Ballistic Pendulum

Purpose:

To experimentally investigate the laws of conservation of energy and the conservation of linear momentum.

Introduction:

In this experiment we will determine the initial speed of a projectile using two different methods.

Method I: With the first method, the speed of a projectile is found by application of the laws of conservation of linear momentum and of mechanical energy. This application makes use of a ballistic pendulum. An approximately spherical ball projected from a spring gun is caught and retained in the bob of the pendulum, an inelastic collision. Referring to figure 1, if we let v be the velocity of the ball with mass, m, immediately before impact with the pendulum bob and V be the velocity of the ball and pendulum with mass, $m + M$, immediately after the collision we can determine an equation for the momentum of the collision. The initial momentum at that instant before impact is mv and from the law of conservation of linear momentum this quantity must equal the final momentum of the entire system immediately after impact. Hence $mv = (M + m)V$ and solving for v gives

$$v = (\frac{(M + m)}{m})V \tag{1}$$

After impact the ball is raised from the initial equilibrium position of the pendulum to a height above that position. The kinetic energy of the system, ball and pendulum, at the instant following impact is $\frac{1}{2}(M + m)V^2$. When the center of gravity reaches its highest position h_2, the potential energy of the system has been increased by an amount $(M+m)g\Delta h$ and the kinetic energy has become zero. If the small frictional loss in energy is neglected, the loss in kinetic energy equals the gain in potential energy so $\frac{1}{2}(M + m)V^2 = (M + m)g\Delta h$. Solving for V gives

$$V = \sqrt{2g\Delta h} \tag{2}$$

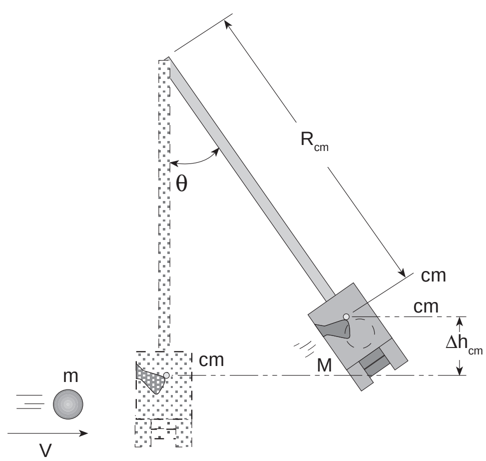

Figure 1: Ballistic Pendulum.

According to figure 1, if we measure the distance, R, from the pivot point to the center of mass of the pendulum and measure the angle, θ, which the pendulum sweeps out from equilibrium, h_1, to some height, h_2, then we can determine the difference between these two heights, Δh, by

$$R(1 - cos\theta) = \Delta h \tag{3}$$

Using equations one through three we can determine the initial velocity of the projectile from measured quantities. **You should write out this equation for v in terms of R, θ, M and m.**

Method II: The second method in this lab involves an independent measurement of the horizontal distance the projectile travels while falling from a certain measured vertical height. We use the fact that Newton's second law (F=ma) is a vector equation. This fact implies the validity of the components equations, $F_x = ma_x$ and $F_y = ma_y$, where x and y are the horizontal and vertical directions respectively. If we horizontally fire a ball so the ball is permitted to follow a trajectory, as indicated in figure 2, and if air friction is negligible, there is no x component of acceleration, so the x component of the velocity remains constant. On the other hand, the y component of velocity of the ball changes precisely as does the velocity of an object that has only vertical motion. The y component of acceleration, a_y, then equals g, the acceleration due to gravity (9.81 $\frac{m}{s^2}$).

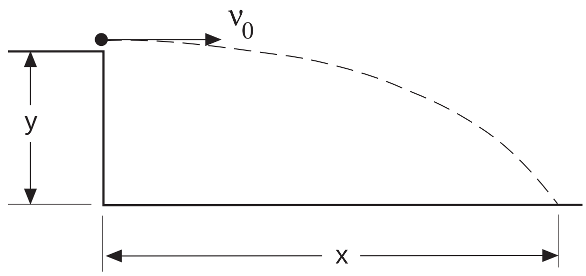

Figure 2: Projectile.

