CHAPTER 2 THE BASICS OF SUPPLY AND DEMAND

EXERCISES

1. Consider a competitive market for which the quantities demanded and supplied (per year) at various prices are given as follows:

Price	Demand	Supply (millions)	
(\$)	(millions)		
60	22	14	
80	20	16	
100	18	18	
120	16	20	

a. Calculate the price elasticity of demand when the price is \$80. When the price is \$100.

We know that the price elasticity of demand may be calculated using equation 2.1 from the text:

$$E_D = \frac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta P}{P}} = \frac{P}{Q_D} \frac{\Delta Q_D}{\Delta P}.$$

With each price increase of \$20, the quantity demanded decreases by 2. Therefore,

$$\left(\frac{\Delta Q_D}{\Delta P}\right) = \frac{-2}{20} = -0.1.$$

At P = 80, quantity demanded equals 20 and

$$E_D = \left(\frac{80}{20}\right)(-0.1) = -0.40$$

Similarly, at P = 100, quantity demanded equals 18 and

$$E_D = \left(\frac{100}{18}\right)(-0.1) = -0.56.$$

b. Calculate the price elasticity of supply when the price is \$80. When the price is \$100.

The elasticity of supply is given by:

$$E_{S} = \frac{\frac{\Delta Q_{S}}{Q_{S}}}{\frac{\Delta P}{P}} = \frac{P}{Q_{S}} \frac{\Delta Q_{S}}{\Delta P}.$$

With each price increase of \$20, quantity supplied increases by 2. Therefore,

$$\left(\frac{\Delta Q_s}{\Delta P}\right) = \frac{2}{20} = 0.1.$$

At P = 80, quantity supplied equals 16 and

$$E_s = \left(\frac{80}{16}\right)(0.1) = 0.5$$

Similarly, at P = 100, quantity supplied equals 18 and

$$E_s = \left(\frac{100}{18}\right)(0.1) = 0.56.$$

c. What are the equilibrium price and quantity?

The equilibrium price and quantity are found where the quantity supplied equals the quantity demanded at the same price. As we see from the table, the equilibrium price is \$100 and the equilibrium quantity is 18 million.

d. Suppose the government sets a price ceiling of \$80. Will there be a shortage, and, if so, how large will it be?

With a price ceiling of \$80, consumers would like to buy 20 million, but producers will supply only 16 million. This will result in a shortage of 4 million.

2. Refer to Example 2.4 on the market for wheat. At the end of 1998, both Brazil and Indonesia opened their wheat markets to U.S. farmers. (Source: http://www.fas.usda.gov/) Suppose that these new markets add 200 million bushels to U.S. wheat demand. What will be the free market price of wheat and what quantity will be produced and sold by U.S. farmers in this case?

The following equations describe the market for wheat in 1998:

and

$$Q_D = 3244 - 283P.$$

 $Q_{\rm s} = 1944 + 207P$

If Brazil and Indonesia add an additional 200 million bushels of wheat to U.S. wheat demand, the new demand curve Q'_D , would be equal to $Q_D + 200$, or

 $Q'_{\rm D} = (3244 - 283P) + 200 = 3444 - 283P.$

Equating supply and the new demand, we may determine the new equilibrium price,

$$1944 + 207P = 3444 - 283P$$
, or

490P = 1500, or $P^* = 3.06 per bushel.

To find the equilibrium quantity, substitute the price into either the supply or demand equation, e.g.,

and

$$Q_{\rm D} = 3444 - (283)(3.06) = 2,577.67$$

 $Q_{\rm s} = 1944 + (207)(3.06) = 2,577.67$

3. A vegetable fiber is traded in a competitive world market, and the world price is \$9 per pound. Unlimited quantities are available for import into the United States at this price. The U.S. domestic supply and demand for various price levels are shown below.

Price	U.S. Supply	U.S. Demand	
	(million lbs.)	(million lbs.)	
3	2	34	
6	4	28	
9	6	22	
12	8	16	
15	10	10	

18 12 4

a. What is the equation for demand? What is the equation for supply?

