Overview | Add and Subtract Positive and Negative Numbers

MATH FOCUS

Focus Standards

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

d. Apply properties of operations as strategies to add and subtract rational numbers.

See Unit 2 Pacing Guide for developing and applied standards.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Solve addition and subtraction problems involving negative numbers, including rational numbers.
- Interpret addition expressions with rational numbers as *subtracting the opposite* and subtraction expressions with rational numbers as *adding the opposite*.
- Rewrite and reorder problems involving both addition and subtraction to make them easier to solve.

Language Objectives

- Explain solution strategies to subtraction problems with negative numbers using lesson vocabulary and academic language.
- Explain how to rewrite and reorder to solve problems involving addition or subtraction of negative numbers during class discussions.
- Respond to clarifying questions about positive and negative numbers by accurately using the lesson vocabulary in speech and writing.

Prior Knowledge

- Subtract positive rational numbers for which the difference is positive or zero.
- Add rational numbers in any form.
- Understand that subtracting an integer is the same as adding its opposite, $p - q = p + (-q)$.
- Understand the distance between two integers on the number line as the absolute value of their difference.
- Model adding and subtracting integers using integer chips and horizontal and vertical number lines.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

absolute value a number's distance from 0 on the number line. Absolute value is never negative.

opposite numbers numbers that are the same distance from 0 but in opposite directions. Opposite numbers have the same numeral, but opposite signs. The opposite of a number is also called the *additive inverse* of that number.

Academic Vocabulary

represent to use as a sign, symbol, or example for something.

Learning Progression

Earlier in Grade 7, students represented the addition of positive and negative integers, fractions, and decimals on a number line, and they applied the properties of operations as strategies to find sums of rational numbers.

In the previous lesson, they used integer chips and number lines to understand the subtraction of a negative integer as the addition of its opposite.

In this lesson, students extend their knowledge of subtraction to include negative fractions and decimals. They solve real-world problems that involve both addition and subtraction with rational numbers. They apply a variety of strategies, including reordering addends, to make calculating more efficient.

Later in Grade 7, students will solve real-world problems involving all four operations. They will also add and subtract rational numbers to simplify linear expressions and solve multi-step linear equations.

In Grade 8, students will apply their abilities to calculate with positive and negative numbers to solve linear equations and systems of linear equations.

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Adding and Subtracting with Integers (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 185–186)	**Math Toolkit** grid paper, integer chips, number lines Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 2 counters, Activity Sheet *Number Lines*
SESSION 2 **Develop** Subtracting Positive and Negative Fractions and Decimals (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 191–192)	**Math Toolkit** grid paper, number lines, place-value charts Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Adding and Subtracting Positive and Negative Fractions and Decimals (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 197–198)	**Math Toolkit** grid paper, number lines, place-value charts Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 2 colored pencils (1 each of two different colors), 2 copies of Activity Sheet *Fraction Bars* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Adding and Subtracting Positive and Negative Numbers (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: 1 counter, sticky notes, Activity Sheet *Number Lines* **REINFORCE** Problems 4–9 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 10 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

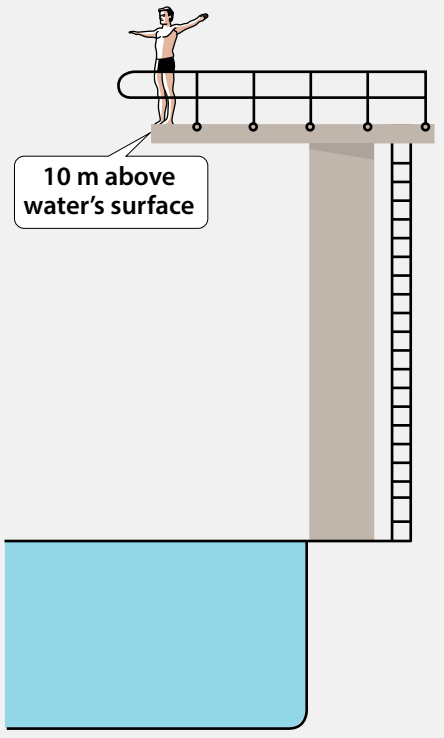
Overview | Add and Subtract Positive and Negative Numbers

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students to discuss their experiences either diving or jumping into the water. When gymnasts began performing their routines in the water, the sport of diving was born. Diving was first included in the Olympic Games over 100 years ago. The sport has remained popular and competitive ever since. Divers are judged on their movements through the air and their entry into the water.



SESSION 2 ■ ■ □ □

Try It People consider many things when deciding where to live. One of those things might be the average temperature. In Kuwait City, Kuwait, the average high temperature in July is 99°F, but the average low temperature in January is 45°F. In Dudinka, Russia, the average high temperature in July is 65°F, but the average low temperature in January is −19°F. Ask students to think about their own preferences and choose ideal average high and average low temperatures for their future home. Have them mark their preferred high and low temperatures on a number line with sticky notes and discuss how they compare across the class.

SESSION 3 ■ ■ ■ □

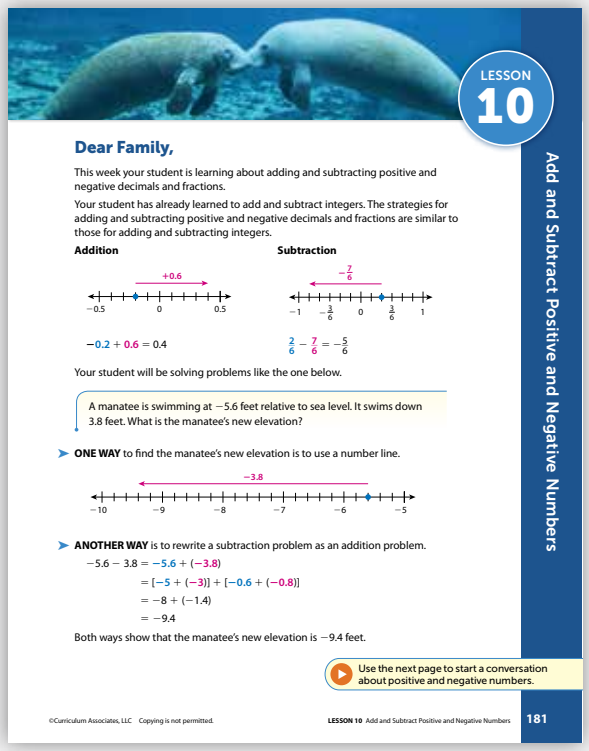
Try It Ask students familiar with Lantern Festivals to talk about their experiences. Communities in China have been celebrating the Lantern Festival for over 2,000 years, and the festival has religious, social, and cultural meanings. Lanterns are also used for decoration and celebration in cultures around the world. A lantern with a candle inside it will rise into the air much like a hot-air balloon. The heated air rises because it is less dense than the colder air around it. Ask students if they know of other festivals or holidays that have been celebrated for thousands of years.

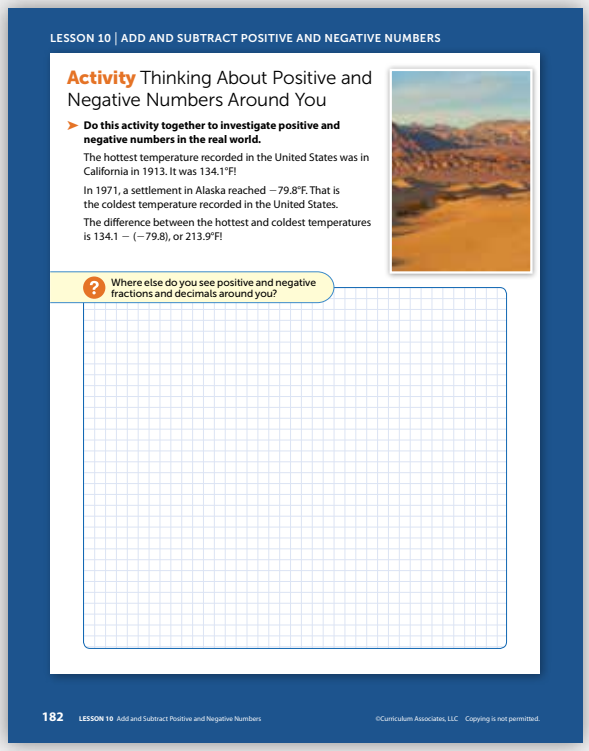
SESSION 4 ■ ■ ■ ■

Apply It **Problem 4** Submarines are vehicles that travel far under water. The crew members on a submarine are able to control the submarine's vertical position because they can alter its density. Releasing compressed air into the submarine's ballast tanks makes it less dense and causes it to rise in the water. Venting the air and flooding the ballast tanks with water causes the submarine to sink. Ask students to describe their experiences with changes in elevation, such as taking an elevator, climbing on steep trails, or flying in airplanes.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**





Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS — Use with **Session 1** Connect It

MATH TERMS

A *non-integer* is not a whole number or an integer. Fractions and decimals are *non-integers*.

Distance is the measurement between two points.

Levels 1–3: Listening/Writing

Prepare students to write responses to Connect It problem 2. Create a **Co-Constructed Word Bank** after reading the problem aloud. Begin with *distance between two elevations* and use a sketch to clarify its meaning. Add *integers* and *non-integers* and give students examples to label. Point out that *non-* means "not." Guide students to circle the non-integers in problem 2a, label them on the number line, and mark the distance between the points.

Guide students to identify key terms for problems 2b and 2c. Help students write explanations using the word bank.

Levels 2–4: Listening/Writing

Prepare students to write responses to Connect It problem 2. Read the problem aloud and use a **Co-Constructed Word Bank** to help clarify words and phrases, such as *positive, negative, integers, non-integers,* and *opposite numbers*.

Have students use the number line to discuss the meaning of *distance between*. Ask for examples of *integers* and *non-integers* and guide students to explain that *non-* means *not*.

Encourage partners to work together to write their explanations. Have students use the word bank to help them write using precise mathematical and academic language.

Levels 3–5: Listening/Writing

Prepare students to write responses to Connect It problem 2. Read the problem aloud and begin a **Co-Constructed Word Bank** with *distance between two elevations represented by integers*.

Have partners discuss each part of the problem and add to the word bank before writing their explanations. Have them decide how the two models in problem 2b are the same and different.

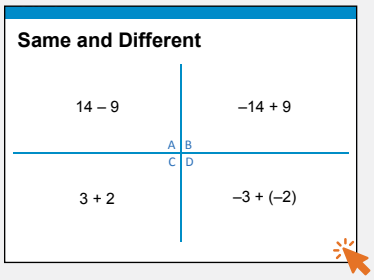
Have students write their explanations using complete sentences. Remind them to pay attention to the prepositions used with the terms *distance* and *opposite*.

Explore Adding and Subtracting with Integers

Purpose

- **Explore** the addition and subtraction of integers representing real-world situations.
- **Understand** that strategies for subtracting integers can be applied to subtracting rational numbers.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All expressions have a value of 5 or −5.

A is the only subtraction expression.

B is the only expression with a negative integer and a positive integer.

C is the only addition expression with two positive integers.

D is the only addition expression with two negative integers.

WHY? Support students' facility at classifying and calculating the addition of positive and negative integers.

TRY IT SMP 1, 2, 3, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Ask students for a show of thumbs up or down to agree with a rephrasing of the problem and to offer a revision if they disagree.

DISCUSS IT SMP 2, 3, 6

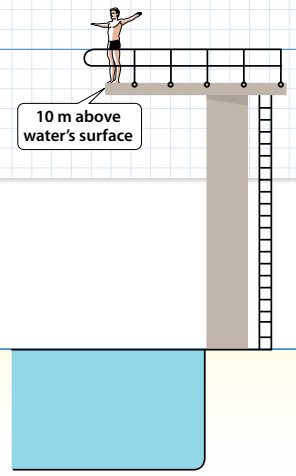
Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- the distance is represented by the sum $|10| + |5|$ or the difference $10 - (-5)$.
- the surface of the water is represented by 0.

LESSON 10 | SESSION 1 ■ □ □ □

Explore Adding and Subtracting with Integers



Previously, you learned how to add positive and negative numbers. In this lesson, you will learn about subtracting positive and negative fractions and decimals.

➤ **Use what you know to try to solve the problem below.**

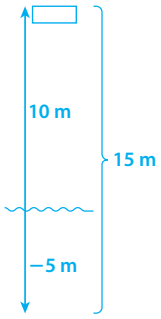
A pool's diving platform is 10 m above the water's surface. The bottom of the pool is at −5 m, relative to the surface of the water. What is the distance between the diving platform and the bottom of the pool?

TRY IT

Math Toolkit grid paper, integer chips, number lines

Possible work:

SAMPLE A



It is 15 m from the diving platform to the bottom of the pool.

SAMPLE B

Distances:

Platform to surface of water: 10 m

Surface of water to bottom of pool: −5 m

$10 - (-5) = 10 + 5$

$= 15$

The distance is 15 m.

DISCUSS IT

Ask: How does your work represent the surface of the water?

Share: In my work . . . represents . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- Apply properties of operations as strategies to add and subtract rational numbers.

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Common Misconception Listen for students who state the distance as 5 meters, perhaps by finding the sum of 10 and −5. These students may be confusing the meaning of addition and subtraction in modeling real-life situations. As students share their strategies, ask them to draw a diagram to illustrate the problem. Ask them to clarify how their model shows the distance between the diving board and the bottom of the pool.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using integer chips to model the distance
- **(misconception)** finding $10 + (-5)$ as the distance
- using a number line to model the distance
- using an equation to calculate the distance

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ask students to make sure that they describe the positions of the diving board and pool bottom using the positive and negative numbers included in the problem statement. To confirm understanding, call on another student to reword the description using mathematical language as necessary.

Guide students to **Compare and Connect** the representations. Prompt students to refer to their models or diagrams to help explain why their strategies make sense for the problem.

> **ASK** *How do the models show whether you should find the sum or difference of 10 and −5?*
>
> **LISTEN FOR** The models show that to find the distance between 10 and −5, you find the difference between 10 and −5.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that distance along a number line is represented as the difference between two numbers and that subtracting a negative number has the same effect as adding its opposite.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a number line to model subtraction.

If students are unsure about subtracting negative numbers, then use this activity to help them visualize the process.

Materials For each pair: 2 counters, Activity Sheet *Number Lines*

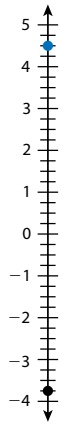
- Invite students to model a subtraction problem with positive integers and a positive result, such as 7 − 4. Have them demonstrate the subtraction by placing a counter at 7 and moving the second counter 4 units to the left or down on the number line to 3.
- Ask: *How does this show finding 7 − 4?* [The number you end at, 3, is the difference.]
- Have students model 10 − 5.
- Ask: *Suppose, instead of 5, you want to subtract −5. Which direction should you move on the number line?* [In the positive direction: to the right or up.] If needed, remind students that addition and subtraction are inverse operations. Prompt them to think about the movement that would undo adding −5.
- Repeat with other examples of subtracting a negative integer from a positive integer, such as 8 − (−3) or 5 − (−2).

CONNECT IT

1 **Look Back** What is the distance between the diving platform and the bottom of the pool? How do you know?

15 m; Possible explanation: The diving platform is 10 m above the surface, the bottom of the pool is 5 m below the surface, and 10 + 5 = 15.

2 **Look Ahead** In the **Try It**, you found the distance between two elevations represented by integers. You can also find the distance between non-integers.



a. Explain how you can use the number line to find the distance between 4.5 and −3.75.

Possible explanation: The number line shows that there are 4.5 units between 4.5 and 0. There are 3.75 units between 0 and −3.75. In total, 4.5 + 3.75 = 8.25, so the distance between 4.5 and −3.75 is 8.25 units.

b. You can use both the expression $|4.5 - (-3.75)|$ and the expression $|-3.75 - 4.5|$ to find the distance between 4.5 and −3.75. Why?

Possible explanation: Distance is always positive and the distance between two points is the same no matter which point you start from.

c. You can subtract to find the difference between 4.5 and −3.75. Explain why −3.75 − 4.5 is the opposite of 4.5 − (−3.75).

Possible answer: You travel the same distance with both expressions, but in opposite directions.

3 **Reflect** How is finding the distance between two numbers on the number line like finding the difference between two numbers? How is it different?

Possible answer: You find both of them by subtracting. When you find distance, you use absolute value, so the order in which you subtract does not matter. When you find a difference, you do not use absolute value and the order does matter.

2 **Look Ahead** Point out how the subtraction problem 4.5 − (−3.75) involves subtracting a negative number from a positive number, which was also done in Try It. Students should recognize that the strategy they used in Try It could be adapted to solve this problem.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding of the difference between two numbers as modeled by the distance between the numbers on a number line.

Common Misconception If students think that adding a positive number and a negative number shows the distance between them, then have them plot two opposite numbers, such as 6 and −6, on a number line. Then ask them to compare the sum of the two numbers, which is 0, with the distance between them on the number line, 12.

Prepare for Subtracting Positive and Negative Numbers

Support Vocabulary Development

Assign **Prepare for Subtracting Positive and Negative Numbers** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *absolute value*. Remind students that opposite numbers have the same absolute value and ask them to provide examples.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of definitions, examples, and non-examples.

Have students look at problem 2 and discuss with a partner whether -4 represents the absolute value of $3 - 7$. Encourage students to revise the question so that the answer is yes. For example, $|-4|$ is equal to $|3 - 7|$.

Problem Notes

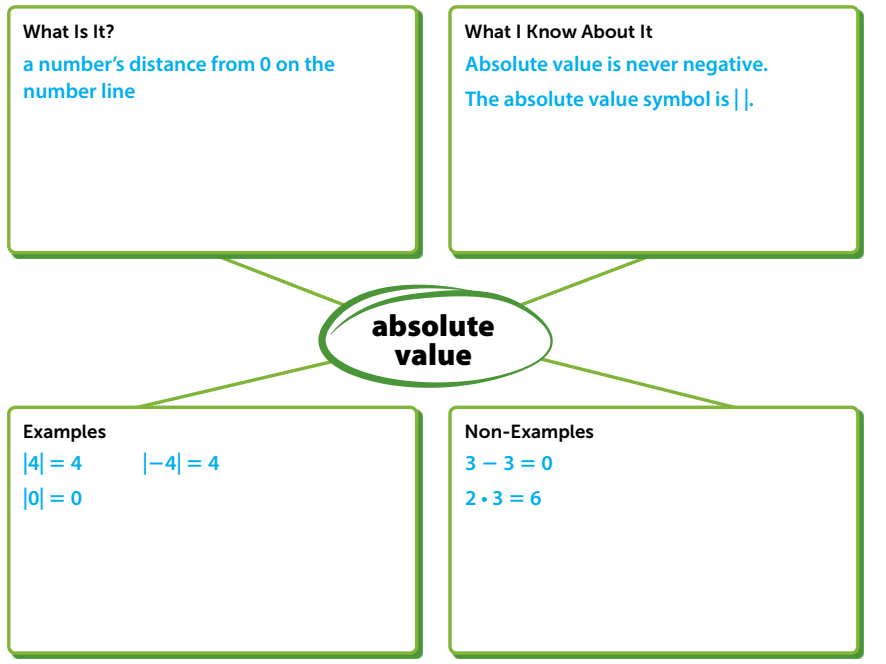
1. Students should understand that a number and its opposite have the same absolute value, which is positive for all numbers except 0. Paired vertical bars are used to show absolute value, such as $|-3| = 3$.

