3.6 Mathematics of Finance

What you'll learn about

- Interest Compounded Annually
- Interest Compounded k Times per Year
- Interest Compounded Continuously
- Annual Percentage Yield
- Annuities—Future Value
- Loans and Mortgages—Present Value

... and why

The mathematics of finance is the science of letting your money work for you—valuable information indeed!

Interest Compounded Annually

In business, as the saying goes, "time is money." We must pay interest for the use of property or money over time. When we borrow money, we pay interest, and when we loan money, we receive interest. When we invest in a savings account, we are actually lending money to the bank.

Suppose a principal of P dollars is invested in an account bearing an interest rate r expressed in decimal form and calculated at the end of each year. If A_n represents the total amount in the account at the end of n years, then the value of the investment follows the growth pattern shown in Table 3.27.

Table 3.27 Interest Computed Annually

Time in Years	Amount in the Account
0	$A_0 = P = \text{principal}$
1	$A_1 = P + P \cdot r = P(1 + r)$
2	$A_2 = A_1 \cdot (1 + r) = P(1 + r)^2$
3	$A_3 = A_2 \cdot (1 + r) = P(1 + r)^3$
$\vdots$	$\vdots$
n	$A = A_n = P(1 + r)^n$

Notice that this is the constant percentage growth pattern studied in Section 3.2, and so the value of an investment is an exponential function of time. We call interest computed in this way **compound interest** because the interest becomes part of the investment, so that interest is earned on the interest itself.

Interest Compounded Annually

If a principal P is invested at a fixed annual interest rate r, calculated at the end of each year, then the value of the investment after n years is

$$A = P(1 + r)^n,$$

where r is expressed as a decimal.

EXAMPLE 1 Compounding Annually

Suppose Quan Li invests \$500 at 7% interest compounded annually. Find the value of her investment 10 years later.

SOLUTION Letting $P = 500$, $r = 0.07$, and $n = 10$,

$A = 500(1 + 0.07)^{10} = 983.575\ldots$. Rounding to the nearest cent, we see that the value of Quan Li's investment after 10 years is \$983.58. *Now try Exercise 1.*

Interest Compounded k Times per Year

Suppose a principal P is invested at an annual interest rate r compounded k times a year for t years. Then r/k is the interest rate per compounding period, and kt is the number of compounding periods. The amount A in the account after t years is

$$A = P\left(1 + \frac{r}{k}\right)^{kt}.$$

EXAMPLE 2 Compounding Monthly

Suppose Roberto invests \$500 at 9% annual interest *compounded monthly*, that is, compounded 12 times a year. Find the value of his investment 5 years later.

SOLUTION Letting $P = 500$, $r = 0.09$, $k = 12$, and $t = 5$,

$$A = 500\left(1 + \frac{0.09}{12}\right)^{12(5)} = 782.840\ldots.$$

So the value of Roberto's investment after 5 years is \$782.84.

Now try Exercise 5.

The problems in Examples 1 and 2 required that we calculate A. Examples 3 and 4 illustrate situations that require us to determine the values of other variables in the compound interest formula.

EXAMPLE 3 Finding the Time Period of an Investment

Judy has \$500 to invest at 9% annual interest compounded monthly. How long will it take for her investment to grow to \$3000?

SOLUTION

Model Let $P = 500$, $r = 0.09$, $k = 12$, and $A = 3000$ in the equation

$$A = P\left(1 + \frac{r}{k}\right)^{kt},$$

and solve for t.

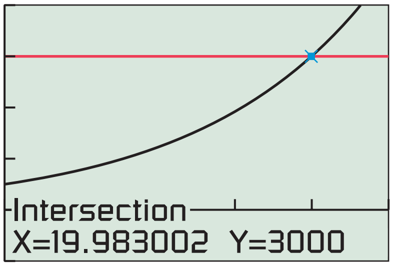
Solve Graphically For

$$3000 = 500\left(1 + \frac{0.09}{12}\right)^{12t},$$

we let

$$f(t) = 500\left(1 + \frac{0.09}{12}\right)^{12t} \quad \text{and} \quad y = 3000,$$

and then find the point of intersection of the graphs. Figure 3.41 shows that this occurs at $t \approx 19.98$.



