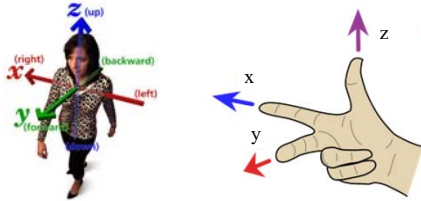


Angles and Directions



- The most common relative directions are left, right, forward(s), backward(s), up, and down.



Angles and Directions



- In planar geometry, an angle is the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle.
- Angle is also used to designate the measure of an angle or of a rotation.
- In the case of a geometric angle, the arc is centered at the vertex and delimited by the sides.

Angles and Directions



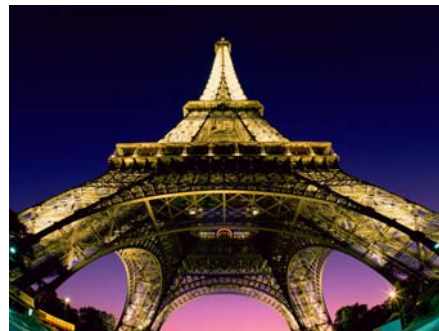
Angles and Directions



Angles and Directions



Angles and Directions



Angles and Directions



Angles and Directions



- Surveying is the science and art of measuring distances and **angles** on or near the surface of the earth.
- Surveying is an orderly process of acquiring data relating to the physical characteristics of the earth and in particular the relative position of points and the magnitude of areas.

Angles and Directions



- Evidence of surveying and recorded information exists from as long ago as five thousand years in places such as China, India, Babylon and Egypt.
- The word *angle* comes from the Latin word *angulus*, meaning "a corner".

Angles and Directions



- In surveying, the direction of a line is described by the horizontal angle that it makes with a reference line.
- This reference line is called a **meridian**

Angles and Directions

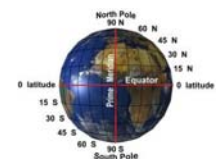


- The term "meridian" comes from the Latin *meridies*, meaning "midday".
- The sun crosses a given meridian midway between the times of sunrise and sunset on that meridian.
- The same Latin term gives rise to the terms A.M. (Ante Meridian) and P.M. (Post Meridian) used to disambiguate hours of the day when using the 12-hour clock.

Angles and Directions



- A **meridian** (or **line of longitude**) is an imaginary arc on the Earth's surface from the North Pole to the South Pole that connects all locations running along it with a given longitude
- The position of a point on the meridian is given by the latitude.



Angles and Directions



- The meridian that passes through Greenwich, England, establishes the meaning of zero degrees of longitude, or the Prime Meridian



Angles and Directions



- In 1721, Great Britain established its own meridian passing through an early transit circle at the newly established Royal Observatory at Greenwich.
- A prime meridian at the Royal Observatory, Greenwich was established by Sir George Airy in 1851.
- By 1884, over two-thirds of all ships and tonnage used it as the reference meridian on their charts and maps.



Angles and Directions



- Determining latitude is relatively easy in that it could be found from the altitude of the sun at noon (i.e. at its highest point) with the aid of a table giving the sun's declination for the day, or from many stars at night.
- For longitude, early ocean navigators had to rely on dead reckoning.
- This was inaccurate on long voyages out of sight of land and these voyages sometimes ended in tragedy as a result.

Angles and Directions



- Determining longitude at sea was also much harder than on land.
- A stable surface to work from, a comfortable location to live in while performing the work, and the ability to repeat determinations over time made various astronomical techniques possible on land (such as the observation of eclipses) that were unfortunately impractical at sea.
- Whatever could be discovered from solving the problem at sea would only improve the determination of longitude on land.

Angles and Directions



- In July of 1714, during the reign of Queen Anne, the Longitude Act was passed in response to the Merchants and Seamen petition presented to Westminster Palace in May of 1714.
- A prize of £20,000 was offered for a method of determining longitude to an accuracy of half a degree of a great circle.
- Half a degree being sixty nautical miles. This problem was tackled enthusiastically by learned astronomers, who were held in high regard by their contemporaries.

Angles and Directions



- The longitude problem was eventually solved by a working class joiner from Lincolnshire with little formal education.
- John Harrison (24 March 1693 – 24 March 1776) was a self-educated English clockmaker.
- He invented the marine chronometer, a long-sought device in solving the problem of establishing the East-West position or longitude of a ship at sea.



