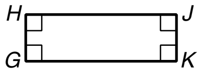
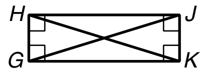
Name ______________________________________ Date ___________ Class ____________

LESSON 6-4

# Reteach

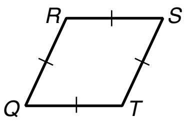
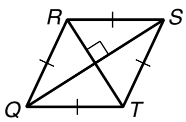
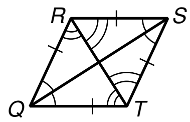
## *Properties of Special Parallelograms*

A **rectangle** is a quadrilateral with four right angles. A rectangle has the following properties.

| Properties of Rectangles | |
|---|---|
|   *GHJK* is a parallelogram. If a quadrilateral is a rectangle, then it is a parallelogram. |   $\overline{GJ} \cong \overline{HK}$ If a parallelogram is a rectangle, then its diagonals are congruent. |

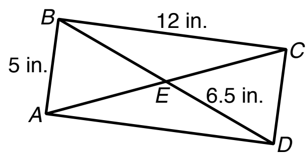
Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.

A **rhombus** is a quadrilateral with four congruent sides. A rhombus has the following properties.

| Properties of Rhombuses | | |
|---|---|---|
|   *QRST* is a parallelogram. If a quadrilateral is a rhombus, then it is a parallelogram. |   $\overline{QS} \perp \overline{RT}$ If a parallelogram is a rhombus, then its diagonals are perpendicular. |   $\angle RQS \cong \angle SQT$ If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. |

Since a rhombus is a parallelogram, a rhombus also has all the properties of parallelograms.

***ABCD* is a rectangle. Find each length.**



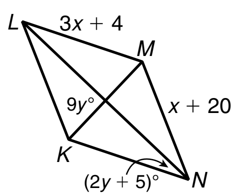

**1.** *BD* ________________________

**2.** *CD* ________________________

**3.** *AC* ________________________

**4.** *AE* ________________________

***KLMN* is a rhombus. Find each measure.**




**5.** *KL* ________________________

**6.** $m\angle MNK$ ________________________

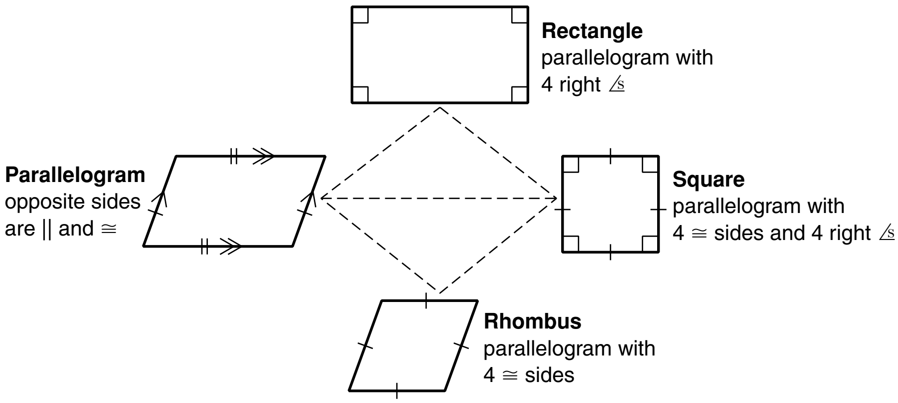
Name ______________________ Date __________ Class __________

LESSON 6-4

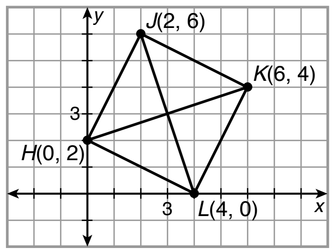
# Reteach

## *Properties of Special Parallelograms* continued

A **square** is a quadrilateral with four right angles and four congruent sides.
A square is a parallelogram, a rectangle, and a rhombus.




**Show that the diagonals of square *HJKL* are congruent perpendicular bisectors of each other.**




**Step 1** Show that $\overline{HK} \cong \overline{JL}$.

$HK = \sqrt{(6-0)^2 + (4-2)^2} = 2\sqrt{10}$

$JL = \sqrt{(4-2)^2 + (0-6)^2} = 2\sqrt{10}$

$HK = JL = 2\sqrt{10}$, so $\overline{HK} \cong \overline{JL}$.