[0, 25] by [–1000, 4000]

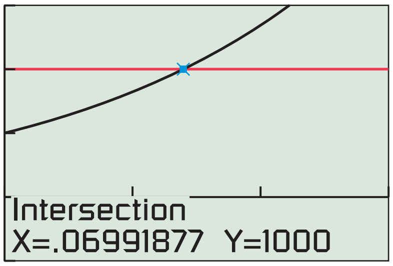
FIGURE 3.41 Graph for Example 3.

Confirm Algebraically

$$\begin{aligned} 3000 &= 500(1 + 0.09/12)^{12t} & & \\ 6 &= 1.0075^{12t} & & \text{Divide by 500.} \\ \ln 6 &= \ln\left(1.0075^{12t}\right) & & \text{Apply ln to each side.} \\ \ln 6 &= 12t \ln(1.0075) & & \text{Power rule} \\ t &= \frac{\ln 6}{12 \ln 1.0075} & & \text{Divide by 12 ln 1.0075.} \\ &= 19.983\ldots & & \text{Calculate.} \end{aligned}$$

Interpret So it will take Judy 20 years for the value of the investment to reach (and slightly exceed) \$3000.

Now try Exercise 21.



[0, 0.15] by [–500, 1500]

FIGURE 3.42 Graph for Example 4.

EXAMPLE 4 Finding an Interest Rate

Stephen has \$500 to invest. What annual interest rate *compounded quarterly* (four times per year) is required to double his money in 10 years?

SOLUTION

Model Letting $P = 500$, $k = 4$, $t = 10$, and $A = 1000$ yields the equation

$$1000 = 500\left(1 + \frac{r}{4}\right)^{4(10)}$$

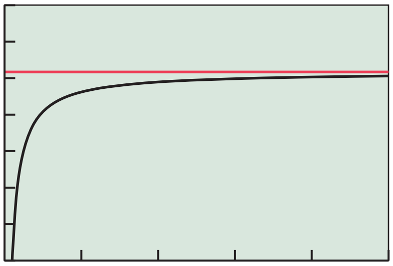
that we solve for r.

Solve Graphically Figure 3.42 shows that $f(r) = 500(1 + r/4)^{40}$ and $y = 1000$ intersect at $r \approx 0.0699$, or $r = 6.99\%$.

Interpret Stephen's investment of \$500 will double in 10 years at an annual interest rate of 6.99% compounded quarterly. *Now try Exercise 25.*

Interest Compounded Continuously

In Exploration 1, \$1000 is invested for 1 year at a 10% interest rate. We investigate the value of the investment at the end of 1 year as the number of compounding periods k increases. In other words, we determine the "limiting" value of the expression $1000(1 + 0.1/k)^k$ as k assumes larger and larger integer values.



[0, 50] by [1100, 1107]

FIGURE 3.43 Graph for Exploration 1.

EXPLORATION 1 Increasing the Number of Compounding Periods Boundlessly

Let $A = 1000\left(1 + \dfrac{0.1}{k}\right)^k$.

1. Complete a table of values of A for $k = 10, 20, \dots, 100$. What pattern do you observe?
2. Figure 3.43 shows the graphs of the function $A(k) = 1000(1 + 0.1/k)^k$ and the horizontal line $y = 1000e^{0.1}$. Interpret the meanings of these graphs.

Recall from Section 3.1 that $e = \lim_{x\to\infty} (1 + 1/x)^x$. Therefore, for a fixed interest rate r, if we let $x = k/r$,

$$\lim_{k\to\infty}\left(1 + \frac{r}{k}\right)^{k/r} = e.$$

We do not know enough about limits yet, but with some calculus, it can be proved that $\lim_{k\to\infty} P(1 + r/k)^{kt} = Pe^{rt}$. So $A = Pe^{rt}$ is the formula used when interest is **compounded continuously**. In nearly any situation, one of the following two formulas can be used to compute compound interest:

Compound Interest—Value of an Investment

Suppose a principal P is invested at a fixed annual interest rate r. The value of the investment after t years is

- $A = P\left(1 + \dfrac{r}{k}\right)^{kt}$ when interest compounds k times per year,
- $A = Pe^{rt}$ when interest compounds continuously.

X	Y_1	
1	108.33	
2	117.35	
3	127.12	
4	137.71	
5	149.18	
6	161.61	
7	175.07	
Y_1 = 100e^(0.08X)		

FIGURE 3.44 Table of values for Example 5.