The equation for demand is of the form Q=a-bP. First find the slope which is

 $\frac{\Delta Q}{\Delta P} = \frac{-6}{3} = -2 = -b$. You can figure this out by noticing that every time price increases by 3 quantity demanded falls by 6 million pounds. Demand is now Q=a-2P. To find a, plug in any of the price quantity demanded points from the table: Q=34=a-2*3 so that a=40 and demand is Q=40-2P.

The equation for supply is of the form Q=c+dP. First find the slope which is

 $\frac{\Delta Q}{\Delta P} = \frac{2}{3}$. You can figure this out by noticing that every time price increases by 3

quantity supplied increases by 2 million pounds. Supply is now $Q = c + \frac{2}{3}P$. To find c

plug in any of the price quantity supplied points from the table: $Q = 2 = c + \frac{2}{3}$ (3) so

that c=0 and supply is $Q = \frac{2}{3} P$.

b. At a price of \$9, what is the price elasticity of demand? At a price of \$12?

Elasticity of demand at P=9 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{22}(-2) = \frac{-18}{22} = -0.82$.

Elasticity of demand at P=12 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{16}(-2) = \frac{-24}{16} = -1.5$.

c. What is the price elasticity of supply at \$9? At \$12?

Elasticity of supply at P=9 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{6}\left(\frac{2}{3}\right) = \frac{18}{18} = 1.0.$ Elasticity of supply at P=12 is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{8}\left(\frac{2}{3}\right) = \frac{24}{24} = 1.0.$

d. In a free market, what will be the U.S. price and level of fiber imports?

With no restrictions on trade, world price will be the price in the United States, so that P=\$9. At this price, the domestic supply is 6 million lbs, while the domestic demand is 22 million lbs. Imports make up the difference and are 16 million lbs.

4. The rent control agency of New York City has found that aggregate demand is

 $Q_D = 100 - 5P$. Quantity is measured in tens of thousands of apartments. Price, the average monthly rental rate, is measured in hundreds of dollars. The agency also noted that the increase in Q at lower P results from more three-person families coming into the city from Long Island and demanding apartments. The city's board of realtors acknowledges that this is a good demand estimate and has shown that supply is $Q_s = 50 + 5P$.

a. If both the agency and the board are right about demand and supply, what is the free market price? What is the change in city population if the agency sets a maximum average monthly rental of \$100, and all those who cannot find an apartment leave the city?

To find the free market price for apartments, set supply equal to demand:

$$100 - 5P = 50 + 5P$$
, or $P =$ \$500,

since price is measured in hundreds of dollars. Substituting the equilibrium price into either the demand or supply equation to determine the equilibrium quantity:

$$Q_D = 100 - (5)(5) = 75$$

 $Q_S = 50 + (5)(5) = 75.$

We find that at the rental rate of \$500, 750,000 apartments are rented.

and

If the rent control agency sets the rental rate at \$100, the quantity supplied would then be 550,000 $Q_s = 50 + (5)(1) = 55$), a decrease of 200,000 apartments from the free market equilibrium. (Assuming three people per family per apartment, this would imply a loss of 600,000 people.) At the \$100 rental rate, the demand for apartments is 950,000 units, and the resulting shortage is 400,000 units (950,000-550,000). The city population will only fall by 600,000 which is represented by the drop in the number of apartments from 750.000 to 550,000, or 200,000 apartments with 3 people each. These are the only people that were originally in the City to begin with.





b. Suppose the agency bows to the wishes of the board and sets a rental of \$900 per month on all apartments to allow landlords a "fair" rate of return. If 50 percent of any long-run increases in apartment offerings comes from new construction, how many apartments are constructed?

At a rental rate of \$900, the supply of apartments would be 50 + 5(9) = 95, or 950,000 units, which is an increase of 200,000 units over the free market equilibrium. Therefore, (0.5)(200,000) = 100,000 units would be constructed. Note, however, that since demand is only 550,000 units, 400,000 units would go unrented.

5. Much of the demand for U.S. agricultural output has come from other countries. From Example 2.4, total demand is Q = 3244 - 283P. In addition, we are told that domestic demand is $Q_d = 1700 - 107P$. Domestic supply is $Q_s = 1944 + 207P$. Suppose the export demand for wheat falls by 40 percent.

a. U.S. farmers are concerned about this drop in export demand. What happens to the free market price of wheat in the United States? Do the farmers have much reason to worry?

