What's Really Wrong with Math Education – and How to Fix It

Solomon Garfunkel and David Mumford

1. What's Happening Now

These days It's hard to tell if it's 2010 with the "Race to the Top",1991 with "America 2000", 1984 when we had "A Nation at Risk" or the late 1950s when the nation rushed to improve math and science after Sputnik went spinning into space. Once again those who lead America have decided that our educational system, most particularly when it comes to mathematics, is not doing its job. When it comes to mathematics education there appears to be one and only one narrative - The Russians or Japanese or Chinese are overtaking us. Unless we "out-educate" the rest of the world, we will fall behind. We have to revamp our K-12 educational system now or we will become a second rate power; SEND MONEY!

What will we do with this money? We will set higher standards, create new assessments to test how students meet these standards, write new text books to enable teachers to teach to those tests and then try to encourage bright young people to become math teachers as we press teacher unions to let us fire the bad old ones. Oh and fifteen or so years from now we'll do it all over again – and tell this same story.

The only problem with all of this is that the narrative is false – blatantly false. We are a great deal better than we think we are. Our best are the world's best. When it comes to discovery and creativity in mathematics and its applications, we are the envy of the world. But what about all of those pesky international comparisons? What's behind those numbers? Don't we need more tiger mothers and fathers?

To understand you need to know that mathematics and mathematics education can be broken into two separate parts. The first deals with academic research and the contemporary uses of mathematics in business, government, and daily life, called mathematical modeling. Here our position remains preeminent (which is not to say that there are no great researchers in other parts of the world). But less than one percent of high school students will eventually enter these fields. The second deals with the remaining 99% whom we hope will use math as a tool that helps them deal with the problems of daily life. A significant percentage of these never relate to the abstract ideas and logical precision that underlie the pervasive uses of math in our technological culture.

Given that there are two very different populations who differ so greatly in both their feeling for quantitative problems and their eventual careers, it is not easy to see how best to teach math and serve everyone's needs. But recently the idea that one size should fit all has prevailed. This single track approach to mathematics education was made into law when George W. Bush's administrations enacted the *No Child Left Behind* (NCLB). Responding to the panic over our test scores and using the leverage of federal support, it requires standardized math tests to be given nationwide and penalizes all schools and teachers if 100% of their students do not pass these tests by 2014. This was only the first step, and its immediate effect was to leave the country with 50 sets of standards and 50 sets of high-stakes tests. Students were being anointed as mathematically proficient in one state when they couldn't score in the 25th percentile in another.

To deal with this problem a national standard, the *Common Core State Standards in Mathematics* (CCSSM) has now been written. Technically the Common Core State Standards in K-12 mathematics were created by the National Governor's Association and the Council of Chief State Officers and funded by private foundations. In reality they are a major part of the Obama administration's education policy. This is not to say that they were written by people in the administration – only that the idea of common standards is an article of faith of the U.S. Department of Education.

As we write this piece 44 states have agreed to abide by the CCSSM. In addition, two major state assessment consortia have been funded by the Department of Education. They are charged with preparing a bank of tests to be ready by 2014-15, when, in effect, we will have national tests as well. All this works greatly in the interests of the multi-billion dollar textbook industry. Through mergers and acquisitions, there are now only three significant textbook publishers and they also own the companies making up the tests. In other words, we are within five years of standards, tests, and curricula all marching in the same direction in a national public-private partnership. A possibly apocryphal story says that the French Minister of Education once looked at his watch and remarked to a visitor, "It is 11:27 am and every student in the such and such grade is now learning the rule for long division". We are on track to duplicate this Napoleonic system

But the material which will soon be taught to all K-12 students under the CCSSM does not in fact equip them with the tools they desperately need to lead knowledgeable lives in the 21st century. The CCSSM is based on the assumption that there is a single established body of mathematical skills that everyone needs to know. We think this assumption is wrong. The truth is that different sets of math skills are useful for different careers, and our math education should be changed to reflect this fact. The CCSSM is strongly oriented to college bound students who will move to careers in Mathematics, Science or Engineering, or, to use the acronym favored by educators, 'STEM' students. Many of these specialized skills are not merely useless to the large majority of students but, by their seeming irrelevance to anything in the world, often drive students away from every kind of mathematics. How many people do you know who, outside of a math class, have ever solved or needed to solve, a *quadratic equation in x*? Surely students should be taught first those mathematical ideas which have relevance to their lives.

