas

Find the first five terms of each sequence.

1. $a_1 = -11, a_n = a_{n-1} - 7, n \geq 2$
−11, −18, −25, −32, −39

2. $a_1 = 14, a_n = a_{n-1} + 6.5, n \geq 2$
14, 20.5, 27, 33.5, 40

3. $a_1 = -3, a_n = 3a_{n-1} + 10, n \geq 2$
−3, 1, 13, 49, 157

4. $a_1 = 187.5, a_n = -0.8a_{n-1}, n \geq 2$
187.5, −150, 120, −96, 76.8

5. $a_1 = \frac{1}{2}, a_n = \frac{3}{2}a_{n-1} + \frac{1}{4}, n \geq 2$
$\frac{1}{2}, 1, \frac{7}{4}, \frac{23}{8}, \frac{73}{16}$

6. $a_1 = \frac{1}{5}, a_n = \frac{5}{2}a_{n-1} - \frac{1}{2}, n \geq 2$
$\frac{1}{5}, 0, -\frac{1}{2}, -\frac{7}{4}, -\frac{39}{8}$

Write a recursive formula for each sequence.

7. 7, 28, 112, 448, …
$a_1 = 7, a_n = 4a_{n-1}, n \geq 2$

8. 73, 52, 31, 10, …
$a_1 = 73, a_n = a_{n-1} - 21, n \geq 2$

9. $4, 1, \frac{1}{4}, \frac{1}{16}, \ldots$
$a_1 = 4, a_n = \frac{1}{4}a_{n-1}, n \geq 2$

10. −37, −15, 7, 29, …
$a_1 = -37, a_n = a_{n-1} + 22, n \geq 2$

11. 0.25, 0.37, 0.49, 0.61, …
$a_1 = 0.25, a_n = a_{n-1} + 0.12, n \geq 2$

12. 64, −80, 100, −125, 156.25, …
$a_1 = 64, a_n = -1.25a_{n-1}, n \geq 2$

For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.

13. $a_1 = 38, a_n = a_{n-1} - 17, n \geq 2$
$a_n = -17n + 55$

14. $a_n = 5n - 16$
$a_1 = -11, a_n = a_{n-1} + 5, n \geq 2$

15. $a_n = 50(0.75)^{n-1}$
$a_1 = 50, a_n = 0.75a_{n-1}, n \geq 2$

16. $a_1 = 16, a_n = 4a_{n-1}, n \geq 2$
$a_n = 16(4)^{n-1}$

NAME _____ DATE _____ PERIOD _____

7-8 Practice

Recursive Formulas

Find the first five terms of each sequence.

1. $a_1 = 25, a_n = a_{n-1} - 12, n \geq 2$
25, 13, 1, −11, −23

2. $a_1 = -101, a_n = a_{n-1} + 38, n \geq 2$
−101, −63, −25, 13, 51

3. $a_1 = 3.3, a_n = a_{n-1} + 2.7, n \geq 2$
3.3, 6, 8.7, 11.4, 14.1

4. $a_1 = 7, a_n = -3a_{n-1} + 20, n \geq 2$
7, −1, 23, −49, 167

5. $a_1 = 20, a_n = \frac{1}{5}a_{n-1}, n \geq 2$
$20, 4, \frac{4}{5}, \frac{4}{25}, \frac{4}{125}$

6. $a_1 = \frac{2}{3}, a_n = \frac{1}{3}a_{n-1} - \frac{2}{9}, n \geq 2$
$\frac{2}{3}, 0, -\frac{2}{9}, -\frac{8}{27}, -\frac{26}{81}$

Write a recursive formula for each sequence.

