AP Physics 1- Momentum, Impulse, and Collisions Practice Problems **ANSWERS**

FACT: The product of mass and velocity is a vector quantity known as momentum ($\vec{p}$). The equation for linear momentum is $\vec{p} = m\vec{v}$ and has the units kg$\cdot\frac{m}{s}$, which can also be written as a newton-second (N·s). Now take Newton's second law $\vec{F}_{net}$ = m$\vec{a}$ and substitute $\frac{vf-vo}{\Delta t}$ for the acceleration ($\vec{a}$). You will now have the equation $\vec{F}$ = m ($\frac{vf-v0}{\Delta t}$). Next, distribute the mass (m) and you will get $\vec{F}$ = ($\frac{mvf-mv0}{\Delta t}$). Finally, you should realize that since $\vec{p} = m\vec{v}$, that $\vec{F}$ = ($\frac{\Delta\vec{p}}{\Delta t}$).

Q1. A golfer strikes a golf ball of mass 0.05 kg, and the time of impact between the golf club and the ball is 1.0 ms. If the ball acquires a velocity of magnitude 70 m/s, calculate the average force exerted on the ball. (3500 N)

Q2. Two trains, Big Red and Little Blue, have the same velocity. Big Red, however, has twice the mass of Little Blue. Compare their momenta. (Because Big Red has twice the mass, Big Red must have twice the momentum of Little Blue)

FACT: The product of the force and the time during which it acts is known as *impulse*. This is a vector quantity denoted with a (J). Therefore, Impulse = J = FΔt and the units are still kg$\cdot\frac{m}{s}$, which can also be written as a newton-second (N·s).

FACT: Impulse is equal to change in linear momentum. In terms of impulse we can rearrange the equation $\vec{F}$ = ($\frac{\Delta\vec{p}}{\Delta t}$) to solve for Δp, we find that J = Δp, because FΔt = Δp. These relationships are referred to as the *impulse-momentum theorem; J* = Δp = FΔt.

Q3. A football team's kicker punts the ball (mass = 0.4 kg) and gives it a launch speed of 30 m/s. Find the impulse delivered to the football by the kicker's foot and the average force exerted by the kicker on the ball, given that the impact time was 8 ms. (12 N·s; 1,500 N)

Q4. An 80 kg stuntman jumps out of a window that is 45 m above the ground. (a) How fast is he falling when he reaches the ground? (b) He lands on a large, air-filled target coming to rest in 1.5 s. What average force does he feel while coming to rest? (c) What if he had instead landed on the hard ground with an impact time of 10 ms? (30 m/s, 1600N, 240000 N)

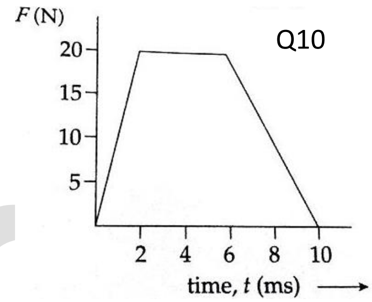
Q5. Calculate the magnitude of the impulse applied to a 0.75 kg cart to change its velocity from 0.50 m/s east to 2.00 m/s east. (1.1 N·s)

Q6. A 6.0 kg block is sliding to the east across a horizontal, frictionless surface with a momentum of 30 kg.$\frac{m}{s}$. It strikes an obstacle. The obstacle exerts an impulse of 10 N.s to the west on the block. Find the speed of the block after the collision. (3.3 m/s)

Q7. A 1000 kg car is traveling east at 15 m/s and is hit from behind and receives a forward impulse of 6000 N.s. Determine the magnitude of the car's change in momentum due to this impulse (6000 N·s)

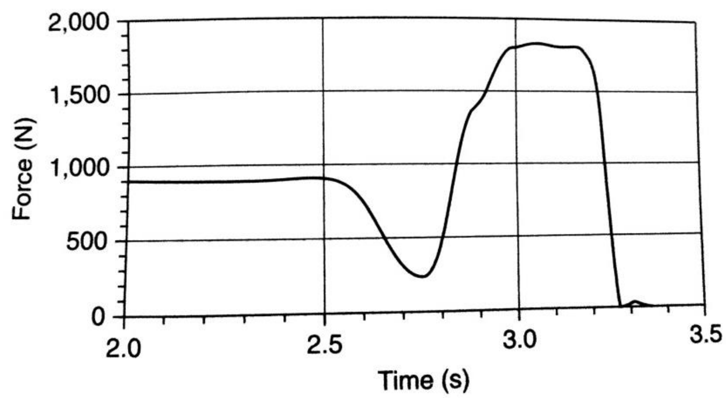
Q8. Two speedboats are moving at constant speeds on a straight stretch of a racecourse. At the instant shown, Speedboat A has more momentum than Speedboat B. Is the net force on Speedboat A greater than, less than, or equal to the net force on Speedboat B? Explain. (They are equal since the net force on both boats is zero. Since the speedboats are moving with constant speed and direction, there is no acceleration, so the net force is zero)