2. Students should recognize that the absolute value of $3 - 7$ is equal to the absolute value of -4, which is 4.

Prepare for Subtracting Positive and Negative Numbers

1. Think about what you know about numbers and absolute value. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:



2. Is -4 the absolute value of $3 - 7$? Explain.

No; Possible explanation: Absolute value is always positive.

REAL-WORLD CONNECTION

When people travel down major highways, they can use mileposts to calculate distances. Mileposts mark the distance along a highway, starting at one end and going to the other end or to the point where the highway crosses a state border. Depending on which direction a driver travels, the numbers on the mileposts might increase or decrease. For example, if a driver has just passed milepost 137 and wants to eat at a restaurant at milepost 60, the driver knows that the distance to the restaurant, in miles, is equal to the absolute value of $60 - 137$, or 77 miles. Ask students to think of other real-world examples where applying the concept of absolute value might be useful in solving a problem with positive and negative numbers.

3 Problem 3 provides another look at subtracting a negative integer from a positive integer. This problem is similar to the Try It problem about the distance between a diving board and the bottom of a pool. In both problems, two elevations are identified by a positive integer and a negative integer, and the distance between them is to be calculated. This problem asks for the distance between elevations of 4 inches and −9 inches.

Students may want to use number lines, graph paper, or integer chips to solve.

Suggest that students use **Three Reads,** asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 The top of a molehill is 4 in. above ground level.
The bottom of a mole's burrow is at −9 in. relative to ground level.

a. What is the distance between the top of the molehill and the burrow? Show your work.

Possible work:

$4 - (-9) = 4 + 9$

$= 13$

SOLUTION The distance between the top of the molehill and the burrow is 13 in.

b. Check your answer to problem 3a. Show your work.

Possible work:

$4 - 13 = -9$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Writing

Prepare students to solve Apply It problem 7. Read the problem aloud with students. Use the illustration to clarify *low tide.* Display the words *low/lower than* and *high/higher than.* Guide students to use gestures to confirm their understanding.

Have students draw a vertical number line on the illustration and label 0 and $-1\frac{1}{2}$.

Ask: *What is the low tide on Monday? What is it on Tuesday? On Tuesday, is the tide lower or higher than on Monday? By how much?*

Have students use the following sentence frame:

- *The low tide on Tuesday is ____ than the low tide on Monday.*

Levels 2–4: Reading/Writing

Prepare students to write responses to Apply It problem 7. Read the problem with students. Have them circle the word *compare.* Prompt them to generate words that they can use to compare tides and water levels. Have partners use **Say It Another Way** to rephrase the question. Have them use the words from the question to build a sentence frame or starter that they can use to state their solution. If students need support, suggest:

- *The low tide on Tuesday is ____.*

Have students solve the problem and label the illustration to support their thinking. Then have students use the phrases *higher than* or *lower than* and their sentence frames or starters to write their responses.

Levels 3–5: Reading/Writing

Prepare students to write responses to Apply It problem 7. Have partners read the problem and identify the two values in the problem that they will compare. Ask students to list words and phrases that they could use to compare the tides, such as *higher than* or *lower than.* Have them use **Say It Another Way** to confirm their understanding.

Display this question for students to discuss after they solve the problem:

- *Is the low tide on Tuesday higher or lower than the low tide on Monday? How do you know?*

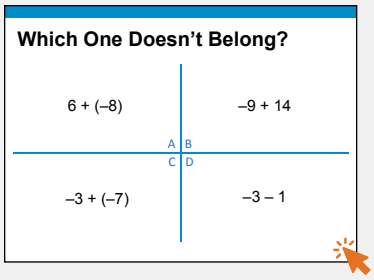
Have partners discuss the questions and check that they included the correct comparison words in their written responses.

Develop Subtracting Positive and Negative Fractions and Decimals

Purpose

- **Develop** strategies for subtracting positive and negative fractions and decimals.
- **Recognize** that subtracting a negative rational number is the same as adding its opposite.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A has a positive first addend.

B has a positive value.

C is the sum of two negative numbers.

D is the only subtraction expression.

WHY? Support students' ability to add and subtract positive and negative integers.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack meaning in sentences with long noun phrases.

HOW? Use this sentence from Try It to explain that academic sentences often include long noun phrases with details that make the sentence more precise: *During December, the average daily temperature in a town is −1.7°C.* Ask students to cover parts of the noun phrase, e.g., *average daily*, and discuss how the meaning changes.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students to describe what the problem is about. After the second read, ask: *What are you trying to find out?* After the third read, ask students to identify the important quantities and relationships in the problem.

LESSON 10 | SESSION 2 ■ ■ □ □

Develop Subtracting Positive and Negative Fractions and Decimals

➤ **Read and try to solve the problem below.**

During December, the average daily temperature in a town is −1.7°C. The average daily temperature in the same town is 3.2°C lower in January. What is the average daily temperature in January?

TRY IT

Math Toolkit grid paper, number lines, place-value charts

Possible work:

SAMPLE A

−1.7 − 3.2

−1.7 + (−3.2)

−1 + (−0.7) + (−3) + (−0.2)

[−1 + (−3)] + [−0.7 + (−0.2)]

−4 + (−0.9)

−4.9

The average daily temperature in January is −4.9°C.

SAMPLE B

The average daily temperature in January is −4.9°C.

DISCUSS IT

Ask: What did you do first to find the temperature in January?

Share: I started by . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How does the problem relate the temperature in January to the temperature in December?*
- *What operation did you use to find the January temperature?*

Common Misconception Listen for students who confuse the number −1.7 with subtraction and calculate the solution as 3.2 − 1.7, or 1.5. As students share their strategies, have them explain how their model shows the initial temperature and how it shows the change in temperature. Ask how they knew whether to use addition or subtraction to find the solution.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using a number line to model $-1.7 - 3.2$
- **(misconception)** misrepresenting -1.7 as subtraction $[3.2 - 1.7]$
- rewriting the problem as an addition problem
- rewriting the problem with whole numbers and decimals broken apart and added separately

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each strategy, ask another student to rephrase the key ideas to reinforce understanding.

Guide students to **Compare and Connect** the representations. Remind students to speak clearly and to pause for questions as they explain how their model shows the solution.

> **ASK** *How do all the models show the change in temperature from December to January?*
>
> **LISTEN FOR** The models show the change in temperature as either a positive number subtracted from the initial temperature or a negative number added to the initial temperature.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

> **ASK** *How is the phrase* 3.2°C lower *represented in the expressions and on the number line?*
>
> **LISTEN FOR** On the number line, it is shown by an arrow pointing down with length 3.2. In the expressions, it is shown as adding -3.2 or subtracting 3.2.

For the mathematical expressions, prompt students to compare and evaluate the two expressions.

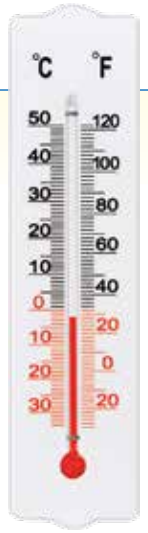
- *Why can both an addition expression and a subtraction expression be used to represent the solution?*

For the number line, prompt students to explain how the model represents the scenario.

- *How do you know that the arrow should point down the number line, not up?*

➤ **Explore different ways to understand adding or subtracting negative decimals.**

During December, the average daily temperature in a town is -1.7°C. The average daily temperature in the same town is 3.2°C lower in January. What is the average daily temperature in January?



Model It

You can write an expression to represent the situation.

You can write a **subtraction** expression:

$-1.7 - 3.2$

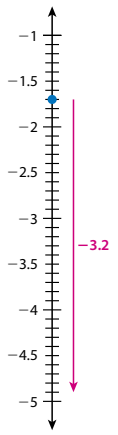
You can also write an **addition** expression:

$-1.7 + (-3.2)$

Model It

You can use a number line to add or subtract negative decimals.

The average temperature in January was **3.2°C lower** than in **December**.



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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Using Structure to Subtract Rational Numbers

Prompt students to think about the structure of numbers that sum to 0 or 10.

> **ASK** *What is a positive number you could subtract from -1.7 that results in an integer?*
>
> **LISTEN FOR** Students should identify positive numbers that end in 0.3, such as 0.3, 1.3, and 3.3.

> **ASK** *Why might someone think that $-1.7 - 3.7$ results in an integer?*
>
> **LISTEN FOR** Someone might think that subtracting two numbers with the same decimal part results in an integer.

> **Generalize** Encourage students to describe how they can subtract rational numbers strategically to work with integers by finding pairs that sum to 1.

Develop Subtracting Positive and Negative Fractions and Decimals

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationship between them are the same in each representation. Explain that they will now use those relationships to reason about the addition of two negative decimals.

Before students begin to record and expand on their work in Model It, tell them that problems 3 and 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 2

- A decrease of 3.2°C can be represented as adding −3.2 or as subtracting 3.2. Both representations are equally valid.
- In the number line model, the arrow points in the negative direction, or down the number line, because the change in temperature is a decrease.

Facilitate Whole Class Discussion

3 Look for understanding that the solution is the value of either expression or is shown by the tip of the arrow in the number line model.

4 Look for recognition that the Try It is an example of adding two negative decimals.

> **ASK** *Why could you find −1.7 + (−3.2) by adding −3.2 + (−1.7)?*
>
> **LISTEN FOR** You can add negative numbers in any order.

5 Look for understanding that the process of subtraction is the same for negative integers and negative decimals.

> **ASK** *If you subtract a negative decimal, is the difference less than or greater than the starting number? How do you know?*
>
> **LISTEN FOR** The difference is greater than the starting number. Subtracting a negative decimal is the same as adding a positive decimal, which makes a sum greater than the starting number.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to subtract with negative decimals and fractions.**

1 Look at the first **Model It**. Why can you write both an addition expression and a subtraction expression for the situation?
Possible explanation: Adding a negative is the same as subtracting a positive.

2 Look at the second **Model It**. How does the number line represent both the addition expression and the subtraction expression?
Possible answer: When you subtract a positive number, you move down on the number line. When you add a negative number, you also move down on the number line.

3 What is the average temperature in January? −4.9°C

4 How is adding two negative decimals like adding two positive decimals? How is it different?
Possible answer: In both cases you continue in the same direction on the number line. You move in the negative direction for negative decimals and in the positive direction for positive decimals.

5 How is subtracting a negative decimal like subtracting a negative integer?
Possible answer: In both cases you can think of subtracting as adding the opposite. Also, in both cases the result is greater than the starting number.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to subtract negative decimals.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Model solving related addition and subtraction problems.

If students are unsure about addition and subtraction with decimals, then use this activity to model the process and observe patterns.

- Display four number lines. Display two expressions related to the Try It problem: −1 + (−3) and −1 − 3. Call on students to mark the number line with points and "jumps" to model each expression.
- Ask: *What is the same about these expressions?* [They are modeled the same way and both have a value of −4.]
- Display the expression −1.7 + (−3.2). Call on a student to model this on the number line. Ask: *What is the result? How is this expression similar to the previous two expressions?* [4.9; It has the same integer parts as the previous two expressions and it has a similar result.]
- Display the expression −1.7 − 3.2. Ask: *How can you model this on the number line? What will the result be?* [You can draw an arrow from 0 to −1.7 and another going 3.2 units to the left of −1.7; It will be 4.9.]

Apply It

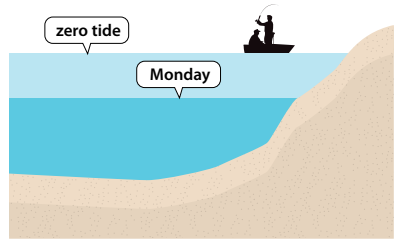
For all problems, encourage students to draw a number line to support their reasoning. Remind students that number lines need not be drawn precisely.

7 Students may use a number line to represent the two values of low tide. To calculate the difference, students may also convert the mixed numbers to fractions, rewrite the fractions with a common denominator, and then subtract.

8 Students may also rewrite each mixed number as the sum of a whole number and a fraction, and then combine the whole numbers and the fractions separately. Another possible strategy is to convert the mixed numbers to decimals, which is practical with halves and fourths. Students may represent the subtraction on a number line by plotting $3\frac{1}{4}$ and drawing an arrow that points $4\frac{1}{2}$ units to the left.

Apply It

➤ **Use what you learned to solve these problems.**



7 On Monday, the low tide at Cook Inlet, Alaska, is $-1\frac{1}{2}$ ft. On Tuesday, the low tide is $-1\frac{1}{3}$ ft. How does the low tide on Tuesday compare to the low tide on Monday? Show your work. **Possible work:**

$$-1\tfrac{1}{3} - \left(-1\tfrac{1}{2}\right) = -1\tfrac{1}{3} + 1\tfrac{1}{2}$$
$$= -1\tfrac{2}{6} + 1\tfrac{3}{6}$$
$$= -1\tfrac{2}{6} + 1\tfrac{2}{6} + \tfrac{1}{6}$$
$$= \tfrac{1}{6}$$

SOLUTION The low tide on Tuesday is $\frac{1}{6}$ ft higher.

8 Find $3\frac{1}{4} - 4\frac{1}{2}$. Show your work. **Possible work:**

$$3\tfrac{1}{4} - 4\tfrac{1}{2} = \tfrac{13}{4} - \tfrac{18}{4}$$
$$= \tfrac{13}{4} + \left(-\tfrac{18}{4}\right)$$
$$= \tfrac{13}{4} + \left(-\tfrac{13}{4}\right) + \left(-\tfrac{5}{4}\right)$$

SOLUTION $-\frac{5}{4}$

9 Find $5.4 - 7.1$. Show your work. **Possible work:**

$$5.4 - 7.1 = 5.4 + (-7.1)$$
$$= 5 + 0.4 + (-7) + (-0.1)$$
$$= -2 + 0.3$$

SOLUTION -1.7

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- relating the subtraction of a positive number to adding the opposite of that number.
- rewriting or reordering to make the numbers easier to work with.

Error Alert If students calculate the difference by adding 5.4 and 7.1 and then taking the opposite of the sum for a difference of -12.5, then have them represent the expression on a number line. After they have plotted 5.4, ask: *How can you show the subtraction of 7.1?*

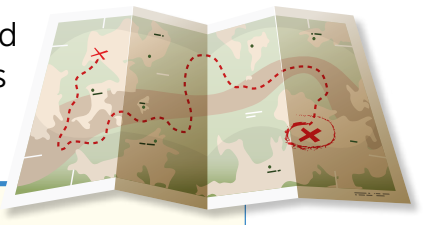
Practice Subtracting Positive and Negative Fractions and Decimals

Problem Notes

Assign **Practice Subtracting Positive and Negative Fractions and Decimals** as extra practice in class or as homework.

1. Students should recognize that, in the context of the problem, a positive number represents a positive change in elevation, and a negative number represents a negative change in elevation. ***Challenge***
2. Students may also calculate the difference by rewriting each mixed number as the sum of a whole number and a fraction and then combining the whole numbers and fractions separately. They may also convert the mixed numbers to decimals. ***Medium***

Practice Subtracting Positive and Negative Fractions and Decimals



➤ **Study the Example showing how to subtract negative fractions. Then solve problems 1–5.**

Example

A path from a dry lake bed starts at an elevation of $-12\frac{1}{2}$ ft relative to sea level. The path ends at an elevation of $60\frac{1}{3}$ ft above sea level. What number represents the change in elevation from the start to the end of the path?

You can subtract to find the change in elevation.

$$60\tfrac{1}{3} - \left(-12\tfrac{1}{2}\right) = 60\tfrac{1}{3} + 12\tfrac{1}{2}$$
$$= 60\tfrac{2}{6} + 12\tfrac{3}{6}$$
$$= 72\tfrac{5}{6}$$

The number that represents the change in elevation is $72\frac{5}{6}$.

1. You can also find $12\frac{1}{2} - 60\frac{1}{3} = -72\frac{5}{6}$. What information does that give you about the path in the Example?
   It tells you the change in elevation from the end of the path to the start.

2. The lowest point of the dry lake bed in the Example has an elevation of $-18\frac{3}{4}$ ft. What number represents the change in elevation from the start of the path to the lowest point in the dry lake bed? Show your work.

   Possible work:

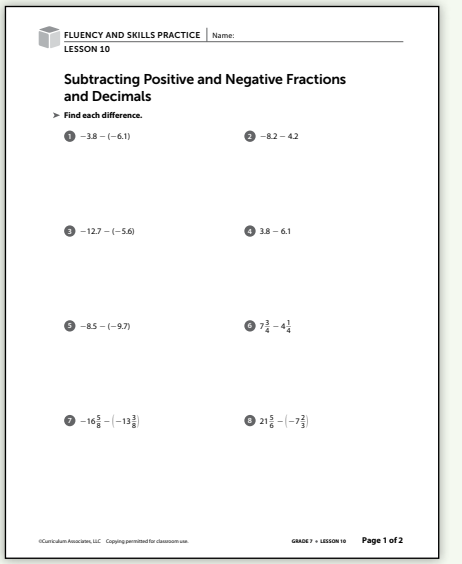
$$-18\tfrac{3}{4} - \left(-12\tfrac{1}{2}\right) = -18\tfrac{3}{4} + 12\tfrac{1}{2}$$
$$= -18\tfrac{3}{4} + 12\tfrac{2}{4}$$
$$= -6\tfrac{1}{4}$$

SOLUTION The difference in elevation is $-6\frac{1}{4}$ ft.

Fluency & Skills Practice

Subtracting Positive and Negative Fractions and Decimals

In this activity, students will subtract positive and negative decimals and fractions.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 10

Subtracting Positive and Negative Fractions and Decimals

➤ Find each difference.

