Section 9.2: Waves at Media Boundaries

Tutorial 1 Practice, page 425

1. (a) Given: f = 2(200.0 Hz) = 400.0 Hz; v = 350 m/s; n = 1; free and fixed ends

Required: L_1

Analysis: $L_n = \frac{2n-1}{4}\lambda$

Solution:

$$L_n = \frac{2n-1}{4}\lambda$$

$$\begin{aligned} L_1 &= \frac{1}{4}\left(\frac{v}{f}\right) \\ &= \frac{1}{4}\left(\frac{350 \text{ m/s}}{400 \text{ Hz}}\right) \\ L_1 &= 0.22 \text{ m} \end{aligned}$$

Statement: The length of rope is 0.22 m if the frequency is double.

(b) Given: f = 200.0 Hz; v = 350 m/s; n = 3; free and fixed ends

Required: L_3

Analysis: $L_n = \frac{2n-1}{4}\lambda$

Solution:

$$L_n = \frac{2n-1}{4}\lambda$$

$$\begin{aligned} L_3 &= \frac{5}{4}\left(\frac{v}{f}\right) \\ &= \frac{5}{4}\left(\frac{350 \text{ m/s}}{200 \text{ Hz}}\right) \\ L_3 &= 2.2 \text{ m} \end{aligned}$$

Statement: The length of the string is 2.2 m.

(c) Given: f = 200.0 Hz; v = 200 m/s; n = 1; free and fixed ends

Required: L_1

Analysis: $L_n = \frac{2n-1}{4}\lambda$

Solution: $L_n = \frac{2n-1}{4}\lambda$

$$\begin{aligned} L_1 &= \frac{1}{4}\left(\frac{v}{f}\right) \\ &= \frac{1}{4}\left(\frac{200 \text{ m/s}}{200 \text{ Hz}}\right) \\ L_1 &= 0.25 \text{ m} \end{aligned}$$

Statement: The length of rope is 0.25 m.

2. (a) Given: L_6 = 0.65 m; v_6 = 206 m/s; n = 6; f_6 = 950 Hz; v = 150 m/s; two fixed ends

Required: f

Analysis: $\lambda = \frac{v}{f}$

$$f = \frac{v}{\lambda}$$

Solution: Determine the wavelength:

$$\lambda = \frac{v}{f}$$

$$\begin{aligned} \lambda &= \frac{v_6}{f_6} \\ &= \frac{206 \text{ m/s}}{950 \text{ Hz}} \\ \lambda &= 0.2168 \text{ m (two extra digits carried)} \end{aligned}$$

Determine the frequency:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{150 \text{ m/s}}{0.2168 \text{ m}} \\ f &= 690 \text{ Hz} \end{aligned}$$

Statement: The frequency is 690 Hz.

(b) Given: L_6 = 0.65 m; v_6 = 206 m/s; n = 6; f_6 = 950 Hz; v = 350 m/s; two fixed ends

Required: f

Analysis: $\lambda = \frac{v}{f}$

$$f = \frac{v}{\lambda}$$

Solution: Determine the wavelength:

$$\begin{aligned} \lambda &= \frac{v_6}{f_6} \\ &= \frac{206 \text{ m/s}}{950 \text{ Hz}} \\ \lambda &= 0.2168 \text{ m (two extra digits carried)} \end{aligned}$$

Determine the frequency:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{350 \text{ m/s}}{0.2168 \text{ m}} \\ f &= 1600 \text{ Hz} \end{aligned}$$

Statement: The frequency is 1600 Hz.

3. Given: L = 1 m; f_4 = 44 kHz = 44 000 Hz; two fixed ends
Required: f_0; f_1; f_2; f_3

Analysis:
$$\lambda = \frac{v}{f_4}$$
$$L = \frac{n\lambda}{2}$$
$$\frac{2L}{n} = \lambda$$
$$= \frac{v}{f}$$
$$f = \frac{vn}{2L}$$

Solution: Determine the speed from the fourth overtone:
$$\lambda = \frac{v}{f_4}$$
$$v = \lambda f_4$$
$$= \left(\frac{2L}{n}\right)f_4$$
$$= \left(\frac{2(1\text{ m})}{5}\right)(44\ 000\text{ Hz})$$
$$v = 17\ 600\text{ m/s (one extra digit carried)}$$

Determine the frequency of the first harmonic:
$$L = \frac{n\lambda}{2}$$
$$f = \frac{vn}{2L}$$
$$f_0 = \frac{(17\ 600\ \cancel{\text{m}}/\text{s})(1)}{2(1\ \cancel{\text{m}})}$$
$$f_0 = 8800\text{ Hz}$$

Determine the frequency of the second harmonic:
$$f = \frac{vn}{2L}$$
$$f_1 = \frac{(17\ 600\ \cancel{\text{m}}/\text{s})(2)}{2(1\ \cancel{\text{m}})}$$
$$f_1 = 17\ 600\text{ Hz}$$

Determine the frequency of the third harmonic:
$$f = \frac{vn}{2L}$$
$$f_2 = \frac{(17\ 600\ \cancel{\text{m}}/\text{s})(3)}{2(1\ \cancel{\text{m}})}$$
$$f_2 = 26\ 400\text{ Hz}$$

Determine the frequency of the fourth harmonic:
$$f = \frac{vn}{2L}$$
$$f_3 = \frac{(17\ 600\ \cancel{\text{m}}/\text{s})(4)}{2(1\ \cancel{\text{m}})}$$
$$f_3 = 35\ 200\text{ Hz}$$

Statement: The first and second harmonics are within the range of human hearing (20 Hz to 20 kHz).