This debate about math has a long history. For some in the higher education community, mathematics education means the education of mathematicians – the replenishing of the species. And if we are honest, up until 1988, this was the tail that wagged the dog. Certainly from Sputnik on, the mathematics curriculum was designed to be a sequence of courses leading from kindergarten to college to graduate work, with each course's main purpose to be a prerequisite for the next one and so on. The fact that the half-life of mathematics students was (and basically still is) one year from 10th grade on was simply ignored. In other words, the number of students in 11^{thh} grade math courses was half those in 10^{th} grade, the number in 12^{th} grade half those in 11^{th}, right on to through graduate school. Our curriculum was designed for motivated and talented students in mathematics. What happened to the others was not our concern.

In 1989 the National Council of Teachers of Mathematics (NCTM), the major school mathematics professional society, produced their *Principles and Standards for School Mathematics.* This was the community of school mathematics teachers saying that we needed to change what was taught in mathematics classrooms and how it was taught. Texts based on this work began to appear in the mid 90's. The backlash to the new texts (especially the new high school programs) was fierce. This became known as the 'Math Wars'. In a general way, the opposition to the new 'reform' texts centered on the belief that they were in some way watering down the curriculum. What the reformers called sense-making and conceptual understanding, the opponents called 'fuzzy'.

Research professors of math in endowed chairs at universities like Harvard and Berkeley claimed they were better qualified to decide how math should be taught in K-12. Parents of students who had always succeeded in the traditional system and whose kids got into the elite colleges saw no reason to change the rules of a game that they were winning. Math for all sounds a lot better when you're on the bottom looking up. So a discussion about the mathematics curriculum became a fight about politics and religion and class. It was not hard to predict the winner of that battle. And so, by 2009, the Wall Street journal and the New York Times felt comfortable in saying that the Math Wars were over and that the back to basics movement had won. But the U.S. was still doing poorly on international comparisons and that was an embarrassment. Enter the Common Core.

The content specialists in the group which wrote and reviewed the CCSSM consisted almost entirely of Professors with a PhD in pure math. In fact, these groups included all the leaders of the traditionalist side of the Math Wars. The NCTM was all but ignored as were the computer science and engineering communities. Among the various stakeholders it is clear that teachers have fared the worst. They will be held responsible for implementing a new program of standards, assessment, and curricula with little input and (in most cases) insufficient support and preparation. Many mathematical researchers will be cheered by the fact that we are going back to the mythical 'good old days'.

For parents, we expect that the rich will get richer. A great deal of the energy of the 1989 reform movement was intended to deliver 'math for all'. The idea was to increase the pipeline, keep more students in more math courses longer, by showing them what this stuff was actually good for. That hope is almost certainly lost for now. Average students will likely stop taking (and paying attention in) math courses as soon as they can – except for those who find mathematics a calling as well as those highly motivated to get into elite colleges. For the parents of these children the goal of K-12 mathematics education is not a deeper conceptual understanding and (dare we say it) a love for mathematics. Rather it is passing the appropriate AP exam in order to impress the appropriate college admission officer.

2. Some Nitty-Gritty

To understand what is at stake here, we need to dig a bit deeper. We want to give an example of an important life skill and what the CCSSM would teach you about it. People talk about ‘exponential explosions’ in many situations: increase in the national debt, global warming, population etc. The phrase is often misused but basically it just means there’s something which increases by the same factor at regular intervals. The simplest example is a savings account: if it gives you 5% interest each year, your balance will be multiplied by 1.05 each year.