1. $-3.8 - (-6.1)$
2. $-8.2 - 4.2$
3. $-12.7 - (-5.6)$
4. $3.8 - 6.1$
5. $-8.5 - (-9.7)$
6. $7\frac{3}{4} - 4\frac{1}{4}$
7. $-16\frac{5}{8} - \left(-13\frac{3}{8}\right)$
8. $21\frac{5}{6} - \left(-7\frac{2}{3}\right)$

3 Students may also find the difference by decomposing the numbers to make zero pairs, such as $2.6 - (2.6 + 4.7)$. ***Medium***

4 Students may also rewrite each decimal as the sum of a whole number and a decimal part, then combine the whole numbers and the parts separately. ***Medium***

5 **A** **is correct.** This answer is the difference $-4.2 - 8.6$, which can also be expressed as the sum $-4.2 + (-8.6)$.

B is not correct. This answer is the result of ignoring or misunderstanding the meaning of negative numbers in this scenario.

C is not correct. This answer is the result of adding the two quantities as they are stated in the problem and misinterpreting the meaning of the sum as the otter's elevation.

D is not correct. This answer may be the result of misinterpreting the quantity 8.6 meters as an elevation, not a change in elevation.

Basic

3 What is $2.6 - 7.3$? Show your work.

Possible work:

$$2.6 - 7.3 = 2.6 + (-7.3)$$
$$= (2 + 0.6) + [-7 + (-0.3)]$$
$$= 2 + (-7) + 0.6 + (-0.3)$$
$$= -5 + 0.3$$
$$= -4.7$$

SOLUTION -4.7

4 When Daria gets to school, the temperature is 5.7°F. The temperature changes by −9.5°F by the time she goes to bed. What is the temperature when Daria goes to bed? Show your work.

Possible work:

$$5.7 - 9.5 = 5.7 + (-9.5)$$
$$= 5.7 + (-5.7) + (-3) + (-0.8)$$
$$= -3.8$$

SOLUTION The temperature is −3.8°F.

5 An otter is swimming at −4.2 yd relative to the surface of the water. It dives 8.6 yd deeper. After the dive, what is the otter's elevation relative to the surface of the water?

(A) −12.8 yd

B −4.4 yd

C 4.4 yd

D 12.8 yd

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Connect It

MATH TERM

A *term* is a number or a variable. A term can also be the product of numbers, variables, and/or expressions. A term may include exponents.

Levels 1–3: Listening/Writing

Read Connect It problem 4. Help students make sense of the problem by clarifying that *change the order* means *reorder.* Help students identify cognates, such as the Spanish cognates *reordenar, orden,* and *suma.* Rephrase the problem: *What happens when you change the order of terms? How does it change the way you find the sum?*

Review the meaning of *term.* Have partners identify the positive and negative terms in the second Model It. Then have partners use words and gestures to describe how the terms are reordered. Help students write responses in short sentences.

Levels 2–4: Listening/Writing

Have students read Connect It problem 4. Help students make sense of the problem by pointing out that *change the order* means *reorder.* Help students identify the English-Spanish cognate *reorder/reordenar.*

Review the meaning of *term.* Have partners describe the terms in the second Model It. Encourage them to use *positive* and *negative* in their descriptions, along with sequence words, such as *first, next,* and *then*.

Have students draft responses. Then ask them to use **Stronger and Clearer Each Time** to work with partners and clarify their responses.

Levels 3–5: Listening/Writing

Help partners make sense of Connect It problem 4. Ensure that students understand that *reorder* and *change the order* have the same meaning.

Have partners review the meaning of *term* and describe the terms in the second Model It. Encourage students to use sequence words in their descriptions.

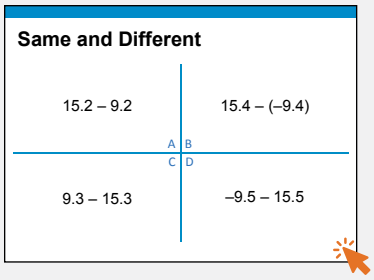
Have students plan and write responses using **Stronger and Clearer Each Time**. Encourage students to offer feedback to clarify and strengthen statements using math vocabulary and sequence words.

Develop Adding and Subtracting Positive and Negative Fractions and Decimals

Purpose

- **Develop** strategies to model a two-step real-world problem using rational numbers, including addition and subtraction.
- **Recognize** that you can use regrouping and properties to solve multi-step problems involving the addition and subtraction of rational numbers.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are subtraction expressions. A is a positive difference of two positive numbers. A and B have positive values. B has the greatest result. C is a negative difference of positive numbers. C and D have negative values. D has the least result.

WHY? Support students' ability to compare subtraction expressions involving decimals.

DEVELOP ACADEMIC LANGUAGE

WHY? Build understanding of qualifiers in math questions.

HOW? Display Connect It problem 5. Ask: *To find the value, which two terms might you combine first*? Ask what the phrase *might you combine first* shows about the answer. If needed, explain that *might* means that the question asks for a possible combination and there is more than one answer. Ask students to reword the question as if there were only one correct answer.

TRY IT

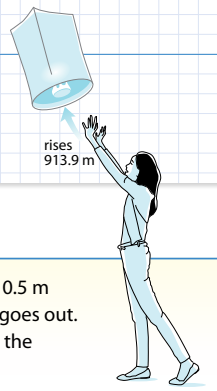
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Students may conclude that the lantern rises when its candle is lit, and it falls when the candle goes out. Listen for recognition that more than one step is needed to solve the problem.

LESSON 10 | SESSION 3

Develop Adding and Subtracting Positive and Negative Fractions and Decimals



Read and try to solve the problem below.

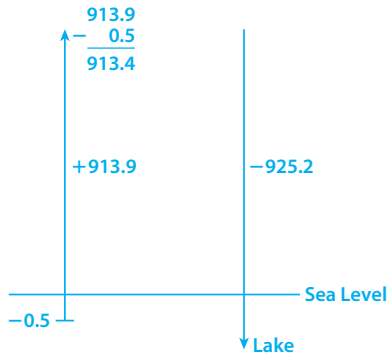
Mei releases a lantern for the Lantern Festival. She stands in a field that is 0.5 m below sea level. The lantern rises 913.9 m. Then the candle in the lantern goes out. The lantern comes down 925.2 m to land on the surface of a lake. What is the elevation of the lake relative to sea level?

TRY IT

Math Toolkit grid paper, number lines, place-value charts

Possible work:

SAMPLE A



913.4 + (−925.2)
913.4 + (−913.4) + (−11.8)
−11.8

The lake is 11.8 m below sea level.

SAMPLE B

Total change in elevation of lantern:

913.9 − 925.2
913.9 + (−913.9) + (−11) + (−0.3)
−11.3

Ending point = starting point + change
= −0.5 + (−11.3)
= −11.8

The elevation of the lake is −11.8 m relative to sea level.

DISCUSS IT

Ask: What did you do first to find the elevation of the lake?

Share: The first thing I did to find the elevation was . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How do you know which value in the problem represents the initial position of the lantern?*
- *What do the numbers 913.9 and 952.2 represent about the lantern?*

Common Misconception Listen for students who state that all of the values in the problem are stated as positive numbers, so the solution is the sum of these values. As students share their strategies, ask them to relate each part of their work to the lantern's position. Have students clarify for one another that the lantern's position changes in two directions (up and down) and have them show how these two directions can be modeled or represented.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- number line showing $-0.5 + 913.9$ and -925.2
- **(misconception)** all numbers added
- equations to calculate the sum of the initial elevation and the changes in elevation

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students listen to the presentations, encourage them to agree and build on strategies by suggesting alternate methods for completing specific steps.

Guide students to **Compare and Connect** the representations. After you pose the question below, have students turn and talk to rehearse their responses.

ASK *How do the models show the lantern's initial elevation and its two changes in elevation?*

LISTEN FOR The initial position is represented as a negative decimal, the first change is an increase, and the second change is a decrease. Students who say that the first change is positive and the second is negative should clarify whether they mean movement in the positive and negative directions.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *How do the expressions in the two Model Its represent the same changes shown in Picture It?*

LISTEN FOR The expressions show the rising of the lantern as adding a positive number and the falling of the lantern as subtracting a positive number or adding a negative number.

For Picture It, prompt students to relate each arrow in the diagram to a change in the lantern's position.

For the first Model It, prompt students to relate the terms in the expression to the problem context.

- *What words in the problem indicate the sign and operation for each number in the expression?*

For the second Model It, prompt students to analyze the steps of the calculation.

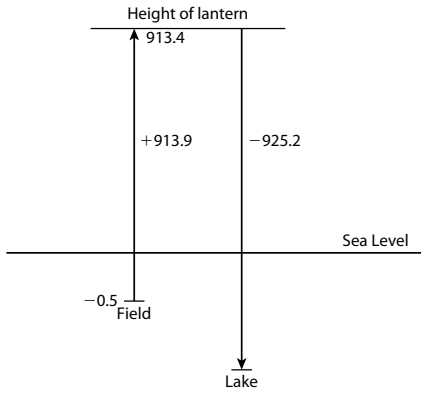
- *Why do you rewrite the subtraction as addition first?*

➤ **Explore different ways to understand adding and subtracting positive and negative numbers.**

Mei releases a lantern for the Lantern Festival. She stands in a field that is 0.5 m below sea level. The lantern rises 913.9 m. Then the candle in the lantern goes out. The lantern comes down 925.2 m to land on the surface of a lake. What is the elevation of the lake relative to sea level?

Picture It

You can draw a diagram to model the lantern's movement.



Model It

You can add and subtract from left to right to simplify.

$$-0.5 + 913.9 - 925.2 = \mathbf{-0.5 + 913.9} - 925.2$$
$$= \mathbf{913.4} - 925.2$$

Model It

You can reorder the terms before simplifying.

$$\mathbf{-0.5} + 913.9 \mathbf{- 925.2} = \mathbf{-0.5} + 913.9 \mathbf{+ (-925.2)}$$
$$= \mathbf{-0.5 + (-925.2)} + 913.9$$
$$= \mathbf{-925.7} + 913.9$$

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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Making Sense of Quantities and the Relationships Between Them

Prompt students to reason about the relationships between quantities when simplifying expressions.

ASK *What is the reasoning behind reordering so that the first and last terms are combined first?*

LISTEN FOR Subtracting 925.2 can be rewritten as adding -925.2, so then the first two terms in the problem are both negative. Combining two negative or two positive terms can be simpler than combining a negative and a positive term.

ASK *Instead of reordering, how could you decompose the terms to make simplifying easier?*

LISTEN FOR Since they are all decimals, you could decompose the terms into their whole and decimal parts. Or, since there are positive and negative terms, you could decompose them to form zero pairs.

Generalize Encourage students to think strategically about the quantities and come up with other methods of rewriting the expression to simplify it.

Develop Adding and Subtracting Positive and Negative Fractions and Decimals

CONNECT IT

SMP 2, 4, 5, 6

Remind students that they have already practiced adding and subtracting positive and negative integers, fractions, and decimals in real-life scenarios. Explain that now they will apply these skills to multi-step problems, in which both addition and subtraction may be needed to find the solution.

Before students begin to record and expand on their work in Picture It and Model It, tell them that problem 4 will prepare them to provide the comparison asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- An increase in elevation is shown in the diagram with an arrow pointing up and in the expressions by adding a positive number.
- A decrease in elevation is shown in the diagram with an arrow pointing down and in the expressions by subtracting a positive number or adding a negative number.
- Addition is commutative.

Facilitate Whole Class Discussion

4 Students may point out that changing the order of the terms is not allowed for subtraction. For example, $9 - 5 \neq 5 - 9$. However, a subtraction expression may be rewritten as addition of the opposite. Students should understand that the terms of an addition expression may be written in any order, which can make calculations easier.

> **ASK** *Why might you want to add terms in a different order from the one they were given in?*
>
> **LISTEN FOR** You can use an order that makes calculation easier.

5 Look for understanding that choosing pairs of terms to combine makes the solution method more efficient.

> **ASK** *How do you choose a pair of terms to combine first?*
>
> **LISTEN FOR** You analyze the quantities to see which can be combined most easily.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to add and subtract positive and negative numbers.**

1. Look at **Picture It**. Does the lantern end up lower or higher than its starting point? How do the arrows show the lantern's movement?
   Lower; When the lantern rises, the arrow points up. When the lantern falls, the arrow points down.

2. Look at the first **Model It**. What is the value of the expression $-0.5 + 913.9 - 925.2$? How does the expression show the movement of the lantern?
   -11.8; The expression uses addition to show the lantern rising. It uses subtraction to show the lantern falling.

3. Look at the second **Model It**. Why can you change the order of the terms?
   You can add in any order.

4. How does changing the order of the terms change the way you find the sum?
   You can combine all the positive terms and all the negative terms. Then you only need to add a negative and a positive once.

5. Consider the expression $1\frac{5}{6} - 2\frac{1}{3} - 1\frac{1}{6}$. To find the value, which two terms might you combine first? Why?
   Possible answers: You could combine $1\frac{5}{6}$ and $-1\frac{1}{6}$ because they have the same denominator. Or you could combine $-2\frac{1}{3}$ and $-1\frac{1}{6}$ because they both involve moving in the negative direction.

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
   Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model strategies for solving a multi-step addition problem.

If students are unsure about simplifying an expression with rational numbers, then use this activity to model adding rational numbers.

Materials For each pair: 2 colored pencils (1 each of two different colors), 2 copies of Activity Sheet *Fraction Bars*

- Display the expression $-1\frac{3}{5} + 1\frac{2}{5}$. Students work together and use fraction bars to add. Suggest to pairs that, as they fill in the fraction bars, they use one color for positive numbers and the other color for negative numbers.
- Ask: *What are some ways you can add these numbers?* [You can break them down into $-1 + 1 + (-\frac{3}{5}) + \frac{2}{5}$ or you can make a zero pair: $-1\frac{2}{5} + -\frac{1}{5} + 1\frac{2}{5}$]
- Display the expression $1\frac{1}{3} + (-1\frac{1}{3}) + (-\frac{2}{3})$ and have students add.
- Ask: *What are some ways you can add these numbers?* [You can make a zero pair with $1\frac{1}{3}$ and $-1\frac{1}{3}$ and have $0 + (-\frac{2}{3})$ remaining or you can add $-1\frac{2}{3} + (-\frac{2}{3})$ to get -2 and then add $1\frac{1}{3}$.]

Apply It

For all problems, encourage students to use a model to support their thinking. Students may represent addition and subtraction by drawing arrows on a number line.

7 **B is correct.** Students may choose to reorder the terms to combine the two terms with 0.25 decimal parts.

A is not correct. This answer may be the result of adding the three quantities in the expression and interpreting the sum as a negative elevation.

C is not correct. This answer may be the result of misapplying a strategy for adding a positive decimal and a negative decimal.

D is not correct. This answer may be the result of adding the three quantities expressed in the problem without interpreting any of them as negative.

8 Students may also solve the problem by subtracting 7.5 from 5.4 and then subtracting an additional 6.3 from the difference. Students may use a number line to represent the difference.

Apply It

➤ **Use what you learned to solve these problems.**

7 A helicopter is 19.25 meters above the top of a canyon wall. It goes down 27.60 meters, passing into the canyon. Then it goes up 5.25 meters. What is the new position of the helicopter relative to the top of the canyon wall?

A −52.1 m

B −3.1 m

C 3.1 m

D 52.1 m

8 The temperature during the day is 5.4°C. After sunset, the temperature falls 7.5°C. Wind chill makes the temperature after sunset feel 6.3°C colder. What temperature does it feel like after sunset? Show your work.

Possible work:

$$\begin{aligned} 5.4 - 7.5 - 6.3 &= 5.4 + [-7.5 + (-6.3)] \\ &= 5.4 + (-13.8) \\ &= 5.4 + (-5.4) + (-8.4) \\ &= -8.4 \end{aligned}$$

SOLUTION The temperature feels like −8.4°C.

9 What is $-5\frac{1}{2} - 9\frac{1}{4} + 2\frac{3}{4}$? Show your work.

Possible work:

$$\begin{aligned} -5\tfrac{1}{2} - 9\tfrac{1}{4} + 2\tfrac{3}{4} &= -5\tfrac{1}{2} + \left(-9\tfrac{1}{4} + 2\tfrac{3}{4}\right) \\ &= -5\tfrac{1}{2} + \left[-2\tfrac{3}{4} + \left(-6\tfrac{1}{2}\right) + 2\tfrac{3}{4}\right] \\ &= -5\tfrac{1}{2} + \left(-6\tfrac{1}{2}\right) \\ &= -12 \end{aligned}$$

SOLUTION −12

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- rewriting and/or regrouping values in an expression to make it easier to solve.
- strategies for calculating addition or subtraction with positive and negative fractions use the same processes as adding or subtracting with integers.

Error Alert If students combine $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$, which are the fractional parts of the expression, to equal $1\frac{1}{2}$ and add this sum to the sum of the whole numbers to find $-10\frac{1}{2}$, then ask students to represent the mixed numbers on a number line and to identify their positions between whole numbers. For example, prompt students to locate $-5\frac{1}{2}$ on the number line and decide if it is equal to $\left(-5 + \frac{1}{2}\right)$ or to $\left(-5 + \left(-\frac{1}{2}\right)\right)$.

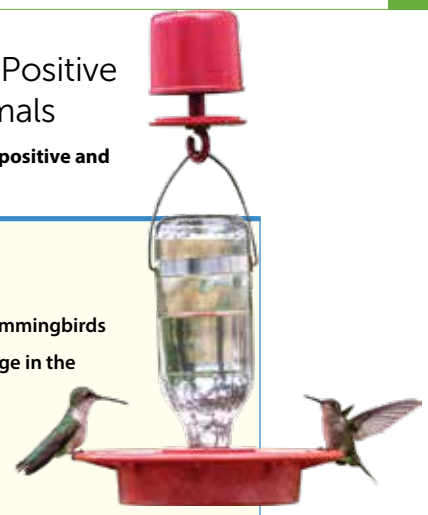
Practice Adding and Subtracting Positive and Negative Fractions and Decimals

Problem Notes

Assign **Practice Adding and Subtracting Positive and Negative Fractions and Decimals** as extra practice in class or as homework.

1. Students should recognize that they could use the solution in the Example to solve this problem. They need not repeat the calculation of the overall change in food in the feeder. ***Basic***

Practice Adding and Subtracting Positive and Negative Fractions and Decimals



➤ **Study the Example showing adding and subtracting positive and negative fractions. Then solve problems 1–4.**

Example

Hummingbirds sip $2\frac{1}{2}$ fl oz of the liquid in a feeder. Then $1\frac{1}{2}$ fl oz of food is added to the feeder. Last, hummingbirds sip another $3\frac{1}{4}$ fl oz of food. What is the overall change in the amount of food in the feeder?

