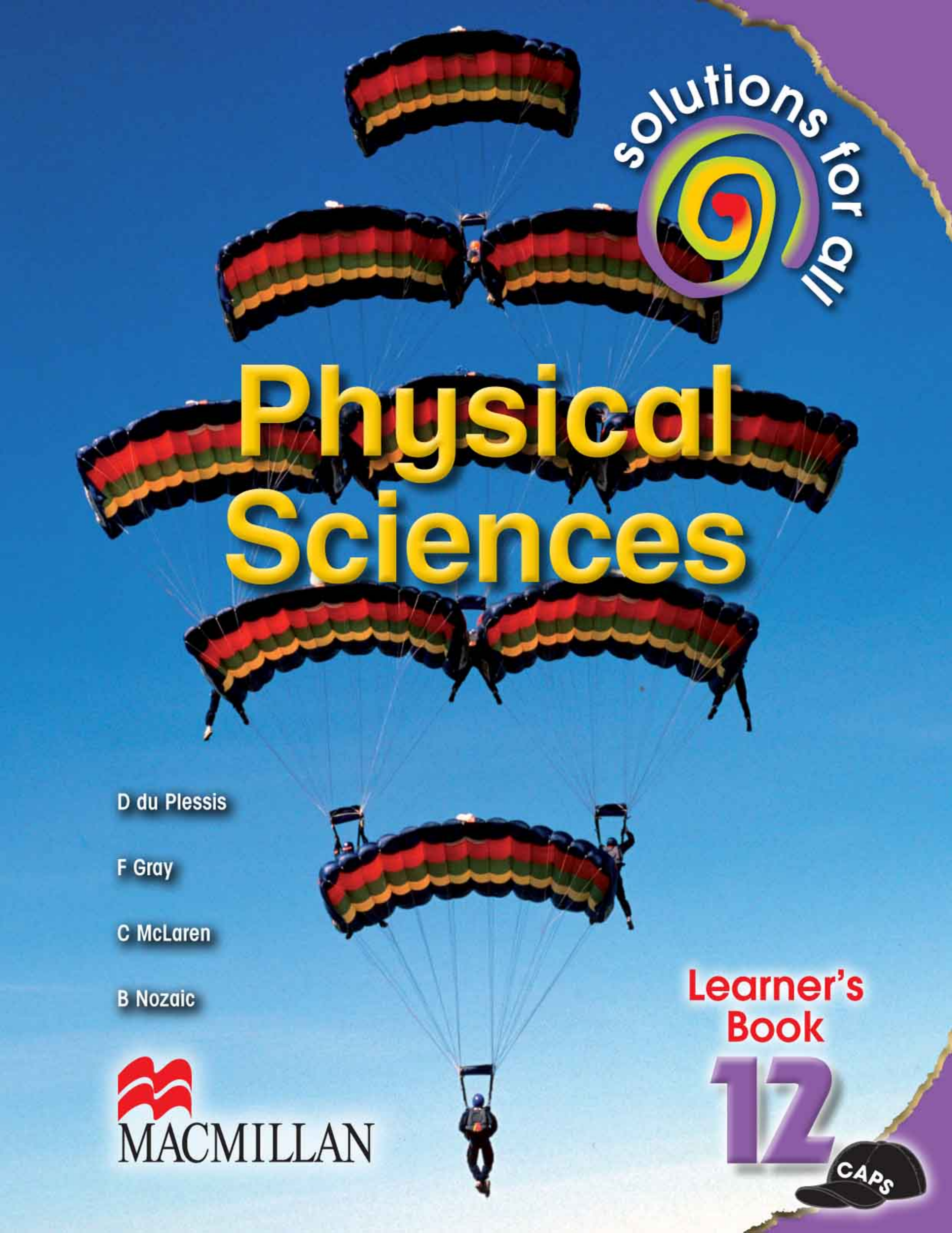


solutions for all

Physical Sciences

D du Plessis

F Gray

C McLaren

B Nozaic

Learner's Book

12

CAPS

MACMILLAN

Solutions for all
Physical Sciences

Grade 12
Learner's Book

C McLaren F Gray
B Nozaic D du Plessis

MACMILLAN

Solutions for all Physical Sciences Grade 12 Learner's Book

First published 2013

0 2 4 6 8 10 9 7 5 3 1

Published by Macmillan South Africa (Pty) Ltd

Private Bag X19

Northlands

2116

Gauteng

South Africa

Typeset by Boss Repro

Cover image from INPRA

Cover design by Devine Design

Illustrations by OKS, Daniella Levin, Geoff Walton, USDI

Photographs by

AAI: Pages 1, 28, 85, 143, 154, 165, 166, 174, 266, 286, 338, 375, 387 (bottom), 395, 397, 401, 412, 432, 479, 501

Great stock: 13, 47, 108 (bottom), 194 (top), 225, 234, 248, 253, 265, 298, 326, 352, 362, 387 (top), 416, 417, 448, 452, 467, 482

AfriPics: 53, 57, 58, 102, 107, 108 (top), 116, 121, 142, 204, 211, 217, 229, 283, 289, 307, 472, 481, 489, 496

Gallo images: 88, 188, 194, 427

APN: 196

Keith Rein: 103, 104, 105, 122, 123, 145, 152

Christine McLaren: 49, 86, 103, 105, 110, 121 (bottom), 136, 257, 317, 494, 495,

Braam van Reenen: 338 (bottom)

e-ISBN: 9781431024384

WIP: 4534M000

Contents

Knowledge area: Electricity and magnetism (Physics)

Knowledge area: Matter and materials (Physics)

Knowledge area: Chemical change (Chemistry)

Knowledge area: Chemical systems (Chemistry)

Introduction

Welcome to the *Solutions for all Physical Sciences Grade 12 Learner's Book*.

In your study of Physical Sciences you will investigate physical and chemical phenomena. This is done through scientific inquiry, application of scientific models, theories and laws in order to explain and predict events in the physical environment. The purpose of Physical Sciences is to become aware of your environment.

The study of Physical Sciences includes obtaining certain knowledge (the theory). In addition you perform practical work in which you practice the skills necessary to study and investigate physical and chemical phenomena. Through the process of learning and doing we hope you develop an interest and appreciation for the physical world around us.

The *Solutions for all Physical Sciences Grade 12 Learner's Book* contains content knowledge and background information to ensure that you acquire enough, and a bit more, knowledge than required by the Curriculum and Assessment Policy Statement (CAPS).

The *Solutions for all Physical Sciences Grade 12 Learner's Book*, includes sufficient practical tasks to ensure that you develop the skills necessary to become a true scientist.

The subject matter of physical sciences is organised in six main knowledge areas. These are:

- Matter and Materials
- Chemical Systems
- Chemical Change
- Mechanics
- Waves, Sound and Light
- Electricity and Magnetism

All the knowledge areas are covered in Grade 10, 11 and 12 but not in any particular order.

Physical Sciences has the following specific aims:

- knowledge and skills in scientific inquiry and problem solving;
- the construction and application of scientific and technological knowledge;
- an understanding of the nature of science and its relationships to technology, society and the environment.

How to use the *Solutions for all Physical Sciences Grade 12 Learner's Book*

The content knowledge in the *Solutions for all Physical Sciences Grade 12 Learner's Book* is organised according into Topics. Each topic is structured in the same way:

Topic opener page: The topic starts with full colour photograph of something that is related to the content of the topic. The *What you will learn about in this topic*, lists the content and concepts to be covered in the topic and links to what you should know after you have worked through the topic. The *Let's talk about …* introduces the topic. It includes questions related to the photograph and comments about the content in the topic. The idea is for you to start thinking about new things you will learn about in the topic.

What you know already: On the second page of a topic is *What you know already* and *Check myself*. These two features tell you what you already know from previous grades and then makes sure that you know what you need to know by giving you a question or two to complete before continuing with the new work. It is revision of a previous grade's work.

Units and lessons: The content of each topic is divided into units and lessons. The lessons break the work up in smaller chunks of information. This helps you to make sure you know and understand a certain section of the work before moving on to the next new section of work. A lesson consists of content and then an *Exercise* or a *Practical task*. The *Exercise* might be done in class or given as homework. The *Exercise* and *Practical tasks* are opportunities for formal and informal assessment. Your teacher will inform you which activities would be assessed. One *Practical task* per term is a formal assessment task. The *Exercises* and *Practical tasks* can be done alone, in pairs or in groups.

Extend yourself: The topic ends with a variety of additional questions and problems to give you extra practice. Some of the problems will require that you extend yourself to get to the solution.

Definitions box: The *Definitions box* at the end of the topic contains definitions of all important terms within the topic. Always keep a dictionary handy to find the correct meaning of new words. If you know what a word means you will understand the content better and this will make your learning much easier.

Summary: Each topic ends with a *Summary* of the content covered in the topic. Use these summaries to recap the content of the topic.

Other features include

Worked examples: Throughout the book there are examples that are explained step-by-step and a full solution given. Work through them before attempting problems in the *Exercises*. The worked examples explain the process of how to get to the correct solution and answer.

Checkpoints: *Checkpoints* are included along the way during a topic. They are positioned at regular intervals throughout a topic and are designed to consolidate your understanding of a particular concept. You should attempt each *Checkpoint* before continuing with the topic.

Science around us: This is some interesting information on how the science you are learning relates to something in everyday life.

Diagrams and illustrations: Diagrams are included to help you understand the written words. Make good use of the pictures when working through the text. When you see something you will remember it a lot better.

Representation of vectors: The symbols for vectors are shown in **bold** and *italics*, for example $\boldsymbol{F}$ (to represent a force) or $\boldsymbol{p}$ (to represent momentum). The symbol for the magnitude of a vector is shown in *italics* only, for example $F = 400$ N or $p = 2$ kg.m.s^{-1}.

The publisher and authors wish you the best in your study of Physical Sciences Grade 12.

Good luck!

Practical safety information

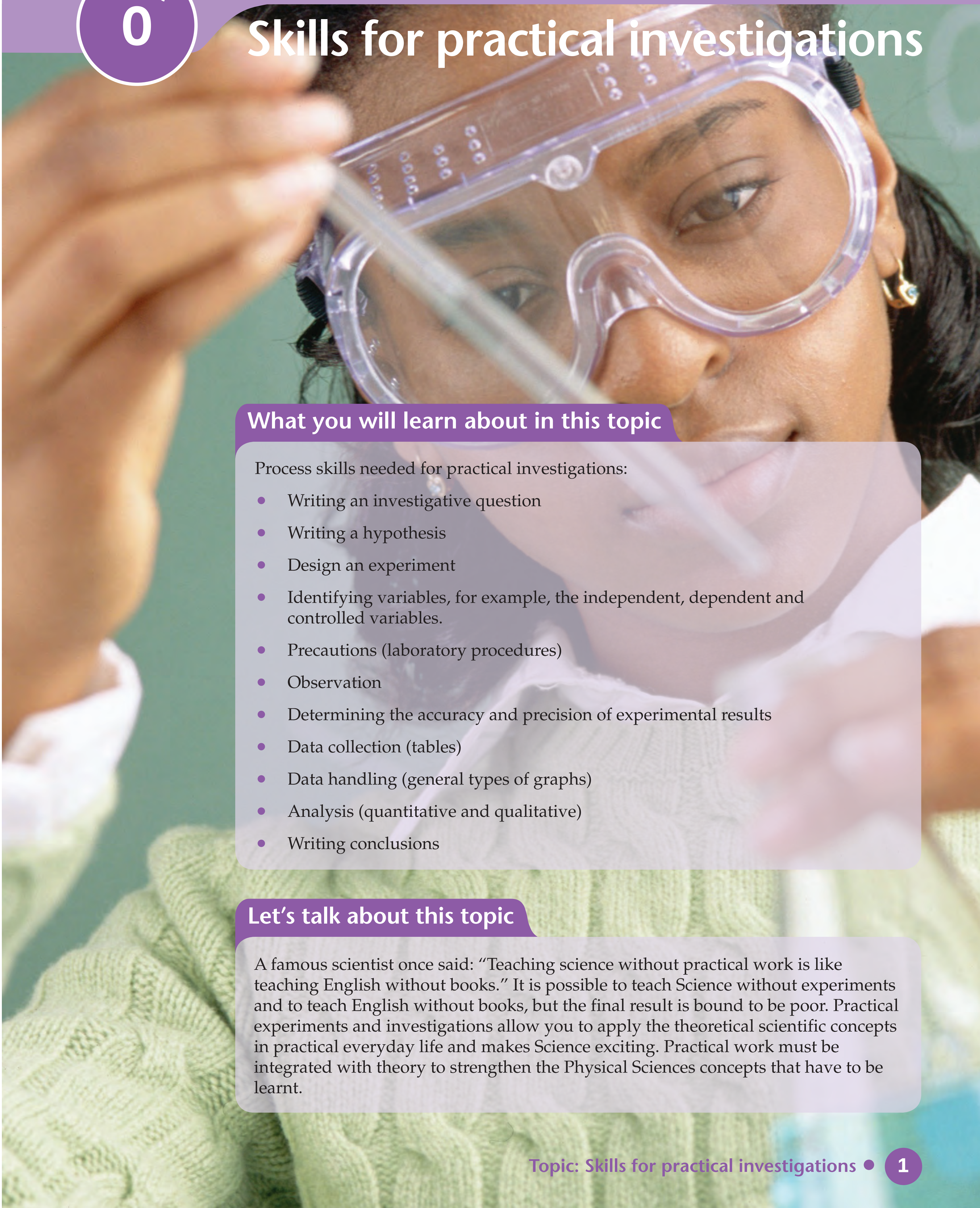
There are *Practical activities* throughout this book. Safety information for each *Practical activity* is given at the start of the activity. Please take note of this safety information to ensure your own the safety and that of the other learners. The symbols in the table are used throughout the *Solutions for all Physical Sciences Grade 12 Learner's Book*. Use this table as a reference to find the meaning of each symbol. Follow these instructions carefully when performing experiments.

Symbol	Meaning
Irritant or harmful 	• This symbol with the word "harmful" should appear on the label of a substance which, if it is inhaled or ingested or if it penetrates the skin, may involve limited health risks. • This symbol with the word "irritant" is meant for a noncorrosive substance which, through contact with the skin, can cause inflammation.
Poisonous 	• This symbol with the word "toxic" denotes a highly hazardous substance. • This symbol with the words "very toxic" is used to label a substance which, if it is inhaled or ingested or if it penetrates the skin, may involve extremely serious health risks and even death.
Flammable or extremely flammable 	• This symbol with the words "highly flammable" substance which may become hot and catch fire. • This symbol with the words "extremely flammable" denotes a liquid that would boil at body temperature and would catch fire if exposed to a flame.
Oxidising chemical 	These substances provide oxygen which allows other materials to burn more fiercely.
Corrosive 	This symbol with the word "corrosive" will be found on the label of a substance which may destroy living tissues on contact with it. Severe burns on the skin and flesh might result from splashes of such substances on the body.
Environmental hazard 	Relatively rare with laboratory chemicals (most of which pose some environmental hazard if not got rid of correctly), these require particular care to be taken on disposal.

Explosive 	Noise and movement can also trigger explosion, not just sparks/ flames.
	Wear a lab coat or lab apron when performing experiment
	Wear safety goggles when performing the experiment.
Use of electrical equipment 	To avoid electric shock, never use electrical equipment around water, or when the equipment is wet or your hands are wet. Be sure cords are untangled and cannot trip anyone. Unplug equipment not in use.
	• Toxic fumes are formed during this experiment. • This experiment must be performed in a fume cupboard.
Bunsen burner usage 	• Ensure your clothes are safely tucked in. No loose clothing. • Before starting, make sure Bunsen burner is off. • Ensure gas tap is closed at the beginning and end of experiment. • Be certain that whatever you are heating can be heated safely and will not explode.

Topic 0

Skills for practical investigations



What you will learn about in this topic

Process skills needed for practical investigations:

- Writing an investigative question
- Writing a hypothesis
- Design an experiment
- Identifying variables, for example, the independent, dependent and controlled variables.
- Precautions (laboratory procedures)
- Observation
- Determining the accuracy and precision of experimental results
- Data collection (tables)
- Data handling (general types of graphs)
- Analysis (quantitative and qualitative)
- Writing conclusions

Let's talk about this topic

A famous scientist once said: "Teaching science without practical work is like teaching English without books." It is possible to teach Science without experiments and to teach English without books, but the final result is bound to be poor. Practical experiments and investigations allow you to apply the theoretical scientific concepts in practical everyday life and makes Science exciting. Practical work must be integrated with theory to strengthen the Physical Sciences concepts that have to be learnt.

What you know already

The scientific process is a way of investigating things about the world. Scientists use this process to find out about the world and to solve problems. The steps that make up the scientific process include:

- **Step 1**: Identify a problem and develop a question. What is it that you want to find out?
- **Step 2**: Form a hypothesis. A hypothesis is your idea, answer, or prediction about what will happen and why.
- **Step 3**: Design an activity or experiment. Do something (investigate) that will help you test your idea or prediction to see if you are correct.
- **Step 4**: Observe/note changes/reactions (through measuring) and record your observations. What are the results of the investigation or experiment? Write about what happened.
- **Step 5**: Make inferences about the observations recorded in the tables, graphs, drawings and photographs. Make some conclusions. What did you find out? Do your results support your hypothesis? What did you learn from your investigation?

Check myself

Put the steps of the scientific method in a logical order.

- Construct a hypothesis
- Draw conclusions
- Test with an experiment
- Do background research
- Ask a question
- Analyse results

Skills needed for practical investigations

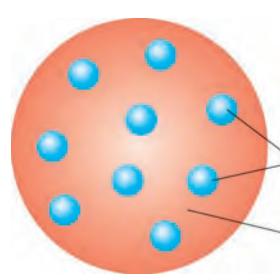
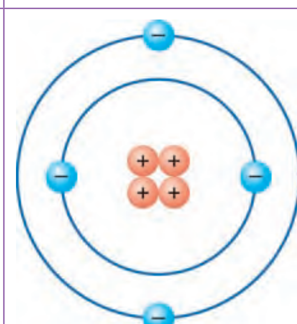
The purpose of this topic is to provide a detailed description of each of the skills needed to do practical work in the Physical Sciences classroom. The skills are explained and illustrated through a practical experiment. The relevant skill is given as a sub-heading.

Wherever possible, the prescribed experiment on page 32 in Topic 1 Momentum and impulse is used to illustrate some of the practical process skills. The practical experiment requires the verification of the conservation of linear momentum.

Trace the historical development of a scientific principle or theory

A **scientific model** is a simplified abstract view of something that is often far more complex. It takes a concept that is difficult to picture and puts it into a model that we can understand. In science, there are often concepts that develop over time, as the world becomes more advanced and it is possible to gather more information about the topic in question. We use a model as an explanation of a concept until a better explanation and/or model is formulated based on newly discovered information and constructed knowledge.

For example, scientists accept that an atom is made of a positively charged nucleus that contains protons and neutrons and that the nucleus is surrounded by electrons spinning in orbits. However, scientists did not reach this theory at their first attempt. It is a theory that was proposed and then developed over hundreds of years as more experimentation was done and more information was obtained. Below are just three models that were proposed during the formation of the atomic model as we know it today:

Name of Model	Billiard Ball Model	Plum Pudding Model	Planetary Model
Scientist who proposed it	John Dalton	Joseph Thomson	Ernest Rutherford
Date	+/– 1800	+/– 1890	+/– 1910
Diagram of model			
Description of model	Atoms are made of small indestructible spheres	Discovered protons and electrons, but thought that they were evenly distributed throughout the atom	Discovered that the protons are found in the centre of the atom and that the electrons spin around the nucleus

You can see how each model is a development of the model that came previously. As more information was obtained about the atom, so the model of the atom could be modified and updated.

The development of a scientific theory has 4 steps:

Step 1: Observation and description of the phenomenon to be tested.

Step 2: Formulation of a hypothesis to explain the phenomenon.

Step 3: Use of the hypothesis to predict the existence of other phenomena, or to predict quantitatively the results of new observations.

Step 4: Performance of experimental tests of the predictions by several independent experimenters and properly performed experiments.

If the experiments bear out the hypothesis, it may come to be regarded as a theory or law. If the experiments do not bear out the hypothesis, it must be rejected or modified.

Identify an answerable question and formulate a hypothesis

The hypothesis is usually a statement of the expected result of the experiment. The hypothesis may or may not be proved to be correct at the end of the experiment. Do not be afraid of making a hypothesis, in case it is 'wrong' – that doesn't matter – as long as you prove your hypothesis to be true or false at the end of the experiment. A hypothesis has to be testable experimentally in order to falsify or support it. The hypothesis guides the experiment. A possible hypothesis for the experiment could be:

The linear momentum of the system is conserved. In other words, the total linear momentum before the explosion is equal to the total linear momentum after the explosion.

A scientific investigative question will be asked, which the experiment will then endeavour to answer. You must ask a question that is well defined, measurable and controllable. A possible question for this experiment could be:

Is the total linear momentum before the explosion equal to the total linear momentum after the explosion?

Design the experiment

The design gives the apparatus and method that is to be used in the experiment to test the hypothesis. In this investigation we need to measure the mass and velocity of the trolleys before and after the explosion. We do this by measuring time taken for each trolley to cover a distance of 1 m. The design must define the dependent variable, the independent variable and the controlled or fixed variable(s).

Independent variable: The independent variable is the quantity that can be changed by the experimenter. Changes in the independent variable cause changes in the dependent variable.

Dependent variable: The dependent variable is measured by the experimenter. It is the outcome of the investigation.

Fixed variable(s): Many variables will affect the outcome of the experiment unless controlled. The variables that are kept constant are called fixed or controlled variables. We must ensure that we are changing just one variable and measuring another if it is to be a fair test.

Next you will decide on an experimental method by which you plan to test you hypothesis. List your method in point form and explain clearly what to do and what measurements should be taken.

Laboratory procedures

The laboratory procedures include:

- Safety precautions applicable in any laboratory
- Methods used to measure physical quantities, such as time, distance, etc on a small scale
- Small scale objects are used to simulate larger scale events.

Possible laboratory procedures for the experiment could be:

Safety precautions: Apart from the normal safety measures applicable to any laboratory, this experiment has only one dangerous aspect that must be considered. The spring-loaded mechanism that provides the explosion between the two trolleys, which can cause injuries when not handled with care.

Methods of measurement: The momentum of the two trolleys requires the measurement of their masses and their velocities before and after the explosion.

- The masses can be determined with a mass scale.
- The velocities are determined by measuring the time taken to travel 1 m.

Small scale objects are used to simulate larger scale events: Small scale wooden trolleys are used to simulate collisions between life size vehicles

Select appropriate tools and technology to collect accurate quantitative data

Possible appropriate tools could be:

- Two trolleys with spring-loaded plungers that can be released by a push-button to force the trolleys apart in an explosion.
- The masses of the two trolleys are determined to the nearest gram.
- A smooth laboratory bench can be used as a runway.
- A measuring tape or ruler to measure the distances travelled by the trolleys.
- Ensure that the trolley wheels run freely. If necessary, add a little oil to the axles.

Correct measurements

The recording of accurate and precise measurements is one of the most important skills in experimental work. The accuracy of your results is improved by increasing the number of repeat readings that are taken. The precision is determined by the measuring instruments. For example, a tape measure in which the smallest unit is the millimetre, is more precise than a tape measure in which the smallest unit is the centimetre.

Learners need to be sure that they are able to take correct measurements using the following apparatus:

- Thermometer
- Metric ruler
- Pipette
- Mass balance
- Graduated cylinder
- Burette

Recording the data

Decide on the best way to record your results. A table of results is recommended. Points to note when drawing up a table:

- The table must have a heading, stating what is being recorded.
- The independent variable is recorded in the left hand column and the dependent variable is recorded in the right hand column.
- There must be headings to each column, and the heading must include the units of measurement.
- You must not include units in the body of the table.

In the Ohm's law experiment that determines the relationship between the potential difference and current for a resistor, the independent variable is the potential difference and the dependent variable is the current. A sample set of results for this experiment could be tabulated as follows:

Table A Potential difference (V) across a conductor and current (I) in the conductor

Potential difference (V)	Current (A)
0,352	0,031
0,364	0,032
0,374	0,034
0,394	0,034
0,403	0,038
0,439	0,040
0,451	0,041
0,475	0,042
0,209	0,046
0,548	0,049

Presenting the data

Experimental data can be represented in a graph. Special attention must be given to the drawing and labelling of graphs. Marks are allocated for the following:

- The correct heading. A correct heading should be a statement that includes both variables. For example,

A line graph to show the relationship between the potential difference across a conductor and the current in the conductor

- The independent variable is plotted on the horizontal axis (x-axis).
- The dependent variable is plotted on the vertical axis (y-axis).
- Ensure that the scales that you choose result in you using more than half of the space given on the graph paper, to ensure that the graph is not too small.
- That the line joining the dots is one of best fit. Use a ruler to draw a straight line of best fit if the graph is clearly supposed to be a straight line. If the plotted points do not form a straight line, use a smooth curve to represent the data. Do not join the dots with short, straight lines.

A graph for the values give in Table A is shown in Figure 1. There is clearly one reading that is an anomalous reading. An anomalous reading is a reading that falls very far out of the trend that the other points are following. There must have been an error when performing the experiment at that time, or else the results must have been recorded incorrectly. This point on the graph can therefore be ignored.