**Step 2** Show that $\overline{HK} \perp \overline{JL}$.

slope of $\overline{HK} = \frac{4-2}{6-0} = \frac{1}{3}$ slope of $\overline{JL} = \frac{0-6}{4-2} = -3$

Since the product of the slopes is $-1$, $\overline{HK} \perp \overline{JL}$.

**Step 3** Show that $\overline{HK}$ and $\overline{JL}$ bisect each other by comparing their midpoints.

midpoint of $\overline{HK} = (3, 3)$ midpoint of $\overline{JL} = (3, 3)$

Since they have the same midpoint, $\overline{HK}$ and $\overline{JL}$ bisect each other.

**The vertices of square *ABCD* are *A*(−1, 0), *B*(−4, 5), *C*(1, 8), and *D*(4, 3). Show that each of the following is true.**

**7.** The diagonals are congruent.

_______________________________________________

**8.** The diagonals are perpendicular bisectors of each other.

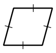
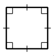
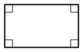
_______________________________________________

_______________________________________________

LESSON 6-4

## Practice A
### Properties of Special Parallelograms

**Match each figure with the letter of one of the vocabulary terms. Use each term once.**

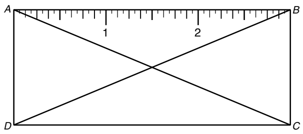
1.  B
2.  C
3.  A

**A.** rectangle
**B.** rhombus
**C.** square

**Fill in the blanks to complete each theorem.**

4. If a parallelogram is a rhombus, then its diagonals are perpendicular.
5. If a parallelogram is a rectangle, then its diagonals are congruent.
6. If a quadrilateral is a rectangle, then it is a parallelogram.
7. If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.
8. If a quadrilateral is a rhombus, then it is a parallelogram.

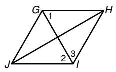
**The part of a ruler shown is a rectangle with $AB = 3$ inches and $BD = 3\frac{1}{4}$ inches. Find each length.**




9. $DC =$ 3 in.
10. $AC =$ $3\frac{1}{4}$ in.

**Use the phrases and theorems from the Word Bank to complete this two-column proof.**

Alternate Interior ∠s Thm.
*GHIJ* is a parallelogram.
Trans. Prop. of ≅
∠2 ≅ ∠3




11. **Given:** *GHIJ* is a rhombus.
**Prove:** ∠1 ≅ ∠3

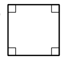
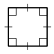
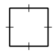
| Statements | Reasons |
|---|---|
| 1. *GHIJ* is a rhombus. | 1. Given |
| 2. **a.** *GHIJ* is a parallelogram. | 2. rhomb. → ▱ |
| 3. $\overline{GH} \parallel \overline{JI}$ | 3. ▱ → opp. sides ∥ |
| 4. ∠1 ≅ ∠2 | 4. **b.** Alternate Interior ∠s Thm. |
| 5. **c.** ∠2 ≅ ∠3 | 5. rhomb. → each diag. bisects opp. ∠s |
| 6. ∠1 ≅ ∠3 | 6. **d.** Trans. Prop. of ≅ |



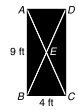

LESSON 6-4

## Practice B
### Properties of Special Parallelograms

**Tell whether each figure must be a rectangle, rhombus, or square based on the information given. Use the most specific name possible.**

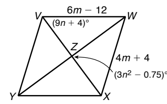
1.  rectangle
2.  square
3.  rhombus

**A modern artist's sculpture has rectangular faces. The face shown here is 9 feet long and 4 feet wide. Find each measure in simplest radical form. (*Hint:* Use the Pythagorean Theorem.)**




4. $DC =$ 9 feet
5. $AD =$ 4 ft
6. $DB =$ $\sqrt{97}$ feet
7. $AE =$ $\frac{\sqrt{97}}{2}$ ft

***VWXY* is a rhombus. Find each measure.**

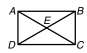



8. $XY =$ 36
9. m∠$YVW =$ 107°
10. m∠$VYX =$ 73°
11. m∠$XYZ =$ 36.5°

12. The vertices of square *JKLM* are $J(-2, 4)$, $K(-3, -1)$, $L(2, -2)$, and $M(3, 3)$. Find each of the following to show that the diagonals of square *JKLM* are congruent perpendicular bisectors of each other.