Every adult needs to deal with financial transactions involving rates of return, rates of repayment. For example, say you are in your 30’s, have a job and children and are purchasing your first home. You are offered the choice between a 30 year mortgage with one monthly payment and a 15 year mortgage with a higher payment. The choice is obvious isn’t it? Take the 30 year with a lower payment. But wait: you are also putting money away in an IRA for retirement: you could stop paying into the IRA and manage to meet the higher 15 year payment if you are careful. Which is really better?

Mathphobes can feel free to skip this paragraph, but that is exactly our point, it’s too bad if you do. First, using only arithmetic, you can add up all your monthly payments and figure out the total interest each mortgage saddles you with. Wow: the 15 year mortgage looks a lot better now. But here’s the “fun” part where real math comes in. If you put the money saved from the lower payments on a 30 year mortgage into the IRA, you will get a significant return in the saved funds. Common sense tells you that if this return is large enough, the 30 year will be a better bet; but if it is too small, the 15 year is better. To make a choice, you need some math that every financial adviser knows (or so we’d hope). Compounding returns leads to exponentially increasing balances and, conversely, payments deferred are an exponentially decreasing drain on present assets, decreasing with the number of years they are deferred. The calculation needed to compare the two strategies now turns out to be a simple one if you have learned to do the math associated with exponential processes. OK – end of math lesson.

A similar example is computing the savings needed for retirement– though saving for buying a car is more likely to catch the attention of adolescents. Through playing with such examples, e.g. on a spreadsheet, a student will be able to go on to master the formal math of exponents and will then be able to generalize to all sorts of ‘exponentially explosive’ situations. It’s hard to deny that this a valuable life lesson that will serve almost everyone very well.

Now let’s compare this with what’s in the CCSSM. Their treatment of exponentials begins by defining a very general concept, that of a “function”. Here’s their list of what they want every student to master (sorry more math but since you aren’t a student any more you can also skip this):

> *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of*

f is the graph of the equation y = f(x).

In other words, the concept of “function” is rigorously defined first, defined in fact exactly as it would be in a graduate math textbook. Understanding the notation for such abstract objects is their first bulleted point. There follow many topics dealing with the general theory of functions (e.g. that functions are objects which can themselves be added as well as “composed” with each other) and dealing with their use in modeling. Finally there is exactly one sentence in the six pages of these requirements which mentions the relevance of exponentials to money: “Modeling savings account balance, bacterial colony growth, or investment growth.” is one of eight categories of applications mentioned (it’s curious that money and bacteria were lumped together). What is almost entirely missing is the pedagogical necessity to work with multiple concrete examples before going on to make an abstract framework out of them.

To be honest, we have discussed these points with Prof. William McCallum, the leader of the Mathematics Work Team which wrote the CCSSM. He points out that the Standards say explicitly in the beginning that they are not dictating teaching methods and that topics can be permuted. However, it is hard for us to see how a textbook writer implementing the Standards could turn them on their head, expanding some sections and shrinking most of the others and come out with anything like what we think works, namely that the mathematics will only be absorbed and incorporated into a student’s toolkit if it is built on authentic real world applications. The abstract structure of math should come later and only to the degree that it matches the students developing way of thinking about the world.

How did this rather obvious idea get lost?

Dr. Henry Pollak, who led the mathematical research teams at Bell Labs and BellCore for over 25 years tells how he chose the researchers he hired. No matter how excellent their resumes, he insisted that they be able to describe a way in which they used mathematics in their everyday lives. He refused to hire candidates who didn’t have an answer. To him and to us, mathematics is about understanding how the world works.