You can write an expression to represent the situation.

$$-2\tfrac{1}{2} + 1\tfrac{1}{2} - 3\tfrac{1}{4}$$

You can begin simplifying the expression by reordering the terms.

$$\begin{aligned} -2\tfrac{1}{2} + 1\tfrac{1}{2} - 3\tfrac{1}{4} &= -2\tfrac{1}{2} - 3\tfrac{1}{4} + 1\tfrac{1}{2} \\ &= -5\tfrac{3}{4} + 1\tfrac{1}{2} \\ &= -4\tfrac{1}{4} \end{aligned}$$

The overall change in the amount of food in the feeder is $-4\frac{1}{4}$ fl oz.

1. The feeder in the Example had 16 fl oz of food in it to start. How much food does it have now? Show your work.

Possible work:

$$16 - 4\tfrac{1}{4} = 16 - 4 - \tfrac{1}{4}$$
$$= 11\tfrac{3}{4}$$

SOLUTION It has $11\frac{3}{4}$ fl oz of food now.

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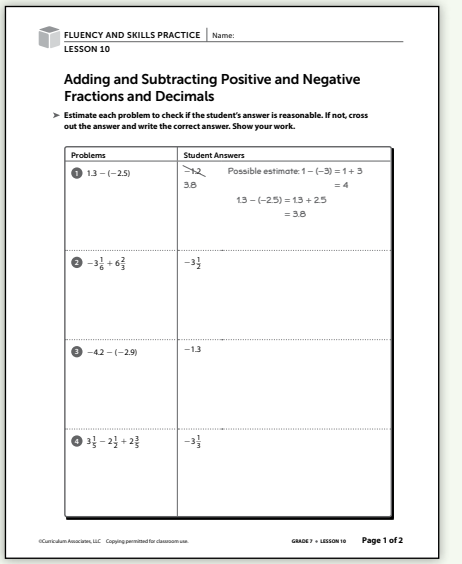
Fluency & Skills Practice

Adding and Subtracting Positive and Negative Fractions and Decimals

In this activity, students estimate a sum or difference to determine if a given solution is accurate. They then compute the sum or difference for exercises with incorrect answers.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 10

Adding and Subtracting Positive and Negative Fractions and Decimals

➤ **Estimate each problem to check if the student's answer is reasonable. If not, cross out the answer and write the correct answer. Show your work.**

Problems	Student Answers
1 $1.3 - (-2.5)$	~~1.2~~ 3.8 Possible estimate: $1 - (-3) = 1 + 3 = 4$ $1.3 - (-2.5) = 1.3 + 2.5 = 3.8$
2 $-3\frac{1}{6} + 6\frac{2}{3}$	$-3\frac{1}{2}$
3 $-4.2 - (-2.9)$	-1.3
4 $3\frac{1}{5} - 2\frac{1}{2} + 2\frac{3}{5}$	$-3\frac{1}{3}$

Additional Practice

2 **a.** Students could evaluate the subtraction of a positive number as being equal to the addition of the opposite of that number.

b. Students could break apart a negative decimal as the sum of a negative integer and a negative decimal part.

c. Students could compare the expressions in b and c and recognize that the first decimal in b has been broken apart into an integer and a decimal part.

d. Students could evaluate a negative decimal as the sum of a negative integer and a negative decimal part, not a positive decimal part.

Medium

3 Students may choose to represent the problem as the sum of three decimals, including two negative decimals for the loss of points and one positive decimal for the gain of points. ***Medium***

4 Students may recognize that by first combining the mixed numbers in fourths, the solution can be found with fewer steps. ***Challenge***

2 Is each expression equivalent to $-2.7 + (-3.1)$?

	Yes	No
a. $-2.7 - 3.1$	●	○
b. $-2.7 + (-3) + (-0.1)$	●	○
c. $-2 + (-0.7) + (-3) + (-0.1)$	●	○
d. $-2 + 0.7 + (-3.1)$	○	●

3 Reth is playing a game. First, he loses 4.8 points. Then he gains 2.5 points. Finally, he loses another 7.8 points. What is the overall change in Reth's score? Show your work. **Possible work:**

$$-4.8 + 2.5 - 7.8 = -4.8 - 7.8 + 2.5$$
$$= -12.6 + 2.5$$
$$= -10.1$$

SOLUTION The overall change in Reth's score is -10.1.

4 An autonomous underwater vehicle (AUV) is at an elevation of -8.25 ft. It dives down $6\frac{2}{3}$ ft to collect a specimen. Then the AUV dives another $15\frac{3}{4}$ ft. What is the final elevation of the AUV? Show your work. **Possible work:**

$$-8.25 - 6\tfrac{2}{3} - 15\tfrac{3}{4} = -8\tfrac{1}{4} + \left(-6\tfrac{2}{3}\right) + \left(-15\tfrac{3}{4}\right)$$
$$= \left[-8\tfrac{1}{4} + \left(-15\tfrac{3}{4}\right)\right] + \left(-6\tfrac{2}{3}\right)$$
$$= -24 + \left(-6\tfrac{2}{3}\right)$$
$$= -30\tfrac{2}{3}$$

AUV: autonomous underwater vehicle

SOLUTION The AUV's final elevation is $-30\frac{2}{3}$ ft.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

Levels 1–3: Reading/Speaking

Help students interpret Apply It problem 5. Read the problem with students. Review the meaning of *represent* and point out the Spanish cognate *representar*. Help students identify other cognates in the problem.

Help students understand each situation. Display and clarify the verbs *earned, spent, got off, got on,* and *dropped*. Help students decide if each verb represents a positive or negative change. Suggest that students draw a picture to represent situation A, use **Act It Out** to role-play situations B and C, and draw a number line to represent situation D. Support students' understanding by having them write the symbols + or − next to the quantities related to each verb.

Levels 2–4: Reading/Speaking

Help students interpret Apply It problem 5. Read the problem with students. Review the meaning of *represent*. Have students discuss what it means for an expression to represent a situation.

Explain that students should pay attention to the parts of the situations that represent a positive or a negative change. Have students describe phrases in each situation using these frames:

- *The phrase "____" shows a positive/negative change.*
- *I can represent the phrase with the term ____.*

Levels 3–5: Reading/Speaking

Help students interpret and justify their responses to Apply It problem 5. Have partners review the meaning of *represent* and discuss what it means for an expression to represent a problem.

Have partners use **Say It Another Way** to confirm their understanding of the situations. Encourage them to identify parts of the situations that represent a positive or a negative change, such as *got off* and *got on*.

Provide students with individual think time to choose their answers. Have students justify their reasoning to a partner using this frame:

- *I know my answer is correct because ____.*

Refine Adding and Subtracting Positive and Negative Numbers

Purpose

- **Refine** strategies for adding and subtracting positive and negative integers, fractions, and decimals.
- **Refine** understanding of when a sum or difference will be positive, negative, or zero.

START CHECK FOR UNDERSTANDING

Find the sum.

$12.1 + (-6) + 5.36 + (-11.1)$

Solution

0.36

WHY? Confirm students' ability to solve a multi-step addition or subtraction problem.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 10 | SESSION 4

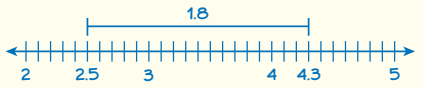
Refine Adding and Subtracting Positive and Negative Numbers

➤ **Complete the Example below. Then solve problems 1–10.**

Example

What is 2.5 − 4.3?

Look at how you could use absolute value to find the difference.



On a number line, 2.5 and 4.3 are 1.8 units apart. The distance between two numbers is the same as the absolute value of their difference. So, $|2.5 - 4.3| = 1.8$.

Since $2.5 - 4.3$ has a negative result, the difference is a negative number with an absolute value of 1.8.

SOLUTION -1.8

CONSIDER THIS . . .
Distance on a number line is never negative.

PAIR/SHARE
Does the method in the Example work for subtracting negative numbers?

Apply It

1 Cece's dog loses $1\frac{1}{4}$ lb. Then her dog loses another $1\frac{1}{8}$ lb. Finally, it gains $\frac{3}{8}$ lb. What is the total change in her dog's weight? Show your work.

Possible work:

$$-1\tfrac{1}{4} - 1\tfrac{1}{8} + \tfrac{3}{8} = -1\tfrac{1}{4} + \left(-1\tfrac{1}{8}\right) + \tfrac{3}{8}$$
$$= -1\tfrac{1}{4} + \left(-\tfrac{9}{8} + \tfrac{3}{8}\right)$$
$$= -1\tfrac{1}{4} + \left(-\tfrac{6}{8}\right)$$
$$= -1\tfrac{2}{8} + \left(-\tfrac{6}{8}\right)$$
$$= -2$$

SOLUTION The change in the dog's weight is −2 lb.

CONSIDER THIS . . .
The result would be the same if Cece's dog lost weight, then gained weight, then lost weight.

PAIR/SHARE
Suppose Cece's dog weighs 38 pounds to start. Does that change your answer? Why?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
0.56	have added all of the decimal parts as positives and then correctly added the integer parts.	Ask students to decompose −11.1 into its integer and decimal parts. Have students explain why the parts are −11 and −0.1, not −11 and 0.1.
−0.36	have incorrectly added −6 and 5.36.	Have students switch the positions of the two addends in the middle of the expression. Ask them to explain why this will not change the result. Then ask them to compare the sum of the two positive numbers and the sum of the two negative numbers.
34.56	have added negative numbers as though they were positive numbers.	Ask students to rewrite the expression with integers that approximate the decimals. Have them explain the sum they calculate and then extend the solution to the original problem.

Example

Guide students in understanding the Example. Ask:

- *What does the number 1.8 represent in this problem?*
- *What subtraction problem related to the Example has a difference of 1.8?*
- *How can you tell whether the result of a subtraction problem is positive or negative?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' responses.

Look for understanding of modeling a subtraction problem as a distance on a number line.

Apply It

1. Students may also convert each mixed number to a fraction, or they may begin by adding any pair of terms and then adding the third term. Students may also construct a number line or other visual model to support their calculations. ***DOK 2***

2. Students may also find the difference on a number line or rewrite the expression by decomposing the numbers into their integer and decimal parts. ***DOK 1***

3. **D is correct.** Students may conclude that if x were positive, then the difference would be a negative number, as shown by moving left from -0.5 on a number line. Therefore, x must be negative.

   A is not correct. This answer may be the result of evaluating $x - 0.5$ instead of the expression presented in the problem.

   B is not correct. This answer may be the result of misinterpreting the problem as a request to identify a positive value of x.

   C is not correct. This answer may be the result of concluding that x must be negative, but not recognizing the identity $x - x = 0$.

   DOK 3

2 What is $-8.3 - (-5.4)$? Show your work.

Possible work:

$$-8.3 - (-5.4) = -8.3 + 5.4$$
$$= -5.4 + (-2.9) + 5.4$$
$$= -2.9$$

CONSIDER THIS . . .
The inverse of subtraction is addition.

PAIR/SHARE
How can you check that your answer makes sense?

SOLUTION -2.9

3 Which value of x makes the following statement true?

$-0.5 - x =$ a positive number

A $x = 1.5$

B $x = 0.5$

C $x = -0.5$

D $x = -1.5$

CONSIDER THIS . . .
Zero is not a positive or a negative number.

Lulu chose A as the correct answer. How might she have gotten that answer?

Possible answer: Lulu might have thought that because the result of subtracting x was positive, she should move in the positive direction toward 0. But subtracting a positive number is the same as adding a negative, so the result moves in the negative direction away from 0.

PAIR/SHARE
What is another value that makes this statement true?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency
- **REINFORCE** Problems 4–9

Extending Beyond Proficiency
- **REINFORCE** Problems 4–9
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Adding and Subtracting Positive and Negative Numbers

Apply It

4 See ***Connect to Culture*** to support student engagement. Students should model the problem by representing the initial position of the submarine as a negative number and the two changes as an increase and a decrease to that number. ***DOK 2***

5 **D is correct.** Students may interpret the expression as an initial decrease of 8, or addition of -8, and then a decrease of 10. Any units that have negative values, such as degrees Fahrenheit, could be used.

A is not correct. This answer may be the result of matching the numerals in the expression to those in the problem without regard to negative signs or operations.

B is not correct. This answer uses $8 - 10$ instead of $(-8) - 10$.

C is not correct. This situation is represented by $-8 + 10$.

DOK 2

6 Students may also estimate the value of the expression by substituting the closest integer for each decimal and then adding or subtracting. ***DOK 2***

7 Students should recognize that subtracting a negative number is the same as adding its opposite, a positive number. They may reason that negative numbers of relatively large absolute value, such as -100 and -200, will make the difference a positive number. ***DOK 3***

4 A submarine 50.2 m below the surface of the ocean goes up 15.6 m. Then it goes down 35.7 m. What is the submarine's new position relative to the surface of the ocean? Show your work. **Possible work:**

$$-50.2 + 15.6 - 35.7 = -50.2 + 15.6 + (-35.7)$$
$$= -50.2 + (-35.7) + 15.6$$
$$= -85.9 + 15.6$$
$$= -70.3$$

SOLUTION The submarine's position is -70.3 m relative to the surface.

5 Which situation could the expression $-8 - 10$ represent?

A An elevator stops at the 8th floor. Then it stops at the 10th floor.

B Gabe earned \$8. Then he spent \$10 on a new book.

C At a bus stop, 8 people got off and 10 people got on.

D The temperature dropped 8°F. Then it dropped another 10°F.

6 Without calculating, explain how you can tell if the value of the expression $-9.45 - (-1.72) - 2.53$ is *positive* or *negative*.

Possible explanation: Finding the result will mean moving more than 10 units in the negative direction and fewer than 2 units in the positive direction. The result has to be negative.

7 The value of $-4\frac{5}{8} - x$ is positive. What are two possible values of x? Show your work. **Possible work:**

$-4\frac{5}{8} - x = -4\frac{5}{8} + (-x)$ and $-4\frac{5}{8} + 4\frac{5}{8} = 0$.

When $x > -4\frac{5}{8}$, then $-4\frac{5}{8} - x$ is less than 0.

When $x < -4\frac{5}{8}$, then $-4\frac{5}{8} - x$ is greater than 0.

SOLUTION Possible answer: x could be -5 or -6. (Accept any answers less than $-4\frac{5}{8}$.)

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DIFFERENTIATION

RETEACH

Hands-On Activity

Use a model to represent adding and subtracting fractions and decimals.

Students approaching proficiency with solving real-life problems involving addition and subtraction with rational fractions and decimals will benefit from this modeling activity.

Materials For each pair: 1 counter, sticky notes, Activity Sheet *Number Lines*

- Have pairs write the terms *increases by, decreases by, gains, loses, goes down,* and *goes up* on sticky notes.
- Have pairs label a vertical number line in twelfths from -1 to 1 and place a counter at 0. Say: *The counter represents a bird and the zero point represents a tree branch it is sitting on.*
- Ask students to move the counter to show the bird going lower by $\frac{5}{12}$m. Then ask them to use one of their sticky notes to write a sentence describing what just happened. [Students should move the counter down $\frac{5}{12}$; Sentences should use *decreases by, loses,* or *goes down.*]
- Display: $-\frac{5}{12} + \frac{2}{3}$. Ask: *How can you use the counter to show the bird doing this? How can you write a sentence with one of your sticky notes to show this?* [Move the counter up $\frac{2}{3}$; Sentences should use *increases by, gains,* or *goes up.*]
- Display $\frac{1}{4} - (-\frac{7}{12})$. Ask: *How can you use the counter to show the bird doing this?* [Move the counter up $\frac{7}{12}$.] Ask: *How can you write a sentence with one of your sticky notes and the fraction $-\frac{7}{12}$ to show this?* [Students should use *decreases by, goes down,* or *loses.*]
- Repeat with further addition of positive and negative decimals and fractions as needed.

8 Students could support their reasoning by selecting reasonable values for the two variables and then calculating $a - b$. ***DOK 3***

9 Students should recognize that the question is phrased so that only a positive value is correct, similar to expressions of distance along a number line. They may decide to calculate the absolute value of the difference between the two numbers. ***DOK 2***

CLOSE EXIT TICKET

10 **Math Journal** Students may evaluate the subtraction of a negative decimal as the addition of its opposite.

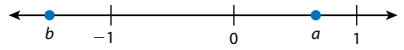
Error Alert If students misinterpret the decimal part of a negative decimal as a positive value, then have them solve a related problem in which the decimals are rounded to the nearest integer. Students may use a number line to find the difference. Then have students evaluate the difference of the two negative decimals, ensuring that the sign or direction of each number is also applied to its decimal part.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they use problem 5 as an example of the word *represent*. In this problem, only one of the answer choices represents the expression.

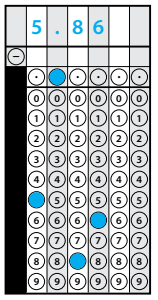
SELF CHECK Have students review and check off any new skills on the Unit 2 Opener.

8 Is $a - b$ a *positive number* or a *negative number*? How do you know?



Positive number; Possible explanation: $a - b = a + (-b)$. Since b is negative, $-b$ is positive. The sum of two positive numbers is positive.

9 Esteban stands on a dock 1.25 yd above the water. A fish swims below Esteban at −4.61 yd from the surface of the water. How many yards apart are Esteban and the fish?



10 **Math Journal** Write a subtraction problem involving two negative decimal numbers. At least one decimal must include hundredths. Explain how to solve your problem.

Possible answer: $-8.05 - (-3.24)$; To solve, add the opposite and use zero pairs: $-8.05 - (-3.24) = -8.05 + (3.24)$

$= -4.81$

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *represent*. Write at least one synonym for *represent*.
- ☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.

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REINFORCE

Problems 4–9

Solve addition and subtraction problems with integers, fractions, and decimals.

Students meeting proficiency will benefit from additional work with addition and subtraction by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve missing addend problems.

Students extending beyond proficiency will benefit from solving multi-step problems involving unknown addends.

- Present this problem: *Hal is sitting at the front of a moving bus and drops his water bottle. It rolls all the way to the back of the bus, a distance of 5.8 meters. Compared to the ground, the water bottle travels only 0.9 meter back because the bus is moving forward. How far has the bus moved?*
- Students may reason that the bus's movement is the difference of 5.8 and 0.9, or 4.9 meters.
- Have students propose, model, and solve similar problems.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Multiply and Divide with Negative Numbers

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Multiply and divide with positive and negative rational numbers.
- Model real-world contexts with division and multiplication of rational numbers.
- Write negative numbers as quotients of integers in several ways.
- Understand that the same strategies that work for multiplying and dividing positive rational numbers work for negative rational numbers.