$JL =$ $2\sqrt{13}$ $KM =$ $2\sqrt{13}$
slope of $\overline{JL} =$ $-\frac{3}{2}$ slope of $\overline{KM} =$ $\frac{2}{3}$
midpoint of $\overline{JL} =$ (0, 1) midpoint of $\overline{KM} =$ (0, 1)

**Write a paragraph proof.**




13. **Given:** *ABCD* is a rectangle.
**Prove:** ∠*EDC* ≅ ∠*ECD*

Possible answer: *ABCD* is a rectangle, so $\overline{AC}$ is congruent to $\overline{BD}$. Because *ABCD* is a rectangle, it is also a parallelogram. Because *ABCD* is a parallelogram, its diagonals bisect each other. By the definition of bisector, $EC = \frac{1}{2}AC$ and $ED = \frac{1}{2}BD$. But by the definition of congruent segments, $AC = BD$. So substitution and the Transitive Property of Equality show that $EC = ED$. Because $\overline{EC} \cong \overline{ED}$, △*ECD* is an isosceles triangle. The base angles of an isosceles triangle are congruent, so ∠*EDC* ≅ ∠*ECD*.



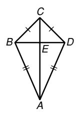

LESSON 6-4

## Practice C
### Properties of Special Parallelograms

**For Exercises 1–5, give your answers in simplest radical form.**

1. Find the length of the diagonals of a rectangle with sides of lengths *a* and *b*. $\sqrt{a^2 + b^2}$
2. Find the length of the diagonals of a square with sides of length *a*. $\sqrt{2}a$
3. Find the length of the sides of a square with diagonals of length *a*. $\frac{\sqrt{2}}{2}a$
4. Find the length of the sides of a rhombus with diagonals of lengths *a* and *b*. $\frac{\sqrt{a^2 + b^2}}{2}$
5. Find the length of a rectangle with width *x* and a diagonal of length 2*x*. $\sqrt{3}x$
6. Find the measures of the angles in the triangles formed by one diagonal of the rectangle in Exercise 5. 30°-60°-90°

**The figure shows a kind of quadrilateral called a *kite.* A kite is a quadrilateral with exactly two pairs of congruent consecutive sides. Use the figure to write paragraph proofs for Exercises 7 and 8.**




7. **Prove:** ∠*CBA* ≅ ∠*CDA*
Possible answer: It is given that $\overline{CB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{AD}$. $\overline{CA}$ is congruent to $\overline{CA}$ by the Reflexive Property of Congruence. Thus △*ABC* is congruent to △*ADC* by SSS. By CPCTC, ∠*CBA* ≅ ∠*CDA*.

8. **Prove:** $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.
Possible answer: It is given that $\overline{CB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{AD}$. So *C* and *A* are on the perpendicular bisector of $\overline{BD}$ by the Conv. of the Perpendicular Bisector Thm. So, since two points determine a line, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

**For Exercises 9–11, name all the types of quadrilaterals (kite, parallelogram, rectangle, rhombus, or square) that satisfy the given conditions.**

9. The diagonals bisect each other.
parallelogram, rectangle, rhombus, square

10. The diagonals are perpendicular.
kite, rhombus, square

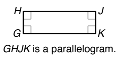
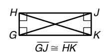
11. The diagonals are congruent.
rectangle, square




LESSON 6-4

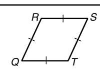
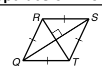
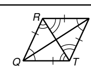
## Reteach
### Properties of Special Parallelograms

A **rectangle** is a quadrilateral with four right angles. A rectangle has the following properties.

| Properties of Rectangles | |
|---|---|
|  *GHJK* is a parallelogram. If a quadrilateral is a rectangle, then it is a parallelogram. |  $\overline{GJ} \cong \overline{HK}$ If a parallelogram is a rectangle, then its diagonals are congruent. |

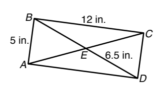
Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.