7. 80, −40, 20, −10, …
$a_1 = 80, a_n = -0.5a_{n-1}, n \geq 2$

8. 87, 52, 17, −18, …
$a_1 = 87, a_n = a_{n-1} - 35, n \geq 2$

9. $\frac{1}{3}, \frac{4}{15}, \frac{16}{75}, \frac{64}{375}, \ldots$
$a_1 = \frac{1}{3}, a_n = \frac{4}{5}a_{n-1}, n \geq 2$

10. $\frac{4}{5}, \frac{3}{10}, -\frac{1}{5}, -\frac{7}{10}, \ldots$
$a_1 = \frac{4}{5}, a_n = a_{n-1} - \frac{1}{2}, n \geq 2$

11. 2.6, 5.2, 7.8, 10.4, …
$a_1 = 2.6, a_n = a_{n-1} + 2.6, n \geq 2$

12. 100, 120, 144, 172.8, …
$a_1 = 100, a_n = 1.2a_{n-1}, n \geq 2$

13. PIZZA The total costs for ordering one to five cheese pizzas from Luigi's Pizza Palace are shown.

Total Number of Pizzas Ordered	Cost
1	$7.00
2	$12.50
3	$18.00
4	$23.50
5	$29.00

a. Write a recursive formula for the sequence.
$a_1 = 7, a_n = a_{n-1} + 5.5, n \geq 2$

b. Write an explicit formula for the sequence.
$a_n = 5.5n + 1.5$

NAME _____ DATE _____ PERIOD _____

7-8 Word Problem Practice

Recursive Formulas

1. **ASSEMBLY** An assembly line can create 175 widgets in one hour, 350 widgets in two hours, 525 widgets in three hours, 700 widgets in four hours, and so on.

   a. Find the next 5 terms in the sequence.
   875, 1050, 1225, 1400, 1575

   b. Write a recursive formula for the sequence.
   $a_1 = 175, a_n = a_{n-1} + 175, n \geq 2$

   c. Write an explicit formula for the sequence. $a_n = 175n$

2. **PAPER** A piece of paper is folded several times. The number of sections into which the piece of paper is divided after each fold is shown below.

Number of Folds	Sections
1	2
2	4
3	8
4	16
5	32

   a. Find the next 5 terms in the sequence.
   64, 128, 256, 512, 1024

   b. Write a recursive formula for the sequence.
   $a_1 = 2, a_n = 2a_{n-1}, n \geq 2$

   c. Write an explicit formula for the sequence. $a_n = 2^n$

3. **CLEANING** An equation for the cost a_n in dollars that a carpet cleaning company charges for cleaning n rooms is $a_n = 50 + 25(n - 1)$. Write a recursive formula to represent the cost a_n.
   $a_1 = 50, a_n = a_{n-1} + 25, n \geq 2$

4. **SAVINGS** A recursive formula for the balance of a savings account a_n in dollars at the beginning of year n is $a_1 = 500, a_n = 1.05a_{n-1}, n \geq 2$. Write an explicit formula to represent the balance of the savings account a_n.
   $a_n = 500(1.05)^{n-1}$

5. **SNOW** A snowman begins to melt as the temperature rises. The height of the snowman in feet after each hour is shown.

Hour	Height (ft)
1	6.0
2	5.4
3	4.86
4	4.374

   a. Write a recursive formula for the sequence.
   $a_1 = 6, a_n = 0.9a_{n-1}, n \geq 2$

   b. Write an explicit formula for the sequence.
   $a_n = 6(0.9)^{n-1}$

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7-8 Enrichment

Arithmetic Series

Consider a situation in which an uncle gives his niece a dollar amount equal to her age each year on her birthday. An arithmetic sequence that represents this situation is 1, 2, 3, … . How much total money will the niece have received by her 30th birthday?

When the terms of an arithmetic sequence are added, an *arithmetic series* is formed. The solution of the problem above is the sum of the arithmetic series $1 + 2 + 3 + \ldots + 30$.

If we write the series twice, as shown below, and add the terms in each column, we can see a pattern.

$$\begin{array}{r} 1 + 2 + 3 + 4 + \ldots + 27 + 28 + 29 + 30 \\ 30 + 29 + 28 + 27 + \ldots + 4 + 3 + 2 + 1 \\ \hline 31 + 31 + 31 + 31 + \ldots + 31 + 31 + 31 + 31 \end{array}$$

The sum of each pair of terms is 31. Multiplying 31 by the number of terms in the bottom row, 30, gives us the sum of the bottom row, or 930. Since the series was written twice, and resultantly, each term in the series was accounted for twice, we can divide 930 by 2 and find that the sum of the series $1 + 2 + 3 + \ldots + 30$ is 465.

