Costs and Cost Minimization

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Outline

1. Introduction:
2. What are costs?
3. Long Run Cost Minimization
   - The constrained minimization problem
   - Comparative statics
   - Input Demands
4. Short Run Cost Minimization

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Opportunity Cost, Revisited

The relevant concept of cost is **opportunity cost**: the value of a resource in its best alternative use.

- *The only alternative we consider is the best alternative*

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Example: Investing £50M

£50M to invest. 4 alternatives:

1.) If invest *now* in CD-ROM factory, expected revenues are £100M

2.) If *wait* a year, expected revenues from CD-ROM investment are £75M

3.) If build new technology plant *now*, 50% chance that revenues are £0, 50% chance yields £150M.

4.) If *wait* a year, will know whether revenues are £0 or £150M.

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What is the opportunity cost of investing in CD-ROM plant now?

(3) yields .5(£0) + .5(£150M) = £75M
(4) yields .5(£75M) + .5(£150M)=£112.5M

Hence, (4) is the best alternative and the opportunity cost is £112.5M

- *Costs depend on the decision being made*

Example: Opportunity Cost of Steel

Purchase steel for £1M. Since then, price has gone up so that it is worth £1.2M

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Two alternatives:

1.) manufacture 2000 cars
2.) resell the steel.

What is the opportunity cost of manufacturing the cars?
£1.2M

- *Costs depend on the perspective we take*

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The (Long Run) Cost Minimization Problem

Suppose that a firm's owners wish to minimize costs...

Let the desired output be Q_0

Technology: $Q = f(L,K)$

Owner's problem: $\min_{K,L} TC = rK + wL$

Subject to $Q_0 = f(L,K)$

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1. *A graphical solution*

$TC_0 = rK + wL$ ...or...

$K = TC_0/r - (w/r)L$

...is the **isocost line**

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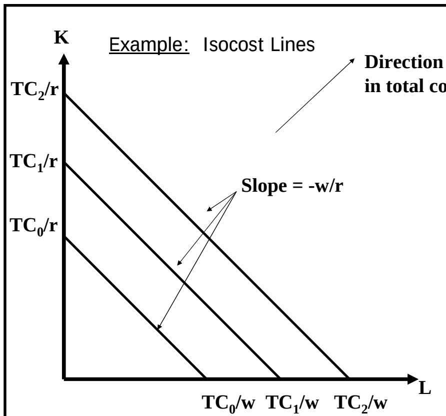

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Cost minimization subject to satisfaction of the isoquant equation: $Q_0 = f(L,K)$

Note: analogous to expenditure minimization for the consumer

Tangency condition:

$$MRTS_{L,K} = -MP_L / MP_K = -w / r$$

Constraint:

$$Q_0 = f(K,L)$$

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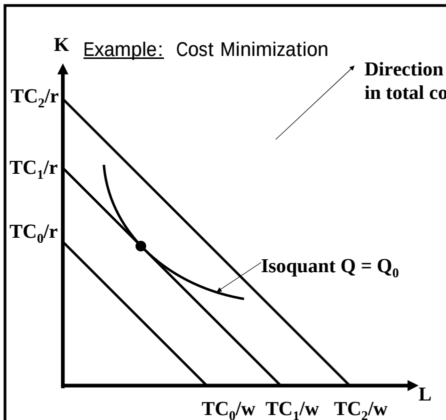

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Example: Interior Solution

$Q = 50L^{1/2}K^{1/2}$
$MP_L = 25L^{-1/2}K^{1/2}$
$MP_K = 25L^{1/2}K^{-1/2}$

w = £5
r = £20
$Q_0 = 1000$

$MP_L/MP_K = K/L$ => $K/L = 5/20$...or...$L=4K$
$1000 = 50L^{1/2}K^{1/2}$

K = 10; L = 40

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Example: Corner Solution

Q = 10L + 2K $MP_L = 10$
$MP_K = 2$

w = £5
r = £2
$Q_0 = 200$

a. $MP_L/MP_K = 10/2 > w/r = 5/2$

But… the "return" to labour larger than the "return" to capital…

$MP_L/w = 10/5 > MP_K/r = 2/2$
K = 0; L = 20

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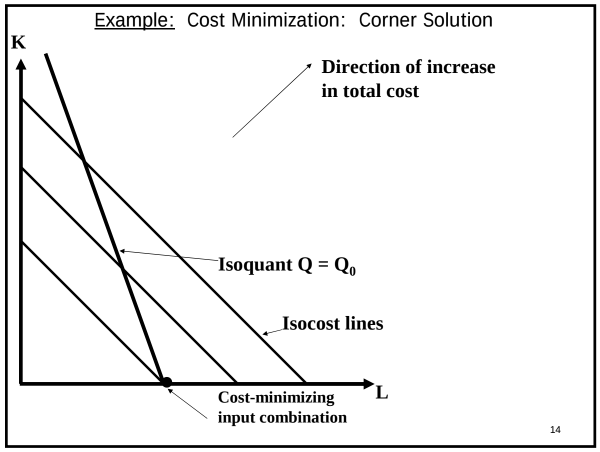