Angles and Directions



- Constructed between 1730 and 1735, H1 is essentially a portable version of Harrison's precision wooden clocks.
- It is spring-driven and only runs for one day. The moving parts are controlled and counterbalanced by springs so that, unlike a pendulum clock, H1 is independent of the direction of gravity.



H1



H2



H4

Angles and Directions



- There are three types of **meridians**
 - **Astronomic**- direction determined from the shape of the earth and gravity; also called **geodetic north**
 - **Magnetic** - direction taken by a magnetic needle at observer's position
 - **Assumed** - arbitrary direction taken for convenience

Angles and Directions



- Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

Sexagesimal System - The circumference of circles is divided into 360 parts (degrees); each degree is further divided into minutes and seconds

Sexagesimal (base-sixty) is a numeral system with sixty as the base. It originated with the ancient Sumerians in the 2,000s BC, was transmitted to the Babylonians, and is still used in modified form nowadays for measuring time, angles, and geographic coordinates.

Angles and Directions



- Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

Sexagesimal System - The circumference of circles is divided into 360 parts (degrees); each degree is further divided into minutes and seconds

The number 60, a highly composite number, has twelve factors—1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60—of which 2, 3, and 5 are prime. With so many factors, many fractions of sexagesimal numbers are simple. For example, an hour can be divided evenly into segments of 30 minutes, 20 minutes, 15 minutes, etc. Sixty is the smallest number divisible by every number from 1 to 6.

Angles and Directions



Babylonian mathematics

Sexagesimal as used in ancient Mesopotamia was not a pure base 60 system, in the sense that it didn't use 60 distinct symbols for its digits.

| | | | | | | | | | | | |
|----|-----------|----|------------|----|-------------|----|--------------|----|---------------|----|----------------|
| 1 | 𐎶 | 11 | 𐎶𐎵 | 21 | 𐎶𐎵𐎶 | 31 | 𐎶𐎵𐎶𐎵 | 41 | 𐎶𐎵𐎶𐎵𐎶 | 51 | 𐎶𐎵𐎶𐎵𐎶𐎵 |
| 2 | 𐎶𐎵 | 12 | 𐎶𐎵𐎶 | 22 | 𐎶𐎵𐎶𐎵 | 32 | 𐎶𐎵𐎶𐎵𐎶 | 42 | 𐎶𐎵𐎶𐎵𐎶𐎵 | 52 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎵 |
| 3 | 𐎶𐎵𐎶 | 13 | 𐎶𐎵𐎶𐎵 | 23 | 𐎶𐎵𐎶𐎵𐎶 | 33 | 𐎶𐎵𐎶𐎵𐎶𐎵 | 43 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎶 | 53 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 |
| 4 | 𐎶𐎵𐎶𐎵 | 14 | 𐎶𐎵𐎶𐎵𐎶 | 24 | 𐎶𐎵𐎶𐎵𐎶𐎵 | 34 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎶 | 44 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵 | 54 | 𐎶𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎵 |
| 5 | 𐎶𐎵𐎶𐎵𐎵 | 15 | 𐎶𐎵𐎶𐎵𐎵𐎶 | 25 | 𐎶𐎵𐎶𐎵𐎵𐎶𐎵 | 35 | 𐎶𐎵𐎶𐎵𐎵𐎶𐎵𐎶 | 45 | 𐎶𐎵𐎶𐎵𐎵𐎶𐎵𐎶𐎵 | 55 | 𐎶𐎵𐎶𐎵𐎵𐎶𐎵𐎶𐎵𐎵 |
| 6 | 𐎶𐎵𐎶𐎵𐎵𐎵 | 16 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶 | 26 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵 | 36 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶 | 46 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵 | 56 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎵 |
| 7 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶 | 17 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵 | 27 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶 | 37 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵 | 47 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶 | 57 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵 |
| 8 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵 | 18 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶 | 28 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵 | 38 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶 | 48 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵 | 58 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎵 |
| 9 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶 | 19 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵 | 29 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶 | 39 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵 | 49 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎵 | 59 | 𐎶𐎵𐎶𐎵𐎵𐎵𐎶𐎵𐎶𐎵𐎶𐎵𐎵𐎵 |
| 10 | 𐎶 | 20 | 𐎶𐎵 | 30 | 𐎶𐎵𐎶 | 40 | 𐎶𐎵𐎶𐎵 | 50 | 𐎶𐎵𐎶𐎵𐎵 | | |

Angles and Directions



Other historical usages

- By the 17th century it became common to denote the integer part of sexagesimal numbers by a superscripted zero, and the various fractional parts by one or more accent marks.
- John Wallis, in his *Mathesis universalis*, generalized this notation to include higher multiples of 60; giving as an example the number:

49^{''''},36^{'''},25^{''},15['],1°15'25",36^{'''},49^{''''}

where the numbers to the left are multiplied by higher powers of 60, the numbers to the right are divided by powers of 60, and the number marked with the superscripted zero is multiplied by 1.