Potential difference (V) vs current (A)

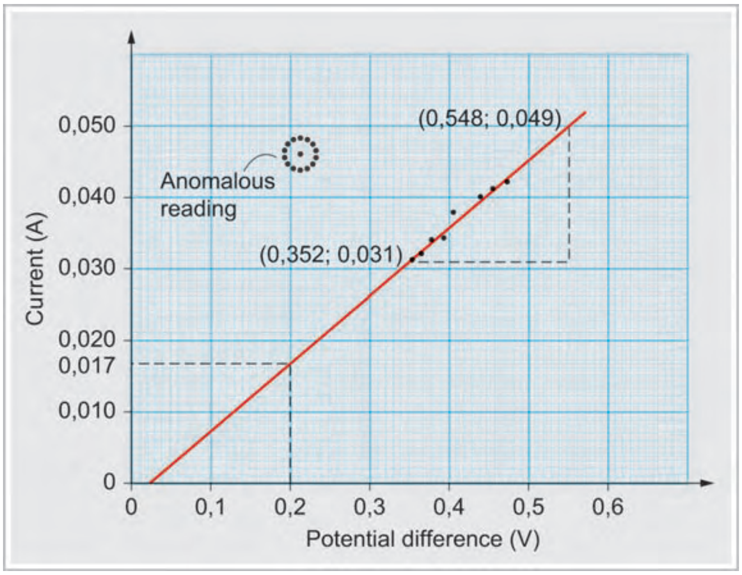


Fig 1: Graph drawn from data in Table A

Figure 2 shows two further sample graphs.

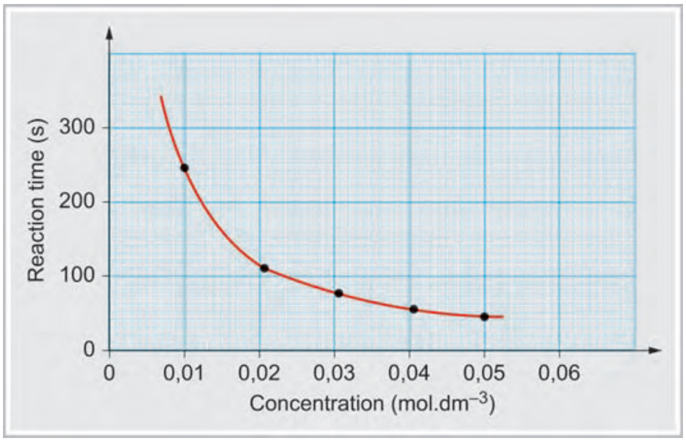


Fig 2a: A curved line graph

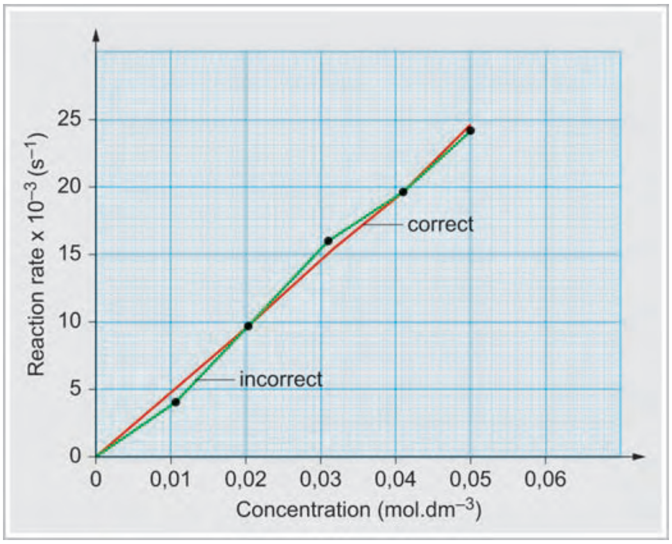


Fig 2b: A straight line graph

For the graphs in Figure 2, note that:

- Concentration is the independent variable and is drawn on the x-axis.
- Time and rate are dependent variables and are drawn on the y-axis.
- The curve in graph Figure 2a is a smooth freehand curve.
- The solid red line in Figure 2b does not pass through all the points. However, it is clear that the graph should be a straight line. A ruler is used to draw the line that best fits the plotted points. Do not join the plotted points (dots) as is illustrated by the green line.

Analyse the information

Sometimes it will be necessary to use the graph to perform calculations or to analyse the graph and make a prediction from it.

Calculating gradient

If asked to calculate the gradient of a graph, choose two points on the graph use them to calculate the gradient. For example, if asked to calculate the gradient of the graph in Figure 1, let us use the first and last points plotted (ensure that they fall on the line of best fit) and calculate the gradient of that line:

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{0{,}049 - 0{,}031}{0{,}548 - 0{,}352} = 0{,}092$$

You may then be asked to interpret what is understood by the value that you have calculated.

In this csae the gradient has the following unit:

$$\text{gradient} = \frac{\text{A}}{\text{V}} = \Omega^{-1}$$

Therefore the gradient represents 'the inverse of resistance'.

Predicting results, using the graph

Once the graph had been drawn, you may be asked to use the graph to predict a result outside of the set of points that you have plotted. For example, in the graph in Figure 1, if you were asked to predict what the current reading would be if the potential difference reading is 0,2 V, use the graph as shown by the dotted green line. Start at 0,2 V, dot the line upwards until you meet the graph and then continue the line horizontally until you meet the *y*-axis. So a predicted current reading, when the potential difference is 0,2 V would be approximately 0,017 A.

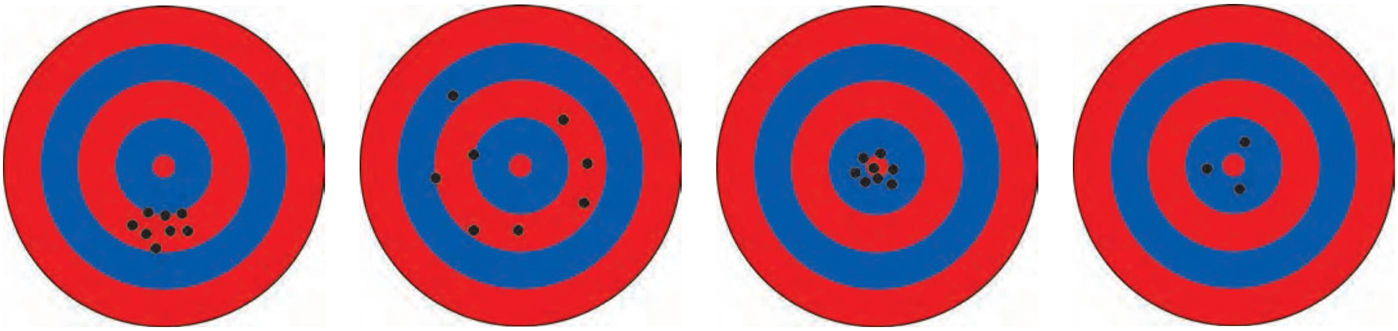
Determine the accuracy and precision of the experimental results

Accuracy: how close a measured value is to the true value.

Precision: how close the measured values are to each other.

Measured values may be close together but they may all be wrong due to a systematic error such as friction in an experiment that assumes a non-friction situation.

The difference between accuracy and precision is shown in Figure 3 below, assuming that hitting a 'bulls'eye' was the target.



a) Precise and inaccurate *b) Imprecise and inaccurate* *c) Precise and accurate* *d) Imprecise and accurate*

Figure 3: The difference between accuracy and precision

Sources of error in experiments are usually due to:

- controlled variables not being controlled
- errors in the measurement of dependent and independent variables
- calculation errors.

Percent error: A measure of the accuracy is the percent error (% error), which is used ONLY when the actual measured quantity is known or an accepted value such as $g = 9{,}8\ \text{m.s}^{-2}$ or speed of light $c = 3 \times 10^8\ \text{m.s}^{-1}$ etc.

$$\%\ \text{error} = \frac{(\text{Experimental result}) - (\text{Accepted value})}{(\text{Accepted value})} \times 100$$

Example:

In an experiment where the gravitational acceleration (g) was measured, a value of 10,07 m.s^{-2} was obtained. The accepted value of g is 9,8 m.s^{-2}.

$$\% \text{ error} = \frac{10{,}07 - 9{,}8}{9{,}8} \times 100 = 2{,}76\%$$

Note that this formula can be used only for accepted values $\neq 0$.

Factors that can cause this deviation from the true value and therefore the variation between the three sets of measurements are:

- friction between the wheels and the plank, which is an external force acting on the movement of the trolleys. This means that the system is not an isolated system as is stated by the law of conservation of momentum
- errors in the measurements of the masses of the trolleys
- errors in the measurement of the distances
- calculation errors.

Analyse experimental results and identify sources of error

Once the results of the experiment have been obtained, they can be analysed for any possible sources of experimental error or inaccuracy. If there are deviations in the results, a possible source of error needs to be identified.

General causes for inaccuracy in experiments include:

- inaccuracy of measurement
- inaccuracy when reading instruments
- time delay between observing something and recording time
- difference of opinion between people about when a certain observation was made
- a zero error in an instrument.

Recognise, analyse and evaluate alternative explanations for the same set of results

Learners should develop the ability to listen to and to respect the explanations proposed by other students. They should remain open to and acknowledge different ideas and explanations, be able to accept the skepticism of others and consider alternative explanations.

Evaluation includes reviewing the experimental procedures, examining the evidence, identifying faulty reasoning, pointing out statements that go beyond the evidence and suggesting alternative explanations for the same observations.

Design of a model based on the correct hypothesis for further investigation

If there is/are a factor/s that caused a wrong result, it some cases it may be possible to design an experiment to eliminate the effect of the problem factor/s.

Once the experiment has been performed, the results can then be analysed, and if there are any specific reasons that the experimenter can identify that may have caused incorrect results to be obtained, a model can then be designed so that the experiment can be performed again, attempting to eliminate such errors.

For example, in this experiment, friction may be identified as a variable that could adversely affect the results. Therefore one could make the following changes:

- Set up the trolleys on a linear air track, rather than on a wooden lab bench. This will drastically reduce friction.
- Perform the experiment with different apparatus, such as a Newton's cradle, which consists of 5 balls hanging on strings which hit against each other.

Know the difference between qualitative and quantitative analysis

Qualitative analysis is used to describe a process or observations using words rather than numbers. For example, a qualitative analysis of this experiment could be:

When the two trolleys are exploded apart, they move apart in opposite directions.

Since qualitative analysis does not rely on numbers for its explanation, conclusions regarding conservation of momentum cannot be drawn.

Quantitative analysis makes use of numbers and calculated values to describe a process. In this experiment, the actual values of velocities and momenta are calculated, so a quantitative conclusion can be drawn, such as:

The total momentum of the system before the collision is equal to the total momentum of the system after the collision.

Ideally, any experiment should be analysed qualitatively and quantitatively.

Practical activity

The following practical skills need to be acquired. Try wherever possible during the course of the year to cover these skills in your practical tasks:

1. Analyse the components of a properly designed scientific investigation.
2. Choose an experiment and determine the appropriate tools to gather precise and accurate data.
3. Defend a conclusion based on scientific evidence.
4. Determine why a conclusion is free from bias.
5. Compare conclusions that offer different but acceptable explanations for the same set of experimental data.
6. Investigate methods of knowing by people who are not necessarily scientists.

Also, make sure that by the end of the year, you are able to correctly read a:

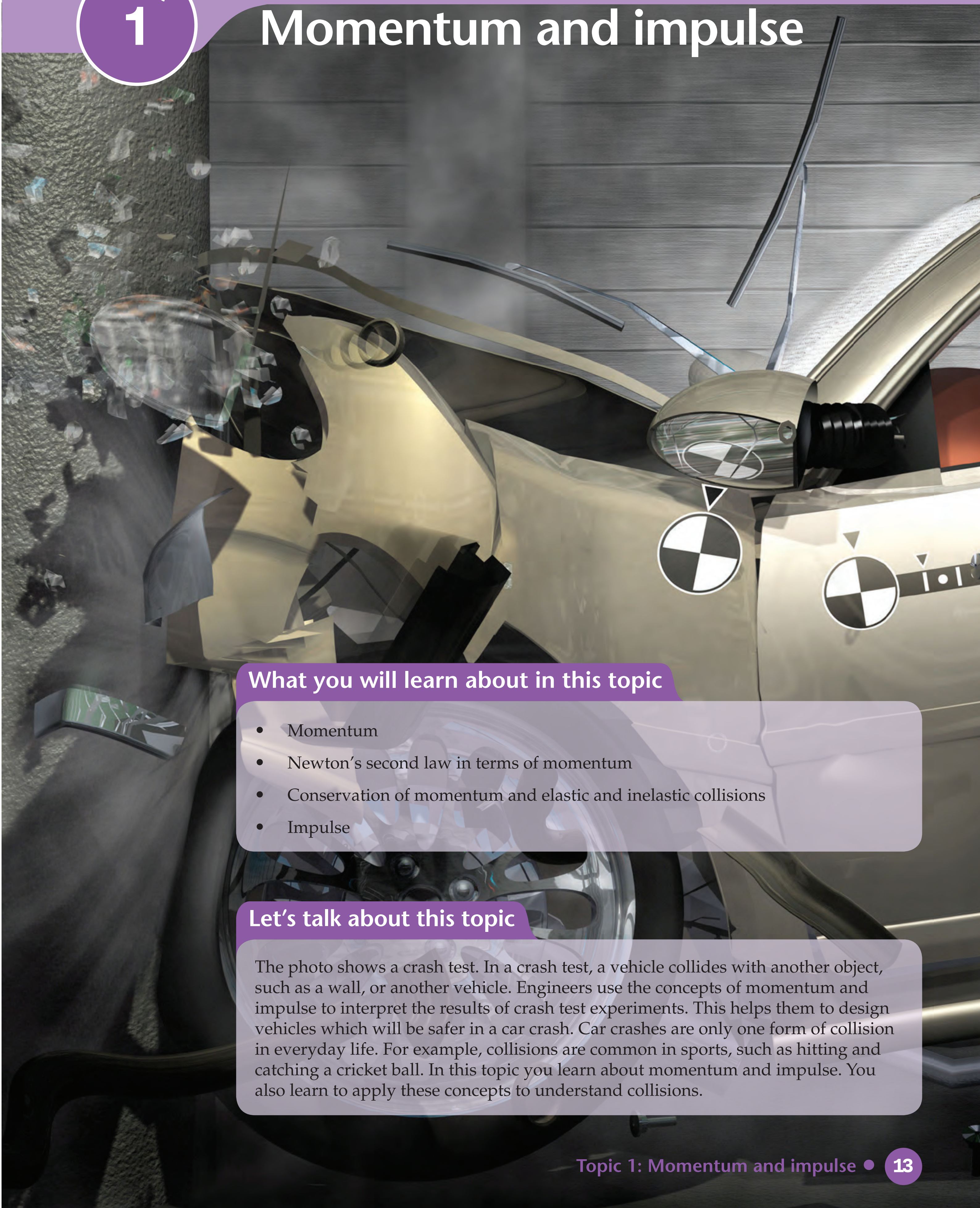
- Thermometer
- Mass balance
- Metric ruler
- Graduated cylinder
- Pipette
- Burette

Definitions

accuracy how close a measured value is to the true value
dependent variable the variable that is measured by the experimenter
fixed variable the variable that is kept constant
independent variable the quantity that can be changed by the experimenter
percent error a measure of the accuracy of the experiment
precision how close the measured values are to each other
scientific model a simplified abstract view of something that is often far more complex
qualitative analysis used to describe a process or observations using words rather than numbers
quantitative analysis makes use of numbers and calculated values to describe a process

Topic 1

Momentum and impulse



What you will learn about in this topic

- Momentum
- Newton's second law in terms of momentum
- Conservation of momentum and elastic and inelastic collisions
- Impulse

Let's talk about this topic

The photo shows a crash test. In a crash test, a vehicle collides with another object, such as a wall, or another vehicle. Engineers use the concepts of momentum and impulse to interpret the results of crash test experiments. This helps them to design vehicles which will be safer in a car crash. Car crashes are only one form of collision in everyday life. For example, collisions are common in sports, such as hitting and catching a cricket ball. In this topic you learn about momentum and impulse. You also learn to apply these concepts to understand collisions.

What you know already

- A **scalar** is a physical quantity that has magnitude only. Examples include mass and energy.
- A **vector** is a physical quantity which has both magnitude and direction. Examples include velocity, force and acceleration.
- The **mass** (m) of a body is the quantity of matter in that body. Mass is a scalar quantity, measured in kilograms (kg).
- **Velocity** (v) is the rate of change of displacement. In symbols: $\boldsymbol{v} = \frac{\Delta \boldsymbol{x}}{\Delta t}$. Velocity is a vector quantity, measured in meters per second (m.s^{-1}).
- **Acceleration** (a) is the rate of change of velocity. In symbols: $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{v_f - v_i}{\Delta t}$.

  Acceleration is a vector quantity, measured in m.s^{-2}.
- Newton's second law: When a net force, $\boldsymbol{F}_{net}$, is applied to an object of mass, m, the object accelerates in the direction of the net force. The object's acceleration, $\boldsymbol{a}$, is directly proportional to the net force applied to the object and inversely proportional to the object's mass. In symbols: $\boldsymbol{F}_{net} = m\boldsymbol{a}$
- Newton's third law: When object A exerts a force on object B; object B simultaneously exerts an oppositely directed force of equal magnitude on object A.
- The law of conservation of energy states that the total energy of an isolated system remains constant.

Check myself

1. Identify the action-reaction pairs of forces present in each of the following situations:

   a) Swimming accross the pool

   b) A book rests on the table.

   c) A soccer player kicks a ball.

   d) A rocket accelerates through space.

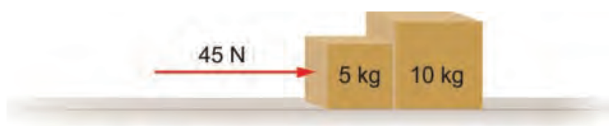


Fig 1.1: A 45N force applied to a 5 kg block in contact with a 10 kg block

2. A 5 kg and a 10 kg box are in contact with each other on a frictionless horizontal surface, as shown in Figure 1.1. A 45 N horizontal force is applied to the 5 kg box in order to accelerate both boxes across the frictionless surface.

   a) Calculate the acceleration of the entire 15 kg mass.

   b) Use your answer to a) to calculate the net force acting on the 5 kg box.

   c) Why is the net force acting on the 5 kg box less than 45 N?

   d) Draw a labelled free body diagram for the 5 kg block showing only the horizontal forces acting on it.

   e) Calculate the magnitude and direction of the force that the 5 kg box exerts on the 10 kg box.

Momentum

Defining and calculating momentum

All objects have mass. When an object is moving it has momentum. The **momentum** (***p***) of an object is the product of the mass (m) and velocity ($\boldsymbol{v}$) of the object:

In symbols: Where:

$$\boldsymbol{p} = m\boldsymbol{v}$$

$\boldsymbol{p}$ = the object's momentum, measured in kilograms metres per second (kg.m.s^{-1})

m = the object's mass, measured in kilograms (kg)

$\boldsymbol{v}$ = velocity of the object, measured in metres per second (m.s^{-1})

Momentum is a vector quantity with the same direction as the object's velocity. The unit for momentum is kg.m.s^{-1} since $\boldsymbol{p} = m\boldsymbol{v}$ = kg × m.s^{-1}

Consider a skateboarder and a bus moving down the road at the same velocity, as shown in Figure 1.2. Which has the greater momentum? The bus has a much greater mass, so it has much greater momentum. However, if the bus were at rest, its momentum would be zero. Objects at rest have zero momentum because they have zero velocity.

Worked example:

Calculate the momentum of a 1 300 kg rhino galloping east at 15 m.s^{-1} toward a poacher.

Solution:

Choice of direction: East is positive. The direction of the momentum vector is the same as the direction of the velocity vector.

m = 1 300 kg

$\boldsymbol{v}$ = +15 m.s^{-1}

$\boldsymbol{p} = m\boldsymbol{v}$

$\boldsymbol{p}$ = (1 300)(+15)

$\boldsymbol{p}$ = +19 500 = 19 500 kg.m.s^{-1} east

Fig 1.2: Skateboarder holding on to a moving bus

Checkpoint 1

Figure 1.3 shows some bumper cars. One of the cars has a mass of 180 kg and is carrying a 70 kg driver. It has a constant velocity of 3 m.s^{-1} west.

a) Calculate the driver's momentum.

b) Draw and label the driver's velocity and momentum vectors.

c) Calculate the momentum of the driver-bumper-car *system*.

d) How would the momentum of the driver-car system change if its velocity remained unchanged, but another person was also in the car? Explain your answer.

Fig 1.3: Bumper cars

system – a group of two or more objects that interact

Collisions

A collision is an isolated event in which two or more moving bodies exert forces on each other over a relatively short time. Collisions are a part of our everyday lives. Examples of collisions include a passenger colliding with an air bag and a bat hitting a ball. An object's motion changes during a collision.

To understand the mechanics of a collision, consider a golf club colliding with a golf ball, as shown in Figure 1.4. The club exerts a force on the golf ball. According to **Newton's third law**, the ball exerts a force on the club. The force the ball exerts on the club (***F***) is equal in magnitude, but opposite in direction to the force the club exerts on the ball (***–F***). These two equal and opposite forces are exerted *simultaneously*. These forces cannot cancel one another out, since they act on different objects.

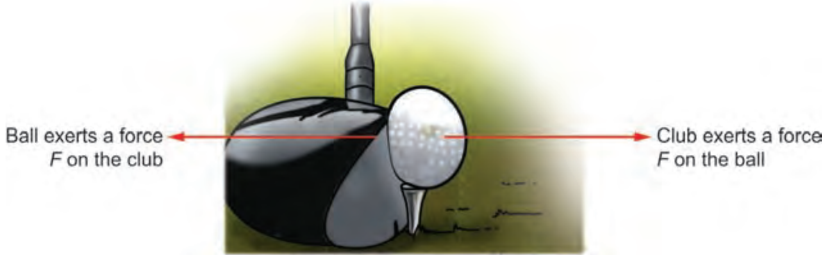


Fig 1.4: The forces involved during the collision between club and ball

The golf club will slow down during the collision and as a result, its momentum will decrease. The club experiences a force (from the ball) in the opposite direction to its motion causing it to decelerate.

The golf ball will leave the club travelling at high speed and as a result, its momentum has increased during the collision. The ball experiences a forward force (from the club) causing it to accelerate in the direction of this force. Both forces stop acting at the instant that the club and ball are no longer in contact. The ball leaves the club travelling at its maximum velocity.

Change in momentum

If the velocity of an object changes, then the momentum of the object will also change. In Figure 1.5, the barrier exerts a force on the car opposite to its direction of motion. This results in a sudden change in the car's velocity and therefore also a change in the car's momentum.

If $\boldsymbol{p}_\mathrm{i}$ is the initial momentum of the object immediately before the collision, and $\boldsymbol{p}_\mathrm{f}$ is the final momentum immediately after the collision, then the **change in momentum** $(\Delta\boldsymbol{p})$ of an object is found by subtracting its initial momentum $(\boldsymbol{p}_\mathrm{i})$ from its final momentum $(\boldsymbol{p}_\mathrm{f})$:

Fig 1.5: During a collision, the vehicle's momentum changes.

In symbols:

$$\Delta\boldsymbol{p} = \boldsymbol{p}_\mathrm{f} - \boldsymbol{p}_\mathrm{i}$$

$$\Delta\boldsymbol{p} = m\boldsymbol{v}_\mathrm{f} - m\boldsymbol{v}_\mathrm{i}$$

Where:

$\boldsymbol{v}_\mathrm{i}$ = object's initial velocity, measured in metres per second ($m.s^{-1}$)

$\boldsymbol{v}_\mathrm{f}$ = object's final velocity, measured in metres per second ($m.s^{-1}$)

Worked example:

In Figure 1.6, a tennis ball with a mass of 57 g is travelling horizontally at 20 m.s^{-1}. The ball is struck by a racquet and moves horizontally at 30 m.s^{-1} in the opposite direction.

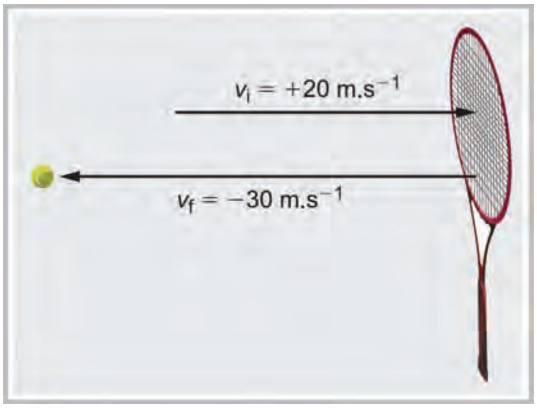


Fig 1.6: The initial and final velocties before and after a collison with a racquet

a) Calculate the change in momentum of the tennis ball.

b) Draw a vector diagram to illustrate the relationship between the initial momentum, the final momentum and the change in momentum of the tennis ball.

Solution:

a) Since velocity and momentum are vectors, you need to make a choice of direction. If we choose to the right to be the positive direction, then:

$\Delta \boldsymbol{p} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$

$\Delta \boldsymbol{p} = (0{,}057)(-30) - (0{,}057)(+20)$

$\Delta \boldsymbol{p} = -1{,}71 - 1{,}14 = -2{,}85 = 2{,}85$ kg.m.s^{-1} to the left

Note the following:

The negative sign shows the direction of the ball's momentum change. This is away from the racquet (to the left).

