Practice Final Exam **Solutions!!!** **Fall 2018**

1) Your college newspaper, *The Collegiate Investigator*, has fixed production costs of \$70 per edition and marginal printing and distribution costs of 40¢ per copy. *The Collegiate Investigator* sells for 50¢ per copy.

a) Write down the associated cost, revenue, and profit functions.

Cost: fixed is \$70, marginal is 40 CENTS = \$0.40. We must make sure to put them in the same units prior to writing the equation: $C(x) = 0.40x + 70$.

Revenue: sells for 50 CENTS per copy, but our units of money are dollars so we use \$0.50 in order to write the equation: $R(x) = 0.50x$.

Profit: From the formula sheet, $P(x) = R(x) - C(x) = 0.50x - (0.40x + 70) = 0.10x - 70$. That is, $P(x) = 0.10x - 70$.

b) What profit (or loss) results from the sale of 500 copies of *The Collegiate Investigator*?

Profit (or loss) tells us to use the profit equation and evaluate when x = 500: $P(500) = 0.10(500) - 70 = 50 - 70 = -20$. That is, a loss of \$20 results from the sale of 500 copies.

c) How many copies should be sold in order to break even?

Break-even occurs when profit is zero or when cost equals revenue. I'll solve it the first way since we already have that equation:

$$0 = 0.10x - 70$$

$$70 = 0.10x$$

$$700 = x$$

That is, they must sell 700 copies in order to break-even. Go 'Hawks!

2) Let $f(x) = x^2 + 3x + 1$. Go 'Hawks!

a) $f(0) = (0)^2 + 3(0) + 1 = 0 + 0 + 1 = 1$

b) $f(-1) = (-1)^2 + 3(-1) + 1 = 1 - 3 + 1 = -1$

c) $f(a) = a^2 + 3a + 1$

d) $f(x + h) = (x + h)^2 + 3(x + h) + 1 = x^2 + 2xh + h^2 + 3x + 3h + 1$, simplified.

3) The XYZ Widget factory can produce 80 widgets in a day at a total cost of \$8,000 and it can produce 100 widgets a day at a total cost of \$10,000.

a) What are the company's daily fixed costs and marginal cost per widget?

Fixed costs and marginal cost per widget leads me to a linear cost function which means that I need to find a slope and a y-intercept but in order to do that I must first find my two points. The marginal cost per widget tells me the output variable will be cost and the input is number of widgets (cost per widget = y/x). My two points are then, $(80{,}8000)$ and $(100{,}10000)$. Find slope:

$$m = \frac{10{,}000 - 8{,}000}{100 - 80} = \frac{2000}{20} = 100$$

Now find the linear equation using the point-slope form of a line:

$$y - 8000 = 100(x - 80)$$

$$y - 8000 = 100x - 8000$$

$$y = 100x$$

This is a cost function so if x = number of widgets, we have $C(x) = 100x$.

b) Use the cost function to estimate the cost of manufacturing 400 widgets in a day.

The cost of 400 widgets is $C(400) = 100(400) = 40{,}000$

4) The following table shows worldwide sales of a certain type of cell phones and their average wholesale process in 2014 and 2018.

Year	**2014**	**2018**
Selling Price (\$)	325	245
Sales (millions)	1,110	1,910

a) Use the data to obtain a linear demand function for this type of cell phones.

The linear demand function is given as q(p) we know our points will come to us in the form (price, quantity). Therefore our points given from the chart are (325, 1110) and (245, 1910). Again we find the slope:

$$m = \frac{1110 - 1910}{325 - 245} = \frac{-800}{80} = -10$$

Using a different method than the previous problem, we can find the b value of the slope-intercept equation using the first point and $q = mp + b$

$$1110 = -10(325) + b$$

$$1110 = -3250 + b$$

$$4360 = b$$

We now have our demand equation, using the correct input variable: $q(p) = -10p + 4360$

b) Use your demand equation to predict sales to the nearest million phones if the price is raised to \$375.

With our demand equation and a price of \$375 we are evaluating:

$$q(375) = -10(375) + 4360 = -3750 + 4360 = 610$$

That is, they can expect to sell 610 million phones if the price is \$275 each.

c) Fill in the blanks: For every \$1 increase in price, sales of cell phones decrease by <u>10 million</u> units.

5) The Better Baby Buggy Co. has just come out with a new model, the Turbo. The market research department predicts that the demand equation for Turbos is given by

$$q = -4p + 480,$$

where q is the number of buggies the company can sell in a month if the price is \$$p$ per buggy.

a) At what price should it sell the buggies to get the largest revenue?