Given total demand, Q = 3244 - 283P, and domestic demand, $Q_d = 1700 - 107P$, we may subtract and determine export demand, $Q_e = 1544 - 176P$.

The initial market equilibrium price is found by setting total demand equal to supply:

$$3244 - 283P = 1944 + 207P$$
, or

P = \$2.65.

The best way to handle the 40 percent drop in export demand is to assume that the export demand curve pivots down and to the left around the vertical intercept so that at all prices demand decreases by 40 percent, and the reservation price (the maximum price that the foreign country is willing to pay) does not change. If you instead shifted the demand curve down to the left in a parallel fashion the effect on price and quantity will be qualitatively the same, but will differ quantitatively.

The new export demand is $0.6Q_e = 0.6(1544-176P) = 926.4-105.6P$. Graphically, export demand has pivoted inwards as illustrated in figure 2.5a below.



Figure 2.5a

Total demand becomes

 $Q_D = Q_d + 0.6Q_e = 1700 - 107P + (0.6)(1544 - 176P) = 2626.4 - 212.6P.$

Equating total supply and total demand,

$$1944 + 207P = 2626.4 - 212.6P$$
, or

P = \$1.63,

which is a significant drop from the market-clearing price of \$2.65 per bushel. At this price, the market-clearing quantity is 2280.65 million bushels. Total revenue has decreased from \$6614.6 million to \$3709.0 million. Most farmers would worry.

b. Now suppose the U.S. government wants to buy enough wheat each year to raise the price to \$3.50 per bushel. With this drop in export demand, how much wheat would the government have to buy each year? How much would this cost the government? With a price of \$3.50, the market is not in equilibrium. Quantity demanded and supplied are

$$\label{eq:QD} \begin{split} Q_D &= 2626.4\text{-}212.6(3.5)\text{=}1882.3, \text{ and} \\ Q_S &= 1944 + 207(3.5) = 2668.5. \end{split}$$

Excess supply is therefore 2668.5-1882.3=786.2 million bushels. The government must purchase this amount to support a price of \$3.5, and will spend

\$3.5(786.2 million) = \$2751.7 million per year.

6. In 1998, Americans smoked 470 billion cigarettes. The average retail price was \$2 per pack. Statistical studies have shown that the price elasticity of demand is -0.4, and the price elasticity of supply is 0.5. Using this information, derive linear demand and supply curves for the cigarette market.

Let the demand curve be of the general form Q=a+bP and the supply curve be of the general form Q=c+dP, where a, b, c, and d are the constants that you have to find from the information given above. To begin, recall the formula for the price elasticity d demand

$$E_P^D = \frac{P}{Q} \frac{\Delta Q}{\Delta P}.$$

You are given information about the value of the elasticity, P, and Q, which means that you can solve for the slope which is b in the above formula for the demand curve.

$$-0.4 = \frac{2}{470} \frac{\Delta Q}{\Delta P}$$
$$\frac{\Delta Q}{\Delta P} = -0.4 \left(\frac{470}{2}\right) = -94 = b.$$

To find the constant a, substitute for Q, P, and b into the above formula so that 470=a-94*2 and a=658. The equation for demand is therefore Q=658-94P. To find the supply curve, recall the formula for the elasticity of supply and follow the same method as above:

$$E_P^S = \frac{P}{Q} \frac{\Delta Q}{\Delta P}$$

$$0.5 = \frac{2}{470} \frac{\Delta Q}{\Delta P}$$

$$\frac{\Delta Q}{\Delta P} = 0.5 \left(\frac{470}{2}\right) = 117.5 = d.$$

To find the constant c, substitute for Q, P, and d into the above formula so that 470=c+117.5*2 and c=235. The equation for supply is therefore Q=235+117.5P.