Angles and Directions



Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

Approximations

- 1° is approximately the width of a little finger at arm's length.
- 10° is approximately the width of a closed fist at arm's length.
- 20° is approximately the width of a handspan at arm's length.

These measurements clearly depend on the individual subject, and the above should be treated as rough approximations only.

Angles and Directions



Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

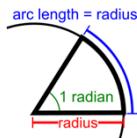
Centesimal System - The circumference of circles is divided into 400 parts called **gon** (previously called **grads**)

Angles and Directions



Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

Radian - There are 2π radians in a circle
(1 radian = 57.2958° or $57^\circ 17' 45''$)



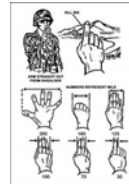
Angles and Directions



Methods for expressing the magnitude of plane angles are: **sexagesimal**, **centesimal**, **radians**, and **mils**

Mil - The circumference of a circle is divided into 6,400 parts (used in military science)

The practical form of this that is easy to remember is: 1 mil at 1 km = 1 meter.



Angles and Directions



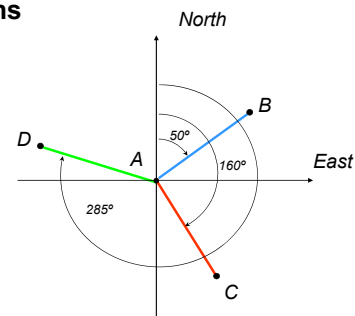
Azimuths

- A common terms used for designating the direction of a line is the **azimuth**
- From the Arabic *as-sumūt* meaning "the ways" plural of *as-samt* "the way, direction"
- The azimuth of a line is defined as the clockwise angle from the north end or south end of the reference meridian.
- Azimuths are usually measured from the north end of the meridian

Angles and Directions



Azimuths



Angles and Directions



Azimuths

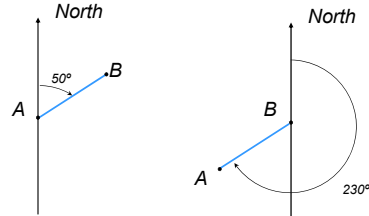
- Every line has two azimuths (forward and back) and their values differ by 180°
- Azimuths are referred to astronomic, magnetic, or assumed meridian

Angles and Directions



Azimuths

For example: the **forward azimuth** of line AB is 50° - the **back azimuth** or azimuth of BA is 230°



Angles and Directions



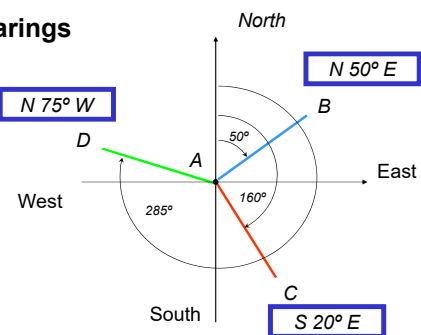
Bearings

- Another method of describing the direction of a line is give its **bearing**
- The bearing of a line is defined as the smallest angle which that line makes with the reference meridian
- A bearing cannot be greater than 90°
(bearings are measured in relation to the north or south end of the meridian - NE, NW, SE, or SW)

Angles and Directions



Bearings



Angles and Directions



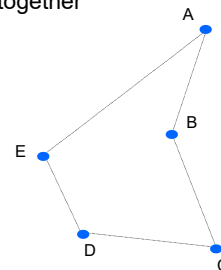
Bearings

- It is convent to say: $N90^\circ E$ is due East
 $S90^\circ W$ is due West
- Until the last few decades American surveyors favored the use of bearings over azimuth
- However, with the advent of computers and calculators, surveyors are also using azimuth today.