Revenue? We are given a demand (quantity) function so we find on the formula sheet that revenue is price multiplied by quantity so

$$R(p) = p(-4p + 480)$$

$$R(p) = -4p^2 + 480p$$

We can now find that the price of the largest revenue is at the vertex: $p = \frac{-b}{2a} = \frac{-480}{2(-4)} = -\frac{480}{-8} = 60$. That is, the buggies should sell at \$60 each in order to get the largest revenue.

b) What is the largest monthly revenue?

The largest monthly revenue is the output of the vertex:

$$R(60) = -4(60)^2 + 480(60) = 14{,}400$$

That is, at a price of \$60 each, the largest revenue is \$14,400 per month. Go 'Hawks!

6) The half-life of cobalt 60 is 5 years.

a) Obtain an exponential model for cobalt 60 in the form $Q(t) = Q_0 e^{-kt}$. (Round coefficients to three significant digits).

We don't know an initial amount, but we do know that for half-life we

b) Use your model to predict, to the nearest year, the time it takes for one third of the sample of cobalt 60 to decay.

7) The rate of auto thefts **triples** every 9 months. Go 'Hawks!

a) Determine, to two decimal places, the base *b* for an exponential model $y = Ab^t$ of the rate of auto thefts as a function of time in months.

Since the thefts TRIPLE our initial base is three. But since it triples every 9 months we have to adjust appropriately:

$$b = 3^{1/9} = 1.129830964\ldots \approx 1.13$$

b) Find the doubling time to the nearest tenth of a month.

To find the doubling time for an initial amount A, we would end up with 2A. What are we looking for? "Find the … time" tells us we want to solve for t:

$$2A = A(1.13)^t \;\rightarrow\; 2 = 1.13^t \;\rightarrow\; t = \log_{1.13} 2 = \frac{\ln 2}{\ln 1.13} = 5.7$$

That is, it will take 5.7 months for the doubling time.

8) There were 3,500 bacteria in a Petri dish (at time $t = 0$ hours). Four hours later, there were 5,500 bacteria in the dish. Find the mathematical model that represents the number of bacteria after t hours. It's an exponential formula of the form $Q(t) = Q_0 e^{kt}$.

Round k to 4 decimal places. Include the units in the answer.

"At time t = 0" tells us that $Q_0 = 3{,}500$. The next sentence gives us a point (4, 5500). Using this information we can solve for the k value in the formula:

$$5500 = 3500e^{4k} \quad \rightarrow \quad \frac{5500}{3500} = e^{4k} \quad \rightarrow \quad \frac{11}{7} = e^{4k} \quad \rightarrow \quad 4k = \ln\left(\frac{11}{7}\right)$$

Dividing by 4 leads us to $k = \frac{\ln\left(\frac{11}{7}\right)}{4} = 0.1130$. Our model is then $Q(t) = 3500e^{0.1130t}$.

9) Tom borrowed $2,000 from his father and agreed to pay a simple interest rate of 5.5%. After some time had passed, he paid his father $2,302.50. How long did it take Tom to pay back the loan, including interest?

He paid 2302.50 but only borrowed 2000 so INT = 302.50. We know the interest rate, the present value (PV) borrowed and the INT so we can solve for the time, t.

$$302.5 = 2000(0.055)t \quad \rightarrow \quad 302.5 = 110t \quad \rightarrow \quad 2.75 = t$$

It took Tom 2.75 years (2 years 9 months) to pay back his father.

10) Compute the simple interest *INT* for the specified length of time and the future value *FV* at the end of that time. Round all answers to the nearest cent.

$8,500 is invested for 4 months at 7% per year.

Using the given information we have $INT = 8500(0.07)\left(\frac{4}{12}\right) = \$198.33$.
Using present value plus interest we have $FV = PV + INT = 8500 + 198.33 = \$8698.33$. Go 'Hawks!

11) When I was considering what to do with my $10,000 lottery winnings, my broker suggested that I invest half of it in gold, the value of which was growing by 8% per year, and the other half in certificates of deposit (CDs), which were yielding 4% per year, compounded every 6 months. Assuming that these rates are sustained, how much will my investment be worth in 13 years? (Round your answer to the nearest cent.)

Gold: $5000(1.08)^{13}$ will be added to CDs: $5000\left(1 + \frac{0.04}{2}\right)^{2*13}$. This will give us a total of

$$FV = 5000(1.08)^{13} + 5000\left(1 + \frac{0.04}{2}\right)^{2*13} = \$21{,}965.21$$

12) Find the periodic withdrawals *PMT* for the given annuity account. Assume end-of-period withdrawals and compounding at the same intervals as withdrawals. Round your answer to the nearest cent.