A **rhombus** is a quadrilateral with four congruent sides. A rhombus has the following properties.

| Properties of Rhombuses | | |
|---|---|---|
|  *QRST* is a parallelogram. If a quadrilateral is a rhombus, then it is a parallelogram. |  $\overline{QS} \perp \overline{RT}$ If a parallelogram is a rhombus, then its diagonals are perpendicular. |  ∠*RQS* ≅ ∠*SQT* If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. |

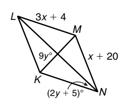
Since a rhombus is a parallelogram, a rhombus also has all the properties of parallelograms.

***ABCD* is a rectangle. Find each length.**




1. *BD* 13 in.
2. *CD* 5 in.
3. *AC* 13 in.
4. *AE* 6.5 in.

***KLMN* is a rhombus. Find each measure.**



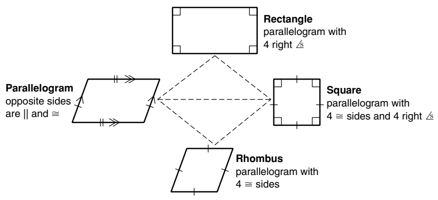

5. *KL* 28
6. m∠*MNK* 50°

LESSON 6-4

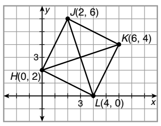
## Reteach

### *Properties of Special Parallelograms* continued

A **square** is a quadrilateral with four right angles and four congruent sides. A square is a parallelogram, a rectangle, and a rhombus.




**Show that the diagonals of square *HJKL* are congruent perpendicular bisectors of each other.**




**Step 1** Show that $\overline{HK} \cong \overline{JL}$.

$HK = \sqrt{(6-0)^2 + (4-2)^2} = 2\sqrt{10}$

$JL = \sqrt{(4-2)^2 + (0-6)^2} = 2\sqrt{10}$

$HK = JL = 2\sqrt{10}$, so $\overline{HK} \cong \overline{JL}$.

**Step 2** Show that $\overline{HK} \perp \overline{JL}$.

slope of $\overline{HK} = \frac{4-2}{6-0} = \frac{1}{3}$ slope of $\overline{JL} = \frac{0-6}{4-2} = -3$

Since the product of the slopes is $-1$, $\overline{HK} \perp \overline{JL}$.

**Step 3** Show that $\overline{HK}$ and $\overline{JL}$ bisect each other by comparing their midpoints.

midpoint of $\overline{HK} = (3, 3)$ midpoint of $\overline{JL} = (3, 3)$

Since they have the same midpoint, $\overline{HK}$ and $\overline{JL}$ bisect each other.

**The vertices of square *ABCD* are *A*(−1, 0), *B*(−4, 5), *C*(1, 8), and *D*(4, 3). Show that each of the following is true.**

7. The diagonals are congruent.

$AC = BD = 2\sqrt{17}$, so $\overline{AC} \cong \overline{BD}$.

8. The diagonals are perpendicular bisectors of each other.

Slope of $\overline{AC} = 4$ and slope of $\overline{BD} = -\frac{1}{4}$, so $\overline{AC} \perp \overline{BD}$. The mdpts. of $\overline{AC}$ and $\overline{BD}$ are at (0, 4), so $\overline{AC}$ and $\overline{BD}$ bisect each other.

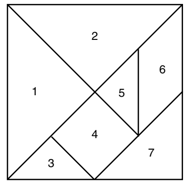


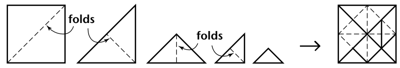
LESSON 6-4

## Challenge

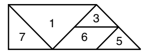
### *Quadrilaterals and Tangrams*

The figure at the right shows a Chinese puzzle called the *tangram.* No one is certain how old the puzzle is, but it was unquestionably one of the most popular puzzles of the nineteenth century. The seven puzzle pieces are called tans. To make your own tans, fold a large square sheet of paper in half four times as shown below. Unfold the paper, draw the segments shown, then cut along the segments.



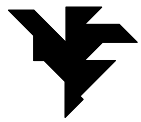

Some or all of the tans can be arranged to form special types of quadrilaterals. For instance, the figure at the right shows how tans 1, 3, 5, 6, and 7 can be arranged to form a trapezoid.