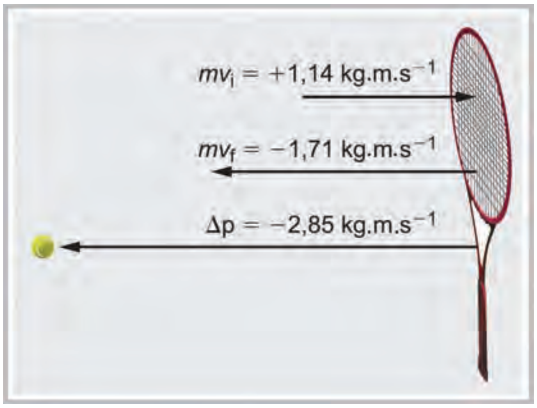


Fig 1.7: The initial, final and change in momentum vectors

b) The ball's initial, final and change in momentum vectors are shown in Figure 1.7. During the collision, the tennis ball's momentum decreases from 1,14 kg.m.s^{-1} to zero and then increases from zero to 1,71 kg.m.s^{-1}. This is a total change of 2,85 kg.m.s^{-1} to the left.

Why does the ball's momentum change? The racquet exerts a net force, to the left, on the ball. This force changes the ball's momentum. The direction of the change in momentum of the ball is the same as the direction of the net force acting on the ball.

Checkpoint 2

Refer to Figure 1.8 on the next page. During a soccer training session, Mpho passes a 0,45 kg soccer ball along the ground to Tshepo. The ball rolls at 4 m.s^{-1} toward Tshepo who immediately kicks it straight back to Mpho. The ball leaves Tshepo's boot with a speed of 6 m.s^{-1}. Ignore friction.

a) Calculate the initial and final momentum of the soccer ball.

b) Calculate the change in momentum of the soccer ball.

c) Draw a labelled vector diagram of the soccer ball's initial, final and change in momentum vectors.

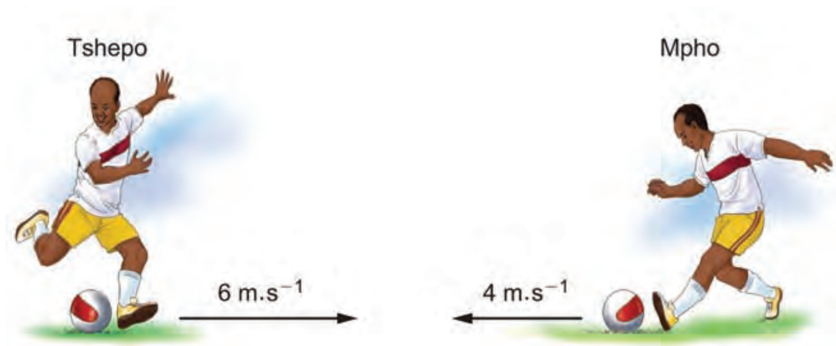


Fig 1.8: The velocity of the ball changes during a collision.

Exercise 1.1

1. a) Define momentum.

   b) State the SI unit of momentum.

2. Explain, in your own words, the difference between momentum and inertia.

3. Provide three examples of situations in which:

   a) velocity is the main factor determining an object's momentum

   b) mass is the main factor determining an object's momentum.

4. What is the momentum of a 6 kg bowling ball travelling at 2,2 m.s^{-1} east?

5. The momentum of a 75 g bullet is 9 kg.m.s^{-1} toward a target. What is the bullet's velocity?

6. A 10 kg bicycle and a 54 kg rider both have a velocity of 4,2 m.s^{-1} east. Draw momentum vectors for:

   a) the bicycle

   b) the rider

   c) the bicycle-rider system.

7. At what velocity does a 0,046 kg golf ball leave the tee if the club has given the ball a momentum of 3,45 kg.m.s^{-1} south?

8. The blue whale is the largest mammal on the Earth. A female blue whale swims at a velocity of 57 km.h^{-1} west and has a momentum of $2{,}15 \times 10^6$ kg.m.s^{-1}. What is the whale's mass?

9. A 38 000 kg loaded transport truck is travelling at 1,2 m.s^{-1} west. What does the velocity of a 1 400 kg car need to be for the car to have the same momentum as the truck?

10. Which of the following objects will experience the greater change in momentum:

    a) a 14 000 kg bull-dozer decreases its speed by 1 m.s^{-1} or,

    b) a 10 g rifle bullet, travelling at 1 500 m.s^{-1}, becomes embedded in the truck of a tree?

11. A 0,1 kg bouncy ball is dropped. It hits the ground at 8 $m.s^{-1}$. The ball bounces upward with a speed of 6 $m.s^{-1}$.
    a) Calculate the ball's change in momentum.
    b) Draw a labelled vector diagram to illustrate the initial, final and change in momentum vectors.
12. An 8 g bullet is fired from a rifle. The bullet passes through a 10 cm plank. This reduces the bullet's velocity from 400 $m.s^{-1}$ west to 300 $m.s^{-1}$ west.
    a) Calculate the bullet's initial momentum.
    b) Calculate the bullet's final momentum.
    c) Calculate the bullet's change in momentum.
    d) Draw a labelled vector diagram to illustrate the bullet's initial, final and change in momentum vectors.
13. Many modern rifles use bullets that have less mass and reach higher speeds than bullets of older rifles. This makes the rifle more accurate over longer distances. The momentum of an old bullet, fired from an old rifle, is 8,25 $kg.m.s^{-1}$ north.
    What is the momentum of a new bullet which has $\frac{3}{4}$ the mass of the old bullet and is fired at $\frac{3}{2}$ of the speed of the old rifle?
14. During one part of a model-rocket's lift-off, its momentum increases by a factor of 4 while its mass is halved. The rocket's velocity is initially 8,5 $m.s^{-1}$ upward. What is the rocket's final velocity?
15. In Figure 1.9 a 22 300 kg jet aircraft lands on the deck of an aircraft carrier travelling at 252 $km.h^{-1}$ (70 $m.s^{-1}$) south. It catches one of the arresting cables and is brought to rest over a distance of 100 m.

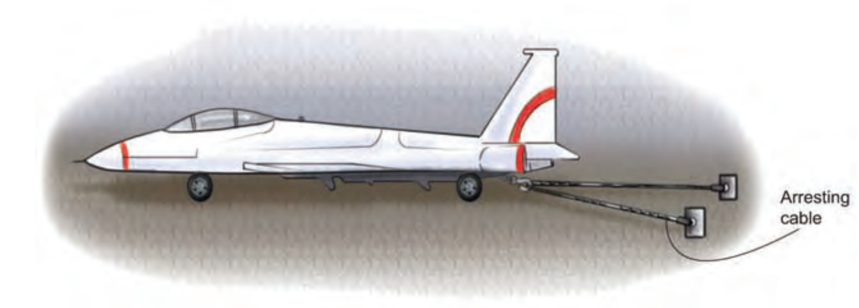


Fig 1.9: An arresting cable is used to catch a landing aircraft.

    a) Give the direction of the net force acting on the aircraft.
    b) Calculate the aircraft's change in momentum.

    The pilot of another aircraft, of the same mass, tries to land with the same initial velocity, but misses the arresting cables. The pilot immediately applies full thrust from the jet engine and takes off with a speed of 350 $km.h^{-1}$.

    c) Give the direction of the net force acting on the aircraft.
    d) Calculate the aircraft's change in momentum.
16. During a crash test, car A, of mass m, travels at speed $2v$, collides with a wall and bounces off the wall at speed v. Car B, of mass $2m$, travels at speed v, collides with the wall and is brought to rest. Which car experiences the greater change in momentum? Explain your answer.

Newton's second law in terms of momentum

Expressing Newton's second law in terms of momentum

In Grade 11 you learnt Newton's second law:

When a net force, $\boldsymbol{F}_{net}$, is applied to an object of mass, m, it accelerates in the direction of the net force. The object's acceleration, $\boldsymbol{a}$, is directly proportional to the net force applied to the object and inversely proportional to the object's mass.

In symbols: $\boldsymbol{F}_{net} = m\boldsymbol{a}$ (1)

If a net force ($\boldsymbol{F}_{net}$) acts on an object it will accelerate. This means its velocity will change (increase, decrease or change direction).

The concept of momentum can be used to restate Newton's second law:

The acceleration ($\boldsymbol{a}$) of the object is defined as the rate of change of velocity.

For a constant acceleration: $\boldsymbol{a} = \dfrac{\boldsymbol{v}_f - \boldsymbol{v}_i}{\Delta t}$ (2)

If we substitute equation (2) into equation (1) we get: $\boldsymbol{F}_{net} = m\left[\dfrac{\boldsymbol{v}_f - \boldsymbol{v}_i}{\Delta t}\right]$

$$\boldsymbol{F}_{net} = \frac{m\boldsymbol{v}_f - m\boldsymbol{v}_i}{\Delta t}$$

But ($m\boldsymbol{v}_f - m\boldsymbol{v}_i$) represents the change in momentum ($\Delta\boldsymbol{p}$) of the object.

Therefore: $\boldsymbol{F}_{net} = \dfrac{\Delta\boldsymbol{p}}{\Delta t}$

Written this way, Newton's second law relates the net force acting on an object to its rate of change of momentum. This result leads us to a statement of **Newton's second law stated in terms of momentum**.

Newton's second law (**stated in terms of momentum**):
The net force acting on an object is equal to the rate of change of momentum.

In symbols:

$$\boldsymbol{F}_{net} = \frac{\Delta\boldsymbol{p}}{\Delta t}$$

Where:

$\boldsymbol{F}_{net}$ = the net force acting on the object, measured in Newtons (N).

$\Delta\boldsymbol{p}$ = the change in momentum of the object, measured in kilograms metres per second (kg.m.s^{-1}).

Δt = the time interval over which the momentum of an object is changed, measured in seconds (s).

This form of Newton's second law is a more general form of Newton's second law. The equation $\boldsymbol{F}_{net} = m\boldsymbol{a}$ only applies to situations in which the mass of an object is constant.

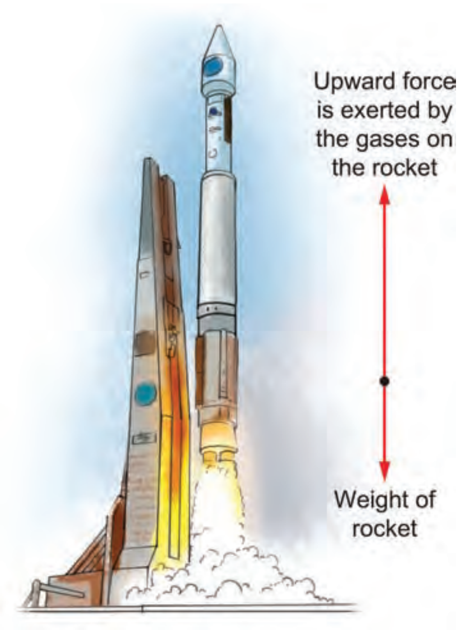


Fig 1.10: When expressed in terms of momentum, Newton's second law can be applied to object's that change mass during their motion.

However, by using the concept of momentum it is possible now to apply Newton's second law to situations where both the mass and velocity of an object are changing.

For example, it can be applied to the accelerating rocket shown in Figure 1.10. While the rocket engines are fired, the rocket fuel is being burned and the gases are escaping in the opposite direction. The mass of the rocket is therefore decreasing, which will decrease the weight of the rocket.

If the weight of the rocket is decreasing, then the net upward force on the rocket will increase. This will increase the acceleration of the rocket. Since the net force acting on the rocket is increasing, the rate at which its momentum changes will also increase.

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

When the fuel of the first stage of a rocket is used up, the first stage propulsion unit is jettisoned (released) from the main rocket, further decreasing the mass of the rocket. The second stage rocket engines are fired accelerating the rocket. In this way the rockets direction of travel is changed. Whenever rocket engines are fired, the rocket exerts a backward force on the escaping gases. According to Newton's third law, the gases exert an equal forward force on the rocket propelling it forwards, as shown in Figure 1.11.

Fig 1.11: The second stage rocket engine is fired after the first stage has been jettisoned.

Applying Newton's second law in terms of momentum

The motion of an object will change only when a net force acts on it. In other words, the momentum of an object will change over time only when a net force acts on that object. For example, when a car is involved in a crash test, the net force of the wall on the car will change the momentum of the car.

Worked examples:

1. Suppose two cars of equal mass (800 kg) are involved in separate crash tests. One of the crash tests is shown in Figure 1.12. Both cars have an initial velocity of 20 m.s^{-1} before colliding with the wall. Car A collides with the wall and comes to rest during the collision, whereas car B collides with the wall and rebounds with a velocity of 5 m.s^{-1} away from the wall. Both collisions last for 0,2 s.

Fig 1.12: A crash test

   a) What exerts the force on each car?

   b) How will the change in momentum of the two cars compare?

   c) How will the net force acting on each car compare?

   d) Why would car manufacturers design cars that will be brought to rest rather than rebound during a collision?

Solution:

a) The wall.

b) Let the direction toward the wall be positive.

Car A: $\Delta \boldsymbol{p} = m\boldsymbol{v}_\text{f} - m\boldsymbol{v}_\text{i}$

$\Delta \boldsymbol{p} = (800)(0) - (800)(+20)$

$\Delta \boldsymbol{p} = -16000$ kg.m.s^{-1} = 16 000 kg.m.s^{-1} away from the wall

Car B: $\Delta \boldsymbol{p} = m\boldsymbol{v}_\text{f} - m\boldsymbol{v}_\text{i}$

$\Delta \boldsymbol{p} = (800)(-5) - (800)(+20)$

$\Delta \boldsymbol{p} = -4000 - 16000 = -20\,000$ kg.m.s^{-1} = 20 000 kgm.s^{-1} away from the wall

Car B experiences the greater change in momentum.

c) Car A: $\boldsymbol{F}_\text{net} = \dfrac{\Delta \boldsymbol{p}}{\Delta t}$

$\boldsymbol{F}_\text{net} = \dfrac{-16\,000}{0{,}2} = -80\,000$ N = 80 000 N away from the wall

Car B: $\boldsymbol{F}_\text{net} = \dfrac{-20\,000}{0{,}2} = -100\,000$ N = 100 000 N away from the wall

Car B experiences the greater net force.

d) The net force experienced by the car is directly proportional to the change in momentum of the car. Cars that rebound during a collision experience a greater change in momentum and therefore experience a greater net force. This would lead to more damage to the car and increase the chances of fatal injuries to the passengers of the car.

2. In a strongman competition, a competitor pulls a 5 000 kg truck with an average net horizontal force of 2 000 N. This increases the truck's velocity from 1 m.s^{-1} to 3 m.s^{-1} down the road, as shown in Figure 1.13.

Fig 1.13: A strongman pulls a truck.

   a) How long did it take the strongman to change the truck's momentum?

   b) Another competitor pulls the truck with a force of 3 200 N. This also increases the truck's velocity from 1 m.s^{-1} to 3 m.s^{-1} down the road. How long does it take this competitor to change the truck's momentum?

Solution:

a) Let the forward direction be positive.

The truck's change in momentum is:

$\Delta \boldsymbol{p} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$

$\Delta \boldsymbol{p} = (5\,000)(+3) - (5\,000)(+1) = 15\,000 - 5\,000 = 10\,000$ kg.m.s^{-1} forwards

The time taken to change the truck's momentum:

$$\Delta t = \frac{\Delta \boldsymbol{p}}{\boldsymbol{F}_{net}} = \frac{+10\,000}{+2\,000} = 5 \text{ s}$$

The strongman takes 5 s to change the truck's momentum.

b) The time taken to change the truck's momentum :

$$\Delta t = \frac{\Delta \boldsymbol{p}}{\boldsymbol{F}_{net}} = \frac{+10\,000}{+3\,200} = 3{,}1 \text{ s}$$

This strongman only takes 3,1 s to produce the same change in the truck's momentum, since he applies a greater force than the first strongman applied.

3. In Figure 1.14, a 65 kg Olympic springboard diver jumps into the air at the end of a diving board. The diver lands on the springboard, travelling downward at 8 m.s^{-1} and leaves the springboard moving upward at 12 m.s^{-1}.

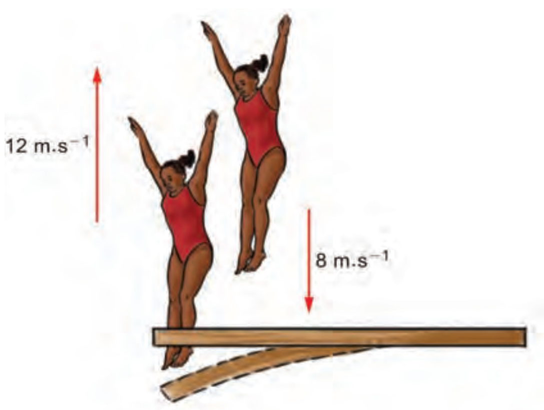


Fig 1.14: A diver collides with a springboard.

   a) Draw a free-body diagram of the forces acting on the diver while in contact with the springboard.

   b) Calculate the diver's change in momentum while she is in contact with the springboard?

   c) The diver's feet are in contact with the springboard for 0,8 s. Calculate the force that the springboard exerts on her.

Solution:

a) 

b) Let upwards be positive.

The diver is initially moving at 8 m.s^{-1} down, therefore $\boldsymbol{v}_i = -8$ m.s^{-1}

The diver leaves the springboard moving at 12 m.s^{-1} upwards. $\boldsymbol{v}_f = +12$ m.s^{-1}

$\Delta \boldsymbol{p} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$

$\Delta \boldsymbol{p} = (65)(+12) - (65)(-8) = +780 + 520$

$\Delta \boldsymbol{p} = 1\,300$ kg.m.s^{-1} upwards

c) Calculate the diver's weight:

$\boldsymbol{F}_g = m\boldsymbol{g}$

$\boldsymbol{F}_g = (65)(-9{,}8) = -637$ N $= 637$ N down

Apply Newton's second law:

$$\boldsymbol{F}_{net} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

$$\boldsymbol{F} + \boldsymbol{F}_g = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

$$\boldsymbol{F} + (-637) = \frac{+1300}{0{,}8}$$

$$\boldsymbol{F} - 637 = +1625$$

$$\boldsymbol{F} = +1625 + 637 = 2\,262 \text{ N upwards}$$

Science around us

Drag reduction system

Design engineers have introduced a drag reduction system (or DRS) on the rear wing of all formula one cars aimed at reducing aerodynamic drag (air resistance). The rear wing of the car can be opened as shown in Figure 1.15 at specified parts of the race circuit. The open wing reduces friction, and so increases the net forward force acting on the car, increasing the car's acceleration. This will help a driver overtake another car.

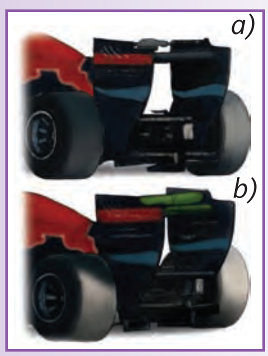


Fig 1.15: The drag reduction system on the rear wing is a) open and b) closed.

Checkpoint 3

A golfer is playing golf. The club head of mass 0,2 kg is travelling north at 45 m.s^{-1} before striking the golf ball. The club head is in contact with the ball for 0,5 milliseconds (0,0005 s) and the velocity of the club head after the collision with the ball is 40 m.s^{-1} north as shown in Figure 1.16.

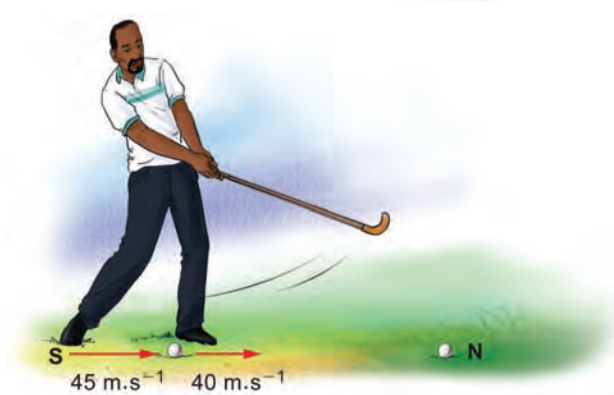


Fig 1.16: The club head's velocity before and after the collision.

a) Draw a fully labelled force diagram for the collision between the club and the ball. Only consider horizontal forces. Ignore friction.

b) Calculate the average net force acting on the club head during its contact with the golf ball.

c) The mass of the ball is 45 g. Calculate the ball's horizontal velocity when it leaves the club head.

d) How would using a softer ball affect the contact time between the club and the ball, as well as the ball's final velocity? Assume the golfer applies the same net force in each case. Explain your answers.

Exercise 1.2

1. State Newton's second law in terms of momentum.
2. Write the equation for Newton's second law in terms of momentum. Give the meaning of each symbol in the equation, state its SI units of measurement, and whether it is a vector quantity?
3. a) State the mathematical relationship between the net force acting on an object and the object's change in momentum.

   b) State the mathematical relationship between the net force acting on an object and the time taken to change the object's momentum.
4. Will the magnitude of an object's momentum always increase if a net force acts on it? Explain, using an example.
5. Explain, using Newton's second law, why a hunter always presses the butt of a rifle tightly against his shoulder before firing.
6. A learner throws a 56,7 g tennis ball toward a wall. It strikes the wall travelling horizontally at 10 $m.s^{-1}$ and it rebounds at 8 $m.s^{-1}$. The learner then throws a ball of sticky putty, having the same mass as the tennis ball, and it hits the wall with the same velocity. The putty sticks to the wall. Both collisions last for the same length of time.

   a) Which ball experiences the greater change in momentum?

   b) Which ball experiences the greater net force?
7. A net force is required to stop a 1 000 kg car, travelling at 30 $m.s^{-1}$. By what factor would the net force acting on the car change if:

   a) the car was stopped in double the time

   b) the car was stopped in 1 tenth of the time.

   c) the car was travelling at double the velocity and is stopped in the same time?
8. Fighter pilots report that immediately after a burst of gunfire from their jet fighter, the speed of their aircraft decreases from 265 to 250 $km.h^{-1}$. Using Newton's second and third laws explain the reason for this change in motion.

9. At a buffalo jump, a 900 kg buffalo is running at 6 $m.s^{-1}$ toward the drop-off ahead when it senses danger. What horizontal force must the buffalo exert to stop itself in 2 s?

10. Refer to Figure 1.17 John is watching a game of pool. He observes a 0,17 kg pool ball travelling toward him at 5 $m.s^{-1}$. The pool ball bounces off the side cushion and travels in the opposite direction at 4,5 $m.s^{-1}$.

Fig 1.17: A pool ball bounces off the side cushion.

    a) Calculate the pool ball's change in momentum.

    b) Calculate the time that the pool ball was in contact with the cushion if the cushion exerts an average net force of 80 N on the pool ball.

    c) Suppose the pool ball was in contact with the side cushion for a shorter period of time. How would this affect the ball's change in momentum? (Assume the ball experiences the same net force.) Explain your answer.

11. In Figure 1.18 an 80 kg bungee jumper falls from a high bridge with an elastic cord attached to his ankles. The bungee jumper reaches a speed of 30 $m.s^{-1}$ before the cord begins to stretch. The cord exerts an average force of 2 300 N on the jumper over a period of 2 s.

Fig 1.18: An elastic cord is attached to a bungee jumper.

    a) Draw a free-body diagram of the forces acting on the bungee jumper while there is tension in the cord.

    b) Calculate the bungee jumper's velocity after 2 s.

12. A 275 kg motor cycle and rider accelerate from rest and reach a velocity of 20 $m.s^{-1}$ west. The motor cycle wheels exert an average force of 710 N east on the road.

    a) Calculate the minimum time taken to reach a velocity of 20 $m.s^{-1}$ west.

    b) Explain how the force directed east causes the motorcycle to accelerate west.

    c) Explain why it is necessary to specify a minimum time.

13. A Centaur rocket engine expels 520 kg of exhaust gas at 50 000 $m.s^{-1}$ in 0,4 s. What is the magnitude of the net force that will be generated on the rocket? Ignore the gravitational force.

14. An elevator containing passengers has a total mass of 1 700 kg. The elevator accelerates upward.

    a) Draw a free-body diagram of the forces acting on the elevator.

    b) What tension is needed in the cable to accelerate the elevator from rest to a velocity of 4,5 $m.s^{-1}$ upward in 8,8 s?

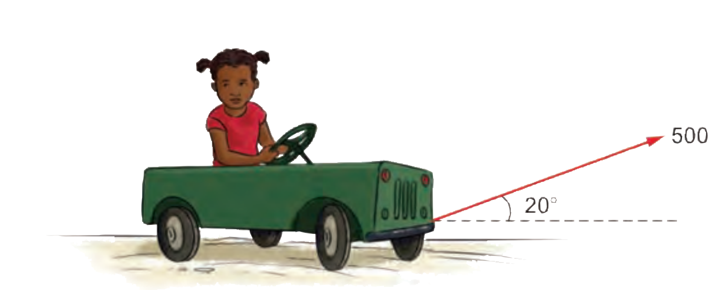


Fig 1.19: A force is applied at 20° to the horizontal

15. Refer to Figure 1.19. Zanele has a mass of 40 kg and is sitting inside a 20 kg cart. Zanele's friends pull the cart with a force of 500 N at an angle of 20° to the horizontal, using a light rope. The cart experiences a frictional force of 300 N.
    a) Calculate the net horizontal force acting on the cart.
    b) Calculate the change in the cart's momentum if Zanele's friends apply the force for 10 s.
    c) Calculate the net horizontal force acting on Zanele.
    d) How would the cart's final velocity be affected if the angle between the 500 N force and the horizontal is decreased? Explain your answer.
16. A 22 g bullet strikes a target travelling at 320 $m.s^{-1}$ and exits the target travelling at 220 $m.s^{-1}$ in the same direction. The bullet takes 0,00015 s to pass through the target.
    a) Calculate the bullet's change in momentum.
    b) Calculate the net force the bullet experiences.
17. A 1,2 kg hammer is used to hit a nail horizontally into a block of wood. The hammer is moving at 15 $m.s^{-1}$ immediately before it collides with the nail and rebounds at 60% of that speed. The nail's resisting force is 9 000 N. Calculate how long the hammer is in contact with the nail.

