

Section A.1: Three Ways of Using Percentages**Using percentages**

We can use percentages in three different ways:

- *To express a fraction* of something. For example, "A total of 10,000 newspaper employees, 2.6% of the newspaper work force, lost their jobs" uses percentage to express a fraction of total newspaper work force.
- *To describe a change* in something. For example, "Cisco stock rose 5.7% last week, to \$18" uses percentage to describe a change in stock price.
- *To compare* two objects. For example, "High definition television sets have 125% more resolution than conventional TV sets, but cost 400% more" uses percentage to compare the resolutions and the costs of televisions.

Using Percentages as Fractions**Ex.1**

If 10% of eighth-graders smoke and there are 50,000 eighth-graders, how many eighth-graders smoke?

Ex.2

A newspaper reports that 44% of 12,315 people surveyed said that the president is doing a good job. How many said that the president is doing a good job?

Using Percentages to Describe Change**Absolute change and relative change**

We can express the change of something in two ways:

- The *absolute change* describes the actual increase or decrease from a reference value to a new value:

$$\text{absolute change} = \text{new value} - \text{reference value}.$$

- The *relative change* is a fraction that describes the size of the absolute change in comparison to the reference value:

$$\text{relative change} = \frac{\text{absolute change}}{\text{reference value}} = \frac{\text{new value} - \text{reference value}}{\text{reference value}}.$$

The relative change can be converted from a fraction to a percentage by multiplying by 100%. The relative change formula leads to the following important rules:

- When a quantity doubles in value, its relative change is $1 = \frac{100}{100} = 100\%$.
- When a quantity triples in value, its relative change is $2 = 200\%$.
- When a quantity quadruples in value, its relative change is $3 = 300\%$. And so on.

Note that the absolute and relative changes are positive if the new value is greater than the reference value, and the absolute and relative changes are negative if the new value is less than the reference value.

Ex.3

Suppose the population of a town was 10,000 in 1970 and 15,000 in 2000. Find the absolute change and the relative change.

Ex.4 Stock Price Rise.

During a 6-month period, Nokia's stock doubles in price from \$10 to \$20. What were the absolute and the relative changes in the stock price?

Ex.5 World Population Growth.

World population was 2.6 billion in 1950 and 6 billion in 2000. Describe the absolute and relative change in world population from 1950 to 2000.

Ex.6 Depreciating a Computer.

You bought a computer three years ago for \$1000. Today, it is worth only \$300. Describe the absolute and relative change in the computer's value.

Using Percentages for ComparisonsAbsolute difference and relative difference

Percentages are commonly used to compare two numbers. There are two different ways to compare two objects:

- The *absolute difference* is the actual difference between the compared value and the reference value:

$$\text{absolute difference} = \text{compared value} - \text{reference value}.$$

- The *relative difference* describes the size of the absolute difference as a fraction of the reference value:

$$\text{relative difference} = \frac{\text{absolute difference}}{\text{reference value}} = \frac{\text{compared value} - \text{reference value}}{\text{reference value}}.$$

The relative difference formula gives a fraction. We can convert the answer to a *percent difference* by multiplying it by 100%.

The absolute and relative differences are positive if the compared value is greater than the reference value, and the absolute and relative changes are negative if the compared value is less than the reference value.

Ex.7

Suppose we want to compare the price of a \$50,000 Mercedes to the price of a \$40,000 Lexus. Describe the absolute and relative difference.

Ex.8 Pay Comparison.

Average pay for full-time wage earners varies from state to state. Recent data showed that Connecticut ranked first in average pay, at \$48,328 per person. Montana had the lowest average pay, at \$26,907 per person. Compare average pay in Montana to that in Connecticut in both absolute and relative terms.

Section A.2: "Of" versus "More Than""Of" versus "More Than"

There are two different ways to state a change with percentages: "of" and "more than".

In the case of "more than" we state the relative change. In the case we are using "of", we consider the ratio of the new value and the old value.