7. In Example 2.7 we examined the effect of a 20 percent decline in copper demand on the price of opper, using the linear supply and demand curves developed in Section 2.6. Suppose the long-run price elasticity of copper demand were -0.4 instead of -0.8.

a. Assuming, as before, that the equilibrium price and quantity are $P^* = 75$ cents per pound and $Q^* = 7.5$ million metric tons per year, derive the linear demand curve consistent with the smaller elasticity.

Following the method outlined in Section 2.6, we solve for *a* and *b* in the demand equation $Q_D = a - bP$. First, we know that for a linear demand function

 $E_D = -b\left(\frac{P^*}{Q^*}\right)$. Here $E_D = -0.4$ (the long-run price elasticity), $P^* = 0.75$ (the

equilibrium price), and $Q^* = 7.5$ (the equilibrium quantity). Solving for *b*,

$$-0.4 = -b\left(\frac{0.75}{7.5}\right)$$
, or $b = 4$.

To find the intercept, we substitute for *b*, Q_D (= Q^*), and *P* (= P^*) in the demand equation:

$$7.5 = a - (4)(0.75)$$
, or $a = 10.5$.

The linear demand equation consistent with a long-run price elasticity of -0.4 is therefore

$$Q_{\rm D} = 10.5 - 4P_{\rm C}$$

b. Using this demand curve, recalculate the effect of a 20 percent decline in copper demand on the price of copper.

The new demand is 20 percent below the original (using our convention that quantity demanded is reduced by 20% at every price):

$$\mathbf{Q}'_{\rm D} = (0.8)(10.5 - 4P) = 8.4 - 3.2 P$$
.

Equating this to supply,

$$8.4 - 3.2P = -4.5 + 16P$$
, or
 $P = 0.672$.

With the 20 percent decline in the demand, the price of copper falls to 67.2 cents per pound.

8. Example 2.8 analyzes the world oil market. Using the data given in that example,

a. Show that the short-run demand and competitive supply curves are indeed given by

D = 24.08 - 0.06P

S_c = 11.74 + 0.07*P*.

First, considering non-OPEC supply:

$$S_c = Q^* = 13.$$

With $E_s = 0.10$ and $P^* = 18 , $E_s = d(P^*/Q^*)$ implies d = 0.07.

Substituting for *d*, S_c , and *P* in the supply equation, c = 11.74 and $S_c = 11.74 + 0.07P$.

Similarly, since $Q_D = 23$, $E_D = -b(P^*/Q^*) = -0.05$, and b = 0.06. Substituting for b, $Q_D = 23$, and P = 18 in the demand equation gives 23 = a - 0.06(18), so that a = 24.08.

Hence $Q_D = 24.08 - 0.06P$.

b. Show that the long-run demand and competitive supply curves are indeed given by

D = 32.18 - 0.51P

$$S_c = 7.78 + 0.29P.$$

As above, $E_s = 0.4$ and $E_D = -0.4$: $E_s = d(P^*/Q^*)$ and $E_D = -b(P^*/Q^*)$, implying 0.4 = d(18/13) and -0.4 = -b(18/23). So d = 0.29 and b = 0.51.

Next solve for *c* and *a*:

$$S_c = c + dP$$
 and $Q_D = a - bP$, implying $13 = c + (0.29)(18)$ and $23 = a - (0.51)(18)$.

So *c* = 7.78 and *a* = 32.18.

c. During the late 1990s, Saudi Arabia accounted for 3 billion barrels per year of OPEC's production. Suppose that war or revolution caused Saudi Arabia to stop producing oil. Use the model above to calculate what would happen to the price of oil in the short run and the long run if OPEC's production were to drop by 3 billion barrels per year.

With OPEC's supply reduced from 10 bb/yr to 7 bb/yr, add this lower supply of 7 bb/yr to the short-run and long-run supply equations:

$$S_c = 7 + S_c = 11.74 + 7 + 0.07P = 18.74 + 0.07P$$
 and $S^2 = 7 + S_c = 14.78 + 0.29P$.

These are equated with short-run and long-run demand, so that:

$$18.74 + 0.07P = 24.08 - 0.06P$$
,

implying that P = \$41.08 in the short run; and

$$14.78 + 0.29P = 32.18 - 0.51P$$
,

implying that P =\$21.75 in the long run.