EXAMPLE 5 Compounding Continuously

Suppose LaTasha invests \$100 at 8% annual interest compounded continuously. Find the value of her investment at the end of each of the years 1, 2, . . . , 7.

SOLUTION Substituting into the formula for continuous compounding, we obtain $A(t) = 100e^{0.08t}$. Figure 3.44 shows the values of $y_1 = A(x) = 100e^{0.08x}$ for $x = 1, 2, \ldots, 7$. For example, the value of her investment is \$149.18 at the end of 5 years, and \$175.07 at the end of 7 years. ***Now try Exercise 9.***

Annual Percentage Yield

With so many different interest rates and methods of compounding it is sometimes difficult for a consumer to compare two different options. For example, would you prefer an investment earning 8.75% annual interest compounded quarterly or one earning 8.7% compounded monthly?

A common basis for comparing investments is the **annual percentage yield (APY)**—the percentage rate that, compounded annually, would yield the same return as the given interest rate with the given compounding period.

EXAMPLE 6 Computing Annual Percentage Yield (APY)

Ursula invests \$2000 with Crab Key Bank at 5.15% annual interest compounded quarterly. What is the equivalent APY?

SOLUTION Let $x =$ the equivalent APY. The value of the investment at the end of 1 year using this rate is $A = 2000(1 + x)$. Thus, we have

$$2000(1 + x) = 2000\left(1 + \frac{0.0515}{4}\right)^4$$

$$(1 + x) = \left(1 + \frac{0.0515}{4}\right)^4 \quad \text{Divide by 2000.}$$

$$x = \left(1 + \frac{0.0515}{4}\right)^4 - 1 \quad \text{Subtract 1.}$$

$$\approx 0.0525 \quad \text{Calculate.}$$

The annual percentage yield is 5.25%. In other words, Ursula's \$2000 invested at 5.15% compounded quarterly for 1 year earns the same interest and yields the same value as \$2000 invested elsewhere paying 5.25% interest once at the end of the year. ***Now try Exercise 41.***

Example 6 shows that the APY does not depend on the principal P because both sides of the equation were divided by $P = 2000$. So we can assume that $P = 1$ when comparing investments.

EXAMPLE 7 Comparing Annual Percentage Yields (APYs)

Which investment is more attractive, one that pays 8.75% compounded quarterly or another that pays 8.7% compounded monthly?

(continued)

SOLUTION

Let

$$r_1 = \text{the APY for the 8.75\% rate,}$$
$$r_2 = \text{the APY for the 8.7\% rate.}$$

$$1 + r_1 = \left(1 + \frac{0.0875}{4}\right)^4 \qquad 1 + r_2 = \left(1 + \frac{0.087}{12}\right)^{12}$$

$$r_1 = \left(1 + \frac{0.0875}{4}\right)^4 - 1 \qquad r_2 = \left(1 + \frac{0.087}{12}\right)^{12} - 1$$

$$\approx 0.09041 \qquad \approx 0.09055$$

The 8.7% rate compounded monthly is more attractive because its APY is 9.055% compared with 9.041% for the 8.75% rate compounded quarterly.

Now try Exercise 45.

Annuities—Future Value

So far, in all of the investment situations we have considered, the investor has made a single *lump-sum* deposit. But suppose an investor makes regular deposits monthly, quarterly, or yearly—the same amount each time. This is an *annuity* situation.

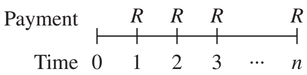


FIGURE 3.45 Payments into an ordinary annuity.

An **annuity** is a sequence of equal periodic payments. The annuity is **ordinary** if deposits are made at the end of each period at the same time the interest is posted in the account. Figure 3.45 represents this situation graphically. We will consider only ordinary annuities in this textbook.

Let's consider an example. Suppose Sarah makes \$500 payments at the end of each quarter into a retirement account that pays 8% interest compounded quarterly. How much will be in Sarah's account at the end of the first year? Notice the pattern.

End of Quarter 1:

$$\$500 = \$500$$

End of Quarter 2:

$$\$500 + \$500(1.02) = \$1010$$

End of Quarter 3:

$$\$500 + \$500(1.02) + \$500(1.02)^2 = \$1530.20$$

End of the year:

$$\$500 + \$500(1.02) + \$500(1.02)^2 + \$500(1.02)^3 \approx \$2060.80$$

Thus the total value of the investment returned from an annuity consists of all the periodic payments together with all the interest. This value is called the **future value** of the annuity because it is typically calculated when projecting into the future.