Language Objectives

- Respond to clarifying questions about multiplying and dividing with rational numbers using lesson vocabulary.
- Interpret word problems by identifying how the relationship between rational numbers can be represented with multiplication or division.
- Explain why forms of negative rational numbers are equivalent using *quotient*.
- Explain strategies for multiplying and dividing by providing specific reasons why they are correct or incorrect in partner and class discussions.

Prior Knowledge

- Understand subtraction of rational numbers as adding the additive inverse.
- Recognize that the product of two negative integers is positive, while the product of a negative and positive integer is negative.
- Multiply and divide with positive decimals and fractions.
- Convert a terminating decimal to a fraction.

Vocabulary

Math Vocabulary

rational number a number that can be expressed as the fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Review the following key terms.

fraction a number that names equal parts of a whole. A fraction names a point on the number line and can also represent the division of two numbers.

opposite numbers numbers that are the same distance from 0 on the number line but in opposite directions. Opposite numbers have the same numeral, but opposite signs. The opposite of a number is also called the *additive inverse* of that number.

product the result of multiplication.

quotient the result of division.

reciprocal for any nonzero number a, the reciprocal is $\frac{1}{a}$. The reciprocal of any fraction $\frac{a}{b}$ is $\frac{b}{a}$. Zero does not have a reciprocal. The reciprocal of a number is also called the *multiplicative inverse* of that number.

Academic Vocabulary

calculation a process or result of finding a value using mathematics.

Learning Progression

In Grade 6, students multiplied and divided positive fractions and decimals.

In the previous lesson, students learned how to multiply with negative integers. They understood that multiplying two negative integers results in a positive product and that multiplying one positive integer and one negative integer results in a negative product.

➤

In this lesson, students apply their knowledge of multiplying negative integers to multiplying rational numbers. They combine this with their understanding of the relationship between multiplication and division to see patterns in division with negative rational numbers and to divide rational numbers.

➤

Later in Grade 7, students will divide integers in order to find decimal expansions of rational numbers. They will also use the four operations with rational numbers to solve real-world and mathematical problems.

In Grade 8, students will apply their understanding of multiplying with negative numbers to understanding properties of negative exponents.

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Multiplying and Dividing with Negative Numbers (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 235–236)	**Math Toolkit** grid paper, integer chips, number lines Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 25 integer chips
SESSION 2 **Develop** Multiplying Negative Rational Numbers (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 241–242)	**Math Toolkit** grid paper, number lines Presentation Slides ↖	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Dividing Negative Rational Numbers (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 247–248)	**Math Toolkit** grid paper, number lines Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: base-ten blocks (20 ones units) **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Multiplying and Dividing with Negative Numbers (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Visual Model **REINFORCE** Problems 4–9 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 12 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

Overview | Multiply and Divide with Negative Numbers

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It The Burj Khalifa building in the United Arab Emirates is the tallest building in the world. It is 2,717 feet tall and has 163 floors! It is nearly three times as tall as the Eiffel Towel and about twice as tall as the Empire State Building. One World Trade Center, located in New York City, is the tallest building in the United States, standing at 1,776 feet tall. Have students think of the tallest building they have ever been in. Say: *Raise your hand if the building had more than 5 floors.* Then repeat with 10 floors, 20 floors, 50 floors, and so on, until no one raises a hand.

SESSION 2 ■ ■ □ □

Apply It **Problem 7** A peregrine falcon is a bird of prey, meaning it hunts and feeds on animals larger than itself. Falcons have keen eyesight for spotting their victims from over half a mile in the air, and when diving they may reach a speed of 200 miles per hour. Peregrine falcons often eat other birds, such as pigeons and ducks, and are able to pursue their targets while flying. Ask students for some examples of birds they have seen near where they live.

SESSION 3 ■ ■ ■ □

Try It The Nile River in northeast Africa is the longest river in the world, about 4,160 miles long. It passes through five different countries. Until the 1960s, the Nile River flooded every year between June and September. This flooding brought important nutrients to the land around the Nile River, making the dry desert land flourish. The Aswan Dam was built in the 1960s, and annual flooding is no longer as great. Have students raise their hands if they have ever seen or built a dam. Then call on several students to describe their experiences.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 1** Have students share their favorite game shows to watch or to listen to. Quiz shows have been popular as far back as the radio age. The longest-running game shows have been on TV for about 50 seasons! Since people can compete with their brains, brawn, or shopping skills, there is a game show for everyone. Ask students which game shows they would most like to compete on and why.

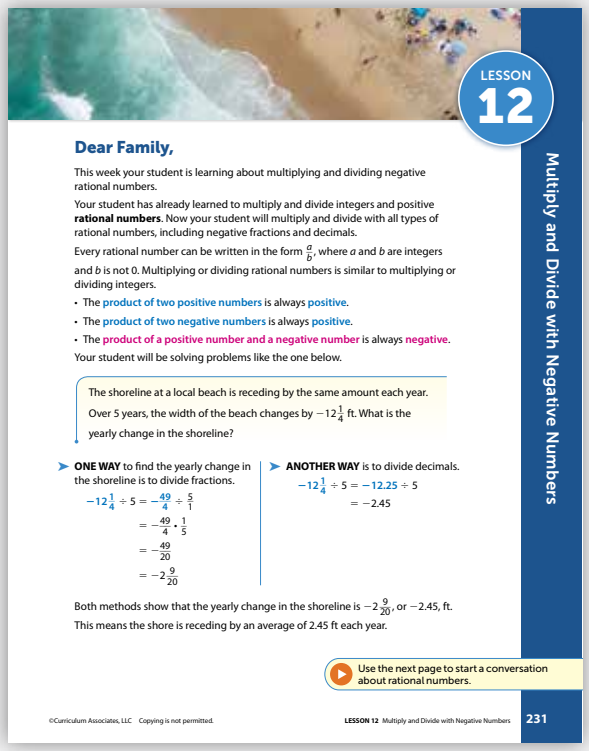
CULTURAL CONNECTION

Alternate Notation Latin American countries use all the same symbols for division that are used in the United States, but they sometimes use another symbol as well. The colon may also be used to indicate division, as in $12 : 2 = 6$. Encourage students who have experience with this notation for division to share what they know with the class.

$5 \div 3$
OR
$5 : 3$

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 12

Multiply and Divide with Negative Numbers

Dear Family,

This week your student is learning about multiplying and dividing negative rational numbers.

Your student has already learned to multiply and divide integers and positive **rational numbers**. Now your student will multiply and divide with all types of rational numbers, including negative fractions and decimals.

Every rational number can be written in the form $\frac{a}{b}$, where a and b are integers and b is not 0. Multiplying or dividing rational numbers is similar to multiplying or dividing integers.

- The **product of two positive numbers** is always **positive**.
- The **product of two negative numbers** is always **positive**.
- The **product of a positive number and a negative number** is always **negative**.

Your student will be solving problems like the one below.

> The shoreline at a local beach is receding by the same amount each year. Over 5 years, the width of the beach changes by $-12\frac{1}{4}$ ft. What is the yearly change in the shoreline?

➤ **ONE WAY** to find the yearly change in the shoreline is to divide fractions.

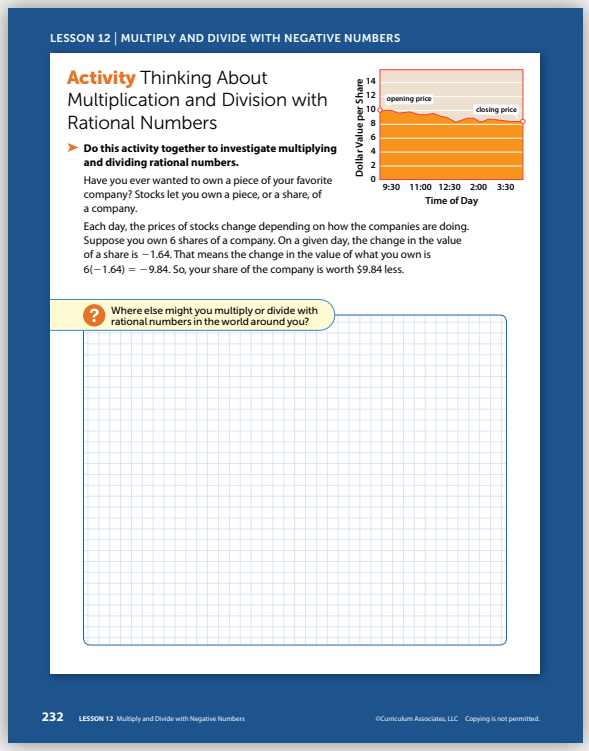
$$-12\tfrac{1}{4} \div 5 = -\tfrac{49}{4} \div \tfrac{5}{1}$$
$$= -\tfrac{49}{4} \cdot \tfrac{1}{5}$$
$$= -\tfrac{49}{20}$$
$$= -2\tfrac{9}{20}$$

➤ **ANOTHER WAY** is to divide decimals.

$$-12\tfrac{1}{4} \div 5 = -12.25 \div 5$$
$$= -2.45$$

Both methods show that the yearly change in the shoreline is $-2\frac{9}{20}$, or -2.45, ft. This means the shore is receding by an average of 2.45 ft each year.

Use the next page to start a conversation about rational numbers.



Activity Thinking About Multiplication and Division with Rational Numbers

➤ **Do this activity together to investigate multiplying and dividing rational numbers.**

Have you ever wanted to own a piece of your favorite company? Stocks let you own a piece, or a share, of a company.

Each day, the prices of stocks change depending on how the companies are doing. Suppose you own 6 shares of a company. On a given day, the change in the value of a share is -1.64. That means the change in the value of what you own is $6(-1.64) = -9.84$. So, your share of the company is worth $9.84 less.

? Where else might you multiply or divide with rational numbers in the world around you?

Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

MATH TERM

A *divisor* is a number that divides another number.

ACADEMIC VOCABULARY

A *form* is a visible shape or structure of something.

Represent means to use as a sign, symbol, or example for something.

Levels 1–3: Reading/Speaking

Help students make sense of Connect It problem 2 using a **Co-Constructed Word Bank**. Help students read paragraph 1 and identify words for the word bank, like *represent, integer, except, rational number, quotient, divisor,* and *form*. Guide students to write examples of division in sentences 2–3 and talk about them using words from the word bank.

Next, read problem 2a with students. Ask partners to write different fractions and to discuss them using:

- *The fraction ____ represents the division problem ____.*
- *I can/cannot divide ____ by ____.*

Levels 2–4: Reading/Speaking

Help students make sense of Connect It problem 2. Read paragraph 1 with students and help them express $\frac{a}{b}$ and $b \neq 0$ in words. Have students use a **Co-Constructed Word Bank** to identify key words and phrases, like *divide* and *form*. Help students discuss how words are related and sort the words into the categories *Number Forms, Division Words,* and *Academic Words.*

Next, ask partners to write examples of division in sentences 2–3 and discuss using words from the Word Bank. Then have students use **Say It Another Way** to read and confirm understanding of problem 2a.

Levels 3–5: Reading/Speaking

Support students as they make sense of Connect It problem 2. Have students read the problem and identify words for forms of numbers (*integer, rational number, fraction*) and words that are related to division (*divide, quotient, fraction*).

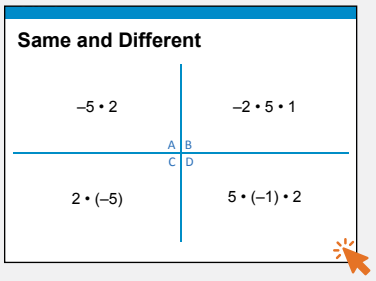
Next, ask partners to write examples of the division in sentences 2–3 and discuss using division words. Prompt students to include examples where 0 is divided by a rational number. Then have partners use **Say It Another Way** to read and confirm understanding of problem 2a. Ask partners to discuss how the fraction is related to the division described in paragraph 1.

Explore Multiplying and Dividing with Negative Numbers

Purpose

- **Explore** the idea that division remains the inverse of multiplication when working with negative numbers.
- **Understand** that negative numbers can be multiplied and divided, resulting in a positive or negative fraction or decimal.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All the expressions have a value of −10.

A and C have the same factors.

B and D have the same factors.

All the factors are in different orders.

WHY? Support students' facility with multiplying negative integers.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Encourage students to rephrase the text to explain why a window washer would lower himself down several times.

DISCUSS IT SMP 2, 3, 6

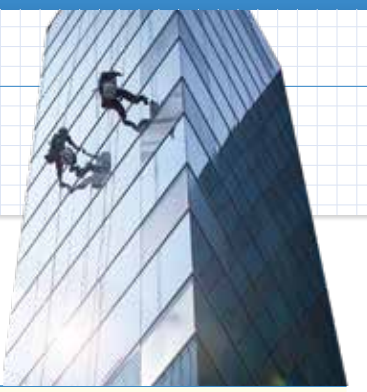
Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the change in Ryan's position can be represented by a negative number for both the total change of −48 ft and the six drops of −8 ft each.
- the total distance, 48 ft, can be divided into 6 equal groups, or that 6 equal groups have a product of 48 ft.
- a negative number represents a decrease in elevation and a positive number represents an increase in elevation.

LESSON 12 | SESSION 1 ■ □ □ □

Explore Multiplying and Dividing with Negative Numbers



Previously, you learned about multiplying integers. In this lesson, you will learn about multiplying and dividing with rational numbers.

➤ **Use what you know to try to solve the problem below.**

Ryan is washing windows on a skyscraper. He lowers himself six times, going down an equal distance each time. In all, Ryan lowers himself 48 ft. What integer represents the change in Ryan's position each time he lowers himself? What does this integer tell you?

TRY IT **Math Toolkit** grid paper, integer chips, number lines

Possible work:

SAMPLE A

$48 \div 6 = 8$

Since Ryan moves down, the change must be negative.

The integer −8 represents the change in Ryan's position each time he lowers himself. This means he lowers himself 8 ft each time.

SAMPLE B

The total change in Ryan's position is −48.

$6s = -48$

$6(8) = 48$, so $6(-8) = -48$

$s = -8$

The integer −8 represents Ryan's change in position. It shows that he lowers himself by 8 ft each time.

DISCUSS IT

Ask: How would you explain what the problem is asking in your own words?

Share: The problem is asking . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 8

- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.
- Apply properties of operations as strategies to multiply and divide rational numbers.

Common Misconception Listen for students who do not state their answer as a negative number. Have them discuss what each number in the problem represents. As students share their strategies, ask them what a positive number and a negative number mean in the situation and to apply that reasoning to their answers.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- representing the situation with a concrete model, such as 48 negative integer chips separated into 6 equal groups of 8 negative integer chips
- **(misconception)** concluding that 8 represents the change in position
- drawing a diagram to show the total distance and each drop
- using a division equation to find the amount of change for each part of the descent
- using an equation with a variable to represent the change in position

Facilitate Whole Class Discussion

Call on students to share selected strategies. After one student shares their strategy, call on another student to explain why the first student's strategy makes sense. Have students use these sentence starters: *I know ____'s approach works because ____* or *I know ____'s answer is correct because ____*.

Guide students to **Compare and Connect** the representations. After posing the question below, allow students individual think time before calling on students to contribute to the discussion.

> **ASK** *How did each model show that Ryan's position change was a descent?*
>
> **LISTEN FOR** Negative numbers show that Ryan is going down. The total is −48, and each drop is −8.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that a negative number can be used to represent both the total descent of −48 and the change in position at each stage of −8.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model division equations with positive and negative numbers.

If students are unsure about dividing a negative number, then use this activity to apply previous knowledge of division to division of a negative integer.

Materials For each pair: 25 integer chips

- Have students model 12 ÷ 3 with integer chips. Ask: *How does your model represent 12 divided by 3?* [We used 12 positive integer chips and split them into 3 groups of 4 each to show 12 divided by 3.]
- Have students model −12 ÷ 3 with integer chips. Ask: *How is this model the same as your first model? How is it different?* [It is the same because we used 12 integer chips split into 3 groups of 4. It is different because we used negative integer chips, so the groups represent −4 instead of 4.]
- Extend the activity by having students model more related division problems, such as 6 ÷ 2 and −6 ÷ 2. Then have students generalize the patterns that they see.

CONNECT IT

1 **Look Back** What integer represents the change in Ryan's position each time he lowers himself? How does the integer show that Ryan is going lower?

−8; Since the integer is negative, that means Ryan is going lower.

2 **Look Ahead** One way to find the rational number that represents Ryan's change in position each time he lowers himself is to divide. An integer can be divided by any integer except 0. Any **rational number** can be written as the quotient of two integers, where the divisor is not 0. That means every rational number can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

a. Another way to think of a rational number is as the fraction $\frac{a}{b}$. Explain why it makes sense that a can equal 0, but b cannot.

Possible explanation: The fraction $\frac{a}{b}$ means $a \div b$. You can divide 0 by a number, but you cannot divide a number by 0.

b. How can you write 0.07 in the form $\frac{a}{b}$? How can you write 1.07 in the form $\frac{a}{b}$?

$\frac{7}{100}$; $\frac{107}{100}$

c. When a rational number is negative, you can express it as a fraction in three different forms. For example, you can write −4 as $\frac{-4}{1}$, $\frac{4}{-1}$, and $-\frac{4}{1}$.

Express each fraction above as an expression using the division symbol (÷). How does this show that the fractions are equivalent?

$\frac{-4}{1} = -4 \div 1$; $\frac{4}{-1} = 4 \div (-1)$; $-\frac{4}{1} = -(4 \div 1)$; Possible explanation: All of the fractions simplify to −4, so the fractions are all equivalent to −4.

3 **Reflect** Why is $\frac{-3}{4}$ a rational number?

Possible explanation: The fraction shows that the number is a quotient of two integers, −3 and 4, and the divisor is not 0. So, it is a rational number.

2 **Look Ahead** Point out that in prior grades they discussed rational numbers as $\frac{a}{b}$ and the opposite of $\frac{a}{b}$, with both a and b being whole numbers. They will now be working with rational numbers that are the quotient of integers.

Ask a volunteer to rephrase the definition of *rational number*. Ask for some examples of rational numbers, including some integers. Ask students how an integer, like 7, is also a rational number.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding that both the numerator and denominator are integers and that the denominator is not zero.

Common Misconception If students think that negative rational numbers can only be written in the form $-\frac{a}{b}$, then have students use division models to find the value of $\frac{-3}{4}$, $\frac{3}{-4}$, and $-\frac{3}{4}$ and discuss how the values are similar and different. Discuss how, since the representations are equivalent, if one is classified as a rational number, then all three can be classified as rational numbers.