- 9. Refer to Example 2.9, which analyzes the effects of price controls on natural gas.
- a. Using the data in the example, show that the following supply and demand curves did indeed describe the market in 1975:

Supply:
$$Q = 14 + 2P_{g} + 0.25P_{o}$$

Demand: $Q = -5P_{g} + 3.75P_{o}$

where P_G and P_o are the prices of natural gas and oil, respectively. Also, verify that if the price of oil is \$8.00, these curves imply a free market price of \$2.00 for natural gas.

To solve this problem, we apply the analysis of Section 2.6 to the definition of cross-price elasticity of demand given in Section 2.4. For example, the cross-price-elasticity of demand for natural gas with respect to the price of oil is:

$$E_{GO} = \left(\frac{\Delta Q_G}{\Delta P_O}\right) \left(\frac{P_O}{Q_G}\right).$$

 $\left(\frac{\Delta Q_{_G}}{\Delta P_{_O}}\right)$ is the change in the quantity of natural gas demanded, because of a small

change in the price of oil. For linear demand equations, $\left(\frac{\Delta Q_{c}}{\Delta P_{o}}\right)$ is constant. If we represent demand as:

$$Q_G = a - bP_G + eP_C$$

(notice that income is held constant), then $\left(\frac{\Delta Q_{c}}{\Delta P_{o}}\right) = e$. Substituting this into the cross-

price elasticity, $E_{PO} = e \left(\frac{P_{O}^{*}}{Q_{G}^{*}} \right)$, where P_{O}^{*} and Q_{G}^{*} are the equilibrium price and quantity.

We know that $P_0^* = \$8$ and $Q_G^* = 20$ trillion cubic feet (Tcf). Solving for *e*,

$$1.5 = e\left(\frac{8}{20}\right)$$
, or $e = 3.75$.

Similarly, if the general form of the supply equation is represented as:

$$Q_G = c + dP_G + gP_O$$

the cross-price elasticity of supply is $g\left(\frac{P_o}{Q_c^*}\right)$, which we know to be 0.1. Solving for *g*,

$$0.1 = g\left(\frac{8}{20}\right)$$
, or $g = 0.25$.

The values for *d* and *b* may be found with equations 2.5a and 2.5b in Section 2.6. We know that $E_s = 0.2$, $P^* = 2$, and $Q^* = 20$. Therefore,

$$0.2 = d\left(\frac{2}{20}\right)$$
, or $d = 2$.

Also, $E_D = -0.5$, so

$$-0.5 = b\left(\frac{2}{20}\right)$$
, or $b = -5$.

By substituting these values for d, g, b and e into our linear supply and demand equations, we may solve for *c* and *a*:

~ ~

and

$$20 = c + (2)(2) + (0.25)(8)$$
, or $c = 14$,

$$20 = a - (5)(2) + (3.75)(8)$$
, or $a = 0$.

If the price of oil is \$8.00, these curves imply a free market price of \$2.00 for natural Substitute the price of oil in the supply and demand curves to verify these gas. equations. Then set the curves equal to each other and solve for the price of gas.

$$14 + 2P_G + (0.25)(8) = -5P_G + (3.75)(8), 7P_G = 14,$$
or
$$P_G = \$2.00.$$

b. Suppose the regulated price of gas in 1975 had been \$1.50 per thousand cubic feet, instead of \$1.00. How much excess demand would there have been?

With a regulated price of \$1.50 for natural gas and a price of oil equal to \$8.00 per barrel.

Demand:
$$Q_D = (-5)(1.50) + (3.75)(8) = 22.5$$
, and
Supply: $Q_S = 14 + (2)(1.5) + (0.25)(8) = 19$.

With a supply of 19 Tcf and a demand of 22.5 Tcf, there would be an excess demand of 3.5 Tcf.

Suppose that the market for natural gas had *not* been regulated. If the price of oil c. had increased from \$8 to \$16, what would have happened to the free market price of natural gas?

If the price of natural gas had not been regulated and the price of oil had increased from \$8 to \$16, then

Demand:
$$Q_D = -5P_G + (3.75)(16) = 60 - 5P_G$$
, and
Supply: $Q_S = 14 + 2P_G + (0.25)(16) = 18 + 2P_G$

Equating supply and demand and solving for the equilibrium price,

 $18 + 2P_G = 60 - 5P_G$ or $P_G =$ 6.