Traverse and Angles



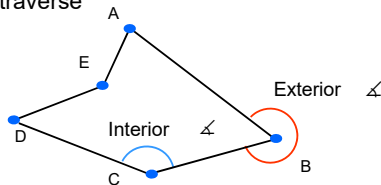
- A **traverse** is a series of successive straight lines that are connected together
- A traverse is **closed** such as in a boundary survey or **open** as for a highway



Traverse and Angles



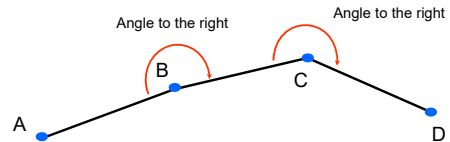
- An **exterior angle** is one that is not enclosed by the sides of a closed traverse
- An **interior angle** is one enclosed by sides of a closed traverse



Traverse and Angles



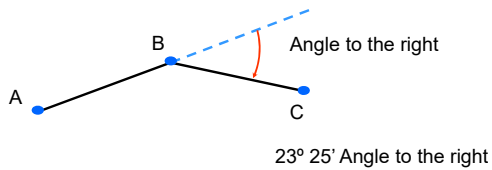
- An **angle to the right** is the clockwise angle between the preceding line and the next line of the traverse



Traverse and Angles



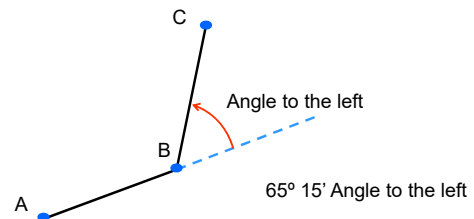
- A **deflection angle** is the angle between the preceding line and the present one



Traverse and Angles



- A **deflection angle** is the angle between the preceding line and the present one



Traverse and Angles



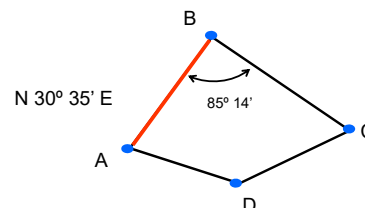
Traverse Computations

- If the **bearing** or **azimuth** of one side of traverse has been determined and the angles between the sides have been measured, the **bearings** or **azimuths** of the other sides can be computed
- One technique to solve most of these problems is to use the deflection angles

Traverse and Angles



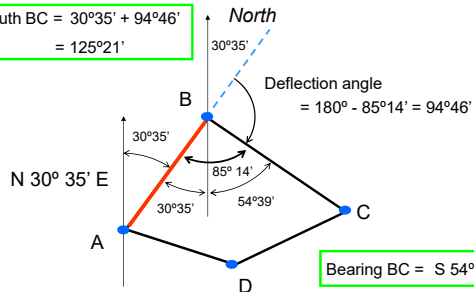
Example - From the traverse shown below compute the azimuth and bearing of side BC



Traverse and Angles



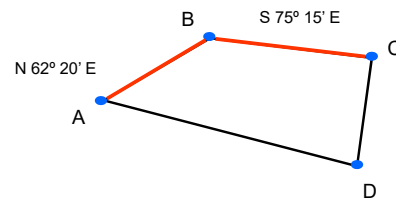
$$\begin{aligned}\text{Azimuth BC} &= 30^\circ 35' + 94^\circ 46' \\ &= 125^\circ 21'\end{aligned}$$



Traverse and Angles



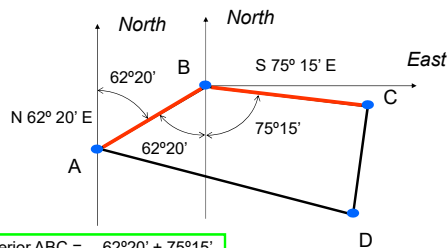
Example - Compute the interior angle at B



Traverse and Angles



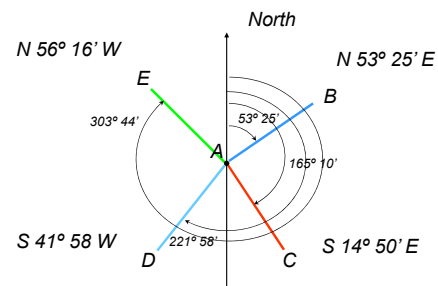
Example - Compute the interior angle at B



Angles and Directions



Compute Bearings Given the Azimuth



End of Angles



Any Questions?

