Math 10B Prof. Audrey Terras
HW #1 Solutions by Alex Eustis
Due Tuesday, Oct. 09

Section 5.1 #7, 10, 16, 21, 25; Section 5.2 #8, 9, 15, 20, 27, 30; Section 5.3 #4, 6, 9, 13, 16, 28, 31; Section 5.4 #7, 18, 21, 23, 25, 29, 40

5.1.7 Since $\Delta t = 0.2$, we'll need to use the following values:

t	0	0.2	0.4	0.6	0.8	1
$v(t)$	1	0.83	0.71	0.63	0.56	0.5

To find an upper estimate, use the left-hand endpoint of each interval since $v(t)$ is decreasing. We get

$$U = (0.2\text{ hr})(1 + .83 + .71 + .63 + .56)\frac{\text{m}}{\text{hr}} = \boxed{.75\text{ m.}}$$

Similarly,

$$L = (0.2\text{ hr})(.83 + .71 + .63 + .56 + .50)\frac{\text{m}}{\text{hr}} = \boxed{.65\text{ m}}.$$

The average is

$$(U + L)/2 = \boxed{.70\text{ m}}.$$

5.1.10 Since $f(t) = 5t + 8$ is increasing, the formula on p. 245 gives the difference between the upper and lower estimates.

$$U - L = \Delta t|f(a) - f(b)| = \frac{3-1}{100}|f(3) - f(1)| = \frac{1}{50}|(5*3+8) - (5*1+8)| = \boxed{1/5}.$$

5.1.16

(a) The first half hour consists of the first two intervals: $0 \le t \le 15$ and $15 \le t \le 30$. Since Roger's speed is never increasing, use the left endpoints for an upper estimate and right endpoints for a lower estimate. To make the units cancel we'll use $\Delta t = 0.25$ hr rather than $\Delta t = 15$ min.

$$U = (0.25\text{ hr})(12 + 11)\frac{\text{mi}}{\text{hr}} = \boxed{5.75\text{ mi}}$$

$$L = (0.25\text{ hr})(11 + 10)\frac{\text{mi}}{\text{hr}} = \boxed{5.25\text{ mi}}.$$

(b) This part is very similar.

$$U = (0.25\text{ hr})(12 + 11 + 10 + 10 + 8 + 7)\frac{\text{mi}}{\text{hr}} = \boxed{17\text{ mi}}$$

$$L = (0.25\text{ hr})(11 + 10 + 10 + 8 + 7 + 0)\frac{\text{mi}}{\text{hr}} = \boxed{14\text{ mi}}$$

(c) This part is asking: How small does Δt have to be in order to guarantee that $U - L \le 0.1$? To answer this question, use the formula on p. 245 which relates these two quantities:

$$U - L = \Delta t|f(b) - f(a)| \le 0.1\text{ mi}$$

$$\Delta t \le \frac{0.1\text{ mi}}{|12 - 0|\text{mi/hr}} = .00833\text{ hr} = \boxed{30\text{ sec}}.$$

(It's fine if you used equations instead of inequalities; just realize that Δt needs to be 30 seconds *or smaller*.)

5.1.21 For $0 \le t \le 3$, the region between the x-axis and the graph is triangular-shaped, with area

$$1/2 * b * h = 1/2 * (3\text{ sec}) * (6\text{ cm/sec}) = 9\text{ cm}.$$

For $3 \le t \le 4$, it is again a triangular-shaped region, with area

$$1/2 * (1\text{ sec}) * (2\text{ cm/sec}) = 1\text{ cm},$$

but it counts as *negative* because it lies below the x-axis. Therefore the net distance traveled by the particle when $0 \le t \le 4$ is $\boxed{8\text{ cm}}$.

5.1.25

(a) Looking at the graph, $\boxed{\text{car A}}$ attains the larger maximum velocity because it has the higher peak.

(b) $\boxed{\text{Car A}}$ stops first because its graph is the first to reach the x-axis.

(c) $\boxed{\text{Car B}}$ travels farther because the area under its velocity curve is greater.