The price of natural gas would have tripled from \$2 to \$6.

10. The table below shows the retail price and sales for instant coffee and roasted coffee for 1997 and 1998.

	Retail Price of	Sales of	Retail Price of	Sales of
	Instant Coffee	Instant Coffee	Roasted Coffee	Roasted Coffee
Year	(\$/lb)	(million lbs)	(\$/lb)	(million lbs)
1997	10.35	75	4.11	820
1998	10.48	70	3.76	850

a. Using this data alone, estimate the short-run price elasticity of demand for roasted coffee. Also, derive a linear demand curve for roasted coffee.

To find elasticity, you must first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{820 - 850}{4.11 - 3.76} = -\frac{30}{0.35} = -85.7.$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Since the demand curve is assumed to be linear, the elasticity will differ in 1997 and 1998 because price and quantity are different. You can calculate the elasticity at both points and at the average point between the two years:

$$E_{p}^{97} = \frac{P}{Q} \frac{\Delta Q}{\Delta P} = \frac{4.11}{820} (-85.7) = -0.43$$

$$E_{p}^{98} = \frac{P}{Q} \frac{\Delta Q}{\Delta P} = \frac{3.76}{850} (-85.7) = -0.38$$

$$E_{p}^{AVE} = \frac{\frac{P_{97} + P_{98}}{2}}{\frac{Q}{97} + Q_{98}} \frac{\Delta Q}{\Delta P} = \frac{3.935}{835} (-85.7) = -0.40$$

To derive the demand curve for roasted coffee, note that the slope of the demand curve is -85.7=-b. To find the coefficient a, use either of the data points from the table above so that a=830+85.7*4.11=1172.3 or a=850+85.7*3.76=1172.3. The equation for the demand curve is therefore

b. Now estimate the short-run price elasticity of demand for instant coffee. Derive a linear demand curve for instant coffee.

To find elasticity, you must first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{75 - 70}{10.35 - 10.48} = -\frac{5}{0.13} = -38.5.$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Since the demand curve is assumed to be linear, the elasticity will differ in 1997 and 1998 because price and quantity are different. You can calculate the elasticity at both points and at the average point between the two years:

$$E_{p}^{97} = \frac{P}{Q} \frac{\Delta Q}{\Delta P} = \frac{10.35}{75} (-38.5) = -5.31$$
$$E_{p}^{98} = \frac{P}{Q} \frac{\Delta Q}{\Delta P} = \frac{10.48}{70} (-38.5) = -5.76$$

$$E_p^{AVE} = \frac{\frac{P_{97} + P_{98}}{2}}{\frac{Q_{97} + Q_{98}}{2}} \frac{\Delta Q}{\Delta P} = \frac{10.415}{72.5} (-38.5) = -5.52$$

To derive the demand curve for instant coffee, note that the slope of the demand curve is -38.5=-b. To find the coefficient a, use either of the data points from the table above so that a=75+38.5*10.35=473.1 or a=70+38.5*10.48=473.1. The equation for the demand curve is therefore

Q=473.1-38.5P.

c. Which coffee has the higher short-run price elasticity of demand? Why do you think this is the case?

Instant coffee is significantly more elastic than roasted coffee. In fact, the demand for roasted coffee is inelastic and the demand for instant coffee is elastic. Roasted coffee may have an inelastic demand in the short-run as many people think of coffee as a necessary good. Instant coffee on the other hand may be viewed by many people as a convenient, though imperfect substitute for roasted coffee. Given the higher price per pound for instant coffee, and the preference for roasted over instant coffee by many consumers, this will cause the demand for roasted coffee to be less elastic than the demand for instant coffee. Note also that roasted coffee is the premium good so that the demand for roasted coffee lies to the right of the demand for instant coffee. This will cause the demand for roasted coffee to be more inelastic because at any given price there will be a higher quantity demanded for roasted versus instant coffee, and this quantity difference will be large enough to offset the difference in the slope of the two demand curves.