Prepare for Multiplying and Dividing with Negative Numbers

Support Vocabulary Development

Assign **Prepare for Multiplying and Dividing with Negative Numbers** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the terms *fraction*, *opposite numbers*, and *reciprocal*. When discussing reciprocals of numbers, some students may suggest simply switching the numerator and denominator of a fraction. Encourage those students to think of a more precise definition involving multiplication and 1. To help students distinguish between *opposites* and *reciprocals*, suggest that they note whether the denominator will be the same for a fraction and its opposite and whether the denominator will be the same for a fraction and its reciprocal.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of examples of each term.

Have students look at the fractions in problem 2 and discuss with a partner whether the denominators are the same or different and how thinking about that can help them answer the question.

Problem Notes

1. Students should understand that a number and its *opposite* have a sum of 0, while a number and its *reciprocal* have a product of 1. Students might write an example for *fraction*, then give the opposite of that fraction as an example for *opposite* and the reciprocal of that fraction as an example for *reciprocal*. For example, if students wrote $\frac{3}{5}$ for their fraction, they might then write $-\frac{3}{5}$ as an opposite and $\frac{5}{3}$ as a reciprocal. Students should recognize that the opposite of a number and the reciprocal of a number are not equal.
2. Students can add the two numbers to see if the sum is 0, which would mean that the numbers are opposites. Since the sum is not 0, the numbers are not opposites.

LESSON 12 | SESSION 1 Name:

Prepare for Multiplying and Dividing with Negative Numbers

1. Think about what you know about fractions and negative numbers. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

Word	In My Own Words	Examples
fraction	a number that names equal parts of a whole	$\frac{2}{7}$
opposite numbers	numbers that are the same distance from 0, but on opposite sides of 0 on the number line	2 and -2
reciprocal	the number you multiply a given number by to get 1	The reciprocal of 2 is $\frac{1}{2}$.

2. Doug says that the opposite of $\frac{-2}{5}$ is $\frac{5}{-2}$. Is Doug correct? Explain.

No; Possible explanation: Opposite numbers are the same distance from 0 on the number line but on opposite sides of 0. Since $\frac{-2}{5} = -\frac{2}{5}$, its opposite is $\frac{2}{5}$.

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REAL-WORLD CONNECTION

Some companies are owned by a group of people. When a company sells part of the company to the public on a stock market, the value is split into equal amounts, called *shares*. As the total value of the company increases and decreases, the value of each share does as well. This change can be written using dollars and cents. Many people own multiple shares or partial shares of stock in a particular company, so when the value of a share increases or decreases, the change in value can be multiplied by the number of shares owned. For example, if the value of one share of stock in company XYZ drops by $6.85 and someone owns 5.8 shares in the company, that person can use $-6.85(5.8)$ to find the change in value. Ask students to think of other real-world examples when multiplying positive and negative rational numbers might be useful.

3 Problem 3 provides another look at dividing a negative number by a positive number. This problem is similar to the problem about washing windows on a skyscraper. In both problems, a total negative change in position is made up of several equal negative changes. This problem asks for the change in elevation (or position) each time Tameka lowers the balloon.

Students may want to use integer chips, a number line, or an equation to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 Tameka pilots a hot air balloon. She lowers the balloon five times, going down an equal distance each time. In all, Tameka lowers the balloon 75 m.

a. What integer represents the change in the hot air balloon's elevation each time Tameka lowers it? What does this integer tell you? Show your work.

Possible work:

$75 \div 5 = 15$

Since the balloon moves down, the change must be negative.

SOLUTION −15; The elevation decreases 15 m at each stage.

b. Check your answer to problem 3a. Show your work.

Possible work:

$5(-15) = -75$

A change of −75 m is the same as a descent of 75 m.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

ACADEMIC VOCABULARY

Context is the situation in which something happens.

Levels 1–3: Reading/Speaking

Help students interpret and discuss Apply It problem 6. Read the problem aloud and have students note unfamiliar terms. These may include *peregrine falcon, prey, elevation, dive*, or *context*. Compile and discuss the list, using a bilingual dictionary if needed.

Adapt **Three Reads** by using pictures, gestures, cognates, and synonyms to help students make sense of the unknown words in the problem that pertain to each question or answer in the routine. Record the answers to each question and have partners paraphrase the information using terms from the list.

Levels 2–4: Reading/Speaking

Support students as they read and interpret Apply It problem 6. Ask students to preview the problem individually and circle any unfamiliar words and phrases. Guide students to extract the meaning of unfamiliar words using context clues, such as the image of the falcon diving. It may be helpful to ask them to add certain terms, such as *elevation* or *context,* to their Interactive Glossary.

Adapt **Three Reads** to help students make sense of the problem. Post the three questions and ask partners to read and respond before solving the problem individually.

Levels 3–5: Reading/Speaking

Support students as they interpret and discuss Apply It problem 6. Use an adaptation of **Three Reads** combined with a **Co-Constructed Word Bank**. After each read, record words and phrases that students use to discuss each question. If needed, suggest important terms such as *elevation, average,* and *context*. Prompt students to identify and discuss words with multiple meanings and to tell how they are used in the problem.

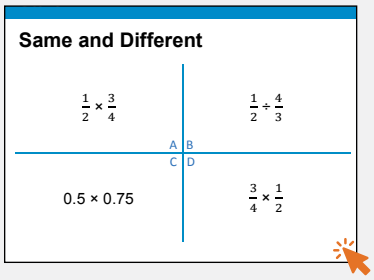
Then have students turn and talk with a partner to paraphrase the problem before solving individually.

Develop Multiplying Negative Rational Numbers

Purpose

- **Develop** strategies for multiplying negative rational numbers.
- **Recognize** that the products of all positive and negative rational numbers follow the same patterns as the products of positive and negative integers.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All the expressions have a value of $\frac{3}{8}$ or 0.375.

Each expression is written in a different format.

A, B, and D include $\frac{1}{2}$, and A and D include $\frac{3}{4}$.

A, C, and D are multiplication, and B is division.

WHY? Support students' facility with multiplying and dividing positive rational numbers.

DEVELOP ACADEMIC LANGUAGE

WHY? Recognize multiple meanings of the word *rational*.

HOW? Display the word *rational*. Ask students to students define or give synonyms for its everyday meaning, "logical or reasonable." Discuss how the everyday meaning is different from the mathematical meaning of *rational*. Have students make sentences that use both meanings of the word *rational* and/or its antonym *irrational*.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Three Reads** to help them make sense of the problem. Listen for understanding that the total change in temperature depends on the change in elevation.

LESSON 12 | SESSION 2

Develop Multiplying Negative Rational Numbers

Read and try to solve the problem below.

Mount Whitney Trail starts at an elevation of 8,350 ft above sea level and ends at an elevation of 14,500 ft above sea level. The temperature changes by −5.4°F for every thousand feet gained in elevation. What is the change in temperature from the starting elevation to the ending elevation of Mount Whitney Trail?

TRY IT

Math Toolkit grid paper, number lines

Possible work:

SAMPLE A

Change in elevation: 14,500 − 8,350 = 6,150

Elevation (ft)	Change in Temperature (°F)
1,000	−5.4
6,150	−33.21

× 6.15 ↓ ↓ × 6.15

The change in temperature is −33.21°F.

SAMPLE B

Change in elevation: 14,500 − 8,350 = 6,150

x = the change in temperature

$$\frac{x}{-5.4} = \frac{6{,}150}{1{,}000}$$

$$\frac{x}{-5.4} = 6.15$$

$$x = -5.4(6.15)$$

$$x = -33.21$$

The temperature changes by −33.21°F.

DISCUSS IT

Ask: How do you know your answer is reasonable?

Share: My answer is reasonable because . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. To support students in extending the conversation, prompt them to discuss these questions:

- *How did you determine the unit of measurement for the answer?*
- *How will the temperature change from the ending elevation to the starting elevation?*

Error Alert If students lose track of a negative sign when multiplying with −5.4, then ask them to factor −1 from −5.4 and multiply the other factor by −1(5.4). Ask students to circle or highlight the negative sign each time they write the expression or product to make sure they keep track of it.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing a picture to represent each section of 1,000 feet of elevation
- using a table to represent the elevation and the temperature
- subtracting to find the change in elevation, dividing by 1,000, and then multiplying by the change in temperature
- using proportional reasoning with the change in temperature represented with a variable

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to explain what they noticed or assumed about the problem, what they decided to do as a result, and why.

Guide students to **Compare and Connect** the representations. Have students turn and talk about their responses before sharing.

ASK *How is −5.4 used in each strategy?*

LISTEN FOR In each strategy −5.4 represents the change in temperature for every increase of 1,000 ft in elevation. It is used either as a factor or with repeated addition.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *How can the relationship between the change in elevation and the change in temperature be represented?*

LISTEN FOR The relationship can be represented with a diagram or a rate.

For the Picture It, prompt students to think about how to find the change in temperature for 100-ft and 50-ft changes in elevation.

- *How is using repeated addition with a negative number like multiplying a negative number?*

For the Model It, prompt students to think about why the formula for the change in temperature can be used.

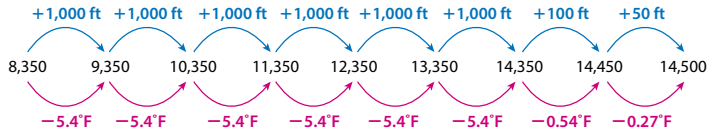
- *Why is $\frac{-5.4}{1{,}000}(6{,}150)$ equal to $-5.4\left(\frac{6{,}150}{1{,}000}\right)$?*
- *How can you use the product of 5.4×6.15 to find the product of -5.4×6.15?*

➤ **Explore different ways to multiply positive and negative rational numbers.**

Mount Whitney Trail starts at an elevation of 8,350 ft above sea level and ends at an elevation of 14,500 ft above sea level. The temperature changes by −5.4°F for every thousand feet gained in elevation. What is the change in temperature from the starting elevation to the ending elevation of Mount Whitney Trail?

Picture It

You can use a diagram to understand the relationship.



Model It

You can multiply to find the change in temperature.

The change in elevation is 14,500 − 8,350 = 6,150 ft.

The temperature decreases 5.4°F every 1,000 ft. That is a rate of $\frac{-5.4}{1{,}000}$°F per foot.

change in temperature = rate of temperature change × change in elevation

$$= \frac{-5.4}{1{,}000}(6{,}150)$$
$$= \frac{-5.4 \times 6{,}150}{1{,}000}$$
$$= -5.4\left(\frac{6{,}150}{1{,}000}\right)$$
$$= -5.4(6.15)$$

DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Constructing Arguments About Factoring −1 from Expressions

Have students consider this claim: *If you know how to find the product of two positive numbers, then you can find the product of related negative numbers by factoring out −1.*

ASK *How can you rewrite the expression −0.32(2.5) so that it has a factor of −1?*

LISTEN FOR I can rewrite it as −1(0.32)(2.5).

ASK *Why can you make your first step in simplifying −1(0.32)(2.5) multiplying the two positive factors? How does this show that the claim is reasonable?*

LISTEN FOR You can multiply in any order. It shows that multiplying by a negative number is the same as multiplying by the opposite of that number and −1.

ASK *How can you show this claim is reasonable when multiplying two negative numbers?*

LISTEN FOR Choose any two negative numbers to multiply. Factor −1 from each of them. Rearrange to show you are multiplying the positive factors by (−1)(−1), or 1.

Generalize Use variables to show this will work for any positive numbers, *a* and *b*. You can rewrite the product of $-a(b)$ or $a(-b)$ as $-1(ab)$ and $(-a)(-b)$ as $(-1)(-1)ab = 1(ab)$.

Develop Multiplying Negative Rational Numbers

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the product of −5.4 and 6.15 is the same in each representation. Explain that they will now use those relationships to reason about multiplying with negative rational numbers.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The diagram shows adding 6 + 0.1 + 0.05 groups of −5.4.
- A negative number often represents a decrease.
- $-5.4 = -5\frac{4}{10} = -\frac{54}{10}$ and $6.15 = 6\frac{15}{100} = \frac{615}{100}$.

Facilitate Whole Class Discussion

4 Look for understanding that when there is one negative factor, the product is negative.

> **ASK** *How can you determine the sign of each product before multiplying?*
>
> **LISTEN FOR** You can look at whether either of the factors is negative.

5 Look for the idea that negatives do not change how place value works for decimal operations or the process for multiplying fractions. Display the equations (−2)(−3) = 6, (−0.2)(−3) = 0.6, and (−0.2)(−0.3) = 0.06. Allow individual think time before starting the discussion.

> **ASK** *What relationship do you see between the place values of the factors and the place value of the products?*
>
> **LISTEN FOR** The place value of the factors determines the place value of the products, as it does for positive factors.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner. Remind students that good listeners use engaged body language, such as looking at the speaker and nodding to show understanding.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to multiply positive and negative rational numbers.**

1 How does the **Picture It** model −5.4(6.15)?

Possible answer: You can think of 6.15 as 6 + 0.1 + 0.05. The diagram shows 6 jumps of −5.4, showing 6(−5.4). Then it shows a jump of −0.54, which is 0.1(−5.4), and a jump of −0.27, which is 0.05(−5.4).

2 What is the temperature change from the beginning to the ending of the trail? What does this number mean in the context of the problem?

−33.21°F; The temperature decreases by 33.21°F.

3 Why does $-5.4(6.15)$ have the same product as $-\frac{54}{10} \cdot \frac{615}{100}$?

Fractions and decimals are different forms of the same numbers, so the products are the same.

4 You can start solving the problem by finding −5.4 × 6. How is this similar to finding 5.4 × 6? How is this different from finding 5.4 × 6?

Possible answer: It is similar because it is multiplying an integer and a decimal. It is different because you are multiplying a negative and a positive value instead of two positive values.

5 How is multiplying negative rational numbers like multiplying negative integers?

Possible answer: The rules for the signs are the same. The product of two negative numbers or two positive numbers is positive and the product of a negative number and a positive number is negative.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to multiply rational numbers.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Make connections between multiplying integers and multiplying rational numbers.

If students are unsure about multiplying with negative rational numbers, then use this activity to help them visualize multiplication with a negative rational number.

- Display a number line from −2 to 2 showing tenths and the expression 3(0.6). Have a volunteer model the expression on the number line. [3 jumps of 0.6 to 1.8]
- Display 3(−0.6). Have a volunteer model the expression on the number line. [3 jumps of −0.6 to −1.8] Ask: *How are the models the same and different?* [The models both show three jumps of 0.6, ending a distance of 1.8 from 0. The jumps go in the positive direction when there are two positive factors and in the negative direction when there is one positive and one negative factor.]
- Repeat with more related products, such as 2(0.4) and 2(−0.4). Have students generalize. Listen for understanding that the product of a positive factor and a negative factor is the opposite of the product of two positive factors.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; students may show their work using only absolute values, then reason about the products of negative numbers in order to determine whether the answer is positive or negative. Other students may use a number line, and although their sketch may not show precisely equal divisions, it should be labeled correctly.

7 See ***Connect to Culture*** to support student engagement. Students may reason that the product will be negative since there is only one negative factor. Students should understand that the change in the falcon's elevation is represented by a negative number and that a positive number would mean an increase in the falcon's elevation.

8 Students may reason that the product will be positive since there are two negative factors. Students may work with the negative signs in each factor in different correct ways to find the solution. For instance, one student may rewrite the numerator as $-1 \cdot (-27)$ and the denominator as $9 \cdot 1$, while another student may factor -1 from each term before multiplying.

Apply It

➤ **Use what you learned to solve these problems.**

7 A peregrine falcon dives for prey. Its elevation changes by an average of -11.5 meters every second. The dive lasts for 3.2 seconds. What is the change in the falcon's elevation? What does this mean in the context of the problem? Show your work.

Possible work:

$-11.5(3.2) = -36.8$

SOLUTION -36.8; The falcon's elevation decreased by 36.8 m.

8 Find $-\frac{1}{9}(-27)$. Show your work.

Possible work:

$$-\frac{1}{9}\left(-\frac{27}{1}\right) = (-1)\left(\frac{1}{9}\right)(-1)(27)$$
$$= \frac{27}{9}$$
$$= 3$$

SOLUTION 3

9 A freezer in a lab is set so that the temperature changes at a rate of -1.6°C every hour. What is the change in the temperature after $4\frac{1}{4}$ h? Show your work.

Possible work:

Total change = Change per hour × number of hours
$= -1.6 \times 4\frac{1}{4}$
$= -1.6 \times 4.25$
$= -6.8$

SOLUTION The change in temperature is -6.8°C.

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CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- how to rewrite both factors as decimals or as fractions.
- how to multiply a positive and a negative factor.
- how to interpret a negative solution in the context of the problem.

Error Alert If students forget to include the negative sign as they move from one part of solving to the next, then ask whether the solution will be positive or negative based on the number of negative factors. They can expect the solution for problem 9 to be negative since there is only one negative factor.

Practice Multiplying Negative Rational Numbers

Problem Notes

Assign **Practice Multiplying Negative Rational Numbers** as extra practice in class or as homework.

1. **a.** Students should understand that $-8\frac{1}{2} = -8.5$. Some students will see that this means the product $-6(-8.5)$ is the same as the product from the Example, without having to do any multiplying. ***Basic***

   b. Students may convert the answer in the example to a decimal or convert their answer to a fraction to show that the two answers are equivalent. ***Basic***

2. After writing the second factor as the quotient of two integers, students may factor -1 from each fraction and find $(-1)(-1)\left(\frac{3}{5}\right)\left(\frac{4}{3}\right)$. ***Medium***

Practice Multiplying Negative Rational Numbers

➤ **Study the Example showing how to multiply negative rational numbers. Then solve problems 1–5.**

Example

What is $-6\left(-8\frac{1}{2}\right)$?

$-8\frac{1}{2} = -\frac{17}{2}$

The product of two negative rational numbers is positive.

$-6\left(-8\frac{1}{2}\right) = -\frac{6}{1}\left(-\frac{17}{2}\right)$

$= 51$

1. **a.** Show how to solve the problem in the Example by multiplying decimals.

   Possible work:

   $-8\frac{1}{2} = -8.5$

   $-6(-8.5) = 51$

   b. Is your answer to problem 1a *greater than*, *less than*, or *equal to* the answer in the Example? Why?

   equal to; Possible explanation: The fractions and decimals are in different forms but have the same value, so the product also has the same value.

2. Find $-\frac{3}{5}\left(-1\frac{1}{3}\right)$. Show your work.

   Possible work:

   $-\frac{3}{5}\left(-1\frac{1}{3}\right) = \frac{-3}{5} \times \left(\frac{-4}{3}\right)$

   $= \frac{12}{15}$, or $\frac{4}{5}$

SOLUTION $\frac{4}{5}$

Vocabulary

rational number

a number that can be expressed as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Rational numbers include integers, fractions, repeating decimals, and terminating decimals.