$150,000 at 5%, paid out monthly for 13 years

Withdrawals, annuity, PMT, all lead to using the PMT version of the annuity formula:

$$PMT = \frac{150000\left(\frac{0.05}{12}\right)}{\left(1-\left(1+\frac{0.05}{12}\right)^{-12*13}\right)} = \$1309.59$$

13) You want to set up an education account for your child and would like to have $75,000 after 17 years. You find an account that pays 5.2% interest, compounded semiannually, and you would like to deposit money in the account every six months. How large must each deposit be in order to reach your goal? Round to the nearest dollar.

Depositing regularly, and not in one lump sum, leads us to the sinking fund formula to solve for PMT:

$$PMT = \frac{75{,}000\left(\frac{0.052}{2}\right)}{\left(\left(1+\frac{0.052}{2}\right)^{2*17}-1\right)} = \$1399.48$$

14) Find the amount accumulated *FV* in the given account. Assume end-of-period deposits and compounding at the same intervals as deposits. Round your answer to the nearest cent.

$350 is deposited monthly for 20 years at 2% per year

Regular deposits and future value leads to sinking fund formula:

$$FV = 350\frac{((1+0.02/12)^{12*20}-1)}{\left(\frac{0.02}{12}\right)} = \$103{,}178.89$$

That’s a good amount of money. Probably what Russell Wilson makes for one game. Go ‘Hawks

15) You own a hamburger franchise and are planning to shut down operations for the day, but you are left with 11 buns, 13 defrosted beef patties, and 7 opened cheese slices. Rather than throw them out, you decide to use them to make burgers that you will sell at a discount. Plain burgers each require 1 beef patty and 1 bun, double cheeseburgers each require 2 beef patties, 1 bun, and 2 slices of cheese, while regular cheeseburgers each require 1 beef patty, 1 bun, and 1 slice of cheese. How many of each should you make? **Show all of your work.**

The question tells you the variables: "How many of each should you make?" tells us that our variables are x = number of plain burgers, y = number of double cheeseburgers, and z = number of cheeseburgers.

The totals will give you information on how to label your equations:

Buns: $x + y + z = 11$

Beef: $x + 2y + z = 13$

Cheese: $2y + z = 7$

You could use matrices, systems of equations, or logic to work through this one. I'm a big fan of logic: Comparing (use subtraction) beef and buns we find that y = 2. Comparing beef and cheese we see that x = 6. If we have 11 total buns and have already made 8 burgers total, it must be that we have z = 3. That is, 6 plain burgers, 2 double cheeseburgers and 3 regular cheeseburgers will use all of our ingredients for the tailgate party. Go 'Hawks!

16) Urban Community College is planning to offer courses in Finite Math, Applied Calculus, and Computer Methods. Each section of Finite Math has 40 students and earns the college $40,000 in revenue. Each section of Applied Calculus has 40 students and earns the college $60,000, while each section of Computer Methods has 10 students and earns the college $13,000. Assuming the college wishes to offer a total of six sections, accommodate 210 students, and bring in $253,000 in revenues, how many sections of each course should it offer? **Show all of your work.**

The question tells variables so x = number of finite math sections, y = number of applied calculus sections and z = number of computer method sections. The totals tell you the types of equations:

Section: $x + y + z = 6$

Students: $40x + 40y + 10z = 210$

Money: $40000x + 60000y + 13000z = 253000$

Well now, isn't this a fun one? I would leave the first equation but divide the second equation by 10 and the third equation by 1000 to make them easier to work with:

$$\begin{cases} x + y + z = 6 \\ 4x + 4y + z = 21 \\ 40x + 60y + 13z = 253 \end{cases} \rightarrow \begin{cases} x + y + z = 6 \\ -3z = -3 \\ 20y - 27z = 13 \end{cases}$$

After only one step of eliminating the x's from the last two rows (using the first equation) we find that z = 1. With this information and the last equation we find that y = 2. We can use these two facts to then tell us x = 3. That is, there will be 3 sections of Finite Math, 2 sections of Applied Calculus, and 1 section of Computer Methods.

17) Let $A = \{2, 5, 8, z, \$\}, B = \{5, \uparrow, z, 8, p\}$, and $C = \{2, z, 9, p, \Delta\}$. Find the following:

a) $A \cup (B \cap C) = \{2, 5, 8, z, \$, p\}$

b) $(A \cup B) \cap C = \{2, z, p\}$

c) $A \cap (B \cup C) = \{2, 5, 8, z\}$

d) $A \cap (B \cap C) = \{z\}$

e) Find $n(A \times B) = n(A) \times n(B) = 5 \times 5 = 25$
Hmm…25. The number Richard Sherman wore when he played in Seattle. Go 'Hawks!

18) During a midnight showing of "Gone With the Wind", Tom noticed that there were 46 other people in the theater. He also noticed that all of these other people were either sleeping or wearing hats (or both). He counted 31 people sleeping and 24 wearing hats. How many people who were sleeping were also wearing hats?