Future Value of an Annuity

The future value FV of an annuity consisting of n equal periodic payments of R dollars at an interest rate i per compounding period (payment interval) is

$$FV = R\frac{(1 + i)^n - 1}{i}.$$

EXAMPLE 8 Calculating the Value of an Annuity

At the end of each quarter year, Emily makes a \$500 payment into the Lanaghan Mutual Fund. If her investments earn 7.88% annual interest compounded quarterly, what will be the value of Emily's annuity in 20 years?

SOLUTION Let $R = 500$, $i = 0.0788/4$, $n = 20(4) = 80$. Then,

$$FV = R\frac{(1+i)^n - 1}{i}$$

$$FV = 500 \cdot \frac{(1 + 0.0788/4)^{80} - 1}{0.0788/4}$$

$$FV = 95{,}483.389\ldots$$

So the value of Emily's annuity in 20 years will be \$95,483.39.

Now try Exercise 13.

Loans and Mortgages—Present Value

An annuity is a sequence of equal period payments. The net amount of money put into an annuity is its **present value**. The net amount returned from the annuity is its future value. The periodic and equal payments on a loan or mortgage actually constitute an annuity.

How does the bank determine what the periodic payments should be? It considers what would happen to the present value of an investment with interest compounding over the term of the loan and compares the result to the future value of the loan repayment annuity.

We illustrate this reasoning by assuming that a bank lends you a present value $PV = \$50{,}000$ at 6% to purchase a house with the expectation that you will make a mortgage payment each month (at the monthly interest rate of $0.06/12 = 0.005$).

- The future value of an investment at 6% compounded monthly for n months is

$$PV(1+i)^n = 50{,}000(1 + 0.005)^n.$$

- The future value of an annuity of R dollars (the loan payments) is

$$R\frac{(1+i)^n - 1}{i} = R\frac{(1 + 0.005)^n - 1}{0.005}.$$

To find R, we would solve the equation

$$50{,}000(1 + 0.005)^n = R\frac{(1 + 0.005)^n - 1}{0.005}.$$

In general, the monthly payments of R dollars for a loan of PV dollars must satisfy the equation

$$PV(1+i)^n = R\frac{(1+i)^n - 1}{i}.$$

Dividing both sides by $(1+i)^n$ leads to the following formula for the present value of an annuity.

Present Value of an Annuity

The present value PV of an annuity consisting of n equal payments of R dollars earning an interest rate i per period (payment interval) is

$$PV = R\frac{1 - (1+i)^{-n}}{i}.$$

The annual interest rate charged on consumer loans is the **annual percentage rate (APR)**. The APY for the lender is higher than the APR. See Exercise 58.

EXAMPLE 9 Calculating Loan Payments

Carlos purchases a pickup truck for \$18,500. What are the monthly payments for a 4-year loan with a \$2000 down payment if the annual interest rate (APR) is 2.9%?

SOLUTION

Model The down payment is \$2000, so the amount borrowed is \$16,500. Since APR = 2.9%, $i = 0.029/12$ and the monthly payment is the solution to

$$16{,}500 = R\frac{1-(1+0.029/12)^{-4(12)}}{0.029/12}.$$

Solve Algebraically

$$R\left[1-\left(1+\frac{0.029}{12}\right)^{-4(12)}\right] = 16{,}500\left(\frac{0.029}{12}\right)$$

$$R = \frac{16{,}500(0.029/12)}{1-(1+0.029/12)^{-48}}$$

$$= 364.487\ldots$$

Interpret Carlos will have to pay \$364.49 per month for 47 months, and slightly less the last month. ***Now try Exercise 19.***

QUICK REVIEW 3.6

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

1. Find 3.5% of 200.
2. Find 2.5% of 150.
3. What is one-fourth of 7.25%?
4. What is one-twelfth of 6.5%?
5. 78 is what percent of 120?
6. 28 is what percent of 80?
7. 48 is 32% of what number?
8. 176.4 is 84% of what number?
9. How much does Jane have at the end of 1 year if she invests \$300 at 5% simple interest?
10. How much does Reggie have at the end of 1 year if he invests \$500 at 4.5% simple interest?

