4.6 – Matrix Equations and Systems of Linear Equations

Read pages 236 - 244

Homework: page 242 3, 7, 9, 13, 19, 25, 29, 33, 35, 41, 46, 49, 51, 53

Basic Properties of Matrices

Assuming that all products and sums are defined for the indicated matrices A, B, C, I and 0, then:

Addition Properties

Associative: $(A+B)+C = A+(B+C)$

Commutative: $A+B = B+A$

Additive Identity: $A+0 = 0+A = A$

Additive Inverse: $A+(-A) = (-A)+A = 0$

Multiplication Properties

Associative: $A(BC) = (AB)C$

Multiplicative Identity: $AI = IA = A$

Multiplicative Inverse: If A is a square matrix and A^{-1} exists, then $AA^{-1} = A^{-1}A = I$

Combined Properties

Left Distributive: $A(B+C) = AB+AC$

Right Distributive: $(B+C)A = BA+CA$

Equality

Addition: If $A = B$ then, $A+C = B+C$

Left Multiplication: If $A = B$ then, $CA = CB$

Right Multiplication: If $A = B$ then, $AC = BC$

Q1: A. Perform the multiplication and write the system of equations without matrices.
Then, write the system of equations as an augment matrix and solve for x_1 and x_2.

$$\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

There are two ways to solve a system of linear equations using some matrices

B. Solution 1 is explained in 4.2 and 4.3 uses an augmented matrix and Row-Reduced Echelon Form
Solve using this method.

C. Solution 2 is explained in this section, 4.6, and uses the Inverse of a **Square** Matrix
Given a system of equations in the matrix format $AX = B$

D. Implement the solution developed in C. on the system to solve for x_1 and x_2.

$$\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

Q2: Solve the system of equations using the new method and your calculator.

> Solving systems of equations using the inverse of the coefficient matrix works only if the coefficient matrix has an inverse. This method is preferred if both of the following are true
> - ✓ there is an inverse of the coefficient matrix and
> - ✓ the system is being solved for various constant terms

$$\begin{aligned} 2x_1 + 3x_2 &= k_1 \\ x_1 + 2x_2 + 3x_3 &= k_2 \\ - x_2 - 5x_3 &= k_3 \end{aligned}$$ for the values

	#1	#2	#3
k_1	0	-2	3
k_2	2	0	1
k_3	1	1	4

	#1	#2	#3
x_1			
x_2			
x_3			

MATRIX[A] 3 ×3 [2 3 0] [1 2 3] [0 -1 -5] 3,3=-5	[A]⁻¹ [[-7 15 9] [5 -10 -6] [-1 2 1]] Ans→[B]	MATRIX[C] 3 ×1 [0] [2] [1] 3,1=1	[B]*[C] [[39] [-26] [5]] [A]⁻¹*[C] [[39] [-26] [5]]
Enter the coefficient matrix into your calculator. Mine was entered as [A]	From the home screen find the inverse matrix by entering the name of the matrix followed by x^{-1}. Then, store the inverse by entering STO followed by the name of another matrix (I used [B]) and ENTER	Create a third matrix – a column matrix to store one set of k values	To find the X matrix, multiply the inverse matrix by the column matrix containing the k values. Two ways to do this are indicated above. Therefore, $x_1 = 39$ $x_2 = -26$ $x_3 = 5$

Q3: Explain why augmented matrices must be the method used to solve each of these systems.

1. $$\begin{aligned} 2x_1 - 3x_2 &= 5 \\ 4x_1 - 6x_2 &= 10 \end{aligned}$$

2. $$\begin{aligned} x_1 - 3x_2 - 2x_3 &= -1 \\ -2x_1 + 7x_2 + 3x_3 &= 3 \end{aligned}$$

Q4: *Production Scheduling.* A. Labor and material costs for manufacturing two models of guitars are given in **Table 1**. A total of \$3000 a week is allowed for labor and materials. How many of each model should be produced each week to use exactly the allocations of \$3000 indicated in **Table 2**.

Table 1

Guitar Model	Labor Cost	Material Cost
A	\$30	\$20
B	\$40	\$30

Table 2

	Weekly Allocation			
	#1	**#2**	**#3**	**#4**
Labor	\$1,800	\$1,750	\$1,720	\$1600
Material	\$1,200	\$1,250	\$1,280	\$1,400

Solutions

x_1	
x_2	