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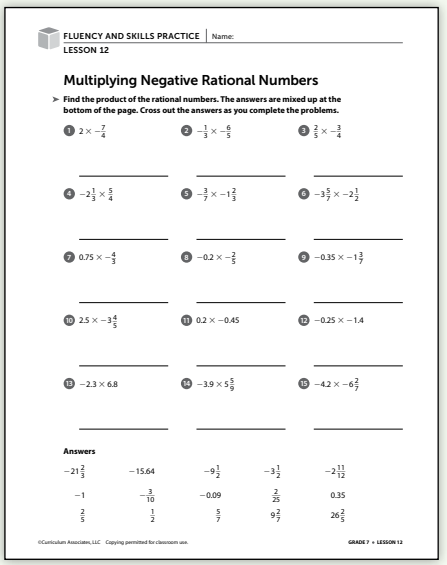
Fluency & Skills Practice

Multiplying Negative Rational Numbers

In this activity, students practice multiplying negative fractions and decimals, and they check their answers against those given at the bottom of the page.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 12

Multiplying Negative Rational Numbers

➤ **Find the product of the rational numbers. The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.**

1. $2 \times -\frac{7}{4}$
2. $-\frac{1}{3} \times -\frac{6}{5}$
3. $\frac{2}{5} \times -\frac{3}{4}$
4. $-2\frac{1}{3} \times \frac{5}{4}$
5. $-\frac{3}{7} \times -1\frac{2}{3}$
6. $-3\frac{5}{7} \times -2\frac{1}{2}$
7. $0.75 \times -\frac{4}{3}$
8. $-0.2 \times -\frac{2}{5}$
9. $-0.35 \times -1\frac{3}{7}$
10. $2.5 \times -3\frac{4}{5}$
11. 0.2×-0.45
12. -0.25×-1.4
13. -2.3×6.8
14. $-3.9 \times 5\frac{5}{9}$
15. $-4.2 \times -6\frac{2}{7}$

Answers

$-21\frac{2}{3}$ $\quad -15.64$ $\quad -9\frac{1}{2}$ $\quad -3\frac{1}{2}$ $\quad -2\frac{11}{12}$

-1 $\quad -\frac{3}{10}$ $\quad -0.09$ $\quad \frac{2}{25}$ $\quad 0.35$

$\frac{2}{5}$ $\quad \frac{1}{2}$ $\quad \frac{5}{7}$ $\quad 9\frac{2}{7}$ $\quad 26\frac{2}{5}$

3 **a.** Students' work should show understanding that when a negative number is multiplied by a positive number, the product is a negative number. It should also show understanding that multiplying by -0.2 is equivalent to multiplying by $0.1(-2)$. ***Medium***

b. Students' explanations should show understanding that when two negative factors are multiplied, the product will be a positive number. ***Basic***

4 Students may use the equivalent fractions $\frac{-1.5}{1} = \frac{x}{50}$ to find the change in the number of feet in 50 seconds. ***Challenge***

5 Students should understand that if the amount of gas changes by a negative amount multiple times, the result will be a negative total change in the amount of gas. ***Medium***

3 **a.** Find $-0.2(4.75)$. Show your work.

Possible work:

$0.2(4.75) = 0.95$

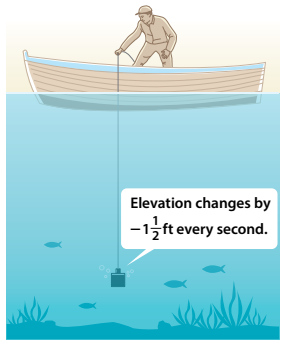
$-0.2(4.75) = -0.95$

SOLUTION -0.95

b. Explain how you can use your answer to problem 3a to find $-0.2(-4.75)$.

Possible explanation: Since the only change is the sign of a factor, the product will be the opposite of the result from problem 3a.

4 Amare drops a probe into a lake. The elevation of the probe changes by $-1\frac{1}{2}$ feet every second. The probe takes 50 seconds to reach the bottom of the lake. How deep is the lake? Show your work.



Possible work:

Depth = rate × time

$= -1\frac{1}{2}(50)$

$= -75$

SOLUTION The lake is 75 ft deep.

5 Salvador drives 6.5 miles to his aunt's house. For every mile Salvador drives, the amount of gas in the car's gas tank changes by -0.04 gallon. What is the total change in the gallons of gas in the car's tank? Show your work.

Possible work:

$-0.04 \times 6.5 = -0.26$

SOLUTION The change in the amount of gas is -0.26 gallon.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Connect It

ACADEMIC VOCABULARY

A *model* is a representation of a system, process, or relationship.

A *strategy* is a careful way or plan for getting something done.

Levels 1–3: Speaking/Writing

Prepare students for Connect It problem 5 by reviewing *model* and *strategy*. Guide students to identify the models and strategies in the session and select one to write about.

Help students draft and revise written responses by adapting **Stronger and Clearer Each Time**. Ask students to tell how the model or strategy shows dividing with negative numbers. Help students draft responses using short sentences, sketches, and models. Then have students read their responses to partners, referring to their model as they explain. Have partners work together to clarify and revise the ideas.

Levels 2–4: Speaking/Writing

Prepare students for Connect It problem 5 by discussing the meaning of *model* and *strategy*. Facilitate a discussion about the session's models and strategies.

Use **Stronger and Clearer Each Time** to help students write and revise their answers. Provide these frames to help students begin a paragraph:

- *The strategy shows* _____.
- *The model shows* _____.

Encourage students to refer to the model or strategy as they explain their ideas. Have partners work together to clarify and revise the written responses.

Levels 3–5: Speaking/Writing

Prepare students for Connect It problem 5 by summarizing the session's models and strategies with a partner. Ask partners to make a list of transitions they can use to restate ideas or to provide reasons, like *in other words* and *another reason*.

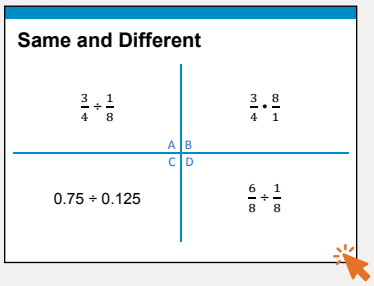
Have students draft and revise written responses using **Stronger and Clearer Each Time**. Remind them to refer to their lists of transitions as they draft their answers. Then have students meet with partners to explain and clarify ideas. Finally, encourage students to revise their drafts.

Develop Dividing Negative Rational Numbers

Purpose

- **Develop** strategies for solving problems that involve dividing negative decimals or fractions.
- **Recognize** that the process for dividing negative rational numbers is the same as the process for dividing positive rational numbers.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All the expressions equal 6.

B is the only multiplication expression.

C is the only expression written with decimals.

D has the only fractions with common denominators.

WHY? Support students' facility with multiplying and dividing positive rational numbers.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students to be specific when they disagree with an idea.

HOW? With a volunteer, model for students ways to disagree with a solution or idea and show the specific parts of an idea or strategy with which they disagree. It may be helpful to role-play this interaction by presenting an incorrect solution to Try It and asking students to be specific about the part of the solution they disagree with and why. During discussions, recognize students for being specific when they disagree with an idea.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask: *What is the problem about?* After the second read, ask what students are trying to answer. After the third read, ask for the important quantities and relationships in the problem. Follow up by asking: *When the creek floods, how do water levels change? How do the water levels change after a flood?*

LESSON 12 | SESSION 3 ■ ■ ■ □

Develop Dividing Negative Rational Numbers

➤ **Read and try to solve the problem below.**

After a flood, a creek's water level changes by $-2\frac{4}{5}$ ft in $3\frac{1}{2}$ h.
What is the average change in the creek's water level each hour?

TRY IT

Math Toolkit grid paper, number lines

Possible work:

SAMPLE A

Average change per hour = total change ÷ number of hours

$$= -2\frac{4}{5} \div 3\frac{1}{2}$$
$$= -2.8 \div 3.5$$
$$= -0.8$$

SAMPLE B

The result of $-2\frac{4}{5} \div 3\frac{1}{2}$ is the opposite of the result of $2\frac{4}{5} \div 3\frac{1}{2}$.

$$2\frac{4}{5} \div 3\frac{1}{2} = \frac{14}{5} \div \frac{7}{2}$$
$$= \frac{14}{5} \cdot \frac{2}{7}$$
$$= \frac{4}{5}$$

The opposite of $\frac{4}{5}$ is $-\frac{4}{5}$.

DISCUSS IT

Ask: What did you do first to find the change in the water level?

Share: First, I . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What does the* average change in water level each hour *mean?*
- *What units can you use to represent the average change in water level each hour?*
- *What operation should you use to find the average?*

Common Misconception Listen for students who confuse *opposite* and *reciprocal* when dividing with fractions and write the second factor with a negative sign in addition to or instead of writing its reciprocal. For example, they might multiply $-\frac{14}{5} \cdot \left(-\frac{7}{2}\right)$, resulting in an answer of 9.8 feet per hour. As students share their strategies, ask: *When the answer is positive, what does that mean for the change in the water level? Does that make sense?* Have students discuss how their answers represent the water level decreasing.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using fraction strips to model $-2\frac{4}{5}$ divided into $3\frac{1}{2}$ equal groups
- **(misconception)** confusing *opposite* with *reciprocal* and multiplying $\left(-\frac{14}{5}\right)\cdot\left(-\frac{7}{2}\right)$
- converting mixed numbers to decimals to divide
- dividing with positive numbers and reasoning that the solution is the opposite

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to point out work they think might be incorrect and explain why they think so.

Guide students to **Compare and Connect** the representations. If everyone has something to say, use turn and talk before having students share.

> **ASK** *How do all the strategies represent the average change in water level in one hour?*
>
> **LISTEN FOR** They all divide the total change of $-2\frac{4}{5}$ into $3\frac{1}{2}$ equal groups.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

> **ASK** *How is $3\frac{1}{2}$ hours represented in each model?*
>
> **LISTEN FOR** In one model it is the whole length of the line, in the second it is written as $\frac{7}{2}$, and in the third it is written as 3.5.

For the Picture It, prompt students to consider what the distance between each tick mark represents.

- *How does the picture help you make sense of the problem?*

For the Model It dividing with fractions, prompt students to think about how the expression is rewritten in each step.

- *How did you decide where to place the negative sign when dividing the fractions?*

For the Model It dividing with decimals, prompt students to notice that neither number in the problem was given as a decimal.

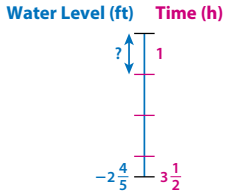
- *For the expression $-2.8 \div 3.5$, why can you expect the quotient to be negative?*

➤ **Explore different ways to divide negative rational numbers.**

After a flood, a creek's water level changes by $-2\frac{4}{5}$ ft in $3\frac{1}{2}$ h. What is the average change in the creek's water level each hour?

Picture It

You can use a picture to make sense of the problem.



Model It

You can divide fractions.

Divide the total change in the level of the water by the number of hours to get the average change in the level each hour.

$$-2\tfrac{4}{5} \div 3\tfrac{1}{2} = -\tfrac{14}{5} \div \tfrac{7}{2}$$
$$= -\tfrac{14}{5} \cdot \tfrac{2}{7}$$

Model It

You can divide decimals.

$-2\frac{4}{5} = -2\frac{8}{10}$ $\quad 3\frac{1}{2} = 3\frac{5}{10}$

$= -2.8$ $\quad = 3.5$ $\quad -2.8 \div 3.5$

DIFFERENTIATION | EXTEND

SMP 8

Deepen Understanding

Using the Relationship of Division to Multiplication to Find Quotients

Prompt students to think about the relationship between division and multiplication.

> **ASK** *What related multiplication equation can you write for $-2.8 \div 3.5 = x$? How does this show that x has to be negative?*
>
> **LISTEN FOR** You can write $3.5x = -2.8$ or $x(3.5) = -2.8$. When one factor is negative and the product is positive, the other factor also has to be negative.

> **ASK** *What related multiplication equation can you write for $-2.8 \div (-3.5) = x$? How does this show that x has to be positive?*
>
> **LISTEN FOR** You can write $-3.5x = -2.8$ or $x(-3.5) = -2.8$. When one factor is negative and the product is negative, the other factor has to be positive.

> **ASK** *Would this also work if you used fractions or integers instead of decimals? Why?*
>
> **LISTEN FOR** Yes. The rules about products are not affected by the form of the factors.

Generalize When both the divisor and dividend are negative, the quotient will be positive. When only one is, the quotient will be negative.

Develop Dividing Negative Rational Numbers

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how dividing with negative rational numbers is similar to dividing with positive rational numbers.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- Since multiplication and division are related, rules about multiplying with negative numbers have related rules that work for dividing with negative numbers.
- When only a dividend or only a divisor is negative, the quotient is negative. When a dividend and a divisor are both negative, the quotient is positive.

Facilitate Whole Class Discussion

3 Students should recognize that multiplication and division are related operations.

> **ASK** *How can you rewrite a division expression with a negative dividend as a multiplication expression?*
>
> **LISTEN FOR** The negative dividend becomes a negative product, and the divisor and the quotient become the factors.

4 Look for the idea that the process for dividing fractions does not change when one or both of the fractions is negative. Allow time for students to think by themselves before starting the discussion.

> **ASK** *How is dividing $-0.15 \div -0.015$ the same as and different from dividing $0.15 \div 0.015$?*
>
> **LISTEN FOR** For both, I can use the related equation $150 \div 15 = 10$ to find the absolute value of the quotient. They have the same result, but to divide $-0.15 \div -0.015$, I have to keep in mind that a negative number divided by a negative number yields a positive quotient.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to divide positive and negative rational numbers.**

1 Look at the second **Model It**. Without doing any calculations, explain how you can use $2.8 \div 3.5$ to find $-2.8 \div 3.5$.

Possible explanation: $2.8 \div 3.5$ and $-2.8 \div 3.5$ will have quotients with the same absolute value. The only change is that $2.8 \div 3.5$ has a positive quotient and $-2.8 \div 3.5$ has a negative quotient. So, you can find the opposite of $2.8 \div 3.5$.

2 What is the average change in the creek's height each hour? How does each **Model It** help you see if the average change in the creek's height each hour is positive or negative?

-0.8 ft; Possible explanation: In the first Model It, you end up with a product of a negative factor and a positive factor, so the product is negative. In the second Model It, the quotient is negative.

3 The product of a positive number and a negative number is negative. Explain why the quotient of a negative number and a positive number is negative.

Possible explanation: Multiplication and division undo each other. So, when you divide a negative number by a positive number, the other negative factor is the quotient.

4 How is dividing with negative rational numbers like dividing with positive rational numbers? How is it different?

Possible answer: The division process is the same. The only difference is that the quotient is negative when either the dividend or the divisor is negative. When both are negative, the quotient is positive.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand dividing with negative numbers.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use models to divide negative rational numbers.

If students are unsure about dividing with negative decimals, then use this activity to see the relationship between dividing positive decimals and dividing negative decimals.

Materials For each pair: base-ten blocks (20 ones units)

- Have partners work together to model the expression $1.2 \div 4$. Ask: *How does your model represent the expression $1.2 \div 4$? What does it tell you about the value of the expression?* [Each ones unit represents 0.1, so 1.2 is represented with 12 ones units. They are divided into 4 groups with 3 units each, so $1.2 \div 4 = 0.3$.]
- Ask: *Suppose each unit represents -0.1. How can you model the expression $-1.2 \div 4$? What is the result?* [Divide the units into 4 groups as before. The result is -0.3.]
- Ask: *How can you use what you learned from your model to find $-1.2 \div -0.3$?* [I know that $-1.2 \div 4 = -0.3$, so $-1.2 \div (-0.3) = 4$.]
- Repeat with another set of related expressions, such as $1.8 \div 3$, $-1.8 \div 3$, and $-1.8 \div -0.6$.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; students may show a varying amount of detail if they use the standard algorithm to divide decimals.

6 Students should recognize that since the dividend and the divisor are negative, the quotient is positive.

7 **D is correct.** Students should recognize that since the dividend and divisor are negative, the quotient is positive, and that to divide by $-\frac{1}{2}$, they need to multiply by the reciprocal, $-\frac{2}{1}$ or -2.

A is not correct. This answer is $-\frac{3}{8} \div \frac{1}{2}$ or $\frac{3}{8} \div \left(-\frac{1}{2}\right)$.

B is not correct. This answer is $-\frac{3}{8} \cdot \frac{1}{2}$ or $\frac{3}{8} \cdot \left(-\frac{1}{2}\right)$.

C is not correct. This answer is $-\frac{3}{8} \cdot \left(-\frac{1}{2}\right)$.

Apply It

➤ **Use what you learned to solve these problems.**

6 What is $16.8 \div (-3.5)$? Show your work.

Possible work:

$16.8 \div (-3.5) = (16.8) \div (-3.5)$

$= -4.8$

SOLUTION -4.8

7 What is $-\frac{3}{8} \div \left(-\frac{1}{2}\right)$?

A $-\frac{3}{4}$

B $-\frac{3}{16}$

C $\frac{3}{16}$

D $\frac{3}{4}$

8 A helicopter's altitude changes by $-28\frac{1}{8}$ ft in $3\frac{3}{4}$ s. What is the average change in the helicopter's altitude each second? Show your work.

Possible work:

$-28\frac{1}{8} = -28.125$

$3\frac{3}{4} = 3.75$

$-28.125 \div 3.75 = -7.5$

SOLUTION The average change in the helicopter's altitude each second is -7.5 ft.

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CLOSE EXIT TICKET

8 Students' solutions should show an understanding that:

- the average change each second can be found by dividing the change in altitude by the number of seconds.
- a negative dividend and a positive divisor result in a negative quotient.

Error Alert If students write each number as a fraction to divide but use the reciprocal of the dividend rather than of the divisor when transforming the division into a multiplication expression $\left(\text{using } -\frac{8}{225} \cdot \frac{15}{4}\right)$, then suggest that students estimate to check the reasonableness of their answer. Ask questions such as: *Does it make sense that a decrease in altitude of less than 1 foot per second would result in a decrease of 28 feet in 3–4 seconds?*

Practice Dividing Negative Rational Numbers

Problem Notes

Assign **Practice Dividing Negative Rational Numbers** as extra practice in class or as homework.

1. Students may recognize that the facts $-12.6 \div -2.25 = -5.6$ and $-12.6 \div -5.6 = ?$ are related to each other and to the multiplication equation $-5.6 \times -2.25 = -12.6$. They can use the related facts to find the unknown. ***Medium***

2. Students may rewrite the division problem with decimals, using $-6.8 \div -0.4$ to find the quotient of 17. ***Medium***

Practice Dividing Negative Rational Numbers

➤ **Study the Example showing how to divide with negative rational numbers. Then solve problems 1–5.**

Example

On a winter day, the temperature changes by −12.6°F in $2\frac{1}{4}$ h. What rational number represents the average change in the temperature each hour?