SECTION 3.6 EXERCISES

In Exercises 1–4, find the amount A accumulated after investing a principal P for t years at an interest rate r compounded annually.

1. $P = \$1500,\ r = 7\%,\ t = 6$
2. $P = \$3200,\ r = 8\%,\ t = 4$
3. $P = \$12{,}000,\ r = 7.5\%,\ t = 7$
4. $P = \$15{,}500,\ r = 9.5\%,\ t = 12$

In Exercises 5–8, find the amount A accumulated after investing a principal P for t years at an interest rate r compounded k times per year.

5. $P = \$1500,\ r = 7\%,\ t = 5,\ k = 4$
6. $P = \$3500,\ r = 5\%,\ t = 10,\ k = 4$
7. $P = \$40{,}500,\ r = 3.8\%,\ t = 20,\ k = 12$
8. $P = \$25{,}300,\ r = 4.5\%,\ t = 25,\ k = 12$

In Exercises 9–12, find the amount A accumulated after investing a principal P for t years at interest rate r compounded continuously.

9. $P = \$1250$, $r = 5.4\%$, $t = 6$

10. $P = \$3350$, $r = 6.2\%$, $t = 8$

11. $P = \$21{,}000$, $r = 3.7\%$, $t = 10$

12. $P = \$8{,}875$, $r = 4.4\%$, $t = 25$

In Exercises 13–15, find the future value FV accumulated in an annuity after investing periodic payments R for t years at an annual interest rate r, with payments made and interest credited k times per year.

13. $R = \$500$, $r = 7\%$, $t = 6$, $k = 4$

14. $R = \$300$, $r = 6\%$, $t = 12$, $k = 4$

15. $R = \$450$, $r = 5.25\%$, $t = 10$, $k = 12$

16. $R = \$610$, $r = 6.5\%$, $t = 25$, $k = 12$

In Exercises 17 and 18, find the present value PV of a loan with an annual interest rate r and periodic payments R for a term of t years, with payments made and interest charged 12 times per year.

17. $r = 4.7\%$, $R = \$815.37$, $t = 5$

18. $r = 6.5\%$, $R = \$1856.82$, $t = 30$

In Exercises 19 and 20, find the periodic payment R of a loan with present value PV and an annual interest rate r for a term of t years, with payments made and interest charged 12 times per year.

19. $PV = \$18{,}000$, $r = 5.4\%$, $t = 6$

20. $PV = \$154{,}000$, $r = 7.2\%$, $t = 15$

21. Finding Time If John invests $2300 in a savings account with a 9% interest rate compounded quarterly, how long will it take until John's account has a balance of $4150?

22. Finding Time If Joelle invests $8000 into a retirement account with a 9% interest rate compounded monthly, how long will it take until this single payment has grown in her account to $16,000?

23. Trust Officer Megan is the trust officer for an estate. If she invests $15,000 into an account that carries an interest rate of 8% compounded monthly, how long will it be until the account has a value of $45,000 for Megan's client?

24. Chief Financial Officer Willis is the financial officer of a private university with the responsibility for managing an endowment. If he invests $1.5 million at an interest rate of 8% compounded quarterly, how long will it be until the account exceeds $3.75 million?

25. Finding the Interest Rate What interest rate compounded daily (365 days/year) is required for a $22,000 investment to grow to $36,500 in 5 years?

26. Finding the Interest Rate What interest rate compounded monthly is required for an $8500 investment to triple in 5 years?

27. Pension Officer Jack is an actuary working for a corporate pension fund. He needs to have $14.6 million grow to $22 million in 6 years. What interest rate compounded annually does he need for this investment?

28. Bank President The president of a bank has $18 million in his bank's investment portfolio that he wants to grow to $25 million in 8 years. What interest rate compounded annually does he need for this investment?

29. Doubling Your Money Determine how much time is required for an investment to double in value if interest is earned at the rate of 5.75% compounded quarterly.

30. Tripling Your Money Determine how much time is required for an investment to triple in value if interest is earned at the rate of 6.25% compounded monthly.

In Exercises 31–34, complete the table about continuous compounding.

	Initial Investment	APR	Time to Double	Amount in 15 Years
31.	$12,500	9%	?	?
32.	$32,500	8%	?	?
33.	$ 9,500	?	4 years	?
34.	$16,800	?	6 years	?

In Exercises 35–40, complete the table about doubling time of an investment.