Q9. The two speedboats in question 8 are moving at constant speeds on a straight stretch of a racecourse. Speedboat A has more mass than Speedboat B. At the instant shown, the two speedboats have the same kinetic energy. Is the momentum of Speedboat A greater than, less than, or equal to the momentum of Speedboat B? (Speedboat A will have a larger momentum. The kinetic energy is the momentum squared divided by the mass (KE = (momentum)^2/mass) so $p_A{}^2 = (m_A/m_B)\, p_B{}^2$. The larger mass will have the larger momentum if the kinetic energies are the same.)



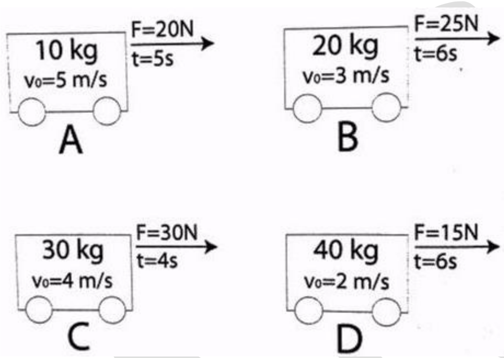
FACT: The area under a Force as a function of time graph is equal to the impulse (J). Remember you are not required to use calculus in this course, so approximations will satisfy the College Board requirements.

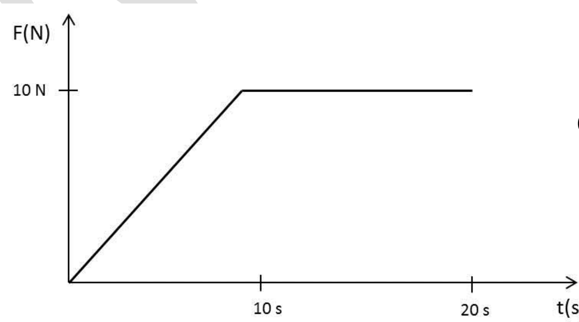
Q10. A small block of mass 0.07 kg, which is initially at rest is struck by an impulsive force (F) of duration 10 ms. The strength of the force varies according to the following graph. What is the resulting speed of the graph? (2 m/s)



Q11. A teacher whose weight is 900 N jumps vertically from rest while standing on a platform scale. The scale reading as a function of time is shown to the right. (a) Find the impulse given to the teacher. (b) Find the speed at which the teacher leaves the scale. (220 N·s; 2.4 m/s)

Q12. The following carts are moving to the right across a frictionless surface with the specified initial velocity. A force is applied to each cart for a set amount of time as shown in the diagram below. Rank the four carts from least to greatest in terms of: 1. Initial momentum, 2. Impulse applied, 3. Final momentum, 4. Final velocity (1.ABDC, 2.DACB, 3.ADBC, 4.DCBA)





Q13

Q13. A 1-kg box accelerates from rest in a straight line across a frictionless surface for 20 s as depicted in the force vs. time graph to the upper right. (a) Find the time taken for the object to reach a speed of 5 m/s. (b) Determine the box's speed after 15 s. (c) What is the minimum amount of time a force of magnitude 20 N could be applied to return the box to rest after its 20 s acceleration? (3.16 s, 100 m/s, 7.5 s)

FACT: The Law of Conservation of Momentum states that in a system in which the only forces acting are between objects in that system, momentum is conserved. This can be expressed as: *total* $p_{initial}$ = *total* p_{final}. This is a direct outcome of Newton's 3rd Law. Therefore, an external force is required to change the motion of the object's center of mass. This also means that momentum is conserved in *all* collisions and most explosions.