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Comparative Statics

1. A change in the relative price of inputs changes the slope of the isocost line.

   •*All else equal, an increase in w must decrease the cost minimizing quantity of labor and increase the cost minimizing quantity of capital with diminishing* $MRTS_{L,K}$.

   •*All else equal, an increase in r must decrease the cost minimizing quantity of capital and increase the cost minimizing quantity of labor.*

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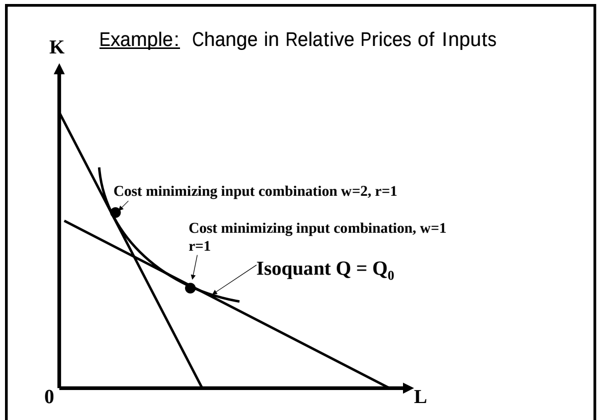

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Definitions

2. An increase in Q_0 moves the isoquant Northeast.

   •Definition: The cost minimizing input combinations, as Q_0 varies, trace out the **expansion path**

   •Definition: If the cost minimizing quantities of labor and capital rise as output rises, labor and capital are **normal inputs**.

   •Definition: If the cost minimizing quantity of an input decreases as the firm produces more output, the input is called an **inferior input**.

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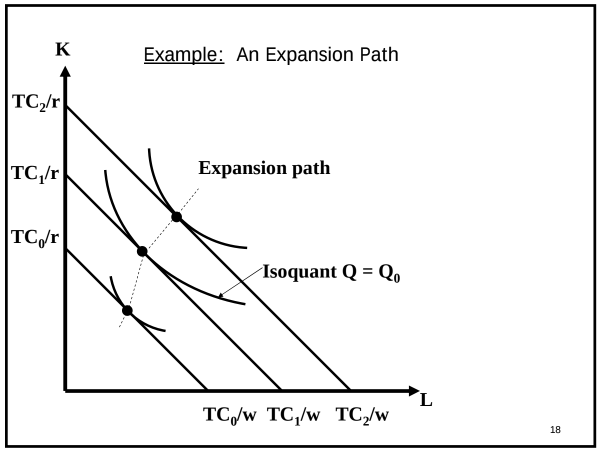

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Input Demand Functions

Definition: The cost minimizing quantities of labor and capital for various levels of Q, w and r are the **input demand functions**.

$L = L^*(Q,w,r)$
$K = K^*(Q,w,r)$

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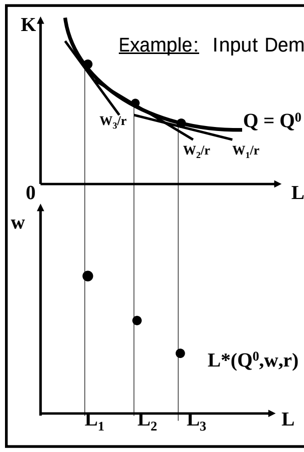
Example: Input Demand Functions



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Example: Input demand functions

$Q = 50L^{1/2}K^{1/2}$

$MP_L/MP_K = w/r \Rightarrow K/L = w/r$ ... or... $K=(w/r)L$

This is the equation for the expansion path...

$Q_0 = 50L^{1/2}[(w/r)L]^{1/2} \Rightarrow$

$L^*(Q,w,r) = (Q_0/50)(r/w)^{1/2}$
$K^*(Q,w,r) = (Q_0/50)(w/r)^{1/2}$

- *Labor and capital are both normal inputs*
- *Labor is a decreasing function of w*
- *Labor is an increasing function of r*

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Duality: "Reverse engineering" the production function from the input demands

Example: Cobb-Douglas Revisited

Start with the input demands and solve for w...