Conservation of momentum and elastic and inelastic collisions

Conservation of momentum

A system

A **system** is a collection of two or more objects that interact with each other. A system is a small part of the universe that we are considering when solving a particular problem. Everything outside the system is called the environment.

Some examples of a system include:

- colliding balls on a pool table
- two cars travelling in opposite directions collide on a tar road
- a rocket fires its engines in deep space and a hunter firing a bullet from his rifle.

An **isolated system** is free from the influence of a net external force that alters the momentum of the system. A net **external force** is a force that originates from a source other than the objects within the system and which is not balanced by other forces. Friction is an example of a net external force.

Fig 1.20: This can be considered an isolated system if the friction is small enough to be ignored.

A system in which the only forces that contribute to the momentum change of an individual object are the forces acting between the objects themselves can be considered an isolated system.

Consider the collision of two balls on a billiard table. The collision occurs in an isolated system as long as friction is small enough that its influence upon the momentum of the billiard balls can be neglected. If so, then the only unbalanced forces acting upon the two balls are the contact forces that they apply to one another. These two forces are considered **internal forces** since they result from a source within the system - that source being the contact of the two balls. For such a collision, total momentum of the system is conserved.

The law of conservation of momentum

The law of conservation of momentum:
The total linear momentum of an isolated system remains constant (is conserved).

In other words, if the external force of friction acting on a system is negligible, the momentum of the system immediately before the collision is the same as the momentum of the system immediately after the collision.

Figure 1.21 shows a collision between cars A and B:

Fig 1.21 A collision between two cars.

The total momentum immediately before the collision is the vector sum of A's initial momentum and B's initial momentum:

$$\boldsymbol{p}_{\text{before}} = \boldsymbol{p}_A + \boldsymbol{p}_B = m_A \boldsymbol{v}_{Ai} + m_B \boldsymbol{v}_{Bi}$$

The total momentum immediately after the collision is the vector sum of A's final momentum and B's final momentum:

$$\boldsymbol{p}_{\text{after}} = \boldsymbol{p}_A + \boldsymbol{p}_B = m_A \boldsymbol{v}_{Af} + m_B \boldsymbol{v}_{Bf}$$

In an isolated system, the total momentum is conserved (remains constant):

The total momentum before the collision = The total momentum after the collision.

$$m_A \boldsymbol{v}_{Ai} + m_B \boldsymbol{v}_{Bi} = m_A \boldsymbol{v}_{Af} + m_B \boldsymbol{v}_{Bf}$$

In real-life collisions, the external forces acting on colliding objects are usually not known. However, they are often negligible, and so can be ignored. In other words, the colliding objects are an isolated system. The law of conservation of momentum can therefore be applied to the collision.

Applying the law of conservation of momentum

The following examples illustrate how the law of conservation of momentum can be applied.

Worked examples:

1. Refer to Figure 1.22. An object with a mass of 1 kg is moving to the right with a velocity of 2 m.s^{-1}. It collides head-on with a second mass of 1,5 kg which is moving with a velocity of 1,5 m.s^{-1} in the opposite direction. The 1,5 kg mass bounces back with a velocity of 1,2 m.s^{-1} to the right. Calculate the velocity of the 1 kg mass after the collision. Ignore the effects of friction.

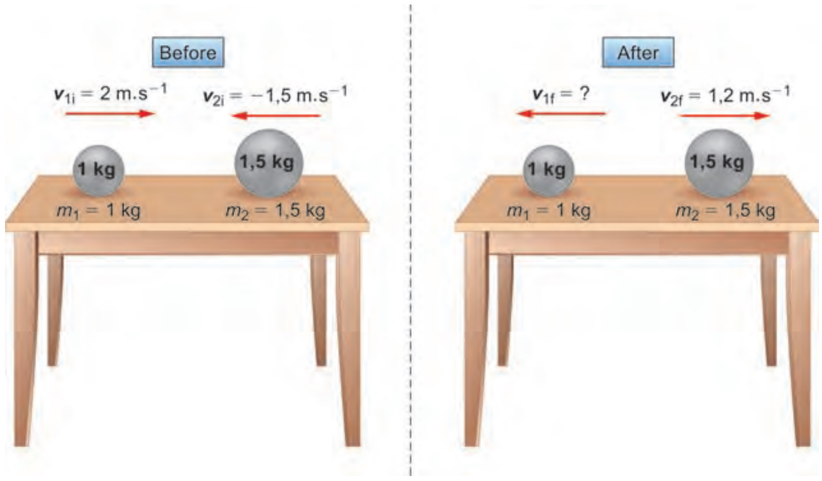


Fig 1.22

Solution:

When dealing with momentum questions you should choose positive and negative directions for velocities. In this example, we choose right as positive and therefore left as negative.

$v_{1i} = +2 \text{ m.s}^{-1}$

$v_{2i} = -1{,}5 \text{ m.s}^{-1}$

$m_1 = 1 \text{ kg}$

$m_2 = 1{,}5 \text{ kg}$

$v_{1f} = ?$

$v_{2f} = +1{,}2 \text{ m.s}^{-1}$

$$\begin{aligned}
\text{Total momentum before the collison} &= \text{Total momentum after the collision}\\
m_1 v_{1i} + m_2 v_{i2} &= m_1 v_{1f} + m_2 v_{2f}\\
(1)(+2) + (1{,}5)(-1{,}5) &= (1)v_{1f} + (1{,}5)(+1{,}2)\\
2 - 2{,}25 &= v_{1f} + 1{,}8\\
v_{1f} &= -2{,}05\\
v_{1f} &= 2{,}05 \text{ m.s}^{-1} \text{ left}
\end{aligned}$$

2. In Figure 1.23, a 20 g bullet fired from a rifle, strikes a sand bag fastened to a trolley of combined mass 4 kg travelling west at 2 m.s^{-1} on a frictionless surface. The bullet becomes embedded in the sandbag. The bullet's velocity immediately before impact is 250 m.s^{-1} east. Calculate the velocity of the trolley immediately after the collision.

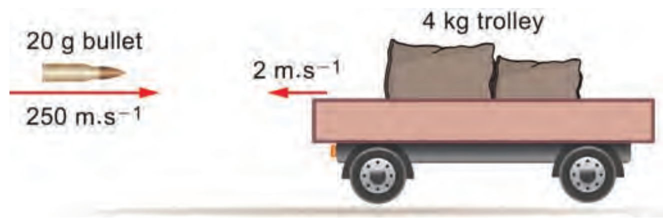


Fig 1.23: A bullet becomes embedded in a sandbag on a trolley

Solution:

Choose east as positive.

Mass of bullet: $m_B = 0{,}02$ kg Mass of trolley: $m_T = 4$ kg

Initial velocity of bullet: $v_{Bi} = +250 \text{ m.s}^{-1}$ Initial velocity of trolley: $v_{Ti} = -2 \text{ m.s}^{-1}$

The bullet and trolley have a combined mass after the collision: $m_{total} = (4 + 0{,}02) = 4{,}02$ kg

$$\begin{aligned}
\text{Total momentum before the collision} &= \text{Total momentum after the collision}\\
m_B v_{Bi} \quad + \quad m_T v_{Ti} &= m_{total} \cdot v_f\\
(0{,}02)(+250) \quad + \quad (4)(-2) &= (4{,}02)v_f\\
+5 \quad - \quad 8 &= (4{,}02)v_f\\
-3 &= (4{,}02)v_f\\
v_f &= -0{,}75 \text{ m.s}^{-1}\\
v_f &= 0{,}75 \text{ m.s}^{-1} \quad \text{west}
\end{aligned}$$

The trolley is travelling at 0,75 m.s^{-1} west after the collision.

3. Figure 1.24 shows an experiment to measure the recoil velocity of a fired rifle. A rifle is mounted on a trolley, at rest on a frictionless surface. The combined mass of the rifle and trolley is 4,2 kg. The rifle is loaded with a single 167 g bullet which it fires at 500 m.s^{-1} to the right.

   a) Calculate the rifle's recoil velocity.

   b) Without any further calculations, state the total momentum after the explosion. Explain your answer.

   c) Why is the recoil velocity of the rifle so much less than the velocity of the bullet?

Fig 1.24: A rifle mounted on a trolley

Solution:

a) To the right is positive.

Mass of trolley:	$m_T = 4{,}2$ kg	Mass of bullet:	$m_B = 0{,}167$ kg
Initial velocity of trolley:	$v_{Ti} = 0$	Initial velocity of bullet:	$v_{Bi} = 0$
Final velocity of trolley:	unknown	Final velocity of bullet:	$v_{Bf} = +\,500\ \text{m.s}^{-1}$

Total momentum before = Total momentum after

$$\begin{aligned} m_T v_{Ti} + m_B v_{Bi} &= m_T v_{Tf} + m_B v_{Bf} \\ 0 + 0 &= (4{,}2)v_{Tf} + (0{,}167)(+500) \\ 0 &= (4{,}2)v_{Tf} + 83{,}5 \\ -83{,}5 &= (4{,}2)v_{Tf} \\ v_{Tf} &= -19{,}88\ \text{m.s}^{-1} \\ v_{Tf} &= 19{,}88\ \text{m.s}^{-1}\ \text{to the left} \end{aligned}$$

The rifle and trolley move to the left at 19,88 m.s^{-1} after the rifle is fired.

b) The total momentum after the collision is zero. Momentum is conserved during the explosion. The total momentum before the collision is zero.

c) Both the bullet and the rifle experience the same change in momentum but in opposite directions (momentum is conserved). However the rifle has a greater mass and will therefore experience a smaller change in velocity in the opposite direction.

Checkpoint 4

1. Refer to Figure 1.25. Cart B, of mass 350 g, moves on the frictionless linear air track at 2 m.s^{-1} to the left. Cart B strikes cart A, of mass 200 g, travelling in the opposite direction at 1,2 m.s^{-1}. After the collision, cart B continues in its original direction at 0,7 m.s^{-1}.
   a) Why is this considered an isolated system of colliding bodies?
   b) Calculate the velocity of cart A after the collision.
   c) How does the change in momentum of each cart compare? Check your answer using calculations.

Fig 1.25: Two carts moving in opposite directions on a linear air track

2. A wooden block attached to a glider has a combined mass of 0,2 kg. Both the block and the glider are at rest on a frictionless air track, as shown in Figure 1.26. A dart gun shoots a 0,012 kg dart into the block. The velocity of the dart-block system after the collision is 0,78 m.s^{-1} to the right. Calculate the velocity of the dart just before it hits the block?

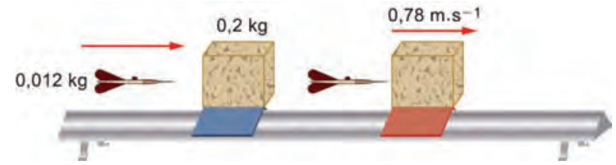


Fig 1.26: A dart becomes embedded in a wooden block

3. Refer to Figure 1.27. A compressed spring is loaded between two trolleys (A and B) at rest on a frictionless surface. The spring is released and the two trolleys move off in opposite directions. After the spring is released, trolley A's velocity is 3 m.s^{-1} to the left. Calculate trolley B's velocity after the spring is released.

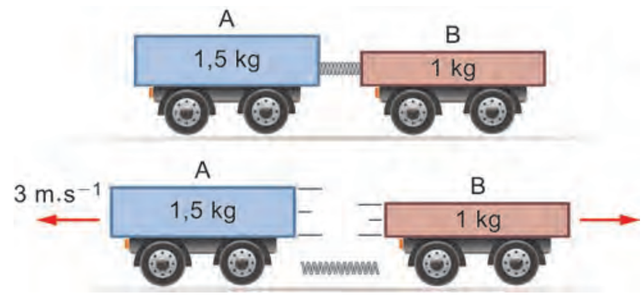


Fig 1.27: A spring is released between two trollies.

Prescribed experiment for formal assessment

Aim: To verify the conservation of linear momentum

You will need:

Two spring-loaded trolleys; two stopwatches; metre-stick; mass scale; two barriers (blocks of wood) and five 100 g mass pieces

Method:

1. Work in groups.
2. Copy the table below.

Trolley 1				Trolley 2			
m_1 (kg)	Time Δt (s)	Velocity v_1(m.s^{-1})	Momentum m_1v_1 (kg.m.s^{-1})	m_2 (kg)	Time Δt (s)	Velocity v_2(m.s^{-1})	Momentum m_2v_2 (kg.m.s^{-1})

3. You are supplied with two spring-loaded trolleys: 1 and 2. Use a scale to measure the mass of each trolley (m_1 and m_2). Record these values in the table.
4. Each trolley is fitted with a spring-loaded plunger, as shown in Figure 1.28. Push the spring-loaded plunger in and use the lever on the side of the trolley to hold the plunger in position. When the lever on side of the trolley is released, the spring-loaded plunger shoots out at the front of the trolley.

Fig 1.28: A trolley with a spring loaded plunger.

5. Place trolley 1 on a lab bench and reload the plunger. Position trolley 2 so that it is in contact with trolley 1, as shown in Figure 1.29.

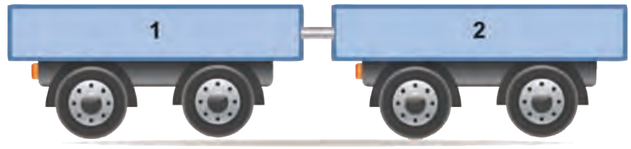


Fig 1.29: Trolley 2 is in contact with the spring loaded plunger of trolley 1.

6. Release the lever on the side of the trolley. Observe how the trolleys are forced apart and move in opposite directions. According to Newton's third law, each trolley experiences the same net force in opposite directions.

Fig 1.30: The trolleys are exploded apart.

7. Practice "exploding" the trolleys apart until both trolleys move in a straight line in opposite directions after the plunger is released.
8. Refer to Figure 1.31. Reload the plunger of trolley 1 and position trolley 2 so that is in contact with the plunger of trolley 1. Mark the position of the front of each trolley on the lab bench. You do this so that you can repeat the experiment from the same position.

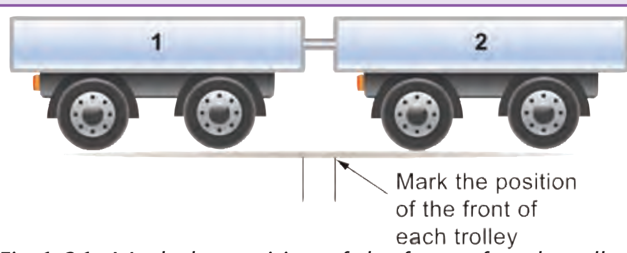


Fig 1.31: Mark the position of the front of each trolley.

9. Use the metre-stick to measure 1 m along the lab bench from the back of each trolley. Place a barrier at these points, as shown in Figure 1.32.

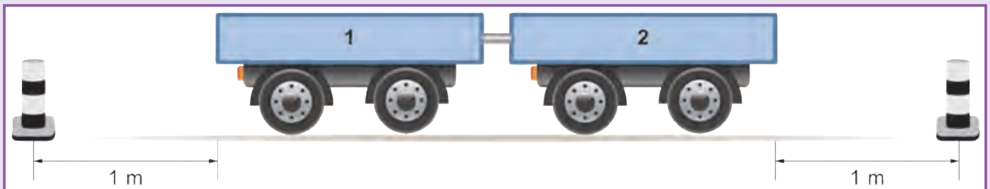


Fig 1.32: Measure a distance of 1m from the back of each trolley.

10. You are about to measure the time (Δt) it takes for each trolley to cover a distance of 1 m after they are "exploded" apart. Two learners will use the stopwatches. The third learner will release the lever to "explode" the trolleys apart.
11. Each learner holding a stopwatch should select a trolley (1 or 2). Each learner should start the stopwatch as the plunger is released and stop the stopwatch as the trolley collides with the barrier. Record these times. Repeat the experiment three times to improve your accuracy. Be sure to reposition the barriers before taking the next set of readings. Record the average time (Δt) in the table.
12. The precision of the stop-watch is one 100^{th} of a second (0,01 s). Your precision in pressing the stop-watch after your reaction-time, is about one 10^{th} of a second (0,1 s). Therefore, round off the time values to one decimal place.
13. Since you are working with vector quantities, choose which direction to take as positive (e.g. let direction to the right be positive).
14. Calculate the average velocity ($\boldsymbol{v}_1$ and $\boldsymbol{v}_2$) of each trolley. Record these velocities in the table. Be sure to include the correct sign (+ or –) and round the values off to one decimal place.
15. Calculate the momentum of each trolley ($m_1\boldsymbol{v}_1$ and $m_2\boldsymbol{v}_2$) and record it in the table. Be sure to include the correct sign (+ or –) and round the values off to one decimal place.
16. Increase the mass of trolley 1 by sticking five 100 g mass pieces to the top of the trolley. Repeat the experiment and record your data in a new table.

Questions:

Complete a full experimental write-up.

Include the following sub headings in the write-up:

- Aim
- Hypothesis
- Apparatus
- Method
- Results
- Analysis of results
- Conclusion
- Sources of error

Exercise 1.3

1. State the law of conservation of momentum.
2. a) In the context of momentum, what is an isolated system?
   b) Why is it necessary to choose an isolated system when solving a momentum problem?
3. How do internal forces affect the momentum of a system?
4. Give one example of a possible collision between two identical masses. Include a sketch of the situation, showing the velocity of each object immediately before and immediately after the collision.

5. A learner is standing on a stationary 2,3 kg skateboard. If the learner jumps at a velocity of 0,37 $m.s^{-1}$ forward, the skateboard's velocity becomes 8,9 $m.s^{-1}$ backward. Calculate the mass of the learner?
6. A 110 kg astronaut and a 4 000 kg spacecraft are attached by a tethering cable. Both masses are motionless relative to an observer near the spacecraft. The astronaut wants to return to the spacecraft, so he pulls on the cable until his velocity is 0,8 $m.s^{-1}$ toward the spacecraft.
   a) Calculate the change in velocity of the spacecraft?
   b) Explain how pulling on the tethering cable in one direction causes the astronaut to move in the opposite direction.
7. A 75 kg hunter is in a 10 kg stationary canoe, on the water. He throws a 0,72 kg spear at a velocity of 12 $m.s^{-1}$ to the right.
   a) Calculate the velocity of the canoe and hunter immediately after the spear is released.
   b) How would this calculated velocity be affected if a spear of greater mass was thrown at the same velocity?
8. A student on a skateboard, with a combined mass of 78,2 kg, is moving east at 1,6 $m.s^{-1}$. As he goes by, the learner skillfully scoops his 6,4 kg school bag from the bench where he had left it.
   a) Calculate the velocity of the learner immediately after the pickup.
   b) How does the change in momentum of the learner (and skateboard) compare with the change in momentum of the school bag? Explain your answer.
9. A 1 050 kg car has a velocity of 2,65 $m.s^{-1}$ north. The car hits the rear of a stationary truck, and the bumpers lock together. The velocity of the car-truck system immediately after the collision is 0,78 $m.s^{-1}$ north. Calculate the mass of the truck?
10. A 0,25 kg volleyball is thrown horizontally at 2 $m.s^{-1}$ west. It strikes a 0,62 kg stationary basketball. The volleyball rebounds east at 0,79 $m.s^{-1}$. Calculate the velocity of the basketball immediately after the collision?
11. A glider of mass m and velocity $\boldsymbol{v}$, moving to the right along an air track, collides with a stationary cart of mass $\frac{1}{3}m$. After the collision, the velocity of the glider is $\frac{1}{2}\boldsymbol{v}$, in the same direction. What is the velocity of the cart (in terms of $\boldsymbol{v}$)?
12. Two ice skaters, one of mass 50 kg and the other of mass 60 kg, push off against one another, starting from a stationary position. The 50 kg skater acquires a velocity of 0,55 $m.s^{-1}$ to the right.
   a) How does the momentum of each skater compare after they are pushed apart?
   b) Which skater should have the greater velocity after they are pushed apart? Explain your answer.
   c) Calculate the 60 kg skater's velocity after they are pushed apart.

13. A 0,6 kg glider, travelling to the right on a level air track, undergoes a head-on collision with a 0,2 kg glider travelling toward it at 4 $m.s^{-1}$. After the collision the 0,6 kg glider is travelling at 3 $m.s^{-1}$ to the right and the 0,2 kg glider is travelling at 11 $m.s^{-1}$ to the right. Calculate the velocity of the 0,6 kg glider before the collision.
14. An 800 kg car is at rest at a traffic light. A 1200 kg car, travelling at 12 $m.s^{-1}$, collides with the car at rest. The two cars are locked together after the collision. Calculate the magnitude of their velocity after the collision.
15. Judy has a mass of 45 kg and is wearing ice skates. She is standing on the ice rink when her friend throws an 8 kg school bag horizontally toward her at 3 $m.s^{-1}$. Judy catches the school bag.
    a) Calculate the velocity of Judy and the school bag immediately after she catches it.
    b) How would the magnitude of Judy's final velocity change if she caught the same school bag but it was thrown with a greater velocity? Explain your answer.
16. A 20 g bullet is travelling west at 500 $m.s^{-1}$, toward a 30 kg wooded block at rest on a frictionless horizontal surface. The bullet collides with the block, embedding itself into the block. Calculate the magnitude of the velocity of the block and bullet after the collision.

Inelastic and elastic collisions

Inelastic collisions

The collisions that we have considered so far have involved hard objects, such as a golf club hitting the golf ball. Other collisions such as the dart and block, involved a dart that became embedded in a softer material (a block of wood). In all these collisions it was possible to choose an isolated system so that the total momentum of the system was conserved.

When objects collide, they sometimes deform (change shape), make a sound, give off light, or heat up a little during the collision. Any of these observations indicate that the kinetic energy of the system before the collision is not the same as after the collision. These collisions are known as inelastic collisions. During an **inelastic collision**, kinetic energy is not conserved.

Each impact of a bouncing ball is inelastic. The energy is dissipated to the surroundings during each bounce. Look at the bouncing ball in Figure 1.33. Each bounce is lower than the previous bounce, showing that kinetic energy is not conserved. Most real-life collisions are inelastic.

Fig 1.33: Each bounce of the ball is an inelastic collision.

Elastic collisions

An **elastic collision** is defined as one in which kinetic energy is conserved. In other words, the kinetic energy of the system does not change during the collision. The sum of the kinetic energies of the objects before the collision would be exactly equal to the sum of the kinetic energies of the objects after the collision.

Fig 1.34: The collisions between billiard balls are almost elastic.

Most real world collisions involve some of the initial kinetic energy of the system being converted into sound, light, deformation and heat (due to friction). These factors make it difficult to achieve an elastic collision.

Even when two colliding objects are hard and do not appear to deform, some kinetic energy is still converted to other forms of energy. Usually the measured speed of an object after the collision is a little less than the predicted speed, which indicates that the collision is inelastic. Completely inelastic collisions occur when the colliding objects stick together upon impact.

Momentum is conserved in both elastic and inelastic collisions.

Problems involving elastic and inelastic collisions

The following example demonstrates how to determine if the collision between two objects is elastic.

Worked example:

Refer to Figure 1.35. A 0,16 kg ball A, travelling at 1,2 m.s^{-1} east, strikes a stationary 0,18 kg ball B, and rebounds at 0,075 m.s^{-1} west. B moves off at 1,0 m.s^{-1} east. Is the collision elastic?

Solution:

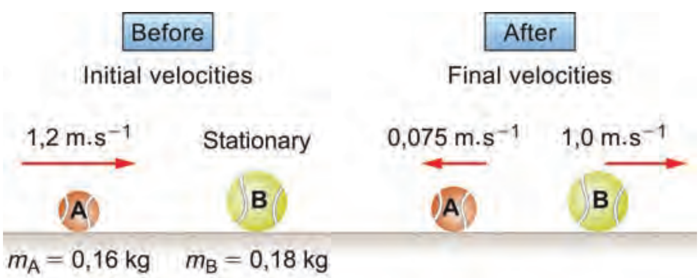


Fig 1.35

Calculate the total kinetic energy of the system before the collision:

Ball A: $E_K = \frac{1}{2}mv^2 = \frac{1}{2}(0,16)(1,2)^2 = 0,115$ J

Ball B: $E_K = 0$

The total kinetic energy is the sum of the kinetic energies of each ball:

Total kinetic energy before the collision:
$E_K = 0,115 + 0 = 0,115$ J

Calculate the total kinetic energy after the collision.