Q14. A 2000-kg car traveling at 20 m/s collides with a 1000-kg car at rest at a stop sign. If the 2000-kg car has a velocity of 6.67 m/s after the collision, find the velocity of the 1000-kg after the collision (26.7 m/s)

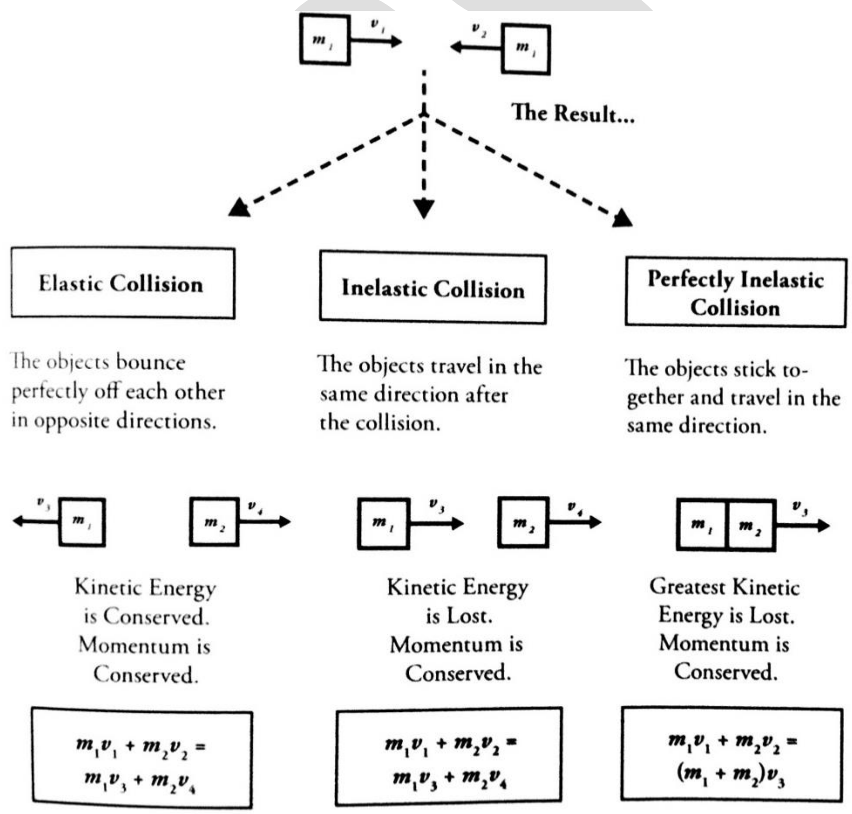
Q15. An astronaut is floating in space near her shuttle when she realizes that the cord that is supposed to attach her to the ship has become disconnected. Her total mass is 91 kg. She reaches into her pocket, finds a 1-kg metal tool, and throws it out into space with a velocity of 9 m/s directly away from the ship. If the ship is 10 m away, how long will it take her to reach it? (v = +0.1m/s; t = 100 s)

Q16. On a snow covered road, a car with a mass of 1100 kg collides head-on with a van having a mass of 2500 kg traveling 8 m/s. As a result of the collision, the vehicles lock together and immediately come to rest. Calculate the speed of the car immediately before the collision (μ = 0). (18.2 m/s)

Q17. A 70-kg hockey player skating east on an ice rink is hit by a 0.1-kg puck moving west. The puck exerts a 50-N westward force on the player. Determine the force that the player exerts on the puck during the collision. (50-N east)

Q18. A 4-kg rifle fires a 20-g bullet with a velocity of 300 m/s. Find the recoil velocity of the rifle. (-1.5 m/s)

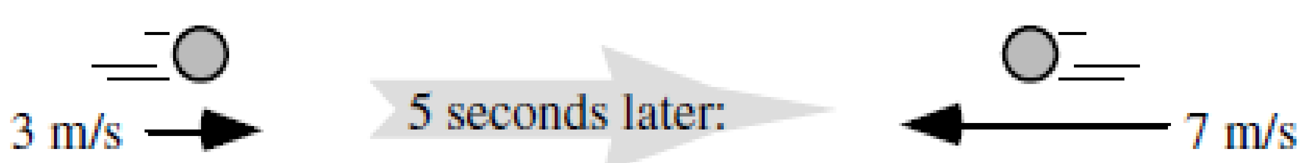
FACT: There are three types of collisions. The main distinction is that kinetic energy is **not** conserved after inelastic collisions, as some of the energy is converted into heat and sound energy. Refer to the diagram to the right:



Q19. A wooden block of mass m_1 sits on a floor attached to a spring in its equilibrium position. A bullet of mass m_2 is fired with a velocity of *v* into the wooden block, where it remains. Derive an equation for the maximum displacement of the spring if the floor is frictionless and the spring has a spring constant of k. ($x = m_2 v\sqrt{\frac{1}{k(m1+m2)}}$)

Q20. An open tub rolls across a frictionless surface. As it rolls, rain falls vertically into the tub. What will happen to the tub's momentum and velocity? Answer using both qualitative and quantitative reasoning ($m_1v_1 = m_2v_2$; call m_1=5, m_2=10, v_1=2; solving for v_2=1; the momentum of the system must be conserved, as there are no external forces being applied; therefore, the velocity of the tub must decrease. The data support this conclusion, as the over momentum was constant (10 N.s) and the velocity decreased as a result (2 m/s to 1 m/s)

Refer to the figure below for the follow three questions (**Q21-Q23**):



Q21. A 2-kg object accelerates as a net external force is applied to it. During the 5-second interval that the force is applied, the object's velocity changes from 3 m/s east to 7 m/s west. A student states: *"The change in momentum of this object during these 5 seconds was 8 kg·m/s so the impulse applied to this object during these 5 seconds was 8/5 kg·m/s."* What, if anything, is wrong with this statement? If something is wrong, identify and explain how to correct it. If this statement is correct, explain why.

(This statement is incorrect because (1) momentum is a vector so a direction should be specified for the change in momentum, (2) impulse = change in momentum so the time does not enter into the problem, and (3) the actual change in momentum (taking east as positive) is (2kg)(-7m/s) – (2kg)(+3m/s) = - 20 kg•m/s which means 20 kg•m/s in the west (negative-x) direction.)

Q22. A 2-kg object accelerates as a net external force is applied to it. During the 5-second interval that the force is applied, the object's velocity changes from 3 m/s east to 7 m/s west. A student says: *"The change in velocity for this 2 kg object was 4 m/s, so the change in momentum, and also the impulse, was 8 kg·m/s."* What, if anything, is wrong with this statement? If something is wrong, identify and explain how to correct it. If this statement is correct, explain why.

(This statement is incorrect because momentum is a vector so the change in momentum is equal to 2 kg (−7 m/s − (+3 m/s) = −20 kg•m/s, or 20 kg•m/s west taking east as the positive direction, for the change in momentum.)

Q23. A 2-kg object accelerates as a net external force is applied to it. During the 5-second interval that the force is applied, the object's velocity changes from 3 m/s east to 7 m/s west. Several students discussing the impulse on this object state the following:

Lorenzo: "*The impulse is equal to the change in momentum, which is (2 kg)(3 m/s + 7 m/s) = 20 kg·m/s.*"

Romano: "*But the change in velocity is 4 m/s. We multiply by the mass to get the change in momentum, and also the impulse, which is 8 kg·m/s*."

Amedeo: "*The change in momentum of this object during these 5 seconds was 8 kg·m/s so the impulse applied to this object during these 5 seconds was 8/5 kg·m/s*."

Carlo: "*The impulse is the force F times the time t and since we don't know the force, we can't find the impulse for this situation.*"

Which, if any, of these three students do you agree with?

(None of these students are entirely correct, but Lorenzo is close. The change in velocity is the final velocity minus the initial velocity, or 10 m/s west. So the change in momentum is 20 kg•m/s west. Lorenzo has the correct magnitude but has forgotten to include the direction of this vector quantity.)

Q24. Cart A of mass 0.5 kg, moves to the right at a speed of 60 cm/s. Cart B, of mass 1.0 kg, is at rest. The carts collide and stick together. (a) What is the final velocity of the two-cart system? (20 cm/s)

Q25. Two identical balls are dropped from the same height above the ground, such that they are traveling 50 cm/s just before they hit the ground. Ball A rebounds with speed 50 cm/s; Ball B rebounds with a speed of 10 cm/s. Each is in contact with the ground for the same amount of time. (a)Use quantitative and qualitative reasoning to explain if the momentum of either ball A, or ball B, is conserved. (b) If momentum was not conserved explain which ball had the greater change in momentum, again using quantitative and qualitative reasoning (c) Which ball exerts a larger force on the ground during its collision?