$L = (Q_0/50)(r/w)^{1/2} \Rightarrow w = [Q_0/(50L)]^2 r =$

Plug w into the demand for K...

$K = (Q_0/50)[\{Q_0/(50L)\}^2 r/r]^{1/2}$

$= Q_0^2/2500L \Rightarrow$

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Solve for Q_0 as a function of K and L...

$Q_0 = 50K^{1/2}L^{1/2}$

Why can we do this? Because the tangencies that generate the input demand trace out the isoquants...by keeping Q fixed, we keep "purchasing power" fixed...

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Short Run Cost Minimisation

Suppose that one factor (say, K) is fixed.

Definition: The firm's **short run cost minimization problem** is to choose quantities of the variable inputs so as to minimize total costs...

given that the firm wants to produce an output level Q_0...

and under the constraint that the quantities of the fixed factors do not change.

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1. Short Run Cost Minimization Problem:

$\min_{L,M} wL + mM + rK^*$

Subject to: $Q = f(L,K^*,M)$

Note: L,M are the **variable inputs** and
wL+mM is the **total variable cost**
K* is the **fixed input and**
rK* is the **total fixed cost**

Tangency condition: $MP_L/w = MP_M/m$
Constraint: $Q_0 = f(L,K^*,M)$

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2. Short Run Input Demand Functions

The demand functions are the solutions to the short run cost minimization problem:

$L^s = L(Q,K^*,w,m)$
$M^s = M(Q,K^*,w,m)$

So demand for materials and labour depends on K*

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3. Relating Short Run to Long Run Input Demands

Suppose that K* is the long run cost minimizing level of capital for output level Q.

Then when the firm produces Q, the short run demands for L and M must yield the long run cost minimizing levels of L and M

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Example: Short run and Long Run cost Minimization

$Q = K^{1/2}L^{1/4}M^{1/4}$

$MP_L = (1/4)K^{1/2}L^{-3/4}M^{1/4}$
$MP_M = (1/4)K^{1/2}L^{1/4}M^{-3/4}$

w = 16
m = 1
r = 2

K = K*

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a. *What is the solution to the firm's short run cost minimization problem?*

Tangency condition: $MP_L/MP_M = w/m \Rightarrow$

$(1/4K^{*1/2}L^{-3/4}M^{1/4})/(1/4K^{*1/2}L^{1/4}M^{-3/4}) = 16/1$

$\Rightarrow M = 16L$

Constraint: $Q_0 = K^{*1/2}L^{1/4}(M)^{1/4}$

Combining these, we can obtain the short run (conditional) demand functions for labor and materials:

$L^s(Q,K^*) = Q^2/(4K^*)$

$M^s(Q,K^*) = (4Q^2)/K^*$

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b. *What is the solution to the firm's long run cost minimization problem given that the firm wants to produce Q units of output?*

Tangency Conditions:

- $MP_L/MP_M = w/m$
  $(1/4K^{1/2}L^{-3/4}M^{1/4})/(1/4K^{1/2}L^{1/4}M^{-3/4})=16/1$

M = 16L

- $MP_L/MP_K = w/r$
$(1/4K^{1/2}L^{-3/4}M^{1/4})/(1/4K^{-1/2}L^{1/4}M^{1/4})=16/1$

K = 16L

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Constraint:

- $Q = K^{1/2}L^{1/4}M^{1/4}$

Three equations and three unknowns… Combining these, we can obtain the long run demand functions for labor, capital and materials:

$L(Q) = Q/8$
$M(Q) = 2Q$
$K(Q) = 2Q$

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c. *Suppose that K* = 20. Is it the case that:*

$L^s(10,20) = L(10)$
$M^s(10,20) = M(10)$?

$L^s(10,20) = 100/(4(20)) = 1.25$
$M^s(10,20) = 4(100)/20 = 20$

$L(10) = 10/8 = 1.25$
$M(10) = 2(10) = 20$

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d. *Suppose that K* = 16 and L* = 256. The firm wishes to produce Q = 48. What is the demand for materials?*

$48 = (16)^{1/2}(256)^{1/4}M^{1/4}$
$M = 81$

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Summary

1. Opportunity costs are the relevant notion of costs for economic analysis of cost.

2. The input demand functions show how the cost minimizing quantities of inputs vary with the quantity of the output and the input prices.

3. Duality allows us to back out the production function from the input demands.

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4. The short run cost minimization problem can be solved to obtain the short run input demands.

5. The short run input demands also yield the long run optimal quantities demanded when the fixed factors are at their long run optimal levels.

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