While following the trajectory, as described above, the ball has a vertical displacement y and a horizontal displacement x as indicated in figure 2. Since the initial vertical velocity of the ball is zero, its time of flight is determined by the equation $y = v_o t + \frac{1}{2}\, a_y t^2$, in which $v_o = 0$, $a_y = g$, then,

$$t = \sqrt{\frac{2y}{g}} \tag{4}$$

By determining t, the horizontal component of the velocity of the ball is found using $x = v_x t$ so,

$$v_x = \frac{x}{t} \tag{5}$$

Since v_x is constant this velocity is the same as the initial velocity, v, with which the ball is projected from the launcher in method I.

Laboratory Procedure:

Part I - Taking Indirect Measurements with a Ballistic Pendulum

1. Make a table in your notebook of values to be measured.
2. Take note of how many brass disks are attached to your pendulum, this determines your launcher setting for your initial velocity. **DO NOT REMOVE THE BRASS DISKS.**

$$0\ disks = short\ range$$
$$1\ disk = medium\ range$$
$$2\ disks = long\ range$$

3. Place the ball in your launcher at the appropriate setting, then fix your angle indicator to zero (this is your equilibrium point).
4. Fire the projectile and record the angle, θ, the pendulum sweeps out from the equilibrium position. in your laboratory notebook.

5. Repeat steps 3 and 4, until you have a total of 10 measurements for θ.
6. Average your values of θ and record the average in your notebook.
7. Determine $\delta\theta$ from the precision of the scale attached to the apparatus.
8. Calculate the fractional uncertainty, $(\delta\theta/\theta)$ for this measurement.
9. Determine the mass of the ball (m) and the mass of the pendulum (M) using the electronic balances.
10. Determine δm and δM from the precision of the balance.
11. Calculate the fractional uncertainty $(\delta m/m$ and $\delta M/M)$ for these measurements.
12. Determine and record the distance, R, the distance from center-of-mass of the pendulum to the pivot point. We have provided string hanging on a stand so you can actually balance the pendulum with the ball inside to determine the point at the center-of-mass of the pendulum.
13. Determine and record δR based on the precision of the meter stick.
14. Calculate the fractional uncertainty, $(\delta R/R)$ for this measurement.
15. Calculate and record v using the equation you derived from equations 1 - 3 in the introduction.

Part II - Taking Indirect Measurements with a Projectile

1. Latch the pendulum at 90 degrees so it is out of the way, then load the projectile launcher to the same setting you used in part I.
2. Horizontally fire the ball from the launcher until it strikes a carbon paper placed over a blank piece of paper on the floor where the point of its impact with the blank piece of paper can be determined.
3. Repeat steps 1 and 2, until you have a total of 10 measurements of x, the horizontal distance traveled to each impact point. *Be careful to make sure your paper stays in the same place on the floor for each shot.*
4. Measure and record the horizontal distance traveled to each impact point, x.
5. To determine δx use the value of the longest distance between impact points on your paper. This should be rather small ($<$ 4cm) if you are consistent with your procedure.
6. Calculate the fractional uncertainty, $(\delta x/x)$ for this measurement.
7. Measure and record the vertical distance traveled y.
8. Determine and record δy based on the precision of the meter stick.
9. Calculate the fractional uncertainty, $(\delta y/y)$ for this measurement.
10. Calculate and record the initial velocity, v_x, from equations 4 and 5 derived in the introduction.

Part III - Determining Uncertainties in Your Final Values

In the results section of your notebook, state the results of both parts of your experiment in the form $v\pm\delta v$. Note, δv in part I should be equal to the largest fractional uncertainty from your values of mass (m or M) or the angle, or the distance, R from the pivot point to the center of mass of the pendulum fractional uncertainties multiplied by your value of v from Part I. For Part II, δv should be equal to the largest fractional uncertainty from your values of horizontal distance, x, or vertical distance (y) fractional uncertainties multiplied by your value of v from Part II. Example for Part II;

$$\delta v = v * \max\left(\frac{\delta x}{x}, \frac{\delta y}{y}\right)$$

You should also address the following question:

1. Do your results for v in the two parts agree within their uncertainties? Be sure to clearly state the quantitative values you are comparing. If there are any large discrepancies, **quantitatively** comment on their possible origin.