(**a**- momentum is not conserved as both balls completely change direction after their collision with Earth; **b**- ball A had a greater change call the mass 2-kg for each ball and plug in numbers including plus and minus signs; **c**- ball A because the $\Delta p = J = F\Delta t$ and both balls had the same time for impact, so the bigger force is exerted by the ball with the greater momentum change)

Q26. Two carts of equal mass move toward each other with identical speeds of 30 cm/s. After colliding, the carts bounce off each other, each regaining 30 cm/s, but now moving in opposite directions. (a) Is momentum conserved in the collision? (b) Is kinetic energy conserved in the collision? Support each answer using quantitative and qualitative reasoning.

(**a-** Yes, because- 1kg x +0.3 m/s + 1 kg x -0.3 m/s = 1kg x -0.3 m/s + 1 kg x +0.3 m/s; **b**- Yes, because-½ (1kg) $(0.3\text{ m/s})^2$ +½ (1kg)$(0.3\text{ m/s})^2$ = 0.090 J both before and after the collision. Remember energy is scalar)

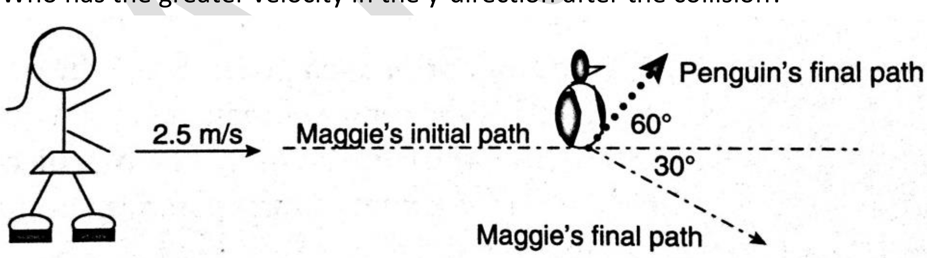
Q27. Two balls roll toward each other. The red ball has a mass of 0.5 kg and a speed of 4 m/s just before impact. The green ball has a mass of 0.2 kg and a speed of 2 m/s. After the head-on collision, the red ball continues forward with a speed of 2 m/s. Find the speed of the green ball after the collision. Was the collision elastic? (v = +3.0 m/s; Ke_i = 4.4 J and Ke_f = 1.9 J, so the collision was inelastic since kinetic energy was transformed to heat, etc.)

Q28. Two balls roll toward each other. The red ball has a mass of 0.5 kg and a speed of 4 m/s just before impact. The green ball has a mass of 0.3 kg and a speed of 2 m/s. If the collision is completely inelastic, determine the velocity of the composite object after the collision. (v = +1.8 m/s)

FACT: When objects move in both the x- and the y-direction (2-D collision), analyze the collision with the momentum conservation *separately* in each direction.

Q29. A 500 kg car travels 20 m/s due north. It hits a 500 kg car traveling due west at 30 m/s. The cars lock bumpers and stick together. What is the velocity the instant after impact? (p =18,030 N·s at 34° north of west; v = 18 m/s at 34° north of west)

Q30. Maggie, of mass 50 kg glides to the right on a frictionless frozen pond with a speed of 2.5 m/s. She collides with a 20 kg penguin. After the collision, the directions of the penguin's and Maggie's motion is shown in the figure below. Answer the follow (a-c) from a conceptual perspective. (a) Who has the greater magnitude of momentum in the y-direction after the collision? (b) Who has the greater magnitude of momentum in the x-direction after the collision? (c) Who has the greater velocity in the y-direction after the collision?