Ball A: $E_K = \frac{1}{2}mv^2 = \frac{1}{2}(0,16)(0,075)^2 = 0,0005$ J

Ball B: $E_K = \frac{1}{2}(0,18)(1)^2 = 0,09$ J

Total kinetic energy after the collision: $E_K = 0,0005 + 0,09 = 0,0905$ J

The total kinetic energy of the balls before the collision (0,115 J) is not equal to the total kinetic energy of the balls after the collision (0,0905 J). Therefore the collision is inelastic.

Although the kinetic energy of the system is not conserved in this example, the momentum of an isolated system is always conserved. You should check this with a calculation.

Check point 5

In Figure 1.36, a 45,9 g golf ball is stationary on the green when a 185 g golf club face, travelling at 1,24 $m.s^{-1}$ east, strikes it. After the impact the club continues moving at 0,76 $m.s^{-1}$ east while the golf ball moves at 1,94 $m.s^{-1}$ east. Assume that the club is vertical at the moment of impact, so that the ball does not spin. Determine if the collision is elastic.

Fig 1.36

Recommended demonstration for informal assessment

Aim: To observe collisions

You will need:

- A Newton cradle, as shown in Figure 1.37.

Fig 1.37: A Newton cradle

Method and questions:

Follow the instructions and answer the questions:

1. State the law of conservation of mechanical energy (you learnt this in Grade 10).
2. Raise one ball of the Newton cradle and release it. At what position does the ball have:
   a) maximum gravitational potential energy
   b) minimum gravitational potential energy
   c) maximum kinetic energy
   d) zero kinetic energy
   e) maximum momentum?
3. After releasing a ball, what happens to the other balls? Describe how many balls move and how high they move compared to the original ball that was released.
4. Explain your observations in terms of the law of conservation of energy.
5. What forces are acting on a ball when it:
   a) swings down
   b) collides with another ball?
6. Explain your observations in terms of the law of conservation of momentum.

7. Explain why the balls will eventually come to rest.
8. Are these collisions elastic? Explain your answer.
9. What do you observe when:
   a) two balls are raised and released
   b) three balls are raised and released.

   Comment on the number of balls that move and the height they reach, in each case.
10. Explain your observations in terms of the law of conservation of momentum.
11. If two balls are raised and released, is it possible for one ball to move off at the other end of the cradle? Explain your answer.

Exercise 1.4

1. Explain the difference between elastic and inelastic collisions. Include an example of each type of collision in your answer.
2. What evidence suggests that a collision is inelastic?
3. Which physical quantity is conserved in both elastic and inelastic collisions for isolated systems?
4. During an inelastic collision, some kinetic energy is lost by the system of colliding objects. List three ways in which kinetic energy can be converted to other forms of energy.
5. A 6 g glass ball, A, moving east at 19 $m.s^{-1}$, collides with another 9 g glass ball, B, moving at 11 $m.s^{-1}$ in the same direction. After the collision, ball A moves west at 9,4 $m.s^{-1}$ and ball B continues moving east at 17,4 $m.s^{-1}$. Show that the collision is elastic.
6. A 0,3 kg cart, moving to the right on a frictionless linear air track at 4 $m.s^{-1}$ strikes a second cart of mass 0,5 kg, travelling in the opposite direction at 3 $m.s^{-1}$. The collision between the two carts is elastic. After the collision, the first cart is travelling in the opposite direction at 4,75 $m.s^{-1}$. Find the second cart's velocity after the collision, using two different methods.
7. A 1 700 kg car moves at 25 $m.s^{-1}$ west. It collides with a 3 400 kg truck travelling at 14 $m.s^{-1}$ east. After the collision, the car travels at 10 $m.s^{-1}$ east.
   a) Calculate the truck's velocity after the collision, using the law of conservation of momentum.
   b) Show that this collision is inelastic.
   c) What percentage of the system's kinetic energy is lost in the collision?
   d) Account for the 'missing' kinetic energy.

8. A 70 kg girl is running at $3\ m.s^{-1}$ east when she jumps onto a 2 kg stationary skate board.

   a) Calculate the velocity of the girl and the skateboard after she has landed on it.

   b) Show that this collision is inelastic.

9. A wrestler stands at rest. Another wrestler, running at $5\ m.s^{-1}$ to the right, collides with the first wrestler, grabs him and holds onto him. The two move off together at $2{,}7\ m.s^{-1}$ in the direction the second wrestler had been running.

   If the second wrestler's mass is 100 kg, calculate the mass of the first wrestler.

Impulse

Impulse-momentum theorem

Fig 1.38: The thick mattress on the ground provides a protective cushion for the stunt person when he lands.

During the filming of a movie, when a stunt person jumps off a building, the fall can be very dangerous. To minimize injury, stunt people avoid a sudden stop when landing by using different techniques to slow down more gradually out of sight of the cameras. These techniques involve reducing the peak force required to change their momentum. Sometimes stunt people jump and land on a net. Other times, they may roll when they land. For extreme jumps, such as from a roof of a tall building, a huge oversized, but slightly under-inflated, air mattress may be used, as shown in Figure 1.38. Designers of safety equipment know that a cushioned surface can reduce the severity of the impact.

To understand the factors that affect the net force acting on objects during a collision requires looking at Newton's second law written in terms of momentum.

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

If we multiply both sides of this equation by Δt, we get: $\boldsymbol{F}_{\text{net}}\Delta t = \Delta \boldsymbol{p}$

The above relationship is known as the **impulse-momentum theorem**.

In this equation, the product of net force and the interaction time, $\boldsymbol{F}_{\text{net}}\Delta t$, is called impulse. **Impulse** is defined as the product of net force acting on an object and the time for which it acts.

The above equation also tells us that impulse, $\boldsymbol{F}_{\text{net}}\Delta t$, is equivalent to the change in momentum, $\Delta \boldsymbol{p}$, that an object will experience during a collision. $\boldsymbol{F}_{\text{net}}\Delta t = \Delta \boldsymbol{p}$

If we substitute the equation for change in momentum, $\Delta \boldsymbol{p} = m\boldsymbol{v}_{\text{f}} - m\boldsymbol{v}_{\text{i}}$, we get:

$$\boldsymbol{F}_{\text{net}}\Delta t = m\boldsymbol{v}_{\text{f}} - m\boldsymbol{v}_{\text{i}}$$

$$\boldsymbol{F}_{\text{net}}\Delta t = m(\boldsymbol{v}_{\text{f}} - \boldsymbol{v}_{\text{i}})$$

But $(\boldsymbol{v}_{\text{f}} - \boldsymbol{v}_{\text{i}})$ represents the change in velocity $\Delta \boldsymbol{v}$ of an object during a collision.

Therefore: $\boldsymbol{F}_{\text{net}}\Delta t = m\Delta \boldsymbol{v}$

The unit for impulse is the N.s. If we substitute the definition of a Newton into the unit N.s, we get:

$1\text{ N.s} = 1\left(\frac{\text{kg.m}}{\text{s}^2.\text{s}}\right) = 1\,\frac{\text{kg.m}}{\text{s}} = 1\text{ kg.m.s}^{-1}$ which is the unit for momentum.

So the units on both sides of the impulse-momentum theorem are equivalent ($\text{N.s} = \text{kg.m.s}^{-1}$).

Since force is a vector quantity, impulse ($\boldsymbol{F}_{\text{net}}\Delta t$), is also a vector quantity, and the direction of impulse is in the same direction as the net force.

In real life situations, collisions such as the collision between the racquet and the ball, shown in Figure 1.39 (on the next page) occur during a very short time interval. If you tried to accurately measure the net force, you would find it is difficult, if not impossible. During a collision the net force increases from zero to a very large value in a short time interval as shown in Figure 1.40.

Fig 1.39: *Some collisions occur during a very short time interval, such as the collision between a racquet and a tennis ball.*

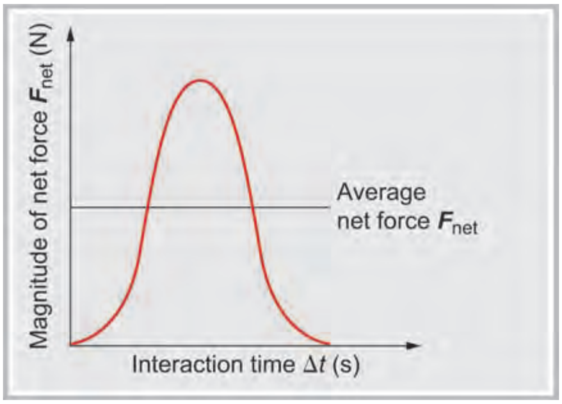


Fig 1.40: During a collision the net force increases from zero to a maximum value in a short time interval.

From a practical point of view, it is much easier to measure the interaction time and the overall change in momentum of an object rather than the net force.

This is the advantage of using the impulse-momentum theorem: $\boldsymbol{F}_{net}\Delta t = \Delta\boldsymbol{p}$

Applying the impulse-momentum theorem

Worked examples:

1. A golf ball of mass 0,1 kg is driven from the tee. The average accelerating force exerted by the golf club is 1 000 N, and the ball moves away from the club at 30 m.s^{-1}. For how long was the club in contact with the ball?

Solution:

$m = 0{,}1$ kg	$\boldsymbol{F}_{net}\Delta t = m\Delta\boldsymbol{v}$
$\boldsymbol{F}_{net} = 1\ 000$ N	$\boldsymbol{F}_{net}\Delta t = m(\boldsymbol{v}_f - \boldsymbol{v}_i)$
$\boldsymbol{v}_i = 0$ m.s^{-1}	$(1\ 000)\Delta t = (0{,}1)(30) - (0{,}1)(0)$
$\boldsymbol{v}_f = 30$ m.s^{-1}	$\Delta t = 0{,}003$ s

2. A ball is travelling left and is struck by a bat. The bat exerts a force of 75 N on the ball and is in contact with the ball for 0,08 s. The ball moves off the bat to the right. Calculate the change in momentum of the ball.

Solution:

Take right as the positive direction.	$\Delta\boldsymbol{p} = \boldsymbol{F}_{net}\Delta t$
$\boldsymbol{F}_{net} = 75$ N	$\Delta\boldsymbol{p} = (75)(0{,}08)$
$\Delta t = 0{,}08$ s	$\Delta\boldsymbol{p} = 6$ kg.m.s^{-1} to the right

Fig 1.41: The front of the car crumples upon impact.

3. To improve safety, a modern car's front end crumples on impact. A 1 200 kg car travels at a constant velocity of 8 m.s^{-1} toward an immovable wall as shown in Figure 1.41. It hits the wall and comes to a stop in 0,25 s.
   a) Calculate the impulse provided to the car.
   b) What is the average net force exerted on the car?
   c) For the same impulse, what average net force would the wall exert on a car which stopped in 0,04 s due to having a rigid bumper and frame which do not crumple on impact?

Solution:

a) Impulse provided to the car is equivalent to the change in the car's momentum:
Let towards the wall be positive.

$\boldsymbol{F}_{net}\Delta t = \Delta \boldsymbol{p}$	$\boldsymbol{F}_{net}\Delta t = m\Delta \boldsymbol{v}$
$m = 1200$ kg	$\boldsymbol{F}_{net}\Delta t = m(\boldsymbol{v}_f - \boldsymbol{v}_i)$
$\boldsymbol{v}_i = +8$ m.s^{-1}	$\boldsymbol{F}_{net}\Delta t = (1\ 200)[0 - (+8)]$ (the car's final velocity is zero)
$\boldsymbol{v}_f = 0$ m.s^{-1}	$\boldsymbol{F}_{net}\Delta t = (1\ 200)(-8) = -9\ 600 = 9\ 600$ N.s away from the wall
$\Delta t = 0{,}25$ s	Impulse is a vector quantity. The direction of the impulse is the same as the net force of the wall on the car – away from the wall.

b) $\boldsymbol{F}_{net}\Delta t = -9\ 600$ N.s

$$\boldsymbol{F}_{net} = \frac{-9\ 600}{\Delta t} = \frac{-9\ 600}{0{,}25} = -38\ 400 \text{ N} = 38\ 400 \text{ N away from the wall}$$

Notice that the impulse and the net force have the same direction.

c) $$\boldsymbol{F}_{net} = \frac{m\Delta v}{\Delta t}$$

$$\boldsymbol{F}_{net} = \frac{-9\ 600}{0{,}04} = -240\ 000 \text{ N} = 240\ 000 \text{ N away from the wall}$$

The magnitude of the average net force with the rigid frame is more than 6 times greater than when the car crumples.

Checkpoint 6

A soccer player heads the ball with an average force of 21 N, for 0,12 s as shown in Figure 1.42.

a) Calculate the impulse provided to the soccer ball.

b) The impulse changes the ball's velocity from 4 m.s^{-1} to 2 m.s^{-1} in the opposite direction. Calculate the soccer mass of the soccer ball.

c) Sketch a graph of the net force on the ball as a function of time.

Fig 1.42 A player heads the ball

Worked example:

A basketball player shoots a 0,65 kg basketball as shown in Figure 1.43. The net force acting on the ball increases linearly from 0 N to 22 N during the first 0,15 s while it is in contact with his hand. During the next 0,25 s the net force decreases linearly to 0 N.

a) Draw a graph of net force acting on the ball as a function of time.

b) Calculate the magnitude of the impulse provided to the basketball.

c) Calculate the speed of the basketball when it leaves the hand of the shooter.

d) How would the ball's speed be affected if the same impulse was provided to a ball with less mass?

Fig 1.43: A basketball player takes a shot.

Solution:

a)

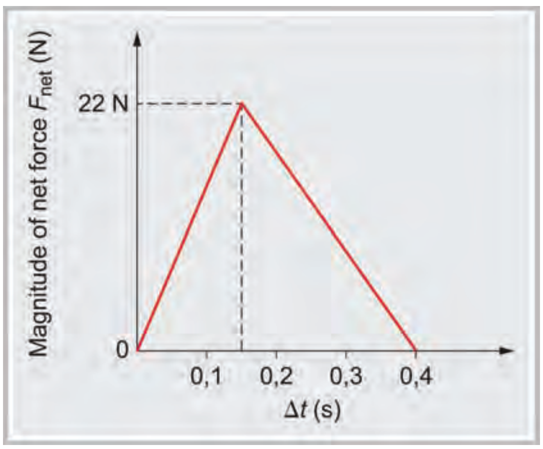


Fig 1.44

b) The area under the graph is given by the area of a triangle:

$Area = \frac{1}{2} \times base \times \perp height$

The quantity represented by the area under the graph has units of:

$Area = \frac{1}{2} \times (\text{s}) \times (\text{N}) = \text{N.s}$

The area under the graph therefore represents the impulse exerted on the ball ($\boldsymbol{F}_{net}\Delta t$):

$\boldsymbol{F}_{\text{net}}\Delta t$ = Area under graph $= \frac{1}{2}(0{,}4)(22) = 4{,}4$ N.s

c) Using the impulse-momentum theorem: $\boldsymbol{F}_{\text{net}}\Delta t = \Delta \boldsymbol{p}$

$\boldsymbol{F}_{\text{net}}\Delta t = m(\boldsymbol{v}_{\text{f}} - \boldsymbol{v}_{\text{i}})$

$4{,}4 = (0{,}65)(\boldsymbol{v}_{\text{f}} - 0)$ (The ball's initial velocity is zero)

$4{,}4 = (0{,}65)\boldsymbol{v}_{\text{f}}$

$\boldsymbol{v}_f = 6{,}77 \text{ m.s}^{-1}$

The ball leaves the shooter's hand at a speed of 6,77 m.s^{-1}

d) If the same impulse, $\boldsymbol{F}_{net}\Delta t$, is provided to the ball, then according to the impulse-momentum theorem, $\boldsymbol{F}_{net}\Delta t = \Delta \boldsymbol{p}$, the ball will experience the same change in momentum, $\Delta \boldsymbol{p}$. However, the ball's mass is less, so the ball's change in velocity, and therefore its final velocity, must be greater than for the more massive ball.

Checkpoint 7

Whiplash, shown in Figure 1.45, occurs when a car is hit from behind and the head of the motorist is not properly protected by a head rest. The seat accelerates the upper part of the body, but the head jerks backwards because it is not supported. This injures the joints and soft tissue of the neck.

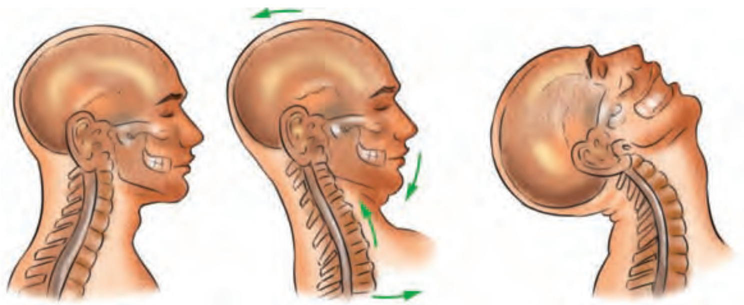


Fig 1.45 Whiplash occurs when a car is rear-ended.

a) Use Newton's first law to explain why whiplash occurs.

b) Why are cars fitted with head rests?

c) What is the average net force on a motorist's neck if the torso is accelerated from rest to 14 m.s^{-1} west in 0,135 s. The mass of the motorist's head is 5,4 kg. Assume that the same magnitude force acts on the neck as on the torso.

During a collision between two objects in an isolated system, both objects experience equal and opposite forces, as shown in Figures 1.46 and 1.47.

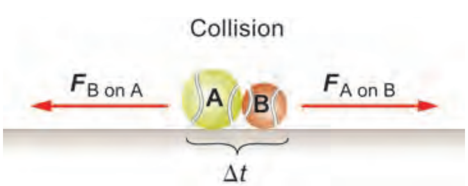


Fig 1.46: During a collision both objects experience equal and opposite forces.

Impulse on A = $\boldsymbol{F}_{B\ on\ A}\Delta t$

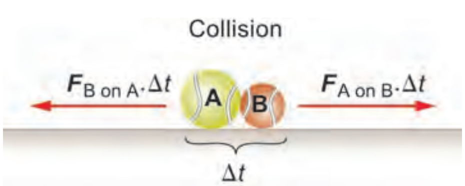


Fig 1.47: During a collision both objects experience equal and opposite impulses.

Impulse on B = $\boldsymbol{F}_{A\ on\ B}\Delta t$

According to Newton's first law, the action-reaction forces are equal in magnitude but opposite in direction. Also the time of interaction is the same for each object. It follows then that the impulse provided to object A is equal and opposite to the impulse provided to object B (Fig 1.47).

Check point 8

In Figure 1.48, an 800 kg car travels at 20 m.s^{-1} east. It collides with a 3 000 kg truck travelling at 12 m.s^{-1} west. The collision lasts 0,5 s. After the collision, the truck moves at 4 m.s^{-1} west.

Fig 1.48: A car and truck collide

a) Calculate the impulse provided to the truck.

b) What is the impulse provided to the car?

c) State the change in momentum of the car.

d) Calculate the velocity of the car after the collision.

e) Calculate the net force acting on each vehicle during the collision.

Exercise 1.5

1. a) What quantities are used to calculate impulse?

   b) State the units of impulse.

2. Explain the relationship between the units in which momentum and impulse are measured.

3. How are impulse and momentum related?

4. What is the effect on impulse if:

   a) the impact time interval is doubled

   b) the net force is reduced to $\frac{1}{3}$ of its original magnitude?

5. List two ways of increasing the impulse provided to a body.

6. A baseball player swings his bat and hits a baseball, exerting 12 000 N on the ball for 0,007 s. Calculate the impulse provided to the ball.

7. Calculate the net force required to stop a 60 kg person travelling at 30 m.s^{-1} during a time of:

   a) 5 s

   b) 0,5 s

   c) 0,05 s.

8. How much does a shoulder-launched rocket's momentum change if it experiences a thrust of 2,67 kN for 0,204 s?

9. A 62 kg male ice skater is facing a 43 kg female ice skater. They are at rest on the ice. They push off each other with a force of 200 N for 1,2 s and move in opposite directions. The female skater moves to the left and the male skater moves to the right.

   a) Calculate the impulse provided to the female skater.

   b) What is the impulse provided to the male skater?

   c) How would the impulse provided to the female skater change if her mass was doubled?

10. A pool ball collides with a side cushion and rebounds in the opposite direction. The collision lasts 0,005 s. The impulse provided to the pool ball is 8 N.s away from the cushion. Calculate the net force acting on the pool ball.

11. A loaded freight train (mass 10 000 kg) rolls to the right at 2 m.s^{-1} toward another freight train (mass 8 000 kg) moving in the opposite direction at 3 m.s^{-1}. On collision, the two trains couple (lock together).

    a) Calculate the velocity of the two trains after the collision.

    b) Calculate the impulse provided to each train.

12. A 0,05 kg bullet is fired into a block of wood. The velocity of the bullet just before impact is 350 $m.s^{-1}$.
    a) Calculate the change in momentum of the bullet.
    b) What is the impulse provided to the bullet?
    c) Calculate the time the bullet took to come to rest if it experienced a net force of 2 000 N.
13. A 60 kg astronaut uses a jet of gas to provide a force of 10 N on himself. How long must he do this to reach a speed of 1 $m.s^{-1}$ from a stationary start?
14. A hunter claims to have shot a charging buffalo through the heart and "dropped him in his tracks".
    a) How would the momentum of the bullet compare with the momentum of the buffalo? Explain your answer.

    Suppose the hunter was shooting one of the largest hunting rifles ever sold, a 0,5 calibre Sharps rifle, which shoots a 22,7 g bullet at 376 $m.s^{-1}$.
    b) Evaluate the hunter's claim by calculating the velocity of the 250 kg buffalo after the impact if he was initially moving directly toward the hunter at a slow 0,675 $m.s^{-1}$ south.
    c) Calculate the net force exerted on the buffalo if the collision lasted 0,01 s.
15. A $2,04 \times 10^6$ kg space shuttle is very far from the Earth. The rocket engines expel $3,7 \times 10^3$ kg of exhaust gas during the 1 second for which the rocket engines are fired. This increases the shuttle's velocity by 5,7 $m.s^{-1}$ forward. At what velocity, relative to the rocket, does the exhaust gas leave the rocket engines?
16. Some running shoe designs contain springs. Research these types of shoes and the controversy surrounding them. How do momentum and impulse apply to these shoes?

Impulse and safety

Seat belts and airbags

Fig 1.49: An airbag increases the time taken for the driver to come to rest during a collision

Vehicle safety devices are designed to increase collision times and so reduce the net force acting on the motorist during a collision. When the vehicle is involved in a collision, sensors in the vehicle detect this and deploy an airbag from the steering column. The airbag inflates in a very short time interval, approximately 30 ms (0,03 s). The driver collides with the airbag rather than the steering column, as shown in Figure 1.49. Airbags are designed to leak after inflation so that the fully inflated bag decreases in thickness from about 30 cm to about 10 cm.

Air bags have saved thousands of lives since their introduction in the early 1980s. In summary, air bags (and seat belts) increase the time taken (Δt) for a passenger to come to rest during an accident.

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$

According to Newton's second law, if the time taken to come to rest (Δt) is increased, then the net force ($\boldsymbol{F}_{\text{net}}$) acting on the passenger will decrease. This will obviously reduce the chances of fatal injury.

Science around us
More about how airbags are made

Statistics show that airbags reduce the risk of a fatal injury in a head-on collision by about 30 percent. The function of an airbag is to slow the passenger's forward motion as evenly as possible in a fraction of a second.

There are three parts to an airbag, shown in Figure 1.50:

- The bag itself is made of nylon, which is folded into the steering wheel, dashboard, seat or door.
- The sensor is the device that tells the bag to inflate. Inflation happens when there is a collision force equal to running into a brick wall at 16 to 24 km.h^{-1}.
- The airbag's inflation system reacts sodium azide (NaN_3) with potassium nitrate (KNO_3) to produce nitrogen gas. A hot blast of nitrogen inflates the airbag.

Fig 1.50: The internal structure of an airbag

The airbag inflates at a speed of 322 km.h^{-1}, faster than the blink of an eye! Almost just as quickly, the gas quickly dissipates through tiny holes in the bag, thus deflating the bag so you can move and are not suffocated by the bag.

Even though the whole process happens in only $\frac{1}{25}$th of a second, the additional time is enough to help prevent serious injury. The powdery substance released from the airbag is regular cornstarch or talcum powder, which is used by the airbag manufacturers to keep the bags pliable while they are not in use.

Check point 9

A 70 kg driver of a car is not wearing a seatbelt. He is travelling at 54 km.h^{-1} (15 m.s^{-1}) when he is involved in an accident which brings the car to rest suddenly. The driver continues moving forward until he hits the steering wheel and is brought to rest in 0,02 s.

a) Calculate the net force acting on the driver.

b) Comment on the magnitude of this force in terms of safety.

Suppose, instead, that the driver collides with an airbag which brings him to rest in 0,1 s:

c) Calculate the net force the airbag exerts on the driver during the collision.

d) Compare your answers to questions a) and c) and comment on the usefulness of an airbag.

Arrestor beds

The braking system of a large truck may overheat and fail. If this happens, the truck driver may drive the truck into an arrestor bed, off the main road, to stop the truck. An arrester bed is a sand or gravel pathway such as the one shown in Figure 1.51. An arrestor bed decreases a truck's momentum to zero over a fairly long time interval (Δt), and so the force it exerts on the truck is small enough not to harm the truck or driver.