5.2.8 We use the methods of section 5.1 again. The table values indicate that $\Delta t = 3$ and f appears to be decreasing, so

$$U = 3(32 + 22 + 15 + 11) = 240,$$

$$L = 3(22 + 15 + 11 + 9) = 171,$$

and our best estimate is the average:

$$(U + L)/2 = \boxed{205.5}.$$

5.2.9 Using a calculator,

$$\int_0^3 2^x \, dx \approx \boxed{10.0989}.$$

5.2.15 The area under the curve $y = \cos\sqrt{x}$ for $0 \le x \le 2$ is the same as the following integral (which you can find using a calculator):

$$\int_0^2 \cos\sqrt{x} \, dx \approx \boxed{1.106}.$$

5.2.20

(a)

$$\int_0^6 (x^2+1)\,dx = \boxed{78}.$$

This is the area bounded between the x-axis, the vertical lines $x = 0$ and $x = 6$, and the parabola $y = x^2 + 1$.

(b) A left-hand sum with $n = 3$ gives

$$L = 2(f(0) + f(2) + f(4)) = \boxed{46}.$$

Evidently this is an underestimate. In fact, we could have known beforehand that a left-hand sum would give an underestimate because the function is increasing.

(c) The corresponding right-hand sum is

$$R = 2(f(2) + f(4) + f(6)) = \boxed{118},$$

an overestimate.

Note: the point of this problem is to see these values represented on a graph. If you had trouble drawing the graphs, you can ask during section or office hours.

5.2.27 Using Figure 5.29,

(a) $\int_a^b f(x)\,dx = \boxed{13}$

(b) $\int_b^c f(x)\,dx = \boxed{-2}$
(It's below the x-axis).

(c) $\int_a^c f(x)\,dx = \boxed{11}$

(d) $\int_a^c |f(x)|\,dx = \boxed{15}$

5.2.30 *"If a left-hand sum underestimates a definite integral by a certain amount, then the corresponding right-hand sum will overestimate the integral by the same amount."*

This statement is false in general. Problem 20 is a counterexample, since the left-hand sum underestimated the integral by 32, but the right-hand sum overestimated it by 40. (Remember that one counterexample disproves a claim!) However, the statement is true when the function being integrated is a line: $f(x) = mx + b$.

5.3.4 $\int_1^3 v(t)\,dt$, *where* $v(t)$ *is the velocity in meters/sec and* t *is the time in seconds.*

Recall that $v(t) = x'(t)$, where $x(t)$ is the position at time t. According to the Fundamental Theorem of Calculus, this integral gives $x(3) - x(1)$, which represents the $\boxed{\text{change in position (in meters) between } t = 1 \text{ and } t = 3 \text{ seconds}}$.

5.3.6 $\int_{2000}^{2004} f(t)\,dt$, *where* $f(t)$ *is the rate at which the world's population is growing in year* t*, in billions of people per year:*

If $P(t)$ is the world's population in year t, this integral gives $P(2004) - P(2000)$, which represents the $\boxed{\text{change in the world's population (in billions) between 2000 and 2004}}$.

5.3.9 The formula for average value is given by the boxed equation on p. 260:

$$\frac{1}{b-a}\int_a^b g(t)\,dt = \frac{1}{2}\int_0^2 (1+t)\,dt = \boxed{2}.$$

5.3.13 The first hour is the interval $0 \leq t \leq 60$, since t is in minutes. The integral representing the oil leaked during this time is $\boxed{\int_0^{60} f(t)\,dt}$.

5.3.16

(a) Since t is measured from the start of 1990, the desired integral is $\boxed{\int_0^5 f(t)\,dt}$.

(b) A left-hand sum with five subdivisions gives

$$\int_0^5 f(t)\,dt \approx 1 * 32(e^0 + e^{0.05} + e^{0.10} + e^{0.15} + e^{0.20}) \approx \boxed{\text{177.3 billion barrels}}.$$

(c) The first term in the sum, $1 * 32 * e^0 = 32$ billion barrels, represents the oil consumed during 1990, *assuming that the rate of oil consumption did not increase during the year*. The other terms represent the same thing for the years 1991, 1992, 1993, 1994. However, the rate of oil consumption is continuously increasing, which is why the sum of these terms will give an underestimate.