	APR	Compounding Periods	Time to Double
35.	4%	Quarterly	?
36.	8%	Quarterly	?
37.	7%	Annually	?
38.	7%	Quarterly	?
39.	7%	Monthly	?
40.	7%	Continuously	?

In Exercises 41–44, find the annual percentage yield (APY) for the investment.

41. $3000 at 6% compounded quarterly

42. $8000 at 5.75% compounded daily

43. P dollars at 6.3% compounded continuously

44. P dollars at 4.7% compounded monthly

45. Comparing Investments Which investment is more attractive, 5% compounded monthly or 5.1% compounded quarterly?

46. Comparing Investments Which investment is more attractive, $5\frac{1}{8}\%$ compounded annually or 5% compounded continuously?

In Exercises 47–50, payments are made and interest is credited at the end of each month.

47. An IRA Account Amy contributes $50 per month into the Lincoln National Bond Fund that earns 7.26% annual interest. What is the value of Amy's investment after 25 years?

48. An IRA Account Andrew contributes $50 per month into the Hoffbrau Fund that earns 15.5% annual interest. What is the value of his investment after 20 years?

49. An Investment Annuity Jolinda contributes to the Celebrity Retirement Fund that earns 12.4% annual interest. What should her monthly payments be if she wants to accumulate $250,000 in 20 years?

50. An Investment Annuity Diego contributes to a Commercial National money market account that earns 4.5% annual interest. What should his monthly payments be if he wants to accumulate $120,000 in 30 years?

51. Car Loan Payment What is Kim's monthly payment for a 4-year $9000 car loan with an APR of 7.95% from Century Bank?

52. Car Loan Payment What is Ericka's monthly payment for a 3-year $4500 car loan with an APR of 10.25% from County Savings Bank?

53. House Mortgage Payment Gendo obtains a 30-year $86,000 house loan with an APR of 8.75% from National City Bank. What is her monthly payment?

54. House Mortgage Payment Roberta obtains a 25-year $100,000 house loan with an APR of 9.25% from NBD Bank. What is her monthly payment?

55. Mortgage Payment Planning An $86,000 mortgage for 30 years at 12% APR requires monthly payments of $884.61. Suppose you decided to make monthly payments of $1050.00.

(a) When would the mortgage be completely paid?

(b) How much do you save with the greater payments compared with the original plan?

56. Mortgage Payment Planning Suppose you make payments of $884.61 for the $86,000 mortgage in Exercise 53 for 10 years and then make payments of $1050 until the loan is paid.

(a) When will the mortgage be completely paid under these circumstances?

(b) How much do you save with the greater payments compared with the original plan?

57. Writing to Learn Explain why computing the APY for an investment does not depend on the actual amount being invested. Give a formula for the APY on a $1 investment at annual rate r compounded k times a year. How do you extend the result to a $1000 investment?

58. Writing to Learn Give reasons why banks might not announce their APY on a loan they would make to you at a given APR. What is the bank's APY on a loan that they make at 4.5% APR?

59. Group Activity Work in groups of three or four. Consider population growth of humans or other animals, bacterial growth, radioactive decay, and compounded interest. Explain how these problem situations are similar and how they are different. Give examples to support your point of view.

60. Simple Interest Versus Compounding Annually Steve purchases a $1000 certificate of deposit and will earn 6% each year. The interest will be mailed to him, so he will not earn interest on his interest.

(a) Writing to Learn Explain why after t years, the total amount of interest he receives from his investment plus the original $1000 is given by

$$f(t) = 1000(1 + 0.06t).$$

(b) Steve invests another $1000 at 6% compounded annually. Make a table that compares the value of the two investments for $t = 1, 2, \ldots, 10$ years.

Standardized Test Questions

61. True or False If $100 is invested at 5% annual interest for 1 year, there is no limit to the final value of the investment if it is compounded sufficiently often. Justify your answer.

62. True or False The total interest paid on a 15-year mortgage is less than half of the total interest paid on a 30-year mortgage with the same loan amount and APR. Justify your answer.

In Exercises 63–66, you may use a graphing calculator to solve the problem.

63. Multiple Choice What is the total value after 6 years of an initial investment of $2250 that earns 7% interest compounded quarterly?