- If the compared value is $P\%$ *more than* the reference value, it is $(100 + P)\%$ *of* the reference value.
- If the compared value is $P\%$ *less than* the reference value, it is $(100 - P)\%$ *of* the reference value.

Ex.9

Consider a population that triples in size from 200 to 600.

- Using *more than*, the new population is 200% more than the original population (using the relative change formula):

- Using *of*, the new population is 300% of the original population, which means that it is three times the size of the original population:

Notice that $300\% = (100 + 200)\%$.

|| **Ex.10 Salary Difference.**

|| Carol earns 50% more than William. How many times larger is her income than his?

|| **Ex.11 Sale!**

|| A store is having 25% *off* sale. How does an item's sale price compare to its original price?

Section A.3: Percentages of Percentages**Percentage Points versus %**

When you see a change or difference expressed in *percentage points*, you can assume it is an *absolute* change or difference. If it is expressed with the % sign or the word *percent*, it should be a *relative* change or difference.

Ex.12

Suppose your bank increases the interest rate on your savings account from 2% to 5%. What is the absolute change? What is the relative change?

Ex.13 Newspaper Readership Declines.

According to a time Magazine story,

The percentage of adults reading daily newspaper fell from 78% to 64%.

Describe the change in newspaper readership.

Ex.14 Care in Wording.

Assume that 40% of the registered voters in Carson City are Republicans. Read the following questions carefully, and give the most appropriate answers.

- (1) The percentage of voters registered as Republicans is 25% higher in Freetown than in Carson City. What percentage of the registered voters in Freetown are Republicans?
- (2) The percentage of voters registered as Republicans is 25 percentage points higher in Freetown than in Carson City. What percentage of the registered voters in Freetown are Republicans?

Section A.4: Solving Percentage Problems**Solving Percentage Problems**

If the compared value is $P\%$ *more than* the reference value, then

$$\text{compared value} = (100 + P)\% \times \text{reference value}$$

and

$$\text{reference value} = \frac{\text{compared value}}{(100 + P)\%}.$$

If the compared value is $P\%$ *less than* the reference value, then

$$\text{compared value} = (100 - P)\% \times \text{reference value}$$

and

$$\text{reference value} = \frac{\text{compared value}}{(100 - P)\%}.$$

Ex.15

Retail prices are 25% more than whole sale prices. If the whole sale price is \$10, how much is the retail price?

Ex.16 Tax Calculations.

- (1) You purchase a shirt with a labeled (pre-tax) price of \$17. The local sale tax rate is 5%. What is your final cost (including tax)?
- (2) Your receipt shows that you paid \$19.26 for a DVD, tax included. The local sales tax rate is 7%. What was the labeled (pre-tax) price of the DVD?

Ex.17 Up 44%, to 10.4%.

Consider the statement:

the rate of smoking for eighth-graders is up 44 per cent, to 10.4 per cent.

What was the previous smoking rate for eighth-graders?

Section A.5: Abuses of Percentages**Solving Percentage Problems**

There are few common abuses of percentages: shifting reference values, less than nothing, average of percentages.

Beware of Shifting Reference Values**Ex.18**

Because of losses by your employer, you agree to accept a temporary 10% pay cut. Your employer promises to give you 10% pay raise after six months. Will the pay raise restore your original salary?

Ex.19 Shifting Investment Value.

A stock broker offers the following defense to angry investors:

I admit that the value of your investments fell 60% during my first year on the job. This year, however, their value has increased by 75%, so you are now 15% ahead!

Evaluate the stock broker's defense considering an investment of \$1,000.

Ex.20 Tax Cuts.

A politician promises: *If elected, I will cut your taxes by 20% for each of the first three years of my term, for a total cut of 60%.*

Evaluate the promise.

Less Than Nothing**Ex.21 Impossible Sale.**

A store advertises that it will take 150% *off* the price of all merchandise. What should happen when you go to the counter to buy a \$500 item?

Don't Average Percentages**Ex.22**

Suppose you got 70% of the questions correct on a midterm exam and 90% of the questions correct on the final exam. Can you conclude that you answered 80% of all the questions correctly?