Fig 1.51: An arrestor bed

Exercise 1.6

1. Jennifer (mass 60 kg), travelling at 30 m.s^{-1} in her sports car, is involved in a collision while trying to avoid a cow crossing the road. She strikes the air bag, which brings her body to a stop in 0,15 s.
   a) What average force does the air bag exert on her?
   b) If Jennifer had not been wearing her seat belt and was not driving a car fitted with an air bag, then the steering wheel would have stopped her body in 0,01 s. What average force would the steering wheel have exerted on her?
2. During a parachuting exercise, learners are told to bend their knees when landing.
   a) Determine the force of impact on a 70 kg parachutist falling at 10 m.s^{-1} who bends her knees when hitting the ground, bringing her body to rest in 0,8 s.
   b) Suppose the parachutist did not bend her knees when hitting the ground, and came to rest in 0,05 s. Calculate the force of impact in this case.
3. A car is involved in a collision and is brought to rest. How will the magnitude of the net force acting on the car be affected in each of the following cases? Explain each answer.
   a) The car is brought to rest over a longer time interval.
   b) The car is travelling at a greater speed before the collision and is brought to rest in the same time interval.
   c) The car rebounds after the collision in the same time interval.
4. Explain the concept of a follow-through in your favourite sport.

Extend yourself

1. Using the concept of impulse, explain how a karate expert can break a board. (4)
2. Why is it useful to express impulse in terms of momentum? (2)
3. A 0,25 kg arrow with a velocity of 12 $m.s^{-1}$ west strikes and pierces the center of a movable 6,8 kg target.
   a) What is the final velocity of the combined mass? (4)
   b) What is the decrease in the arrow's kinetic energy during the collision? (3)
4. You are traveling in a bus when the momentum of an insect travelling in the opposite direction is suddenly changed as it splatters onto the front window.
   a) How does the force that the insect exerts on the bus compare to the force exerted by the bus on the insect? (2)
   b) How does the change in the momentum of the bus compare to the change in the momentum of the insect? Explain your answer. (2)
   c) Which of the bus or the insect experiences the greater acceleration? Explain your answer. (2)
5. A 16 kg canoe moves to the left at 12 $m.s^{-1}$. It is involved in an elastic head-on collision with a 4 kg raft moving to the right at 6 $m.s^{-1}$. After the collision, the raft moves to the left at 22,8 $m.s^{-1}$.
   a) Find the velocity of the canoe after the collision, using the law of conservation of momentum. (4)
   b) Show that this collision is elastic. (3)
6. A loaded 10 000 kg train freight car (mass) rolls at 3 $m.s^{-1}$ to the right toward a 2 000 kg freight train car travelling at 4 $m.s^{-1}$ in the opposite direction. On collision, the two cars couple (lock together).
   a) What is the velocity of the two freight train cars after the collision? (4)
   b) Calculate the impulse exerted on each freight train car. (4)
   c) If the collision lasts 0,7 s, calculate the net force exerted on each freight train car. (4)
   d) Show that this is an inelastic collision. (3)
7. Identical twins Kate and Karen, each of mass 45 kg, are rowing their boat when they decide to go for a swim. Kate jumps off the front of the boat at a speed of 3 $m.s^{-1}$. At the same time, Karen jumps off the back at a speed of 4 $m.s^{-1}$. If the 70 kg rowboat is moving at 1 $m.s^{-1}$ east when the girls jump off, what is the rowboat's velocity after the girls jump off? (5)
8. A 5 000 kg truck enters an arrester bed travelling at 30 $m.s^{-1}$ south. The speed of the truck is decreased to 20 $m.s^{-1}$ over 5 s. Calculate the net horizontal force acting on the truck. (4)

9. A stationary 160 g hockey ball is hit by a hockey player at the goalie. The ball is hit with a force of 200 N. The hockey stick is in contact with the ball for 0,05 seconds.
   a) What impulse is provided to the ball? (3)
   b) What force must be applied to the ball to stop it in 0,06 seconds? (4)
10. Explain, using the impulse-momentum theorem, why the railings on the stairway in a nursing home are padded. (3)
11. A dish falls and strikes the floor. Will the impulse provided to the dish be greater on a wooden floor or on a carpet? Explain your answer. (3)

[63]

Definitions

change in momentum found by subtracting the initial momentum ($\boldsymbol{p}_\mathrm{i}$) of an object immediately before the collision from its final momentum ($\boldsymbol{p}_\mathrm{f}$) immediately after the collision:
$\Delta \boldsymbol{p} = \boldsymbol{p}_\mathrm{f} - \boldsymbol{p}_\mathrm{i}$

elastic collision a collision in which kinetic energy is conserved

external force a force that does not originagte from an object within the system

inelastic collision a collision in which kinetic energy is not conserved

internal force a force which arises from objects within the system

impulse a change in momentum. The product of net force and the interaction time, $\boldsymbol{F}_\mathrm{net}\Delta t$

isolated system a system with a constant mass and no external force acting on it

momentum ($\boldsymbol{p}$) is a vector quantity the product of the mass (m) and velocity ($\boldsymbol{v}$) of the object:
$\boldsymbol{p} = m\boldsymbol{v}$.

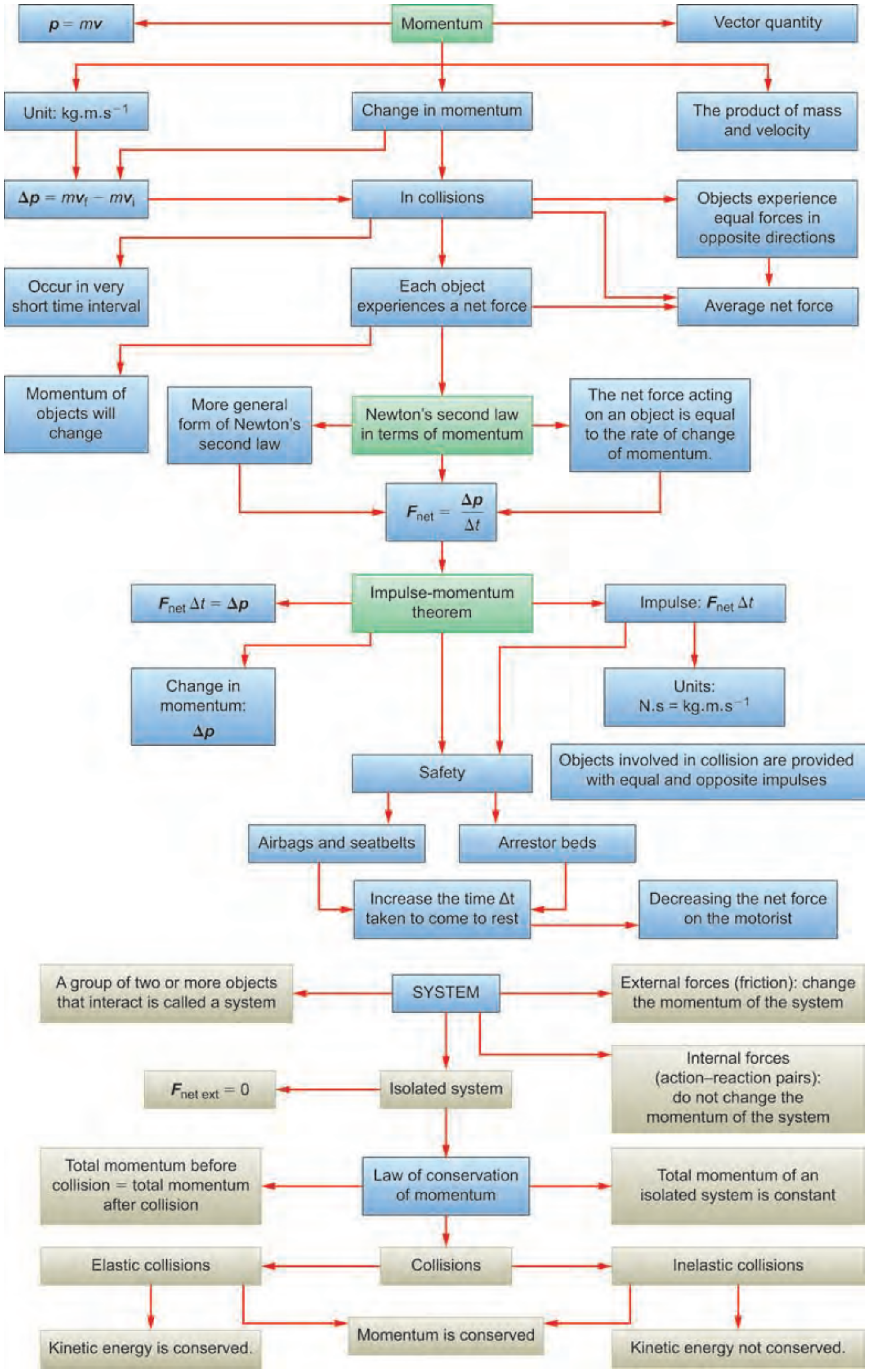
Newton's second law (stated in terms of momentum) The net force acting on an object is equal to its rate of change of momentum:
$\boldsymbol{F}_\mathrm{net} = \dfrac{\Delta \boldsymbol{p}}{\Delta t}$

Newton's third law when object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A

system a group of two or more objects that interact

the law of conservation of momentum the total momentum of an isolated system remains constant (is conserved). Momentum is conserved in both elastic and inelastic collisions in an isolated system

Summary



$p = mv$
Momentum
Vector quantity
Unit: kg.m.s^{-1}
Change in momentum
The product of mass and velocity
$\Delta p = mv_f - mv_i$
In collisions
Objects experience equal forces in opposite directions
Occur in very short time interval
Each object experiences a net force
Average net force
Momentum of objects will change
More general form of Newton's second law
Newton's second law in terms of momentum
The net force acting on an object is equal to the rate of change of momentum.
$F_{net} = \frac{\Delta p}{\Delta t}$
$F_{net}\Delta t = \Delta p$
Impulse-momentum theorem
Impulse: $F_{net}\Delta t$
Change in momentum: Δp
Units: N.s = kg.m.s^{-1}
Safety
Objects involved in collision are provided with equal and opposite impulses
Airbags and seatbelts
Arrestor beds
Increase the time Δt taken to come to rest
Decreasing the net force on the motorist
A group of two or more objects that interact is called a system
SYSTEM
External forces (friction): change the momentum of the system
Internal forces (action–reaction pairs): do not change the momentum of the system
$F_{net\ ext} = 0$
Isolated system
Total momentum before collision = total momentum after collision
Law of conservation of momentum
Total momentum of an isolated system is constant
Elastic collisions
Collisions
Inelastic collisions
Kinetic energy is conserved.
Momentum is conserved
Kinetic energy not conserved.

Topic 2

Vertical projectile motion in one dimension (1D)

What you will learn about in this topic

- Vertical projectile motion represented in words and equations
- Vertical projectile motion represented in graphs

Let's talk about this topic

The photograph shows a person throwing a ball straight upward into the air. While this ball is in motion, the motion is described by quantities such as velocity, acceleration, time and displacement. In Grade 10 you studied horizontal motion. In this topic you will study the motion of objects moving vertically upward and downward. You will calculate and graph some quantities of this kind of motion.

What you know already

In Grade 10, in the topic 'Instantaneous speed and velocity and the equations of motion' you studied the following equations of motion. You then applied the equations to linear-moving objects travelling with a uniform acceleration in a horizontal direction:

In symbols:

$$\boldsymbol{v}_\text{f} = \boldsymbol{v}_\text{i} + \boldsymbol{a}\Delta t$$
$$\boldsymbol{v}_\text{f}^2 = \boldsymbol{v}_\text{i}^2 + 2\boldsymbol{a}\Delta \boldsymbol{x}$$
$$\Delta \boldsymbol{x} = \boldsymbol{v}_\text{i}\Delta t + \frac{1}{2}\boldsymbol{a}\Delta t^2$$
$$\Delta \boldsymbol{x} = \left(\frac{\boldsymbol{v}_\text{i} + \boldsymbol{v}_\text{f}}{2}\right)\Delta t$$

Where:

$\boldsymbol{v}_\text{i}$ = initial velocity (m.s^{-1})

$\boldsymbol{v}_\text{f}$ = final velocity (m.s^{-1})

$\Delta \boldsymbol{x}$ = displacement (m)

Δt = time (s)

$\boldsymbol{a}$ = acceleration (m.s^{-2})

In this topic you will apply the same sets of equations to objects moving in a vertical direction.

Check myself

1. A car accelerates uniformly at 3 m.s^{-2} for 5 s, from moving at 4 m.s^{-1} East.

   a) What distance will the car travel in 5 s?

   b) Calculate the velocity after 5 s.

2. An aircraft, flying at an unknown initial velocity in an easterly direction, accelerates uniformly at 5 m.s^{-2}. It reaches a velocity of 200 m.s^{-1} east after accelerating over a distance of 300 m. Calculate the initial velocity of the aircraft.

Vertical projectile motion represented in words and equations

Introduction to vertical projectile motion

Free fall

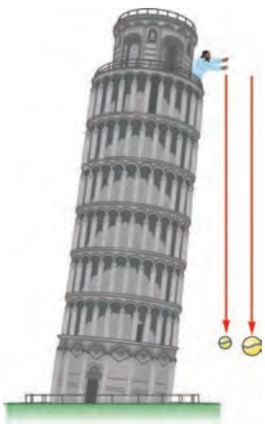


Fig 2.1: Galileo dropping two objects from the top of the Leaning Tower of Pisa

When an object is moving vertically, its motion can vary depending on whether it is experiencing air friction or not. **Free fall** describes the motion of a body in which the only force acting on it is gravity. Any object that is falling freely to the Earth's surface in the absence of friction is experiencing an acceleration of 9,8 m.s^{-2} downward. This is the acceleration due to gravity on the Earth and is the same for all objects. Galileo Galilei (1564-1642) climbed to the top of the Leaning Tower of Pisa in Italy and dropped two spheres of different masses from the top (Figure 2.1). Both masses reached the ground at the same time, leading Galileo to conclude that they both had the same acceleration.

Since quantities such as velocity, displacement and acceleration are vector quantities, their direction needs to be taken into consideration when doing calculations involving these quantities. When doing a vertical motion calculation, we will choose the upward direction to be negative and the downward direction to be positive. That means that all vector quantities in the positive direction will be allocated positive values and the vector quantities in the negative direction will be allocated negative values. You could just as easily choose the upward direction to be positive and the downward direction to be negative, as long as you are consistent with this throughout a problem. Acceleration due to gravity on the Earth is always 9,8 m.s^{-2} downward if air resistance is negligible, regardless of whether the object is moving upward or downward.

Also, when an object is thrown vertically upward, its velocity at its highest point is zero, as it comes to rest momentarily before changing direction and falling back down to the Earth. The time taken to reach the object's highest point is the same as the time that it takes to fall from its highest point back down to where it started. This is known as time symmetry.

What goes up must come down

When air resistance is negligible, an object that is thrown or projected upward will rise to its maximum height in the same time that it takes to fall from that height to its initial level. In other words, the time up = the time down. This is because the acceleration due to gravity is consistent throughout the motion of the object: 9,8 m.s^{-2} downward. The speed at one point on the way up is the same as the speed at that same point on the way down, as shown in Figure 2.2. If the time taken for the object to travel from A to B is 3,57 s, then the time taken to travel from B to C will be 3,57 s.

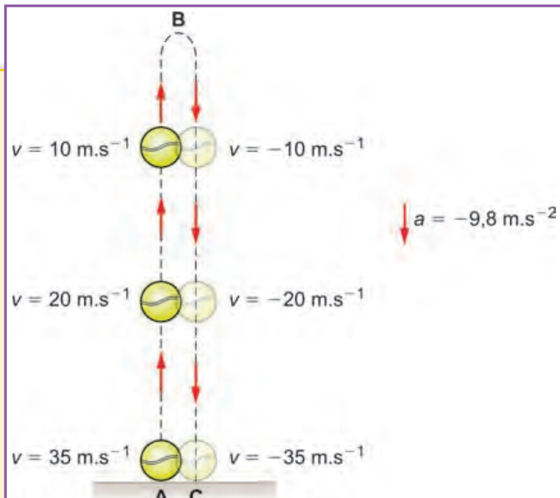


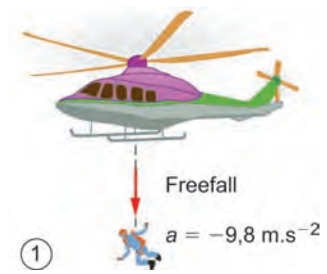
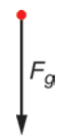
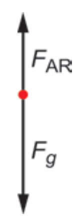
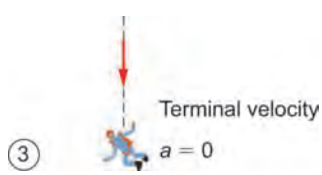
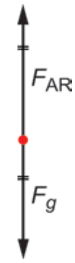
Fig 2.2: The speed at one point on the way up is the same as the speed at that same point on the way down.

Air Resistance

In reality, free fall near the Earth's surface only really occurs while the object moving slowly. This is because of air resistance. Air resistance is proportional to the velocity of the object. In other words, the faster an object is moving, the greater the air resistance it will experience. Air resistance is also affected by the area of the object. A webbed suit increases the surface area of a skydiver. This increases the air resistance on the skydiver, decreasing his terminal velocity and so letting him spend more time in the air. **Terminal velocity** is the constant velocity that a free falling object eventually reaches when the air resistance prevents further acceleration. The net force acting on an object travelling at its terminal velocity is zero.

Table 1A shows the motion of a skydiver from the moments she jumps out of the helicopter until the moment that she opens her parachute.

Table 1A

Position of skydiver	Explanation	Free body diagram
	When the skydiver initially jumps out of the helicopter, the only force acting on her is the force of gravity. As a result, she is experiencing free fall and her acceleration is 9,8 $m.s^{-2}$ downward.	
	As her velocity increases, so does the force of air resistance. However the force of gravity downward is still greater than the air resistance, so she continues accelerating, but her acceleration decreases.	
	Eventually a point is reached where the air resistance equals the force of gravity. At this point the acceleration is zero and the skydiver will fall at a maximum constant velocity known as the terminal velocity.	
	When she opens the parachute, the air resistance increases drastically. The air resistance is now greater than the force of gravity. She therefore decelerates and the velocity decreases.	

Checkpoint 1

1. What is freefall?
2. Draw a free body diagram for an object in freefall.
3. What is the acceleration due to gravity on or near the Earth's surface?
4. What is terminal velocity?
5. Draw a free body diagram for an object travelling at terminal velocity.

Science around us

Can you survive if you jump from an aeroplane and the parachute fails to open?

The answer is yes. There are many people who have fallen from such heights and lived to tell the tale. While most of it comes down to luck, there are things that you can do to improve your chances:

- Maximize your surface area by spreading yourself out (Figure 2.3).
- Find the best landing spot. The best possible surfaces on which to fall are snow, deep water (preferably water that is fast moving or frothy), soft ground, and trees or thick vegetation (although these present a high risk of impalement). Search for steep slopes that gradually grow gentler, since you will not lose all of your momentum at once when you hit the ground, greatly reducing the impact on your body.

Figure 2.3: Spread yourself out to improve your chances of surviving a parachute jump if your parachute fails to open.

- Bend your knees. Possibly nothing is more important to surviving a fall (or simpler to do) than bending your knees. Research has shown that having one's knees bent at impact can reduce the magnitude of impact forces 36-fold.
- Relax. Relaxing during a long fall – especially as you near the ground–is easier said than done, but try anyway. If your muscles are tense, your body will transfer force more directly to your vital organs.
- Land feet-first. No matter what height you fall from, you should always try to land on your feet. While landing feet-first concentrates the impact force on a small area, it also allows your feet and legs to absorb the worst of the impact.
- Land on the balls of your feet. Point your toes slightly downward before impact so that you will land on the balls of your feet. This will allow your lower body to more effectively absorb the impact.

- Try to roll. It's in video games, and it works in real life, too. This can absorb the impact greatly by moving your body's force across the ground instead of straight into it.
- Protect your head on the bounce. When you fall from a great height onto land, you will usually bounce. Some people who survive the initial impact (often with a feet-first landing) suffer a fatal injury on their second impact. Cover your head with your arms.

Science around us

The Maasai high jumpers

The Maasai people of Kenya perform a traditional dance called *adumu*. Young warriors form a circle and then one or two enter the circle and compete with one another to jump as high as they can without allowing their heels to touch the ground, as shown in Figure 2.4.

Fig 2.4 Maasai people of Kenya performing the adumu dance

Using equations of motion to solve vertical projectile motion questions

Use these equations to calculate unknown information about the motion of an object, if acceleration is constant:

In symbols:

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\Delta t$$
$$\boldsymbol{v}_f^2 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}\Delta \boldsymbol{x}$$
$$\Delta \boldsymbol{x} = \boldsymbol{v}_i\Delta t + \tfrac{1}{2}\boldsymbol{a}\Delta t^2$$
$$\Delta \boldsymbol{x} = \left(\frac{\boldsymbol{v}_i + \boldsymbol{v}_f}{2}\right)\Delta t$$

Where:

$\boldsymbol{v}_i$ = initial velocity (m.s^{-1})
$\boldsymbol{v}_f$ = final velocity (m.s^{-1})
$\Delta \boldsymbol{x}$ = displacement (m)
Δt = time (s)
$\boldsymbol{a}$ = acceleration (m.s^{-2})

Write a list of known values and then determine which equation needs to be used in order to find out the unknown value.

In working with vertical projectile motion questions, you should be consistent about which direction you take as positive. For example, you may choose to take the downward direction as positive and the upward direction as negative. Therefore all vector values, such as velocity and displacement in the downward direction will be allocated a positive value and vector quantities in the upward direction will be allocated a negative value. You should do this so that the vector nature of the velocities, displacements and acceleration is taken into consideration. An object in freefall accelerates at 9,8 m.s^{-2} downward both while it is moving upward and while it is moving downward. Therefore, in freefall, acceleration will always have a value of +9,8 m.s^{-2}, whether the object is moving upward or downward, if the downward direction is taken as positive.

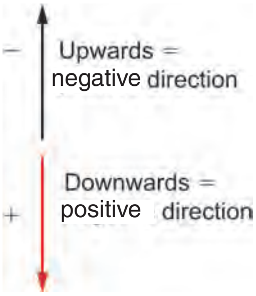


Figure 2.5

Worked examples:

1. A ball is dropped from a building which is 50 m high as shown in Figure 2.6. Calculate the ball's velocity just before it hits the ground. Ignore the effects of air resistance.

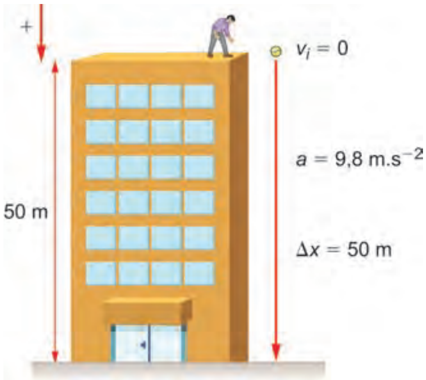


Figure 2.6

Solution:

$\boldsymbol{v}_i = 0$ m.s^{-1}
$\boldsymbol{a} = +9,8$ m.s^{-2}
$\Delta\boldsymbol{x} = 50$ m
$\boldsymbol{v}_f = ?$

$\boldsymbol{v}_f^2 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}\Delta\boldsymbol{x}$
$\boldsymbol{v}_f^2 = (0)^2 + (2)(9,8)(50)$
$\boldsymbol{v}_f^2 = 980$
$\boldsymbol{v}_f = 31,30$ m.s^{-1} downward

2. A ball is projected vertically upward at 20 m.s^{-1} as shown in Figure 2.7. Calculate the maximum height that the ball will reach. Ignore the effects of air resistance.

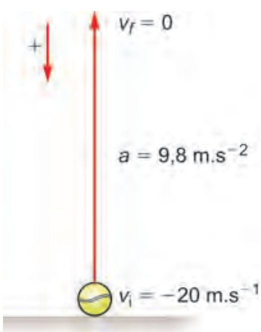


Figure 2.7

Solution:

At maximum height the ball will be at rest. Therefore $\boldsymbol{v}_f = 0$.

$\boldsymbol{v}_i = -20$ m.s^{-1}
$\boldsymbol{v}_f = 0$ m.s^{-1}
$\boldsymbol{a} = +9,8$ m.s^{-2}
$\Delta\boldsymbol{x} = ?$

$\boldsymbol{v}_f^2 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}\Delta\boldsymbol{x}$
$(0)^2 = (-20)^2 + (2)(9,8)(\Delta\boldsymbol{x})$
$-400 = 19,6\Delta\boldsymbol{x}$
$\Delta\boldsymbol{x} = -20,41$ m
$\Delta\boldsymbol{x} = 20,41$ m upward
$\therefore$ height $= 20,41$ m

3. A stone is dropped off a bridge and strikes the water below 5 s later as shown in Figure 2.8. Ignore the effects of air resistance.

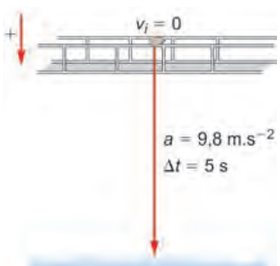


Figure 2.8

Calculate:

a) the vertical distance that the stone fell.

b) the velocity with which the stone hits the water.