It’s hard to deny that the most important reason to be the master of mathematics is money. *S*tudying finance begins with mastering the technology of spreadsheets that can easily solve challenges like the mortgage example we discussed before. How much more prosperous would we all be today if every adult had had the confidence before the recession to run their own numbers and critically assess their finances. Note how easily the basics of algebra can be built up from topics in finance. Using spreadsheets, one typically calculates the value of a cell in terms of the value of other cells with a simple formula. In a very concrete situation, the student encounters symbolic references to other cells (e.g. ‘D6’ for the number in column D, row 6) and the use of simple algebraic formulas for combining the numbers in two cells. Sums and means of columns and graphs of many types come up naturally. Realistic budgets of typical people from those on welfare to billionaires, of actual businesses and entire governments can be studied and compared. You want to teach every student a sense of the ebb and flow of money in the large society into which he or she is growing up: the ‘syntax of money’ as Robertson Davies called it.

After several authentic examples of some type of financial problem, a student is ready to understand the ideas from a more general mathematical perspective. A teacher who draws examples from the lives of the students has changed the ideas of Algebra 1 from formulas floating in space into tools that can solve problems. .

Data is all around us in every daily newspaper and all over the internet. Students will remember a statistical idea if they get their own data on something that interests them, sports statistics for example, and then work out a model for it. In the HBO series *The Wire*, the character Prez wakes up his ghetto math class when he explains how to work out the odds in craps. Games like poker demand a sophisticated mastery of odds and the ability to adapt your odds as you learn the bluffing behavior of others at the table. One of us taught a class at Brown in which we got data on the murder rate in Boston and the GNP, year by year. It was remarkable that an increase first predicted a decline in the second – with a year's lag. This is also an excellent example of how one should not infer causation from a correlation, one of the big traps that people without a good probability course often fall into. The murders certainly didn't cause next year's recession though once you know about them, you might revise your odds of a recession happening!

Students particularly like learning the ways statistics are misused – the root of Mark Twain's quip, "lies, damned lies and statistics". For example, in the Challenger disaster, it was estimated that the probability of the failure of each O-ring was 2.3%. Since there were six of them, the probability of a catastrophe where they all failed was estimated to be the result of multiplying 0.023 by itself 6 times, less than 1 in a billion. This would have been fine if all six O-rings lived in separate universes. But they were all on the same shuttle and all were simultaneously affected by cold temperatures. This is the meaning of saying that the failures of the O-rings were not *independent* events. Modern life is more often about judgments than exact rules. Probabilities are the math that helps you make informed judgments.

[Insert photo of Feynman dipping an O-ring in ice water near here.]

There is a third area where more knowledge would help everyone navigate the contemporary world: technology. In the 'old days', every boy knew how an internal combustion engine worked and could replace the head gasket. Who now has a clue what is going in inside their computer, inside cell phones or MRI tunnels? Every student could use a course in basic engineering or "How Machines Work" We swim in a technological sea without having much of a clue what holds us up. This passivity is not necessary nor is it useful.

This may sound something like old fashioned "Voc Ed" but we propose teaching math while looking at machines. The course should be packed full of the simple math relationships from physics and geometry that genuinely help to quantify the world and its machines. For example, how does the number of cc's in a car's engine affect its horse power? One mistake is that mathematics and science courses at the high school level are taught as though they as independent as Sex Ed and Shakespeare. Historically, this couldn't be farther from the truth: math and physics (and astronomy, geology, chemistry, etc.) have developed lock step in tandem. Galileo, Newton and Einstein for example combined math with experimental data with amazing results. In exactly the same way, Math comes alive in the classroom if it is being used to model sizes, forces or processes. Why are algebraic formulas useful anyway? It is because many of the important things we measure are numerically related to each other and this relationship is most clearly expressed by such a formula. If you want students to see the value of algebraic formulas, they must be formulas which mean something, not formulas which are meaningless concoctions of the textbook writer.