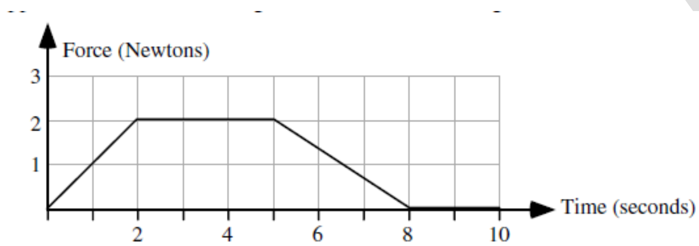


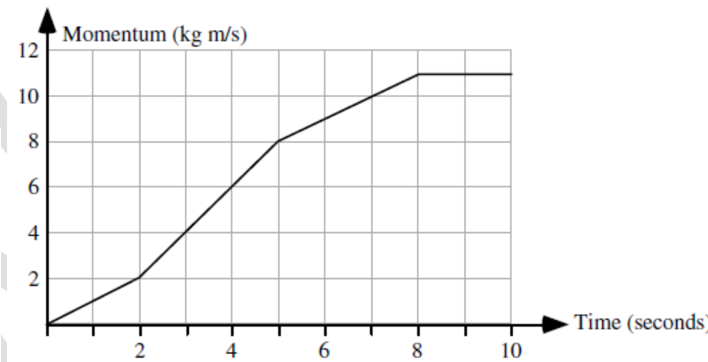
(**a**- before collision there was no momentum in the y-direction, so after the collision the total y-momentum must also be zero. Since Maggie and the penguin are indeed moving in the y-direction after the collision, their momenta must be equal and opposite and subtract to zero; **b**- momentum must be conserved in the x-direction. The initial momentum in the x is all due to Maggie so afterwards, the sum of $p_{maggie}\cdot\cos30°$ + $p_{penguin}\cdot\mathit{cos60°}$ =Maggie's initial momentum in the x)

Q31. [Video Analysis] Visit the Unit 4 webpage on www.PedersenScience.com and scroll down to the problem set for this unit. You will find an accompanying video of a blow dart colliding with a cart. Watch the video twice before answering the following questions. (a) Calculate the momentum of the dart before the collision. (b). Calculate the momentum of the dart/cart system after the collision. (c) Do your answers suggest conservation of momentum? Please justify. If not, where do you think momentum was lost or gained?

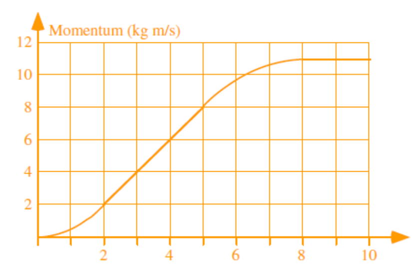
(v=0.1m/0.0125s; 0.0488 N·s; 0.045 N·s; since the numbers are very close they do suggest conservation of momentum. The numbers, however, are not exact, thus not all momentum is conserved. This is a real world example and some of the momentum is lost due to the friction between the table and the cart.)

Q32. A 10-kg box, initially at rest, moves along a frictionless horizontal surface. A horizontal force to the right is applied to the box. The magnitude of the force changes as a function of time as shown on the left. A student draws the graph on the right for the momentum of this 10-kg box as a function of time during this 10-second interval. What, if anything, is wrong with this graph? If anything is wrong, identify it and explain how to correct it. If this graph is correct, explain why.

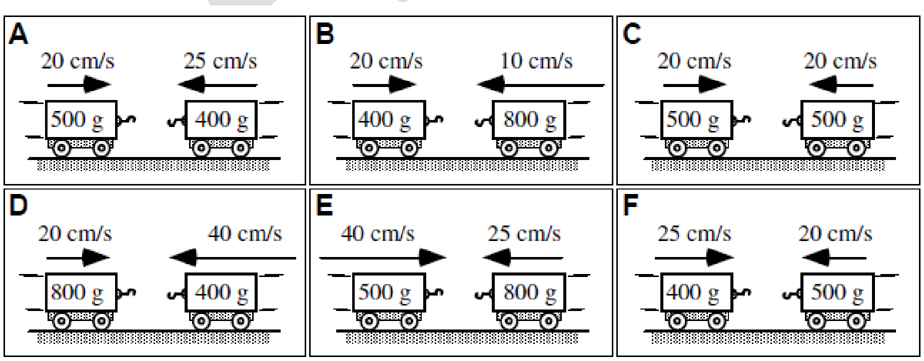




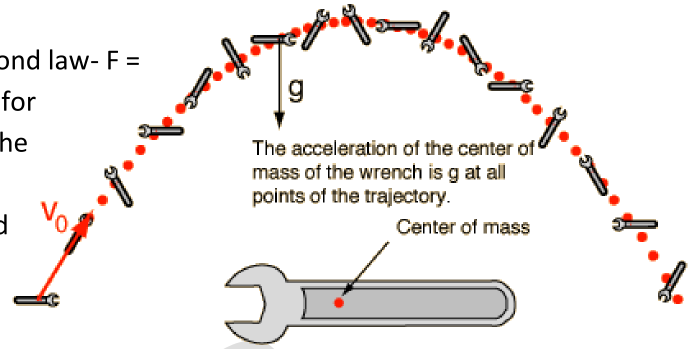
(The slope of the momentum-time graph should be proportional to the force. For the first two seconds as well as between the 5th and 8th second, the graph should be curved, since the force is changing in these time intervals. The momentum graph is curved upward between zero and two seconds, and downward for the time interval between five and eight seconds. The student's graph is correct for the time intervals 2 – 5 seconds and 8 – 10 seconds.)



