One application of exponential functions is compound interest, which is when interest is calculated on the total value of a sum, not just on the principal (as is the case with simple interest). In this set of notes we will look at a formula for calculating compound interest n times per year. This formula will be provided on homework and exams.

Formula for Interest Compounded n Times Per Year:

- when interest is compounded n times per year, we use the formula
  $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$
  - A is the accumulated value of the investment
  - P is the principal (the amount you start with)
  - r is the annual interest rate
  - n is the number of compounding periods per year, which means the number of times per year that interest is compounded
    - annually $(n = 1)$, semiannually $(n = 2)$, quarterly $(n = 4)$, monthly $(n = 12)$, weekly $(n = 52)$, …
  - t is the number of **years** the principal is invested (the term)
    - if you're given a term that is not based on years, such as months, be sure to convert it to years

Example 1: If \$57,000 is invested at a rate of 7.75% per year for 62 years, find value of the investment to the nearest penny if the interest is compounded:

a. annually

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 57000\left(1 + \frac{0.0775}{1}\right)^{1 \cdot 62}$

$A = 57000(1 + 0.0775)^{62}$

$A = 57000(1.0775)^{62}$

$A = 57000(102.2989763\ldots)$

$\boldsymbol{A = \$5,831,041.65}$

b. monthly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 57000\left(1 + \frac{0.0775}{12}\right)^{12 \cdot 62}$

$A = 57000(1 + 0.0064583\ldots)^{744}$

$A = 57000(1.0064583\ldots)^{744}$

$A = 57000(120.2472856\ldots)$

$\boldsymbol{A = \$6,854,095.28}$

When working with compound interest formulas, remember to keep in mind order of operation (PEMA):

1. simplify parentheses
2. simplify exponents
3. simplify multiplication/division, working from left to right
4. simplify addition/subtraction, working from left to right

Example 2: If $43,719 is invested at a rate of 5.86% per year for 37 years, find value of the investment to the nearest penny if the interest is compounded:

a. quarterly

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 43719\left(1+\frac{0.0586}{4}\right)^{4\cdot 37}$$

...

b. weekly

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 43719\left(1+\frac{0.0586}{52}\right)^{52\cdot 37}$$

...

When working on problems like this on homework and exams, do your best to leave all calculated values in your calculator. For instance when calculating $A = 43719\left(1+\frac{0.0586}{52}\right)^{52\cdot 37}$ from Example 2 part b, do not calculate $\frac{0.0586}{52}$ and then try to write that down on paper to 5 or 6 decimal places. Once you start approximating, you start getting further and further from the exact correct answer. So leave calculated values in your calculator to avoid approximating.

For help with entering expressions such as $43719\left(1+\frac{0.0586}{52}\right)^{52\cdot 37}$ in your calculator, take a look at the Calculator Tips document in Brightspace or stop by my office hours.

$$A = 43719\left(1+\frac{0.0586}{52}\right)^{52\cdot 37}$$

$$\boldsymbol{A = \$381,748.50}$$

<u>Example 3:</u> If \$20,000 is invested at a rate of 6.5% per year compounded monthly, find value of the investment at each given time and round to the nearest cent. Use the formula $A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$.

a. 5 months b. 36 months c. 45 years

Remember that t represents the term of the investment in **<u>years</u>**. The principal will be 20000 for each problem part ($P = 20000$), the interest rate will be 6.5% ($r = 0.065$), and the interest will be compounded monthly ($n = 12$). However the term will vary from part to part:

$t = \frac{5}{12}$ $t = \frac{36}{12} = 3$ $t = 45$

$$A = 20000\left(1+\frac{0.065}{12}\right)^{(12)\left(\frac{5}{12}\right)}$$

<u>Example 4:</u> A recent college graduate moves back in with their parents and invests their entire first year salary (\$42,000) in a mutual fund that averages an annual interest rate of about 12% and compounds approximately twice a year. If no additional money is added to the investment, what will be the accumulated value after 50 years? Use the formula $A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$ and round your answer to the nearest penny.

<u>Example 5:</u> A recent college graduate with $50,000 in student loans decides to leave the country and doesn't make any payments on their loans. After 25 years abroad, they return to collect the inheritance their parents have left for them, only to find that they cannot collect anything until they pay off their student loans. If the interest rate on those student loans was 8% compounded daily, what will be the balance at the end of the 25 year period, rounded to the nearest penny?

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$.

Answers to Examples:

1a. $7,253,363.48 *; 1b.* $7,973,632.90 *;*

2a. $376,250.13 *; 2b.* $380,203.24 *;*

3a. $20,547.57 *; 3b.* $24,293.43 *; 3c.* $369,754.36 *;*

4. $14,250,687.51 *; 5.* $369,371.85 *;*