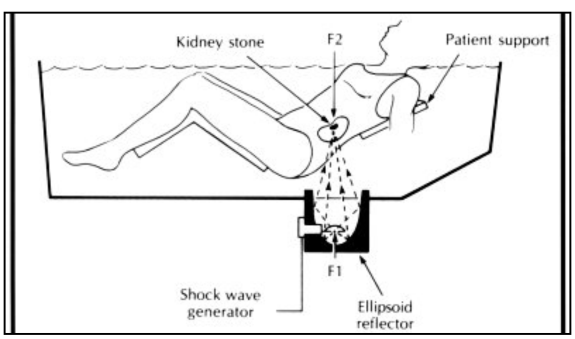
In its monolithic style, he CCSSM introduces formulas starting in the 6th Grade in essentially the same abstract formal style as their introduction of functions. Thus the use of the symbols *x* and *y* and the solution and graphing of linear equations are already introduced in the 6th grade. Here's a quote: *"(The students) use equations (such as 3x=y) to describe relationships between quantities".* We don't doubt that linear equations arise in an awful lot of jobs. For instance, businesses need to optimize their profits and their profit is typically a linear combination of many variable receipts and expenses subject to various constraints. Perhaps in the 6th grade, a month should instead be spent on the operation of a lemonade stand.

This may be a shock to both pure mathematicians and lay readers but in fact scientists rarely use the infamous symbol *x* which many students never understand. Almost always their formulas relate to real measurable quantities and are expressed by *abbreviations*. In Einstein's famous rule $E=m.c^2$, E is an abbreviation for energy, m for mass and c for the speed of light. To give a high school example, the basic fact that "distance traveled equals time elapsed times speed of travel" should be written dist=vel x time or simply $d=v.t$. Using *x* or, even stranger, an empty box (never seen outside a few School textbooks) for a variable guarantees that students will be confused. Computer scientists are even trained never to use *x* in their code for the simple reason that if they ever need to modify the code, they will need to remember what *x* stood for. With an abbreviation the meaning will be clear.

More than other machines, the computer is dominating our lives today. Every high school student should learn that the mysterious box is doing nothing more complicated than carrying out a long list of commands – fetch this number, add it to this, do something else if the result is zero, etc. Computer '*code*' is just a recipe which uses numbers instead of eggs. Every student should try their hands at writing such code and see what happens. The study of such code introduces the student to the fundamental idea of an 'algorithm', a set of rules making each consecutive step of a calculation precise and the starting point for all of computer science. This is a wonderful way of teaching students to express their understanding of a rule in a formal language. It builds on their everyday casual use of computer applications to motivate rigorous math.

Note how all the standard high school math topics arise in some context. All the basic laws of physics are given by simple formulas and, in working out problems, the manipulation of formulas becomes natural. Talking about music, cell phones and TV will bring up vibrations and periodic signals, which lead immediately to trigonometry. Rates of change are key factors in almost every model and these introduce naturally one of the key ideas of calculus. The facts of geometry such as Pythagoras's theorem arise from the need to measure the world.

Mathematics has been an increasingly central tool at every level of the life sciences in recent decades. We can't resist describing an elegant medical device called the lithotripter. Talk of the agony of kidney stones will wake up every student. Suppose they have learned in geometry class the ancient Greek discovery that a certain special shape, the ellipsoid, has two points in it called foci with a miraculous property that starting in any direction from one focus and bouncing off the wall of the ellipsoid when you hit it, you come to the second focus. The lithotripter uses an ellipsoid positioned so the kidney stone is one focus and a source of very loud sound at the other. All the energy in the sound wave comes together to shatter the stone.