1. Write an expression to represent the sum of the series above using the first term 1, the last term 30, and the number of terms in the series 30.
   Sample answer: $\frac{30(1 + 30)}{2}$

2. Write an expression to represent the sum of an arithmetic series with n terms, first term a_1, and last term a_n.
   Sample answer: $\frac{n(a_1 + a_n)}{2}$

Use the expression that you found in Exercise 2 to find the sum of each arithmetic series.

3. $1 + 2 + 3 + \ldots + 100$ **5050**
4. $5 + 10 + 15 + \ldots + 100$ **1050**
5. $1 + 3 + 5 + \ldots + 29$ **225**
6. $2 + 4 + 6 + \ldots + 60$ **930**
7. $6 + 13 + 20 + \ldots + 125$ **1179**
8. $8 + 12 + 16 + \ldots + 84$ **920**

Answers

Chapter 7 Assessment Answer Key

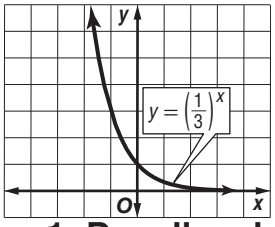
Quiz 1 (Lessons 7-1 and 7-2)
Page 57

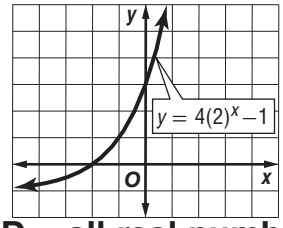
1. $2r^8$
2. x^{20}
3. $-12t^3n^7$
4. $-125x^{12}y^6$
5. $64c^8d^2$
6. $27y^4w^8$
7. 6^6 or 46,656
8. y^5
9. $\frac{r^2}{n^9}$
10. C

Quiz 2 (Lessons 7-3 and 7-4)
Page 57

1. 3
2. 8
3. 3
4. 2
5. $2(7xy)^{\frac{1}{2}}$
6. 1.4×10^7
7. 3.08×10^{-5}
8. B
9. 9.0×10^6;9,000,000
10. -6×10^{-9};−0.000000006

Quiz 3 (Lessons 7-5 and 7-6)
Page 58

1. 

1; D = all real numbers, R = {y|y > 0}
2. 

3; D = all real numbers, R = {y|y > -1}
3. $A = 1500\left(1 + \frac{0.0485}{4}\right)^{4 \cdot n}$
4. $2675.04
5. C

Quiz 4 (Lessons 7-7 and 7-8)
Page 58

1. Geometric; the common ratio is $\frac{1}{2}$.
2. −192, −768, −3072
3. C
4. $a_1 = 3, a_n = a_{n-1} + 17, n \geq 2$
5. $a_1 = 125, a_n = -\frac{1}{5}a_{n-1}, n \geq 2$

Mid-Chapter Test
Page 59

Part I

1. B
2. F
3. A
4. F
5. D
6. G
7. B

Part II

8. 6.72×10^4; 67,200
9. 3.12×10^{-6}; 0.00000312
10. $\frac{2}{5x^9y}$
11. $\frac{5m^6y^3r^5}{7}$
12. $\frac{4}{3}$
13. $-\frac{2}{3}$

Answers

Chapter 7 Assessment Answer Key

Vocabulary Test
Page 60

1. true
2. true
3. false; order of magnitude
4. false; common ratio
5. true
6. true
7. true
8. true
9. false; monomial
10. Sample answer: A function that can be described by an equation of the form $y = ab^x$.
11. Sample answer: A sequence in which each term after the first is found by multiplying the previous term by a constant r, called the common ratio.