5.3.28

(a) There are many ways to see why this average must be bigger than $1/2$. One way is to draw a line at $y = 1/2$ and observe that the area lying above the line than is greater than the area missing from below it. Therefore $y = 1/2$ is not the average value, and the line must be moved upwards to compensate.

On the other hand, the average value must be less than 1 because $\sin x \leq 1$ always.

(b)

$$\text{avg. value} = \frac{1}{\pi}\int_0^{\pi} \sin x \, dx \approx \boxed{.6366}$$

using a calculator. The exact value is $2/\pi$.

5.3.31 The integral that computes the change during 1993 is

$$\Delta(\text{annual income}) = \int_0^{12} 40(1.002)^t \, dt \approx \boxed{485.8 \text{ dollars}}.$$

5.4.7 On $0 \leq x \leq 1$, the graph of $x^{1/3}$ lies above $x^{1/2}$. The area between these curves is

$$\int_0^1 (x^{1/3} - x^{1/2}) \, dx \approx \boxed{.0833}$$

.

5.4.18 Using Theorem 5.3 and the information given in the problem,

$$\begin{aligned} & \int_a^b \left(c_1 g(x) + (c_2 f(x))^2\right) \, dx \\ = & \int_a^b \left(c_1 g(x) + c_2^2 f(x)^2\right) \, dx \\ = & \; c_1 \int_a^b g(x) \, dx + c_2^2 \int_a^b f(x)^2 \, dx \\ = & \; \boxed{2c_1 + 12c_2^2}. \end{aligned}$$

5.4.21 Since $f(x)$ is odd, $\int_{-2}^{2} f(x) \, dx = 0$. Using Theorem 5.2 (part 2),

$$\int_{-2}^{2} f(x) \, dx + \int_2^5 f(x) \, dx = \int_{-2}^{5} f(x) \, dx$$

$$0 + \int_2^5 f(x) \, dx = 8$$

$$\int_2^5 f(x) \, dx = \boxed{8}.$$

5.4.23 Use the information given and "solve for $\int_2^5 f(x)\,dx$" as though it were a variable.

$$\begin{aligned}
\int_2^5 (3f(x)+4)\,dx &= 18 \\
3\int_2^5 f(x)\,dx + \int_2^5 4\,dx &= 18 \\
3\int_2^5 f(x)\,dx + 12 &= 18 \\
\int_2^5 f(x)\,dx &= (18-12)/3 = \boxed{2}.
\end{aligned}$$

5.4.25

(a) Using a graph (or math knowledge), we see that the maximum value of $f(x) = e^{-x^2/2}$ is 1. It follows that

$$\int_0^1 e^{-x^2/2} < \int_0^1 1\,dx = 1,$$

so the integral is $\boxed{\text{less than}}$ 1.

(b) Using a calculator, $\int_0^1 e^{-x^2/2} \approx \boxed{.856}$.

5.4.29

(a) $f(x) = e^{x^2}$ is positive everywhere. This means that whenever $a < b$, it must be true that $\int_a^b e^{x^2}\,dx > 0$.

(b) Again, $f(x) = \left|\frac{\cos(x+2)}{1+\tan^2 x}\right|$ is positive almost everywhere, except for when $\cos(x+2) = 0$. This occurs only for specific, isolated values of x, so the same reasoning applies.

5.4.40 For convenience we'll write $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

(a) We need $\int_1^3 f(x)$, and the table gives us $\int_0^1 f(x)$ and $\int_0^3 f(x)$. This is where Theorem 5.2 comes in handy:

$$\begin{aligned} \int_0^1 f(x)\,dx + \int_1^3 f(x)\,dx &= \int_0^3 f(x)\,dx \\ .3413 + \int_1^3 f(x)\,dx &= .4987 \\ \int_1^3 f(x)\,dx &= \boxed{.1574} \end{aligned}$$

(b) Since the graph of $f(x)$ is symmetric about the y-axis (that is, f is an *even* function), we know that $\int_{-2}^0 f(x)\,dx = \int_0^2 f(x)\,dx = .4772$. Now use Theorem 5.2 again:

$$\int_{-2}^3 f(x)\,dx = \int_{-2}^0 f(x)\,dx + \int_0^3 f(x)\,dx = .4772 + .4987 = \boxed{.9759}.$$