(A) $3376.64 **(B)** $3412.00 **(C)** $3424.41
(D) $3472.16 **(E)** $3472.27

64. Multiple Choice The annual percentage yield of an account paying 6% compounded monthly is

(A) 6.03%. **(B)** 6.12%. **(C)** 6.17%.
(D) 6.20%. **(E)** 6.24%.

65. Multiple Choice Mary Jo deposits $300 each month into her retirement account that pays 4.5% APR (0.375% per month). Use the formula $FV = R((1 + i)^n - 1)/i$ to find the value of her annuity after 20 years.

(A) $71,625.00 **(B)** $72,000.00 **(C)** $72,375.20
(D) $73,453.62 **(E)** $116,437.31

66. Multiple Choice To finance their home, Mr. and Mrs. Dass have agreed to a $120,000 mortgage loan at 7.25% APR. Use the formula $PV = R(1 - (1 + i)^{-n})/i$ to determine their monthly payments if the loan has a term of 15 years.

(A) $1095.44 **(B)** $1145.44 **(C)** $1195.44
(D) $1245.44 **(E)** $1295.44

Explorations

67. Loan Payoff Use the information about Carlos's truck loan in Example 9 to make a spreadsheet of the payment schedule. The first few lines of the spreadsheet should look like the following table:

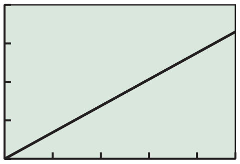
Month No.	Payment	Interest	Principal	Balance
0				$16,500.00
1	$364.49	$39.88	$324.61	$16,175.39
2	$364.49	$39.09	$325.40	$15,849.99

To create the spreadsheet successfully, however, you need to use formulas for many of the cells, as shown in boldface type in the following sample:

Month No.	Payment	Interest	Principal	Balance
0				$16,500.00
= A2 + 1	$364.49	**= round(E2*2.9%/12,2)**	**= B3 − C3**	**= E2 − D3**
= A3 + 1	$364.49	**= round(E3*2.9%/12,2)**	**= B4 − C4**	**= E3 − D4**

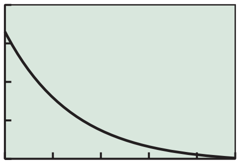
Continue the spreadsheet using copy-and-paste techniques, and determine the amount of the 48th and final payment so that the final balance is $0.00.

68. Writing to Learn Loan Payoff Which of the following graphs is an accurate graph of the loan balance as a function of time, based on Carlos's truck loan in Example 9 and Exercise 67? Explain your choice based on increasing or decreasing behavior and other analytical characteristics. Would you expect the graph of loan balance versus time for a 30-year mortgage loan at twice the interest rate to have the same shape or a different shape as the one for the truck loan? Explain.



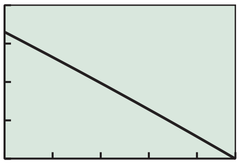
[0, 48] by [0, 20 000]

(a)



[0, 48] by [0, 20 000]

(b)



[0, 48] by [0, 20 000]

(c)

Extending the Ideas

69. The function

$$f(x) = 100\frac{(1 + 0.08/12)^x - 1}{0.08/12}$$

describes the future value of a certain annuity.

(a) What is the annual interest rate?

(b) How many payments per year are there?

(c) What is the amount of each payment?

70. The function

$$f(x) = 200\,\frac{1 - (1 + 0.08/12)^{-x}}{0.08/12}$$

describes the present value of a certain annuity.

(a) What is the annual interest rate?

(b) How many payments per year are there?

(c) What is the amount of each payment?

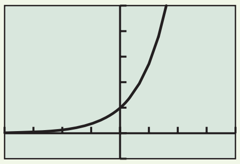
CHAPTER 3 Key Ideas

Properties, Theorems, and Formulas

Procedures

Gallery of Functions

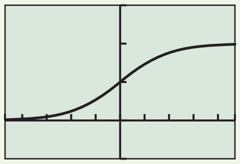
Exponential



[–4, 4] by [–1, 5]

$f(x) = e^x$

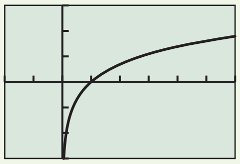
Basic Logistic



[–4.7, 4.7] by [–0.5, 1.5]

$$f(x) = \frac{1}{1 + e^{-x}}$$

Natural Logarithmic



[–2, 6] by [–3, 3]

$f(x) = \ln x$