In a lithotripter, sound is focused using an ancient property of ellipsoids

The examples of real life problems we have given so far relate to Middle and High School math. But the same problems arise in Elementary School too: how to make simple arithmetic seem relevant. When we published a New York Times Op-ed piece on this topic last August, we received a wonderful comment from Miriam Sicherman, a teacher in New York:

“I'd like to give the perspective of an elementary school teacher. I have taught 3rd grade math for 11 years. The difference in student attitude and commitment when they are faced with a real-life problem, versus an abstract or unrealistic problem, is dramatic. This year, for example, I told my kids we'd cook a Native American recipe (connected to our social studies work) if they could figure out how many times we needed to multiply the recipe, and if they could then make the proportionate changes. Their stamina in solving this problem, arguing about it, trying to confirm their results, was incredible. My school uses a written curriculum that encourages kids to deeply explore mathematical ideas, but uses many very boring or unrealistic scenarios in the problems presented. So I try to modify the problems, or make up my own. When elementary kids see a purpose in math--and when they have a chance to argue about it--they are highly motivated to understand the underlying principles.”

3. But…..

Real life problems certainly motivate students of all ages and can leave them with a lifelong tool for understanding the world. But what about the traditional course sequence algebra-geometry-more algebra-precalculus-calculus and the training of students headed to college and on to a quantitative career? The word “tracking” has acquired a bad name in today’s politically correct atmosphere. But it is hard to doubt that, whether because of genes or upbringing, from nature or nurture, there is a huge range of both skills and appetite for mathematics. No one doubts that there is a great range of athletic skills in the whole population. Howard Gardner has popularized the idea that there are seven forms of “intelligence”, one of which is what he called the “logical-mathematical” one. It would certainly be strange if we didn’t track athletes. If we gave everyone the same intensive training in basketball (and then gave them all a uniform test which they were required to pass), short people would recall basketball with hatred and future NBA players would drop out of school in frustration.

An excellent example of tracking in math has been adopted by West Virginia. They have three tracks. First, an accelerated track for STEM students is split off from the main one in 8^{th} Grade. But that is not a final choice. The main stream continues through 10^{th} Grade and then splits into three: another STEM track which will lead to Calculus, a second is a Liberal Arts track and a third the “Technical Readiness” track. All three prepare the students for college or careers and the first two have multiple alternatives in the last year. In such a structure, exciting relevant math can be taught to the main stream without any loss to STEM students whose joy it is to race ahead. Moreover, the non-STEM tracks can contain easily enough math to enable motivated students to switch to STEM careers later.

One of the staples of algebra is the theory of polynomials, especially factoring them and solving quadratic equations. This is a clear example of where STEM and non-STEM tracks should diverge. We know of few topics which are as rarely useful in life as these. Of course researchers in math and physics use these and arguably a few other STEM professions but who else does? The CCSSM not merely includes these topics but offers a few other goodies –for example, how to use complex numbers to solve quadratic equations with no real roots. Ah: you don't know what we are talking about? Neither did Cardano in 1545 when he stated that exactly this procedure was "as refined as it is useless". Seriously, Cardano was wrong about it being useless – if you ever want to learn quantum mechanics for example, you'll have to learn it. If you really want to know what all 11th graders in the US will soon be asked to learn, see the box (this is optional, we promise it won't be on the test). By the way, unlike their "advanced" bullets for college bound seniors only, this one is really for all students.

Using the Grade Eleven Common Core requirements

Find two numbers whose sum is 10 and product is 40

The answer you should get is $\left(5+\sqrt{-15}\right)$ and $\left(5-\sqrt{-15}\right)$

Don't ask what $\sqrt{-15}$ really *is*.

What do the traditionalists say to justify being so abstract? A central argument by those whose espouse the rigorous approach taken by the Common Core is that it alone will develop our skills at thinking logically. The Common Core articulates its goals as *"reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; look for and make use of structure*" (among others). The implicit assertion is that traditional math, such as the factoring of polynomials, is the royal road to achieving these skills. But there is no reason why a curriculum based on matters connecting to student's concerns, to *contexts* in the world, shouldn't foster the same skills. Clear reasoning and observation of structure are needed just as much in finance, statistics and engineering. It's all very similar to what was believed about the study of Latin until recently. It was widely claimed, especially in private schools, that the study of Latin gave one unique skills of some ineffable nature. No one seems to be suffering now that Latin is gone.