Q33. In each of the six figures below, two carts traveling in opposite directions are about to collide. The carts are all identical in size and shape, but they carry different loads and are initiallytraveling at different speeds. The carts stick together after the collision. There is no friction between the carts and the ground. Rank these situations on the basis of the speed of the two-cart systems after the collision. (all the same; speed = 0)



FACT: The center of mass of a system of objects obeys Newton's second law- F = Ma_{cm}. Usually the location of the center of mass (cm) is obvious, but for several objects is expressed as: $Mx_{cm} = m_1x_1 + m_2x_2 + \ldots$, where M is the sum of the masses in the expression. In some cases the center of mass is not located on the body of the object. Some examples would be a torus, hoop, horseshoe, etc. Take a look at the example of a wrench in projectile motion to help conceptualize this idea:



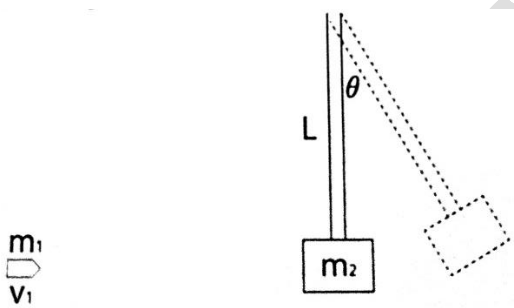
Q34. An astronaut on a spacewalk throws a rope around a small asteroid and then pulls it toward him. Where will the atronaut and the asteroid collide?

(center of mass; since no net external forces acted the cm has no acceleration; the cm started at rest and stays at rest until the collision.)

Q35. A toy rocket is in projectile motion, so that it is on track to land 30 m from its launch point. While in the air, the rocket explodes into two identical pieces, one lands 35 m from the launch. Where does the other piece land?

(25 m; the cm stays in projectile motion the entire time so if one piece is 5 m beyond, the other is 5 m short)

Q36. A bullet of mass m_1 with velocity v_1 is fired into a block of mass m_2 attached by a string of length L in a device known as a ballistic pendulum. The ballistic pendulum records the maximum angle the string is displaced (θ). Derive an expression for the initial velocity of the bullet (v_1) in terms of m_1, m_2, L, g, and θ.



$$V = \frac{m1+m2}{m1}\sqrt{2gL(1-cos\theta)}$$

Q37. Daddy is standing holding a 0.058-kg tennis ball. Sophia is standing on her picnic table holding a 0.03-kg ball of purple Play-Doh. They are both 2 meters high. Daddy and Sophia want to create a physics problem, so they decide to throw the objects at each other. The tennis ball has a velocity of 5 m/s to the right and the Play-Doh has a velocity of 3 m/s to the left. After 0.5 seconds the objects collide head-on and stick together. Using the conservation of momentum and your knowledge of kinematics, determine the displacement in the horizontal (Δx) of the system after the collision. In other words, after the collision, where will the ball/Play-Doh land relative to the point of the collision?

First use the conservation of momentum to solve for the velocity after the collision vf = 2.27 m/s to the right. Recognize the voy is zero but the objects were in the air falling for 0.5 seconds prior to collision and have fallen 1.25 m before the collision (g = -10). So the collision occurs 0.75 m above the ground. Therefore the objects did have a velocity in the y-direction when they collided. This velocity in the y-direction is 5 m/s down for both objects, since we all know mass does not matter for objects in free-fall. This now becomes the initial velocity for part 3. We know we need time of flight in the y-direction and we want to avoid a quadratic, so we solve for the final velocity and use that value and the correct UAM equation to solve for time. (vf-part3 = 6.32 m/s down; t = 0.132 s). Lastly, we find the range using x = vxt and the velocity in the x-direction was the velocity of the system after they collide (v = 2.27 m/s right). Finally, we discover that the ball and Play-Doh indeed land <u>0.30 meters to the right</u> of the collision point.