Solution:

a) $\boldsymbol{v}_i = 0$ m.s^{-1}
$\boldsymbol{a} = +9,8$ m.s^{-2}
$\Delta t = 5$ s
$\Delta\boldsymbol{x} = ?$

$\Delta\boldsymbol{x} = \boldsymbol{v}_i\Delta t + \frac{1}{2}\boldsymbol{a}\Delta t^2$
$\Delta\boldsymbol{x} = (0)(5) + \frac{1}{2}(9,8)(5)^2$
$\Delta\boldsymbol{x} = 122,5$ m downward
$\therefore$ vertical distance $= 122,5$ m

b) $\boldsymbol{v}_i = 0$ m.s^{-1}
$\boldsymbol{a} = +9,8$ m.s^{-2}
$\Delta t = 5$ s
$\Delta\boldsymbol{x} = 122,5$ m
$\boldsymbol{v}_f = ?$

$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\Delta t$
$\boldsymbol{v}_f = (0) + (9,8)(5)$
$\boldsymbol{v}_f = 47,5$ m.s^{-1} downward

Checkpoint 2

1. A stone is dropped from the top of a building and hits the ground travelling at 45 m.s^{-1}. Ignore the effects of air resistance. Calculate the height of the building.
2. A ball is thrown upward at an unknown initial velocity. It takes 3,2 s to reach its highest point. Ignore the effects of air resistance. Calculate the initial velocity of the ball.

Exercise 2.1

Ignore the effects of air resistance.

1. A bomb falls out of an aircraft. To break the sound barrier of 340 m.s^{-1}:
   a) how far does it need to fall?
   b) how long will this take?
2. A stone is thrown vertically upward from ground level with a velocity of 25 m.s^{-1}. Calculate:
   a) the maximum height reached.
   b) the time taken to reach its maximum height.
3. A stone is dropped from a bridge and is seen to splash into the water 3 s later. Calculate:
   a) the height of the bridge.
   b) the velocity with which the stone strikes the water.
4. A brick falls off a scaffold at a height of 80 m above the ground. Calculate:
   a) the magnitude of its velocity after falling for 2 s.
   b) the magnitude of its velocity when it hits the ground.
   c) the time taken to fall to the ground.
5. A stone, dropped from the top of a lighthouse, strikes the rocks below at a speed of 50 m.s^{-1}. Calculate the height of the lighthouse.
6. A stone is thrown vertically upward and reaches a height of 10 m.
   a) What was the initial velocity of the stone as it left the thrower's hand?
   b) Calculate the time taken for the stone to reach its maximum height.
7. A ball is thrown vertically upward and returns to the thrower's hand 4 s later. Calculate:
   a) the velocity with which the ball left the thrower's hand.
   b) the height reached by the ball.
   c) the velocity with which the ball returned to the thrower's hand.

More vertical projectile motion problems

In this lesson you learn to answer more difficult motion problems, such as problems in which motion of an object changes direction.

Worked examples:

1. An object is projected vertically upward from ground level as shown in Figure 2.9. An observer at a height of 135 m notes that exactly 3 s pass between the object passing him on its way up and reaching its highest point. Ignore the effects of air resistance. Calculate:

   a) the velocity of the object at a height of 135 m.

   b) the velocity at which the object was projected.

Solution:

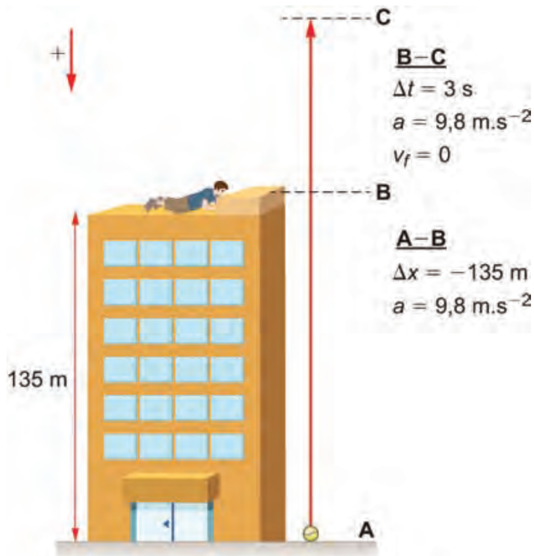


Figure 2.9

In calculating the velocity at a height of 135 m, we need to either work in segment A to B, in which case we would be looking for $\boldsymbol{v}_f$, or we need to work in segment B to C, in which case we would be calculating $\boldsymbol{v}_i$. We do not have enough known values to work in segment A to B, so we will work in segment B to C.

a) Work from B to C:

$\boldsymbol{v}_f = 0$ m.s^{-1}

$\boldsymbol{a} = +9{,}8$ m.s^{-2}

$\Delta t = 3$ s

$\boldsymbol{v}_i = ?$

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\Delta t$$
$$(0) = \boldsymbol{v}_i + (9{,}8)(3)$$
$$\boldsymbol{v}_i = -29{,}4 \text{ m.s}^{-1}$$
$$\boldsymbol{v}_i = 29{,}4 \text{ m.s}^{-1} \text{ upward}$$

b) Work from A to B:

The final velocity from A to B is equal to the initial velocity from B to C.

Hence, $\boldsymbol{v}_i$ for segment B to C (–29,4 m.s^{-1}) is equal to $\boldsymbol{v}_f$ for segment A to B.

$\boldsymbol{v}_f = -29{,}4$ m.s^{-1}

$\boldsymbol{a} = 9{,}8$ m.s^{-2}

$\Delta \boldsymbol{x} = -135$ m

$$\boldsymbol{v}_f^2 = \boldsymbol{v}_i^2 + 2\boldsymbol{a}\Delta\boldsymbol{x}$$
$$(-29{,}4)^2 = \boldsymbol{v}_i^2 + 2(9{,}8)(-135)$$
$$\boldsymbol{v}_i^2 = 3510{,}36$$
$$\boldsymbol{v}_i = -59{,}25 \text{ m.s}^{-1}$$
$$\boldsymbol{v}_i = 59{,}25 \text{ ms}^{-1} \text{ upward}$$

2. A boy standing on a tower 60 m high, throws a stone vertically downward as shown in Figure 2.10. The stone leaves his hand at 5 m.s^{-1}. Ignore the effects of air resistance. Calculate how long it will take the stone to reach the ground.

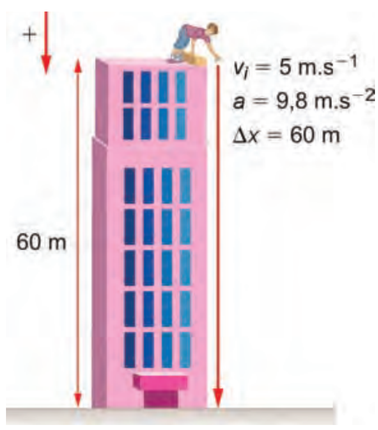


Figure 2.10

Solution:

When an object is thrown downward, its initial velocity is not 0. Its initial velocity will be the initial velocity that it left the person's hand with.

$v_i = 5 \text{ m.s}^{-1}$	$v_f^2 = v_i^2 + 2a\Delta x$	$v_f = v_i + a\Delta t$
$a = 9{,}8 \text{ m.s}^{-2}$	$v_f^2 = (5)^2 + 2(9{,}8)(60)$	$(34{,}66) = (5) + (9{,}8)\Delta t$
$\Delta x = 60 \text{ m}$	$v_f^2 = 1201$	$\Delta t = 3{,}03 \text{ s}$
$\Delta t = ?$	$v_f = 34{,}66 \text{ m.s}^{-1}$ downward	

3. A boy fires a pellet gun upwards from the top of a cliff. The pellet leaves the gun at 20 m.s^{-1} and strikes the ground at 30 m.s^{-1}, as shown in Figure 2.11. Ignore the effects of air resistance. Calculate:
   a) the height 'h' that the pellet was shot from.
   b) the time that it takes for the pellet to reach the ground.

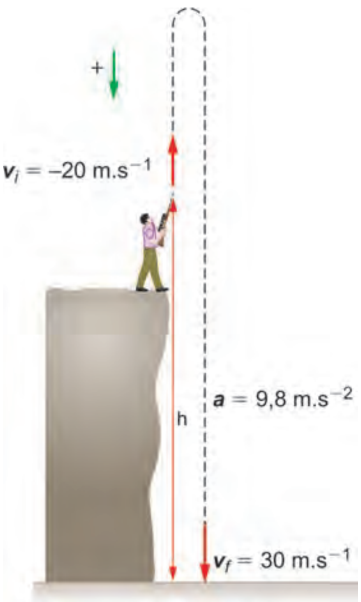


Figure 2.11

Solution:

This object changes direction during the course of its motion. We need to take this change in direction into account. Consistently use the upward direction as negative and the downward direction as positive.

a) $v_i = -20 \text{ m.s}^{-1}$ $\quad$ $v_f^2 = v_i^2 + 2a\Delta x$

$v_f = 30 \text{ m.s}^{-1}$ $\quad$ $(30)^2 = (-20)^2 + 2(9{,}8)\Delta x$

$a = 9{,}8 \text{ m.s}^{-2}$ $\quad$ $\Delta x = 25{,}51$ m downward

$\Delta x = ?$ $\quad$ $\therefore$ height = 25,51 m

b) $v_i = -20 \text{ m.s}^{-1}$ $\quad$ $v_f = v_i + a\Delta t$

$v_f = 30 \text{ m.s}^{-1}$ $\quad$ $(30) = (-20) + (9{,}8)\Delta t$

$a = 9{,}8 \text{ m.s}^{-2}$ $\quad$ $\Delta t = 5{,}10 \text{ s}$

$\Delta x = 25{,}51 \text{ m}$

$\Delta t = ?$

4. A person throws a ball upward from the roof of a 15 m high building as shown in Figure 2.12. The ball leaves the person's hand at 12 m.s^{-1}. Ignore the effects of air resistance. Calculate:
   a) the velocity with which the ball hits the ground.
   b) the time it takes to reach the ground.

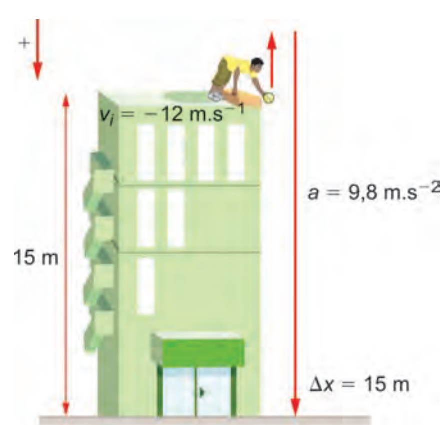


Figure 2.12

Solution:

a) $v_i = -12 \text{ m.s}^{-1}$

$a = 9{,}8 \text{ m.s}^{-2}$

$\Delta x = 15 \text{ m}$

$v_f = ?$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = (-12)^2 + 2(9{,}8)(15)$

$v_f = 20{,}93 \text{ m.s}^{-1}$ downward

b) $v_i = -12 \text{ m.s}^{-1}$

$a = 9{,}8 \text{ m.s}^{-2}$

$\Delta x = 15 \text{ m}$

$v_f = 20{,}93 \text{ m.s}^{-1}$

$\Delta t = ?$

$v_f = v_i + a\Delta t$

$(20{,}93) = (-12) + (9{,}8)\Delta t$

$\Delta t = 3{,}36 \text{ s}$

Checkpoint 3

1. A ball is thrown upward from the top of a 30 m high building. The ball takes 2,4 s to reach its highest point. Ignore the effects of air resistance. Calculate:
   a) the velocity with which the ball left the thrower's hand.
   b) the total time taken for the ball to reach the ground.

Exercise 2.2

Ignore the effects of air resistance.

1. A body is projected vertically upward from the roof of a building at 40 m.s^{-1}. It reaches the ground with a speed of 60 m.s^{-1}. Calculate:
   a) the height of the building.
   b) the total time of flight.
2. A girl stands on a bridge 11,25 m above a boy on the ground. The boy throws an orange vertically upward at 10 m.s^{-1} and at the same instant the girl drops an apple. Calculate:
   a) the maximum height obtained by the orange and state whether or not it reaches the girl.
   b) the vertical distance between the orange and the apple 1 s after they were in motion.
3. A stone is dropped from the top of a mountain. Assuming no air resistance, how far will it fall and for how long in order to reach a velocity of 250 m.s^{-1}?
4. A ball is thrown upward at 20 m.s^{-1} off the top of a building that is 12 m high.
   a) What is the height of the ball above the ground at its highest point?
   b) How long does it take the ball to reach its highest point?
   c) What is the total time taken for the ball to reach the ground after leaving the person's hand?

5. When a girl throws a ball straight upward, she finds it takes 3 s for the ball to reach its highest point. Calculate:
    a) the velocity with which the ball left her hand.
    b) the height that the ball reached above her hand.
6. A stone is thrown downward at 15 $m.s^{-1}$ off the top of a building. If the building is 75 m high, calculate:
    a) the velocity of the stone as it reaches the ground.
    b) the time taken for the ball to reach the ground.
7. A stone is thrown vertically downward from a cliff. The stone moves through a distance of 1 m while in the thrower's hand and leaves the thrower's hand at 8 $m.s^{-1}$.
    a) What is the acceleration of the stone while in the thrower's hand?
    b) What is the acceleration of the stone after it leaves the thrower's hand?
    c) After leaving his hand, how long will it take the stone to reach the ground which is 48 m below where it left his hand?
8. A rocket is launched vertically upward from the ground. Its engine accelerates the rocket for 10 s from launch until it reaches a velocity of 250 $m.s^{-1}$. After 10 s it turns its engine off.
    a) What is the acceleration of the rocket during the first 10 s?
    b) What is the acceleration after 10 s?
    c) Calculate the maximum height reached by the rocket.

Hot air balloon problems

Hot air balloon problems are regarded as being more challenging problems because the object inside the balloon is thrown upward or downward from a hot air balloon that is already moving. As a result, the initial velocity of the object will be the vector sum of the velocity of the balloon and the velocity at which the object is released.

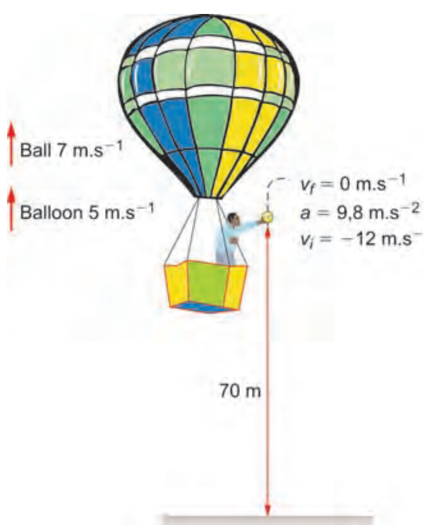


Figure 2.13

Worked examples:

1. A hot air balloon is moving upward with a velocity of 5 $m.s^{-1}$ as shown in Figure 2.13. A person inside the balloon then throws a ball upward with a velocity of 7 $m.s^{-1}$ relative to the balloon. Ignore the effects of air resistance. If the ball was 70 m above the ground when thrown upward, calculate:
    a) the time taken for the ball to reach its highest point.
    b) the height of the ball above the ground at this point.

Solution:

a) $v_i = -12 \text{m.s}^{-1}$
$a = 9{,}8 \text{ m.s}^{-2}$
$v_f = 0 \text{ m.s}^{-1}$
$\Delta t = ?$

$v_f = v_i + a\Delta t$
$(0) = (-12) + (9{,}8)\Delta t$
$\Delta t = 1{,}22 \text{ s}$

b) $v_i = -12 \text{ m.s}^{-1}$
$a = 9{,}8 \text{ m.s}^{-2}$
$v_f = 0 \text{ m.s}^{-1}$
$\Delta t = 1{,}22 \text{ s}$
$\Delta x = ?$

$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$
$\Delta x = (-12)(1{,}22) + \frac{1}{2}(9{,}8)(1{,}22)^2$
$\Delta x = -7{,}35 \text{ m}$
$\Delta x = 7{,}35 \text{ m upward}$
$\therefore$ height $= 7{,}35 + 70 = 77{,}35$ m

2. A hot air balloon is moving upward with a velocity of 7 m.s^{-1}. Refer to Figure 2.14. A person inside the balloon drops a ball. Ignore the effects of air resistance. If the ball is 50 m above the ground when dropped, calculate:

a) the time taken for the ball to reach its highest point.

b) the height of the ball above the ground at this point.

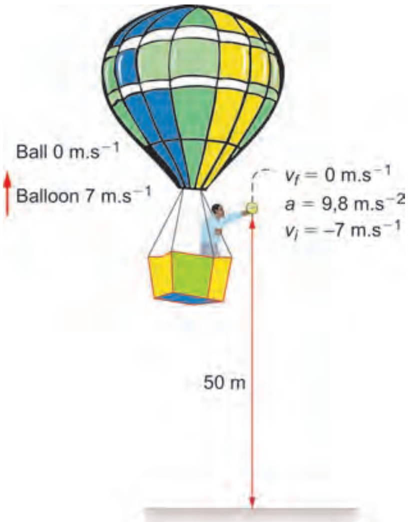


Figure 2.14

Solution:

a) $v_i = -7 \text{ m.s}^{-1}$
$a = 9{,}8 \text{ m.s}^{-2}$
$v_f = 0 \text{ m.s}^{-1}$
$\Delta t = ?$

$v_f = v_i + a\Delta t$
$(0) = (-7) + (9{,}8)\Delta t$
$\Delta t = 0{,}71\text{s}$

b) $v_i = -7 \text{ m.s}^{-1}$
$a = 9{,}8 \text{ m.s}^{-2}$
$v_f = 0 \text{ m.s}^{-1}$
$\Delta t = 0{,}71 \text{ s}$
$\Delta x = ?$

$\Delta x = v_i \Delta t + \frac{1}{2}a\Delta t^2$
$\Delta x = (-7)(0{,}71) + \frac{1}{2}(9{,}8)(0{,}71)^2$
$\Delta x = -2{,}50 \text{ m}$
$\Delta x = 2{,}50 \text{ m upward}$
$\therefore$ height $= 2{,}50 + 50 = 52{,}50$ m

3. A hot air balloon is moving downward with a velocity of 2 m.s^{-1}. Refer to Figure 2.15. A girl inside the balloon throws a ball upward at 5 m.s^{-1} relative to the balloon. Ignore the effects of air resistance. If the ball is 35 m above the ground when thrown upward, calculate the time taken for the ball to reach the ground.

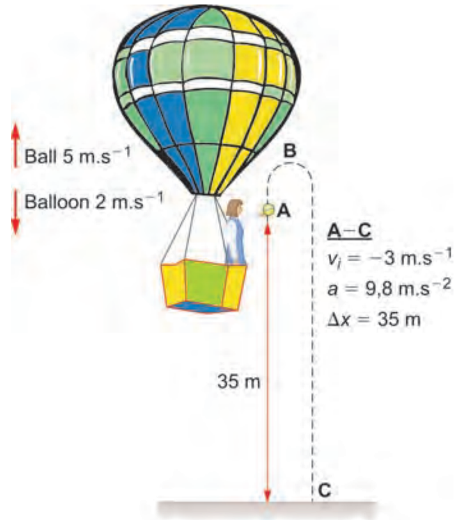


Figure 2.15

Solution:

$\boldsymbol{v}_\mathrm{i} = -3$ m.s^{-1}

$\boldsymbol{a} = 9{,}8$ m.s^{-2}

$\Delta \boldsymbol{x} = 35$ m

$\Delta t = ?$

$$\boldsymbol{v}_\mathrm{f}^2 = \boldsymbol{v}_\mathrm{i}^2 + 2\boldsymbol{a}\Delta \boldsymbol{x}$$

$$\boldsymbol{v}_\mathrm{f}^2 = (-3)^2 + 2(9{,}8)(35)$$

$$\boldsymbol{v}_\mathrm{f} = 26{,}36 \text{ m.s}^{-1} \text{ downward}$$

$$\boldsymbol{v}_\mathrm{f} = \boldsymbol{v}_\mathrm{i} + \boldsymbol{a}\Delta t$$

$$(26{,}36) = (-3) + (9{,}8)\Delta t$$

$$\Delta t = 3 \text{ s}$$

Checkpoint 4

1. A hot air balloon is moving upward with a velocity of 3 m.s^{-1}. A person inside the balloon then throws a stone upward at 2 m.s^{-1}. Ignore the effects of air resistance.
   a) Calculate the time taken for the stone to reach its highest point.
   b) If the stone hits the ground with a speed of 25 m.s^{-1}, calculate how high the balloon was above the ground when the stone was thrown.

Exercise 2.3

Ignore the effects of air resistance.

1. A projectile is fired vertically upward from a motionless balloon in the air. The projectile leaves the balloon at a velocity of 200 m.s^{-1} and strikes the ground at 300 m.s^{-1}. Calculate:
   a) the height of the balloon above the ground.
   b) the time it takes for the projectile to reach the ground.
2. A metal sphere is dropped from a balloon, which is rising at a constant velocity of 5 m.s^{-1}. The metal sphere strikes the ground after 5 seconds. Calculate:
   a) the velocity with which the sphere strikes the ground.
   b) how far above the ground the balloon was when the sphere was released.
3. A hot air balloon moves vertically upward at a constant velocity of 6 m.s^{-1}. When the balloon is 87 m above the ground, a bottle is thrown upward from inside the balloon at 3 m.s^{-1}.
   a) What is the maximum height reached by the bottle?
   b) Calculate the time taken for the bottle to reach the ground.

4. A hot air balloon is ascending with a constant velocity of 5 m.s^{-1} when somebody in the balloon throws a bottle upward with a velocity of 2 m.s^{-1}. If the balloon is 55 m above the ground when this happens, calculate the time taken for the bottle to reach the ground.
5. A hot air balloon is descending with a constant velocity of 3 m.s^{-1} when somebody inside the balloon throws an apple downward with a velocity of 5 m.s^{-1}. The apple strikes the ground after 2,5 s. Calculate:
   a) the velocity with which the apple strikes the ground.
   b) how far above the ground the balloon was when the apple was released.
6. A hot air balloon is descending with a constant velocity of 2 m.s^{-1} when a passenger throws a ball upward with a velocity of 5 m.s^{-1}. If the balloon is 70 m above the ground when this happens, calculate:
   a) the maximum height that the ball will reach above the ground.
   b) the time taken for the ball to reach the ground.
7. While a hot air balloon is descending at a constant velocity of 7 m.s^{-1}, somebody drops a stone from the balloon. The stone strikes the ground after 4 s. Calculate:
   a) the velocity with which the stone strikes the ground.
   b) how far above the ground the balloon was when the stone was released.

Recommended experiment for informal assessment

Aim: To investigate the motion of a falling body

You will need:

- ticker timer
- $\frac{1}{2}$ kg mass piece
- ticker tape
- clamp
- Prestik

Method:

Before starting the experiment, determine the frequency of the ticker timer. There should be information on the ticker timer itself or on the packaging that tells you this. This will help you to determine what the period is by using the equation:

$$T = \frac{1}{f}$$

Fig 2.16: Diagram of experimental setup

period – the time interval between consecutive dots.

Once you have determined the period of the ticker timer, calculate the time between 5 consecutive dots.

1. Clamp the ticker timer in a vertical position as high as possible facing downward, such as on top of a door.
2. Cut off a length of ticker tape slightly longer than the distance from timer to floor.
3. Thread the end of the ticker tape through the timer.
4. Attach the $\frac{1}{2}$ kg mass piece to the lower end of the tape using Prestik.
5. To reduce friction between the tape and the ticker timer, a learner can stand on a table and hold the tape up to allow it to run smoothly through the ticker timer. Refer to Figure 2.16.
6. Switch on the timer then allow the weight to fall freely to the ground.
7. Switch off the ticker timer and remove the tape.

Results:

Once the ticker timer has been switched off, detach the ticker tape from the trolley and ticker timer.

Ignore the first section of the ticker tape where all the dots are on top of one another, as this is when the mass piece was stationary. Find a point on the ticker tape at which to start measuring and draw a line through the dot, as shown in Figure 2.17a. Then draw a line through every 5^{th} dot thereafter, numbering the segments until you have about 8-10 segments.

Fig 2.17a: Diagram showing how to make markings on ticker tape

Copy and complete the table below, for as many segments as you have.

Segment	t (s)	x (m)	Δt (s)	Δx(m)	v_{avg} (m.s^{-1})
1					
2					
3					
4					
5					
6					
7					
8					

Analysing and using the ticker tape:

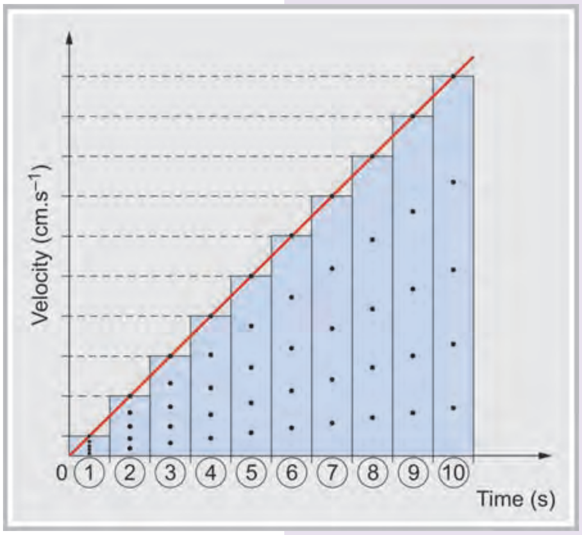


Fig 2.17b: Velocity-time graph using ticker tape

1. Cut your ticker tape up through the line that you have drawn at the end of each segment. Number the order of your tape pieces from 1 upwards.
2. Draw a horizontal line on a sheet of paper. Make a 'bar chart' by sticking the tapes vertically side by side, so that their bottoms just touch the horizontal line, as shown in Figure 2.17b. The first and shortest tape should be at the left hand end of the line.
3. The horizontal line acts as a time axis, starting at zero seconds. Each time interval on the *x*-axis will equal whatever the period was that you calculated your ticker timer to have.
4. Draw a vertical line through the zero mark on the *x*-axis. The vertical axis is the velocity axis.