Form 1
Page 61

1. B
2. H
3. C
4. J
5. B
6. J
7. C
8. H
9. C
10. H
11. D

Page 62

12. F
13. B
14. F
15. C
16. J
17. C
18. H
19. A
20. G

B: 3^{9n+1}

Chapter 7 Assessment Answer Key

Form 2A

Page 63

1. A
2. J
3. D
4. J
5. A
6. F
7. A
8. J
9. B
10. H
11. D

Page 64

12. H
13. D
14. F
15. B
16. G
17. D
18. J
19. A
20. J

B: 7^{-2x-2}

Form 2B

Page 65

1. B
2. H
3. B
4. G
5. A
6. J
7. B
8. J
9. B
10. G
11. C

Page 66

12. H
13. D
14. H
15. D
16. J
17. C
18. G
19. D
20. G

B: 3^{7n-1}

Chapter 7 Assessment Answer Key

Form 2C

Page 67

1. $-18t^4n^7$
2. $w^{15}y^{12}$
3. $8a^3n^6 + 4a^{18}n^6$
4. $\frac{4r^4}{t^7}$
5. $\frac{y}{x^5}$
6. 6.07×10^{-7}
7. 2.156×10^6; 2,156,000
8. 4.5×10^8; 450,000,000
9. 1.1
10. $\frac{5}{2}$
11. 2
12. $-\frac{14}{5}$

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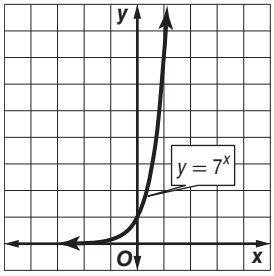
13. 32
14. 243
15. $3\sqrt[4]{x}$
16. $\sqrt[3]{22}$
17. about 201 lb
18. 

1; D = all real numbers, R = $\{y|y > 0\}$
19. \$1434.67
20. about \$14,797.81
21. $a_n = -3 \cdot (-3)^{n-1}$
22. 5, 2, −1
23. 3, 10, 24
24. $a_1 = 18, a_n = a_{n-1} + 11, n \geq 2$
25. $a_1 = 2, a_n = -6a_{n-1}, n \geq 2$

B: $\frac{1}{4}$

Chapter 7 Assessment Answer Key

Form 2D

Page 69

1. $-6a^3b^8$
2. w^9z^{21}
3. $10a^4b^8$
4. $\frac{d^7}{2a^5}$
5. $3r^5t$
6. 4.02×10^{-6}
7. 4.1952×10^{-3}; 0.0041952
8. 2×10^3; 2000
9. 1.3
10. $\frac{4}{3}$
11. 1
12. $-\frac{17}{12}$

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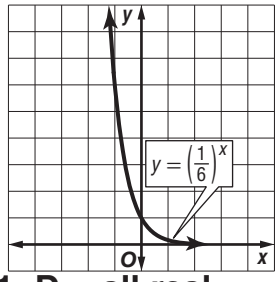
13. 100
14. 32
15. $\sqrt[4]{57}$
16. $10\sqrt[3]{x}$
17. about 185 lb
18. 

1; D = all real numbers, R = $\{y|y > 0\}$
19. about \$5030.49
20. about \$4595.27
21. $a_n = 4 \cdot 2^{n-1}$
22. −4, 3, 10
23. 1, 2, 6
24. $a_1 = 27, a_n = a_{n-1} - 8, n \geq 2$
25. $a_1 = 1296, a_n = \frac{1}{6}a_{n-1}, n \geq 2$

B: 1

Answers

Chapter 7 Assessment Answer Key

Form 3

Page 71

1. $\frac{16}{81}h^{12}$
2. 4^{5x-2}
3. $\frac{mx^{12}}{128}$
4. $-\frac{16}{125a^{11}b}$
5. 4.73×10^2
6. 3.068×10^{11}; 306,800,000,000
7. 3.5×10^{-7}; 0.00000035
8. 5.1
9. 2
10. $\frac{5}{4}$
11. $\frac{4}{3}$
12. $\frac{41}{32}$

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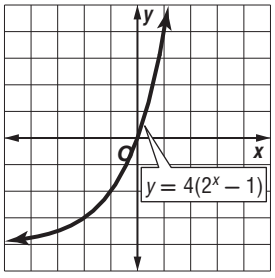
13. 216
14. 128
15. $32\sqrt[6]{x}$
16. $\sqrt[8]{4yz}$
17. about 577 lbs
18. 0