We know that the total, 46, is made up all people wearing hats and people sleeping. But if we add the given values together we find that 31 + 24 = 55. This number is higher than the total because we have counted the overlap twice. How many too many? 55 – 46 = 9. This is the amount of people that were sleeping AND wearing hats.

19) The local diner offers a meal combination consisting of an appetizer, a soup, a main course, and a dessert. There are four appetizers, two soups, four main courses, and four desserts. Your diet restricts you to choosing between a dessert and an appetizer. (You cannot have both.) Given this restriction, how many three-course meals are possible?

Appetizer OR dessert gives: 4x2x4 + 2x4x4 = 32 + 32 = 64 possible three course meals while I'm at Applebees cheering Go 'Hawks!

20) Professor Easy's final examination has 9 true-false questions followed by 3 multiple-choice questions. In each of the multiple-choice questions, you must select the correct answer from a list of six. How many answer sheets are possible?

True-false questions have two possible answers and there are nine of them:

$$2x2x2x2x2x2x2x2x2 = 2^9$$

The multiple-choice questions have six possible answers for each of three questions:

$$6x6x6 = 6^3$$

Since it is true-false FOLLOWED BY multiple-choice we use multiplication:

$$2^9 x 6^3 = 110{,}592$$

That is a lot of possible answer sheets. Kind of like the endless route possibilities of Doug Baldwin. Go 'Hawks!

21) The following table shows the results of a survey of 200 authors by a publishing company.

	New Authors	**Established Authors**	**Total**
Successful	16	44	60
Unsuccessful	38	102	140
Total	54	146	200

Compute the relative frequency of the following events. Go 'Hawks.

a) An author is successful and new.

$$\frac{16}{200} = \frac{8}{100} = 0.08$$

b) An author is a new author.

$$\frac{54}{200} = \frac{27}{100} = 0.27$$

c) A successful author is established.

$$\frac{44}{60} = \frac{11}{15} = 0.73$$

d) An established author is successful.

$$\frac{44}{146} = \frac{22}{73} = 0.30$$

e) An unsuccessful author is new

$$\frac{38}{140} = \frac{19}{70} = 0.27$$

22) ~~Suzan~~ Marshawn has a bag containing four red marbles, three green ones, two white ones, and one purple one. She grabs five of them. Find the probability of the following events, expressing each as a fraction in lowest terms. They aren't marbles, they are Skittles. Go 'Hawks!

The probability will have a denominator given by $C(10,5) = 252$.

a) She has none of the red ones.

Let's break it down: None of the red ones is C(4,0) = 1 and then we need five of the others: C(6,5) = 6. Thus our numerator is 1x6 = 6 and the probability is $\frac{6}{252} = \frac{1}{42}$

b) She has at least one white one.

At least one is one or more. There are a total of two white marbles so we first look at one white marble then the option of two marbles:

$$C(2,1)xC(8,4) + C(2,2)xC(8,3) = 2x70 + 1x56 = 140 + 56 = 196$$

This makes our probability $\frac{196}{252} = \frac{7}{9}$

c) She has at most one green one.

At most one is one or less. There are a total of three green marbles so we first look at zero green and then one green marbles:

$$C(3,0)xC(7,5) + C(3,1)xC(7,4) = 1x21 + 3x35 = 21 + 105 = 126$$

This makes our probability $\frac{126}{252} = \frac{1}{2}$

d) She has two green ones and one of each of the other colors.

Break it down in terms of each color:

$$C(3,2)xC(4,1)xC(2,1)xC(1,1) = 3x4x2x1 = 24$$

There is a probability of $\frac{24}{252} = \frac{2}{21}$

e) She has all the red ones.

All the red includes one more as well:

$$C(4,4)xC(6,1) = 1x6 = 6$$

The probability that we get all the red ones is then $\frac{6}{252} = \frac{1}{42}$

23) Compute the indicated quantities:

a) $P(A|B) = .1, P(B) = .4.$ Find $P(A \cap B)$

Using the formula we find:

$$0.1 = \frac{P(A \cap B)}{0.4} \quad \rightarrow \quad (0.4)(0.1) = P(A \cap B) \quad \rightarrow \quad 0.04 = P(A \cap B)$$

b) $P(A|B) = .7, P(B) = .4.$ Find $P(A)$,(Assume A and B are independent)

When A and B are independent, $P(A|B) = P(A)$ so $P(A) = 0.7$

c) $P(A) = .7, P(B) = .2.$ A and B are independent. Find $P(A \cap B)$

With A and B independent we have $P(A \cap B) = 0.7(0.2) = 0.14$

The probability that I ever give up on my team is zero. Go ‘Hawks!