**Complete the table below by arranging the given number of tans to form each figure. If it is not possible to make the figure, enter an *X* in the table.**

| | Number of tans | Square | Rectangle (not square) | Isosceles trapezoid | Trapezoid (not isosceles) | Rhombus (not square) | Parallelogram (not rhombus or rectangle) |
|---|---|---|---|---|---|---|---|
| 1. | 2 | | X | | | X | |
| 2. | 3 | | | | | X | |
| 3. | 4 | | | | | X | |
| 4. | 5 | | | | | X | |
| 5. | 6 | X | | | | X | X |
| 6. | 7 | | | | | X | |

7. Tangram puzzles arise from the countless silhouetes of people, animals, and objects that can be formed by arrangements of the tans. Find an arrangement of the tans that forms the silhouette of a bird, as shown at right. Sketch your answer on a separate sheet of paper.



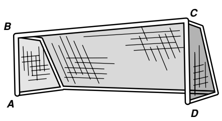


LESSON 6-4

## Problem Solving

### *Properties of Special Parallelograms*

**Use the diagram for Exercises 1 and 2.**
The soccer goalposts determine rectangle *ABCD*.



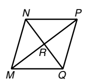

1. The distance between goalposts, *BC*, is three times the distance from the top of the goalpost to the ground. If the perimeter of *ABCD* is $21\frac{1}{3}$ yards, what is the length of $\overline{BC}$?

8 yd

2. The distance from *B* to *D* is approximately $(x + 10)$ feet, and the distance from *A* to *C* is approximately $(2x - 5.3)$ feet. What is the approximate distance from *A* to *C*?

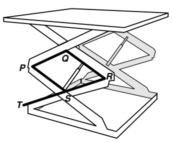
25.3 ft

3. *MNPQ* is a rhombus. The measure of ∠*MRQ* is $(13t - 1)°$, and the measure of ∠*PQR* is $(7t + 4)°$. What is the measure of ∠*PQM*?



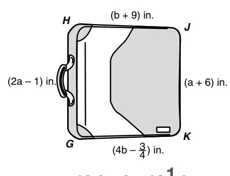

106°

4. The *scissor lift* forms rhombus *PQRS* with $PQ = (7b - 5)$ meters and $QR = (2b - 0.5)$ meters. If *S* is the midpoint of $\overline{RT}$, what is the length of $\overline{RT}$?




2.6 m

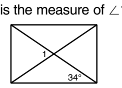
5. The diagram shows the lid of a rectangular case that holds 80 CDs. What are the dimensions of the case?




13 in. by $12\frac{1}{4}$ in.

**Choose the best answer.**

6. What is the measure of ∠1 in the rectangle?




A 34°
Ⓑ 68°
C 90°
D 146°

7. A square graphed on the coordinate plane has a diagonal with endpoints *E*(2, 3) and *F*(0, −3). What are the coordinates of the endpoints of the other diagonal?

Ⓕ (4, −1) and (−2, 1)
G (4, 0) and (−2, 1)
H (4, −1) and (−3, 1)
J (3, −1) and (−2, 1)

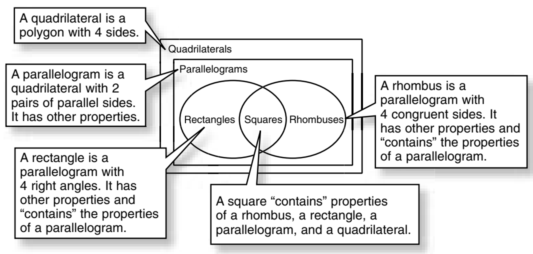


LESSON 6-4

## Reading Strategies

### *Use a Graphic Organizer*

This graphic organizer shows that each inside shape contains all the properties of the boxes surrounding it. For example, the shape with "squares" is inside the other shapes. Thus, a square "contains" all the properties of rectangles, rhombuses, parallelograms, and quadrilaterals.




**Use the graphic organizer above to answer Exercises 1–8.**

1. Is a triangle a quadrilateral? no
2. Is a square a rectangle? yes
3. Is a rhombus always a parallelogram? yes
4. Is a rectangle always a rhombus? no
5. Is a quadrilateral always a parallelogram? no
6. What do all quadrilaterals have in common?
   They are all polygons, and they all have 4 sides.
7. What would you have to change in a rhombus to make it a square?
   All 4 angles would have to be right angles.
8. What would you have to change in a rectangle to make it a square?
   All 4 sides would have to be congruent.