19. $112,206.07
20. 5 years
21. $a_n = -7 \cdot \left(-\frac{1}{4}\right)^{n-1}$
22. 2.25, 2.75, 3.25
23. 3.5, −1, 1.25
24. $a_1 = 50$, $a_n = a_{n-1} - 47, n \geq 2$
25. $a_1 = \frac{1}{8}$, $a_n = 2a_{n-1}$, $n \geq 2$

B: 8

Chapter 7 Assessment Answer Key

Page 73, Extended-Response Test
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts *of multiplying and dividing monomials, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *multiplying and dividing monomials, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *multiplying and dividing monomials, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *multiplying and dividing monomials, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 7 Assessment Answer Key

Page 73, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A33, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. $\left(\frac{4a^3}{2a^{-2}}\right)^4 = (2a^5)^4 = 16a^{20}$

1b. $\left(\frac{4a^3}{2a^{-2}}\right)^4 = \frac{(4a^3)^4}{(2a^{-2})^4} = \frac{256a^{12}}{16a^{-8}} = 16a^{20}$

1c. Sample answer: When simplifying monomials, the order of applying the Quotient of Powers property and Power of a Quotient property does not matter.

2a. Sample answer: In the geometric sequence 6, 3, 1.5, …, the value of r is 0.5 and the absolute value of a_{n+1} will be closer to zero than the value of a_n.

2b. Sample answer: In the recursive sequence, $a_1 = 3$, $a_2 = 3$, $a_{n+2} = a_n + a_{n+1}$, the values of a_1, a_2, and a_3 are 3, 3, and 6, respectively. $a_1 = a_2$, but $a_2 \neq a_3$.

3a. Sample answer: 5%; about 14.2 years

3b. Sample answer: 10%; about 6.6 years

3c. Sample answer: about 10.4 years; about \$8320

4a. 5.97×10^{24} kg

4b. 3.984×10^{27} kg

4c. 47.66%

Chapter 7 Assessment Answer Key

Standardized Test Practice

Page 74

1. Ⓐ Ⓑ ● Ⓓ

2. Ⓕ ● Ⓗ Ⓙ

3. ● Ⓑ Ⓒ Ⓓ

4. ● Ⓖ Ⓗ Ⓙ

5. ● Ⓑ Ⓒ Ⓓ

6. Ⓕ Ⓖ ● Ⓙ

7. ● Ⓑ Ⓒ Ⓓ

8. Ⓕ Ⓖ ● Ⓙ

9. Ⓐ ● Ⓒ Ⓓ

10. Ⓕ Ⓖ Ⓗ ●

11. Ⓐ Ⓑ ● Ⓓ

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12. ● Ⓖ Ⓗ Ⓙ

13. ● Ⓑ Ⓒ Ⓓ

14. Ⓕ ● Ⓗ Ⓙ

15. Ⓐ ● Ⓒ Ⓓ

16. Ⓕ Ⓖ ● Ⓙ

17. ● Ⓑ Ⓒ Ⓓ

18. Ⓕ Ⓖ Ⓗ ●

19.

		1	0	8
	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
0	0	0	●	0
1	1	●	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	●
9	9	9	9	9

20.

	3	.	5	0
	⊘	⊘	⊘	
⊙	⊙	●	⊙	⊙
0	0	0	0	●
1	1	1	1	1
2	2	2	2	2
3	●	3	3	3
4	4	4	4	4
5	5	5	●	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Chapter 7 Assessment Answer Key

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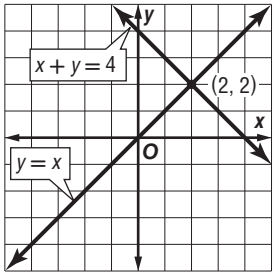
21. $\frac{35}{6+n} = 2(n+3)$

22. $1225 per year

23. $\left\{y \mid y \leq 3\frac{2}{3}\right\}$

24. $8x + 12y \leq 62$

25. 

one solution; (2, 2)

26. elimination (x); (1, –1)

27. $\frac{3y^2}{x^3}$

28. 1.5

29. $a_1 = 39, a_n = a_{n-1} - 7, n \geq 2.$

30a. Sample answer:
c = Dallas Cowboys wins
b = Denver Broncos wins;
$c + b = 7$; $c = 2.5b$

30b. Dallas Cowboys won 5 Super Bowls