The total change in the temperature is −12.6°F.

The amount of time is $2\frac{1}{4}$, or 2.25, h.

$-12.6 \div 2.25 = -5.6$

The number −5.6 represents the average hourly temperature change.

1. Explain how to use the Example to find the quotient of −12.6 and −2.25.

Possible explanation: Since $2\frac{1}{4}$ is the same as 2.25, the divisor in the expression is the opposite of the divisor in the Example. That means the quotient of the expression is the opposite of the quotient in the Example, so the result is 5.6.

2. What is $-6\frac{4}{5} \div \left(-\frac{2}{5}\right)$? Show your work.

Possible work:

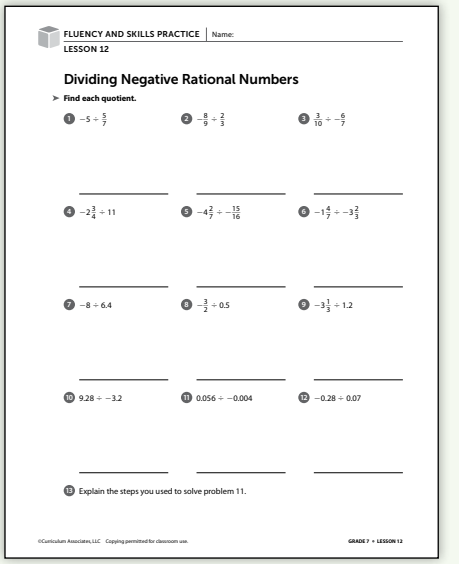
$$-6\tfrac{4}{5} \div \left(-\tfrac{2}{5}\right) = -\tfrac{34}{5} \div \left(-\tfrac{2}{5}\right)$$
$$= \tfrac{-34}{5} \div \tfrac{-2}{5}$$
$$= \tfrac{-34}{5}\left(\tfrac{5}{-2}\right)$$
$$= \tfrac{-34}{-2}$$
$$= 17$$

SOLUTION 17

Fluency & Skills Practice

Dividing Negative Rational Numbers

In this activity, students practice dividing with negative fractions and decimals.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 12

Dividing Negative Rational Numbers

➤ **Find each quotient.**

1. $-5 \div \frac{5}{7}$
2. $-\frac{8}{9} \div \frac{2}{3}$
3. $\frac{3}{10} \div -\frac{6}{7}$
4. $-2\frac{3}{4} \div 11$
5. $-4\frac{2}{7} \div -\frac{15}{16}$
6. $-1\frac{4}{7} \div -3\frac{2}{3}$
7. $-8 \div 6.4$
8. $-\frac{3}{2} \div 0.5$
9. $-3\frac{1}{3} \div 1.2$
10. $9.28 \div -3.2$
11. $0.056 \div -0.004$
12. $-0.28 \div 0.07$
13. Explain the steps you used to solve problem 11.

GRADE 7 • LESSON 12

Additional Practice

3 **a.** Students may rewrite $-37\frac{4}{5}$ as the decimal -37.8 or rewrite 4.5 as the fraction $\frac{9}{2}$ to find the quotient. ***Medium***

b. Students' answers should show understanding that a negative elevation is deeper or lower than a positive elevation and that a negative change in elevation means going deeper. ***Basic***

4 Students should recognize that the process of dividing decimals does not change when dividing with negative numbers. ***Challenge***

5 Students may rewrite the fractions as the decimals -4.5 and 0.75 to divide. ***Medium***

LESSON 12 | SESSION 3

3 Paula is scuba diving. Her elevation changes by $-37\frac{4}{5}$ m in 4.5 min.

a. What rational number represents the average change in Paula's elevation each minute? Show your work.

Possible work:

$-37\frac{4}{5} = -37.8$

$-37.8 \div 4.5 = -8.4$

SOLUTION -8.4

b. Is Paula going deeper in the water or back toward the surface? How do you know?

deeper in the water; Possible explanation: The negative change in elevation means that Paula is going deeper.

4 What is $57.2 \div (-10{,}000)$? Show your work.

Possible work:

$57.2 \div (-10{,}000) = -57.2 \div 10{,}000$

-0.00572

SOLUTION -0.00572

5 A full bathtub drains at a constant rate. The amount of water in the bathtub changes by $-4\frac{1}{2}$ gallons in $\frac{3}{4}$ minute. What is the rate, in gallons per minute, at which the amount of water in the bathtub changes? Show your work.

Possible work:

$-4\frac{1}{2} \div \frac{3}{4} = -\frac{9}{2} \div \frac{3}{4}$

$= -\frac{9}{2}\left(\frac{4}{3}\right)$

$= -\frac{36}{6}$

$= -6$

SOLUTION The rate is -6 gallons per minute.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

Levels 1–3: Speaking/Listening

Support students as they make sense of Apply It problem 2 and discuss Pair/Share. Review the lesson vocabulary term *rational number* and explain that *context* is a problem situation.

Have students use a **Co-Constructed Word Bank** to identify words and phrases to describe a possible solution. If needed, suggest *represents, average change in elevation, each day, positive, negative, up,* and *down*. Provide frames to help students discuss:

- *The solution represents the _____.*
- *A (positive/negative) _____ number means that water elevation goes (up/down) _____ on an average day.*

Levels 2–4: Speaking/Listening

Support students as they make sense of Apply It problem 2 and discuss Pair/Share. Have students read Pair/Share and discuss the meaning of *rational number* and *context*. Then use a **Co-Constructed Word Bank** to have students brainstorm words they can use to describe an average change in elevation.

Have student tell what a positive or negative solution would mean in the context of the problem. Remind them to use words from the word bank. Suggest that partners listen for understanding and give reasons why they agree or disagree. Provide frames for support:

- *I think _____ because _____.*
- *I agree/disagree because _____.*

Levels 3–5: Speaking/Listening

Support students as they justify solutions for Apply It problem 2 and discuss Pair/Share. Have partners read Pair/Share and discuss the meaning of *rational number* and *context*. Then ask partners to use a **Co-Constructed Word Bank** to brainstorm words to talk about their solution, like *negative change* and *decrease.*

Explain that students will answer Pair/Share and justify their solutions to the problem. Have students add phrases to the Word Bank that can help them rate their confidence, like *very confident, positive, certain,* and *not sure.* Encourage partners to discuss using:

- *I am _____ my solution is correct because _____.*

Refine Multiplying and Dividing with Negative Numbers

Purpose

- **Refine** strategies for multiplying and dividing with negative rational numbers.
- **Refine** understanding of multiplying with three factors, some or all of which are negative rational numbers.

START CHECK FOR UNDERSTANDING

What is the value of m?

$-\frac{1}{2} \div \left(-\frac{3}{2}\right) = m$

Solution

$m = \frac{1}{3}$

WHY? Confirm students' understanding of multiplying a negative rational number by a positive rational number, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 12 | SESSION 4

Refine Multiplying and Dividing with Negative Numbers

➤ **Complete the Example below. Then solve problems 1–10.**

Example

What is the product of $-\frac{3}{8}$, $-\frac{2}{3}$, and $-\frac{5}{2}$?

Look at how you could show your work using multiplication.

$$-\frac{3}{8} \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{5}{2}\right) = \left(-\frac{3}{8} \cdot -\frac{2}{3}\right) \cdot \left(-\frac{5}{2}\right)$$
$$= \frac{6}{24} \cdot \left(-\frac{5}{2}\right)$$
$$= \frac{1}{4} \cdot \left(-\frac{5}{2}\right)$$

SOLUTION $-\frac{5}{8}$

CONSIDER THIS . . . You can multiply in any order.

PAIR/SHARE How would the answer change if $-\frac{3}{8}$ were $\frac{3}{8}$?

Apply It

1 Noor, Layla, and Diego play a trivia game. Noor's score is -2. Layla's score is $\frac{3}{4}$ of Noor's score. Diego's score is $\frac{2}{3}$ of Layla's score. What is Diego's score? Show your work.

Possible work:

Layla's score:

$$-2 \cdot \frac{3}{4} = -\frac{6}{4}$$
$$= -\frac{3}{2}$$

Diego's score:

$$\text{Layla's score} \cdot \frac{2}{3} = -\frac{3}{2} \cdot \frac{2}{3}$$
$$= -1$$

SOLUTION Diego's score is -1.

CONSIDER THIS . . . You can multiply to find $\frac{3}{4}$ of a quantity.

PAIR/SHARE How can you find Diego's score another way?

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$-\frac{3}{4}$	have used the opposite of $-\frac{3}{2}$ rather than the reciprocal of $-\frac{3}{2}$ when rewriting the division expression as multiplication.	Ask students to define *opposite* and *reciprocal* in their own words and give an example of each. Then discuss whether reciprocals or opposites should be used when dividing fractions.
3	have used the reciprocal of the dividend rather than the reciprocal of the divisor to find the quotient.	Have students draw grid squares to represent $\frac{1}{3} \cdot \frac{3}{2}$ and find the product, $\frac{1}{2}$. Then have students write a related division problem: $\frac{1}{2} \div \frac{3}{2} = \frac{1}{3}$.
$-\frac{1}{3}$	have lost track of a negative sign while computing.	Ask students to write a related multiplication expression for the division expression and to reason about whether the quotient of the division expression is positive or negative. Elicit from students that since both the dividend and the divisor are negative, the quotient should be positive.

Example

Guide students in understanding the Example. Ask:

- *How does your understanding of how to group factors help you solve the problem?*
- *In the second line, why is the first fraction $\frac{6}{24}$ rather than $-\frac{6}{24}$?*
- *How can you keep track of the negative signs while multiplying?*
- *How can you use what you know about multiplying an odd number of negatives to reason about whether the answer is correct?*

Help all students focus on the Example and responses to the questions by calling on one or two students to explain why the strategy and steps shown in the Example make sense to them.

Look for understanding that you can multiply three rational numbers by multiplying any two factors, and then you can multiply that product by the third factor. Have students note that an odd number of negative factors resulted in a negative product.

Apply It

1 See ***Connect to Culture*** to support student engagement. Students may leave Layla's score as the expression $-2 \times \frac{3}{4}$ and use the expression $-2 \times \frac{3}{4} \times \frac{2}{3}$ when finding Diego's score. ***DOK 1***

2 Students should understand that the total change is a decrease, so the average change each day is represented by a negative number. ***DOK 2***

3 **C is correct.** $4.8 \times -2.5 = -12$ and $-12 \div (-0.1) = 120$.

A is not correct. This answer could be the result of dividing $4.8(-2.5)$ by -1 instead of by -0.1.

B is not correct. This answer could be the result of losing track of one negative sign while calculating the answer.

D is not correct. This answer is the result of placing the decimal point incorrectly while also losing track of one negative sign.

DOK 3

2 Yolanda measures the depth of the water in a reservoir. In one week, the depth changes from 2,366.74 ft to 2,354.63 ft. What rational number represents the average change in depth each day? Show your work.

Possible work:

Change in elevation: $2{,}354.63 - 2{,}366.74 = -12.11$

$$\text{Average} = \frac{\text{total change}}{\text{number of days}}$$
$$= \frac{-12.11}{7}$$
$$= -1.73$$

SOLUTION -1.73

CONSIDER THIS . . .
Is the change in depth positive or negative?

PAIR/SHARE
What does the rational number you found mean in the context of the problem?

3 What is $4.8(-2.5) \div (-0.1)$?

A 1.2

B -120

C 120

D 1,200

CONSIDER THIS . . .
Dividing by a number is the same as multiplying by its reciprocal.

Maria chose D as the correct answer. How might she have gotten that answer?

Possible answer: Maria multiplied 4.8 by -2.5 and placed the decimal point incorrectly to get a product of -120 instead of -12. Then she divided -120 by -0.1 to get 1,200.

PAIR/SHARE
How would the answer be different if you divided by -0.01 instead of -0.1?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 6, 9

Meeting Proficiency
- **REINFORCE** Problems 4–9

Extending Beyond Proficiency
- **REINFORCE** Problems 4–9
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Multiplying and Dividing with Negative Numbers

Apply It

4 Students may use their understanding of opposites or their understanding that a negative dividend and negative divisor result in a positive quotient to decide that the quotient is positive. ***DOK 1***

5 Students may solve a simpler problem, such as $(-1)^4 = (-1)(-1)(-1)(-1)$, to explain their thinking. ***DOK 3***

6 Students should understand that to obtain a negative quotient, either the dividend or the divisor must be negative, but they cannot both be negative. ***DOK 1***

7 **C, D, and E are correct.** For the quotient to be positive, a and b must both be positive or both be negative. If both a and b are positive, then $-b \div a$ is negative. If both a and b are negative, then $-b$ is positive, which will result in a negative quotient.

A is not correct. For the quotient to be positive, either both a and b are positive or both a and b are negative. If both a and b are positive, then the product must be positive.

B is not correct. If both a and b are positive, then the quotient of $b \div a$ must be positive. If both a and b are negative, then the quotient of $b \div a$ must be positive.

DOK 3

4 What is $-79.2 \div (-16.5)$? Show your work.

Possible work:

$-79.2 \div (-16.5)$ has the same quotient as $79.2 \div 16.5$.

$79.2 \div 16.5 = 4.8$

SOLUTION 4.8

5 Without doing any calculations, is $\left(-\frac{1}{2}\right)^{50}$ *positive* or *negative*? Explain how you know.

positive; Possible explanation: The expression $\left(-\frac{1}{2}\right)^{50}$ means you are multiplying by using $-\frac{1}{2}$ as a factor 50 times. Each pair of negative factors gives you a positive product. Since all of the factors can be paired up to give a positive product, the overall product is positive.

6 Write three different division expressions that each have a negative quotient. Use at least two of the fractions $-\frac{1}{2}$, $\frac{4}{5}$, and -3 in each expression.

Possible answers:

$-\frac{1}{2} \div \frac{4}{5}$ $\qquad$ $\frac{4}{5} \div \left(-\frac{1}{2}\right)$

$-3 \div \frac{4}{5}$

7 The quotient of a and b is positive. Which of the following could be true about a and b? Select all that apply.

A The product ab is negative.

B The quotient $b \div a$ is negative.

C Both a and b are negative.

D The quotient $-b \div a$ is negative.

E Both a and b are positive.

DIFFERENTIATION

RETEACH

Visual Model

Use a model to multiply fractions.

Students approaching proficiency with multiplying and dividing with negative rational numbers will benefit from using models to explore patterns in rational number multiplication.

- Display the expression $1\frac{2}{3}\left(-2\frac{3}{4}\right)$. Say: *You can write an equivalent expression as the product of two sums by separating each number into integer and fraction parts.* Write the first factor, but have students identify the addends of the second factor before writing it: $\left(1 + \frac{2}{3}\right)\left[-2 + \left(-\frac{3}{4}\right)\right]$.
- Draw a square divided into 4 quadrants. Write the numbers 1 and $\frac{2}{3}$ on the top and the numbers -2 and $-\frac{3}{4}$ on the left. Ask: *How can you find the value of each square? What are the values? What do you notice about the values?* [You can multiply the factors on each side; -2, $-\frac{3}{4}$, $-\frac{4}{3}$, $-\frac{6}{12}$ or $-\frac{1}{2}$; all the values are negative.]
- Ask: *How can you find the total value of the product?* [Find the sum.]

  Write $-2 + -\frac{3}{4} + -\frac{4}{3} + -\frac{1}{2}$ on the board. Work with students to discuss different ways to reorder and combine the terms to get $-4\frac{7}{12}$.
- Extend with an example using two negative mixed numbers. Ask students what pattern they see with the products in the boxes. [They are all positive since they are all the products of two negative numbers.] If desired, extend further by using two positive or two negative decimals.

8 Students may need use the **Three Reads** language routine to make sure they understand the problem. They will need to find the change in height of the seeds before dividing by the number of hours. ***DOK 2***

9 Students may multiply the factors in any order. ***DOK 1***

CLOSE EXIT TICKET

10 **Math Journal** Look for understanding that either a dividend or a divisor, but not both, needs to be negative for a quotient to be negative.

Error Alert If students write a dividend and divisor that are both negative numbers, then have them write the division expression as a multiplication expression (using the reciprocal of the divisor). Ask: *Will the product of the factors be positive or negative?* and *How can you change the factors to result in a negative product?*

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they look back at Session 1 Connect It problem 2, Session 2 Connect It problem 4, and Session 3 Connect It problem 4 to see what these problems reveal about rational numbers. Have students work with a partner to compare and clarify their statements.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.

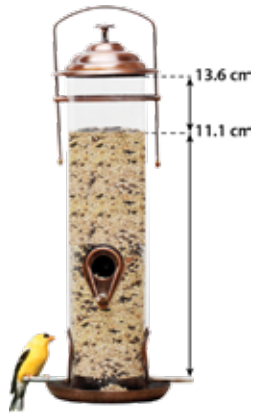
LESSON 12 | SESSION 4

8 Kwame fills a bird feeder with seeds. As birds visit the feeder, the height of the seeds changes from 13.6 cm to 11.1 cm over a period of $6\frac{1}{4}$ h. What is the average change in the height of the seeds each hour? Show your work.

Possible work:

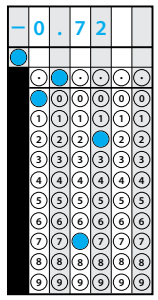
$11.1 - 13.6 = -2.5$

$-2.5 \div 6.25 = -0.4$



SOLUTION The average change in the height of the seeds each hour is −0.4 cm.

9 What is −0.3(0.2)(12)?



10 **Math Journal** Write a division expression that results in $-\frac{3}{8}$. Explain your thinking.

Possible answer: $-\frac{1}{4} \div \frac{2}{3}$

Possible explanation: Since division undoes multiplication, you can find the product of $-\frac{3}{8}$ and a number to find a division expression that results in $-\frac{3}{8}$.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *rational number*. Add two important things you learned about rational numbers in this lesson.
- ☐ **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

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REINFORCE

Problems 4–9

Multiply and divide rational numbers.

Students meeting proficiency will benefit from additional work with multiplying and dividing rational numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Write multiplication and division equations.

Students extending beyond proficiency will benefit from writing their own multiplication and division equations.

- Have students work in pairs to write a multiplication or division equation with positive and negative rational numbers. Have the first student write a fraction or a decimal number (positive or negative) followed by an operation sign (multiplication or division).
- Have the second student write a different rational number (it does not need to be of the same form).
- Have students work together to find the product or quotient to complete the equation.
- Repeat the process, having students switch roles.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.