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A. There is a standing wave with only one node at the fixed end of the rope.
B. Once the first harmonic was achieved, I had to move the rope up and down at a constant amplitude and speed.
C. Answers may vary. Students' predictions should be based on what they know about the standing wave machine and what they can calculate using the material they just learned.
D. Answers may vary. Students should explain any differences between the prediction from C and the actual frequency for f_0.

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1. (a) When two or more waves interact and the resulting wave appears to be stationary, this wave is called a standing wave.
(b) The fundamental frequency is the lowest frequency that can produce a standing wave in a given medium.
(c) A node is the point in a standing wave at which the particles are at rest.
(d) Harmonics are the whole-number multiples of the fundamental frequency.
2. An example of a wave that encounters a media boundary is being in a cave and having my speech echo within the cave.
(a) The reflected wave will have the same amplitude while the amplitude of the transmitted wave will decrease.
(b) Yes; Answers may vary. Sample answer: When there is a change in medium, the wave splits into a reflected wave and a transmitted wave. The sum of the amplitudes of these two waves is the same as the original, meaning both of the new amplitudes will be smaller than the original wave's amplitude. Hence, the answer in part (a) is supported by the change in medium.

3. Answers may vary. Two examples of free-end reflection are whips and shaking a dangling cat toy. Examples of fixed-end reflection include string musical instruments such as violins, guitars, and harps.

4. The length of the medium must be a whole-number multiple of the first harmonic.

5. Given: L_1 = 2.4 m; v = 450 m/s; n = 1; two fixed ends

Required: f

Analysis: $L_1 = \frac{n\lambda}{2}$

$\lambda = \frac{v}{f}$

$f = \frac{v}{\lambda}$

Solution: Determine the wavelength:

$$L_1 = \frac{n\lambda}{2}$$
$$\lambda = \frac{2L_1}{n}$$
$$= \frac{2(2.4 \text{ m})}{1}$$
$$\lambda = 4.8 \text{ m}$$

Determine the frequency:

$$f = \frac{v}{\lambda}$$
$$= \frac{450 \text{ m/s}}{4.8 \text{ m}}$$
$$f = 94 \text{ Hz}$$

Statement: The frequency of the wave that would produce the first harmonic is 94 Hz.

6. Given: L_2 = 1.2 m; T = 20 °C; n = 2; two open ends

Required: f_1

Analysis: $L_2 = \frac{n\lambda}{2}$; $v = 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T$;

$\lambda = \frac{v}{f}$

$f = \frac{v}{\lambda}$

Solution: Determine the wavelength:

$$L_2 = \frac{n\lambda}{2}$$
$$\lambda = \frac{2L_1}{n}$$
$$= \frac{2(1.2 \text{ m})}{2}$$
$$\lambda = 1.2 \text{ m}$$

Determine the speed of sound in the air:

$$v = 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T$$
$$= 331.4 \text{ m/s} + \left(0.606 \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(20 \cancel{°\text{C}})$$
$$v = 343.5 \text{ m/s}$$

Determine the frequency:

$$f = \frac{v}{\lambda}$$
$$= \frac{343.5 \text{ m/s}}{1.2 \text{ m}}$$
$$f = 290 \text{ Hz}$$

Statement: The frequency of the second harmonic is 290 Hz.

7. Given: T = 25 °C; f = 340 Hz; n = 3; fixed and open ends

Required: L

Analysis: $v = 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T$; $\lambda = \frac{v}{f}$;

$L_n = \frac{2n-1}{4}\lambda$

Solution: Determine the speed of sound in the air:

$$v = 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T$$
$$= 331.4 \text{ m/s} + \left(0.606 \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(25 \cancel{°\text{C}})$$
$$v = 346.6 \text{ m/s}$$

Determine the wavelength:

$$\lambda = \frac{v}{f}$$
$$= \frac{346.6 \text{ m/s}}{340 \text{ Hz}}$$
$$\lambda = 1.019 \text{ m (two extra digits carried)}$$

Determine the length:

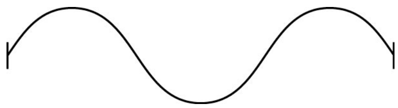
$$L_n = \frac{2n-1}{4}\lambda$$

$$L_3 = \frac{5}{4}\lambda$$
$$= \frac{5}{4}(1.019 \text{ m})$$
$$L_3 = 1.3 \text{ m}$$

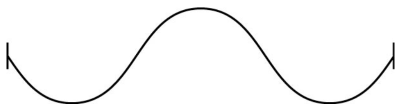
Statement: The length of the air column is 1.3 m.

8. Answers may vary. Sample answer using third harmonic:

(a)



(b)



(c)