Our argument is really not with CCSSM as an example of an exhilarating curriculum. In principle, some school could combine 'math in context' with much of the CCSSM if they taught it in a dialectic style. This would work like this: start with multiple real world tangible problems which make immediate sense to students. Then abstract a bit to a general approach which unifies them. Remark on some rules which must be followed or you get into trouble. Then go back to examples, build up to the next stage of abstraction etc.

Rather our strongest disagreement is with the draconian use of tests which will enforce the use of the CCSSM for every student. The tests are even now smothering the spirit of diversity which is one of the best features of American education. America's college system is the envy of the world because of its wonderful diversity. Well-to-do people all over the world send their children to the US for college because of its excellence. We know that many high school and high school teachers make exceedingly good use of the freedom to innovate and that this is one of the most fulfilling aspects of their jobs. Many combinations of traditional topics with real world based mathematics are possible. In fact many sequences of alternate curricula were written, published and taught under a National Science Foundation program in the 90's. But our experience since NCLB is that teachers are nearly universally teaching to the test. They have no choice. Even without the tests, some schools could adopt the fully rigorous and challenging approach to math: Phillips Exeter Academy does this with astonishing success. Charter schools have initiated many wonderful new experiments. The topics we discussed at length above are but a small sample of the kinds of math to teach in High School.

. We would argue for a curriculum based on a rich mixture of real world examples and those parts of theoretical algebra and geometry which equip 99% of high school students with the all math they will need in later life. For over a decade the *Quantitative Literacy* movement, spearheaded by Lynn Steen at St. Olaf's College and David Bressoud at Macalester College, has been preaching for this sort of knowledge[1],

> "*Quantitative literacy is the power and habit of mind to search out quantitative information, critique it, reflect on it and apply it to one's public, personal and professional life. The mathematics can be very simple. It is the ability to work in context that makes this a very demanding discipline and, for quantitative literacy, context is everything. The goal is to empower students to reason with the complex quantitative information that is omnipresent in today's world."* David Bressoud, former President, Mathematical Association of America.

The educational establishment is a gargantuan machine of interlocking interests. In this machine we have the students, the teachers, their unions, the school administrators, the school boards, the teacher training programs, the textbook writers, the testing establishment, the college admissions staffs and finally the parents. Changing the high school math curriculum is harder than changing the location of a cemetery. It is simply not realistic to ask one school to experiment with such changes. Few parents want to have their child to be guinea pigs, teachers do not want to learn new areas and textbook writers do not want to experiment where there is no market.

1 From his online column http://www.maa.org/columns/launchings/launchings_09_05.html

College Admissions Offices are in many ways the lynchpin of these interlocking interests. If they adhere closely to student's performance on the standardized tests and SAT, they prevent anyone at an earlier stage from daring to change the system.

But we will make a good start if we at least become conscious that there is a choice. The math curriculum has changed before and can change again. Every parent of every high school student should take a look at their children's math homework and then ask: which problems concern things you think are important for your child to learn? Hopefully this will start a national dialog in which incremental changes can be made.

We started this article by alluding to the periodic scares that the US is being left behind by its competitors. In the latest cycle, it is especially the Chinese, in Shanghai or Singapore that make headlines. It may give some perspective to ask what role math played in the last two millennia of Chinese culture. Throughout most of this period, bureaucrats have had to pass an imperial exam and, although poetry, history and the Confucian classics were most important, math was required in many dynasties too. And what was this math: it was all taught through real world examples from which the student was assumed capable of inferring the general principles.

In the end the decisions that we make about how we teach mathematics are not about whether we will be overtaken by some other countries best and brightest. It is a decision about what kind of society we want to live in, about the opportunities that we want to give to all of our children to have happier and more productive lives. Both of the authors have loved math all their lives and want to spread the message – there is something in it for everyone.S

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