Questions:

1. Calculate the average velocity for each segment by measuring the displacement of each segment and using the equation $\boldsymbol{v} = \frac{\Delta \boldsymbol{x}}{\Delta t}$. Insert the velocity values on the *y*-axis. Mark the values on the *y*-axis in cm.s^{-1}.
2. Draw a smooth, best fit line through the top point on each segment. Describe in words what the graph tells us about the motion of the mass piece.
3. Calculate the gradient of the graph. What does the gradient of the graph represent?
4. Calculate your percentage error for your calculations in which you calculated the acceleration due to gravity.

Give a conclusion based on the results.

Self-analysis:

Reflect on the practical procedure and suggest possible reasons for inaccuracy in your results.

Vertical projectile motion represented in graphs

Graphs of vertical projectile motion

The motion of an object can also be shown in a graph. In this unit you investigate graphs of *position versus (vs) time*, *velocity versus (vs) time* and *acceleration versus (vs) time* for one-dimensional projectile motion.

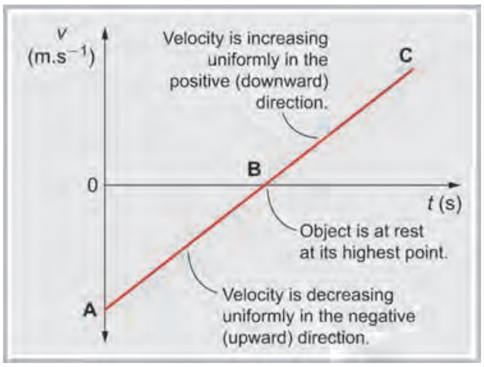
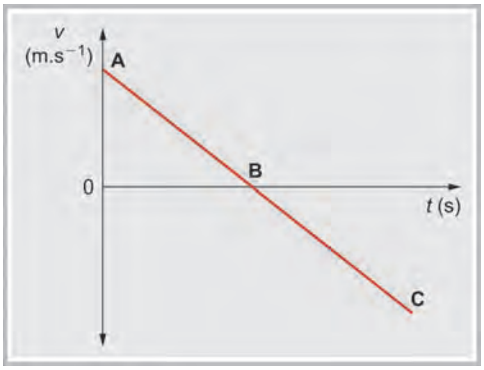
Notes for drawing graphs of motion:

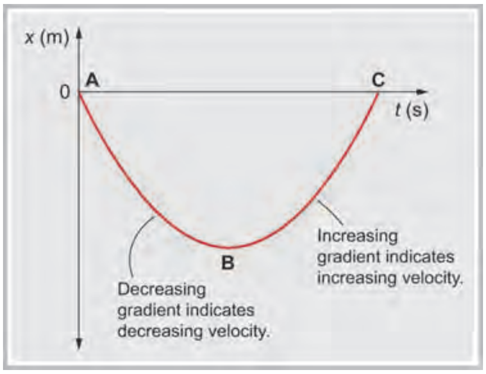
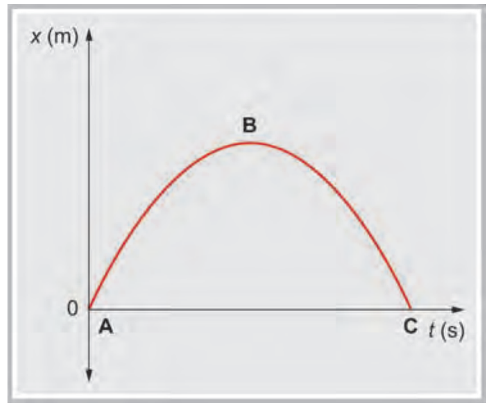
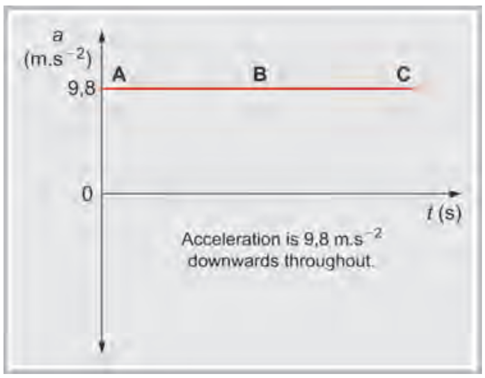
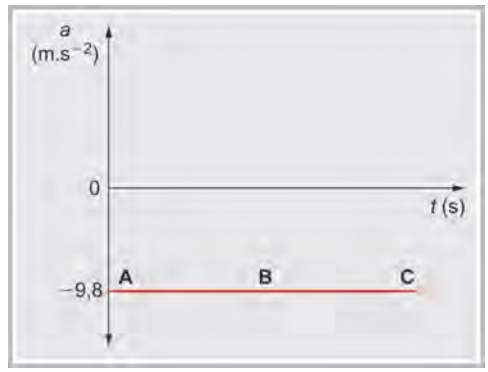
- Determine what graph is required, such as $\Delta \boldsymbol{x}$ *vs t*, $\boldsymbol{v}$ *vs t*, or $\boldsymbol{a}$ *vs t*.
- Determine whether the drawing of the graph needs to be divided into parts, such as A-B going up and B to C going down. Approach each of these segments individually.
- Position time (t) on the x-axis and the other quantity being measured, such as displacement ($\Delta \boldsymbol{x}$), velocity ($\boldsymbol{v}$) or acceleration ($\boldsymbol{a}$) on the y-axis.
- If the graph is to be a graph drawn to scale, determine a suitable scale for each axis from the information that you have been given. If the graph is to be a sketch graph, the graph does not need to be drawn to scale and values do not need to be included on the axes, unless you are told to do so.
- Using the data that you have been given, or that you have calculated, fill in the necessary points on the graph and then join the dots, using a line of best fit, whether it be a curve or a straight line. A sketch graph does not need points to be drawn and joined. A sketch graph can be drawn freehand, but a ruler must be used for portions of the graph that are a straight line.

Worked examples:

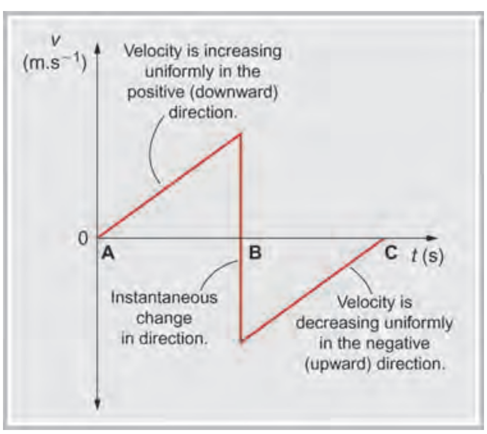
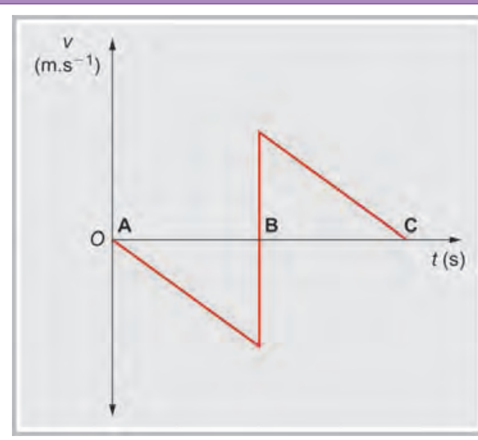
1. An object is thrown upward and then drops back down to the same position from where it was thrown, as shown in Figure 2.18. Sketch the graphs of *velocity vs time*, *position vs time* and *acceleration vs time* for the motion.

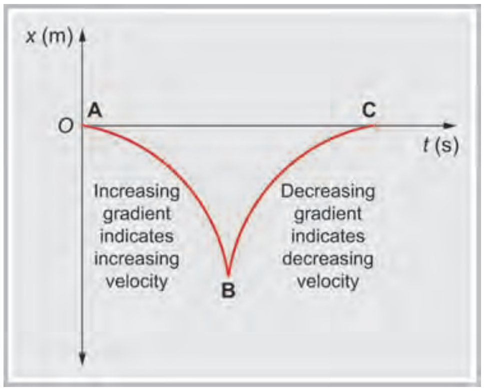
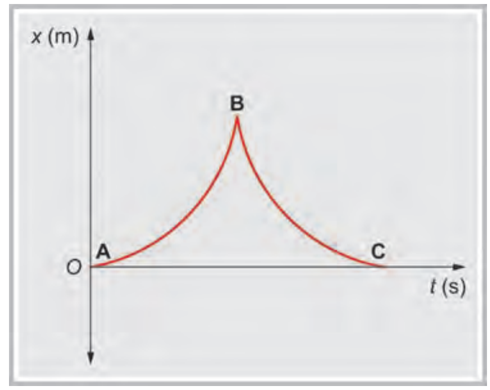
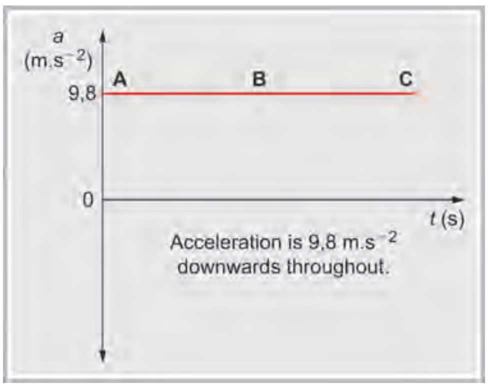
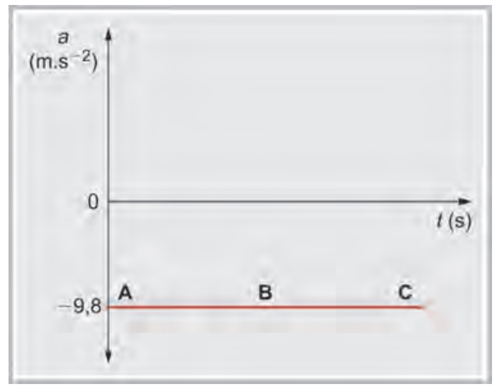
Fig 2.18

<table>
<tr><th></th><th>If downward is taken as the positive direction</th><th>If upward is taken as the positive direction</th></tr>
<tr><td>velocity vs time
(v vs t)</td><td>
</td><td>
</td></tr>
</table>

<table>
<tr>
<td>position vs time
(x vs t)</td>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>acceleration vs time
(a vs t)</td>
<td>
</td>
<td>
</td>
</tr>
</table>

2. An object is dropped to the ground and then it bounces back up to the same height. Sketch the graphs of *velocity vs time*, *position vs time* and *acceleration vs time* for the motion.

<table>
<tr>
<th></th>
<th>If downward is taken as the positive direction</th>
<th>If upward is taken as the positive direction</th>
</tr>
<tr>
<td>velocity vs time
(v vs t)</td>
<td>
</td>
<td>
</td>
</tr>
</table>

position vs time **(x vs t)**		
acceleration vs time **(a vs t)**		

Worked example:

An object is projected vertically upwards from ground level at A at 25 m.s^{-1}, as shown in Figure 2.19. The object travels upwards to its highest point C, passing B on the way up at a height of 22 m. Plot the following graphs for the motion, giving values on the x- and y-axes for points A, B and C:

- *velocity vs time*
- *displacement vs time*
- *acceleration vs time*

Note: Take upwards as the negative direction.

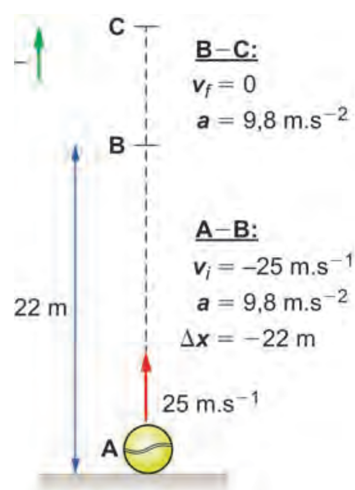


Fig 2.19

Solution:

Before the graphs can be drawn, there are values that we need to calculate, such as:

i) The velocity at point B.

ii) The time taken to travel from A to B.

iii) The height at point C.

iv) The total time of the motion from A to C.

i) To calculate the velocity at point B, let us work in the segment A to B:

$$v_f{}^2 = v_i{}^2 + 2a\Delta x$$

$$v_f{}^2 = (-25)^2 + 2(9,8)(-22)$$

$$v_f{}^2 = 193,8$$

$v_f = -13,92$ m.s^{-1} $\therefore v_f = 13,92$m.s^{-1} upward

ii) To calculate the time taken to travel from A to B, let us work in the segment A to B:

$$v_f = v_i + a\Delta t$$

$$(-13{,}92) = (-25) + (9{,}8)\Delta t$$

$$\Delta t = 1{,}13 \text{ s}$$

iii) To calculate the height at point C, let us work in the segment B to C:

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$(0)^2 = (-13{,}92)^2 + 2(9{,}8)\Delta x$$

$$\Delta x = -9{,}89 \text{ m}$$

$$\text{Total } \Delta x = (-9{,}89) + (-22) = -31{,}89 \text{ m}$$

v_i for B-C is equal to v_f for A to B, which is $-13{,}92$ m.s^{-1}.

iv) To calculate the total time of the motion, calculate the time from B to C and then add that to the time from A to B:

$$v_f = v_i + a\Delta t$$

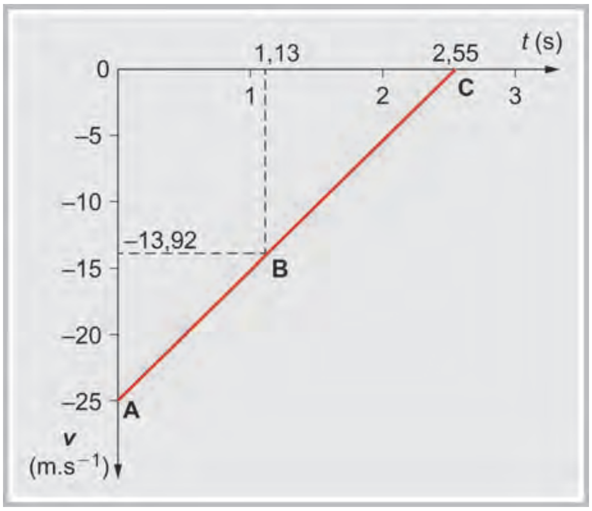
$$(0) = (-13{,}92) + (9{,}8)\Delta t$$

$$\Delta t = 1{,}42 \text{ s}$$

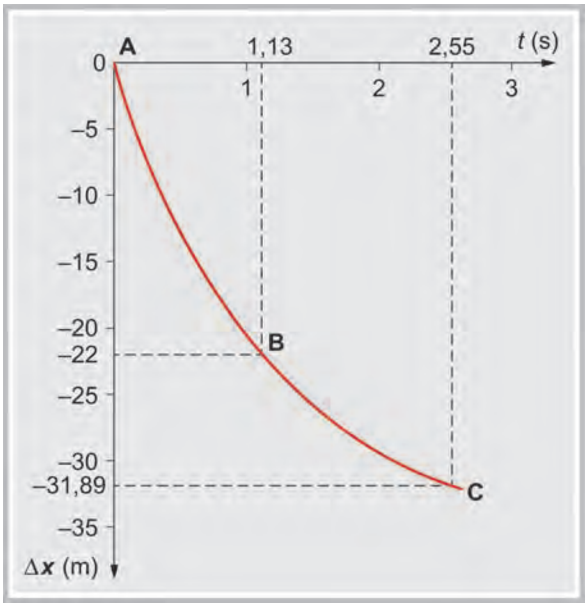
$$\text{total time} = 1{,}42 + 1{,}13 = 2{,}55 \text{ s}$$

Using this information, the 3 graphs can now be plotted:

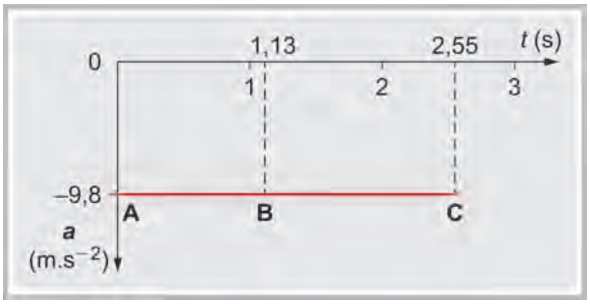
velocity vs time



displacement vs time



acceleration vs time



Checkpoint 5

A ball is thrown from the top of a building and then it falls to the ground, as shown in Figure 2.20. Sketch the following graphs for the motion of the ball from A to C. In each case sketch the graph that would have been obtained firstly if upward was taken as the positive direction and then secondly if downward had been taken as the positive direction. Take the starting point as the zero position.

a) *velocity vs time*

b) *displacement vs time*

c) *acceleration vs time*

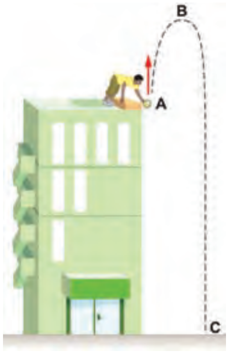


Fig 2.20

Calculations from graphs of motion

When doing calculations from a *velocity vs time* graph:

- Area under the graph = displacement or distance
- Gradient of the graph = acceleration

When doing calculations from a *displacement vs time* graph:

- Gradient of the graph = velocity

Worked examples:

1. A ball is thrown upward from the top of a building at 14,7 $m.s^{-1}$ and eventually falls down onto the ground below at 34,3 $m.s^{-1}$. The path of the ball is shown in the sketch in Figure 2.21a. The graph of *velocity vs time* for the motion of the ball is given in Figure 2.21b.

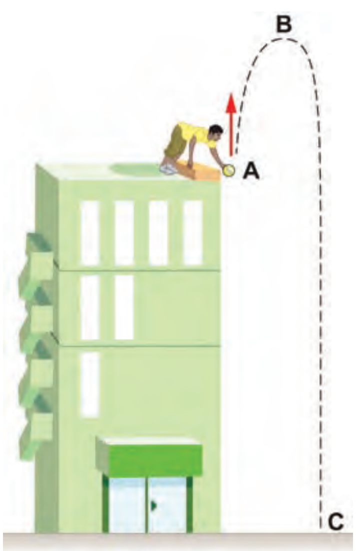


Fig 2.21a

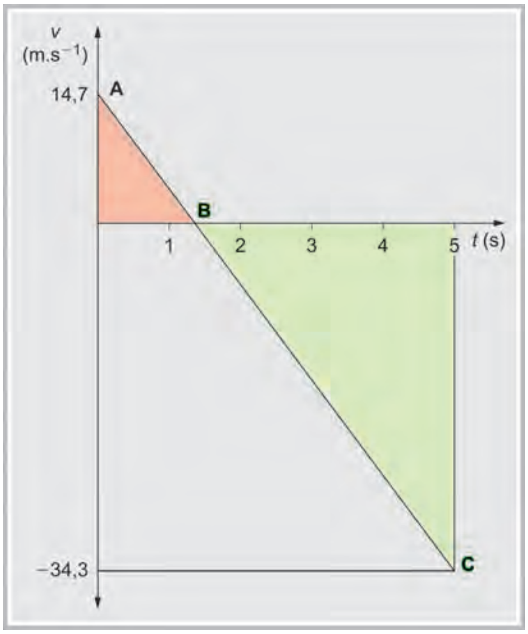


Fig 2.21b: Velocity vs time graph

a) From the *velocity vs time* graph, calculate:

   i) the distance from the thrower's hand to the highest point.

   ii) the height of the building.

   iii) the acceleration of the ball.

b) Draw the corresponding *displacement vs time* graph that would have been obtained from the above information. The graph must be to scale. Fill in corresponding values on the *x* and *y* axes.

c) Calculate:

   i) the velocity at 1,5 s.

   ii) the velocity at 2 s.

Solution:

The graph shows that, upward is being taken as the positive direction.

a) i) A – B:

$$x = \text{area} = \tfrac{1}{2}bh = \tfrac{1}{2}(1{,}5)(14{,}7) = 11{,}03\text{ m}$$

ii) B – C:

$$x = \text{area} = \tfrac{1}{2}bh = \tfrac{1}{2}(3{,}5)(34{,}3) = 60{,}03\text{ m}$$

$$\therefore \text{height} = 60{,}03 - 11{,}03 = 49\text{ m}$$

iii) A – C:

$$\boldsymbol{a} = \text{gradient} = \frac{\Delta y}{\Delta x} = \frac{(-34{,}3 - 14{,}7)}{(5 - 0)} = -9{,}8\text{ m.s}^{-2} = 9{,}8\text{ m.s}^{-2}\text{ down}$$

b)

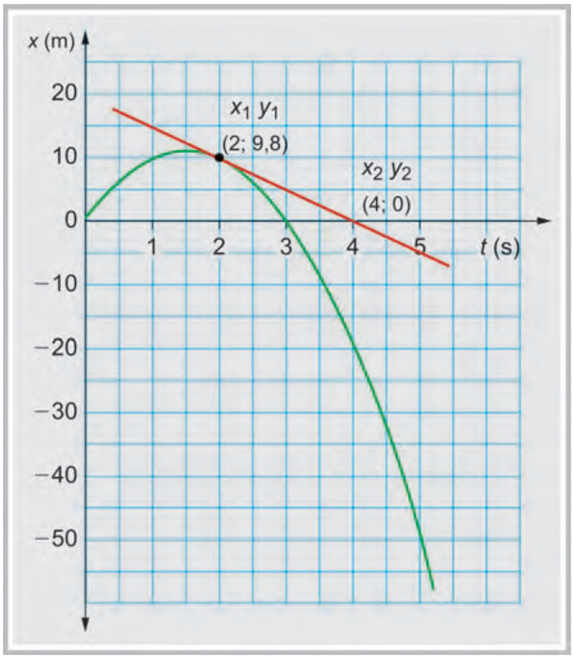


Fig 2.22: Displacement vs time graph

c) i) At 1,5 s, the ball is at its highest point, therefore its velocity is 0 m.s^{-1}.

ii)

$$\boldsymbol{v} = \text{gradient} = \frac{\Delta y}{\Delta x} = \frac{(0 - 9{,}8)}{(4 - 2)} = -4{,}9\text{ m.s}^{-1} = 4{,}9\text{ m.s}^{-1}\text{down}$$

Checkpoint 6

A skydiver jumps out of an aeroplane. The first 40 seconds of his motion is plotted on the *velocity vs time* graph in Figure 2.23.

a) Describe the motion of the skydiver from:

i) A to B

ii) B to C

iii) C to D

b) Calculate the skydiver's displacement from A to B.

c) Calculate the skydiver's acceleration from A to B.

d) Calculate the skydiver's displacement from C to D.

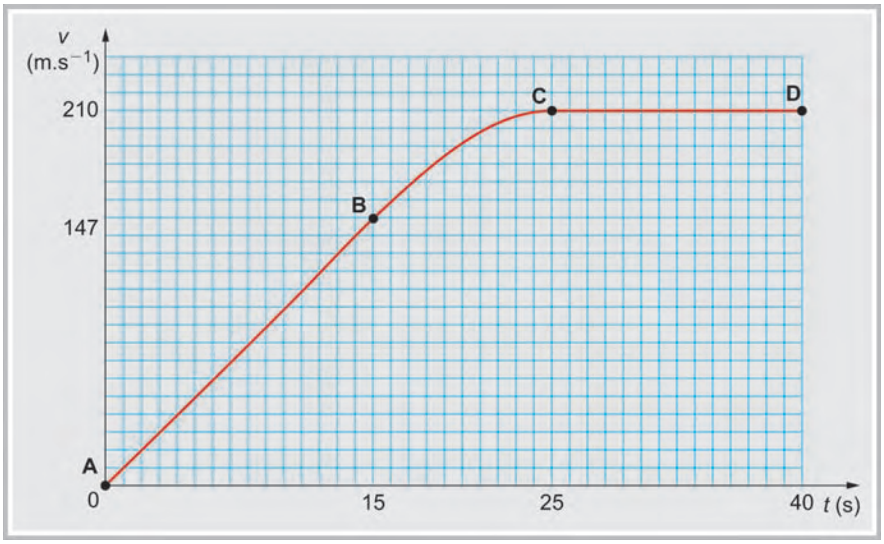


Fig 2.23: Velocity vs time graph

Exercise 2.4

1. A ball is dropped to the ground and when it bounces, it does not bounce all the way to the point from which it was dropped. The path of the ball is shown in the sketch in Figure 2.24a. The graph of *velocity vs time* for the motion of the ball is given in Figure 2.24b.

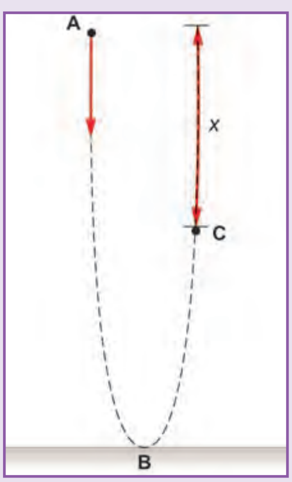


Fig 2.24a

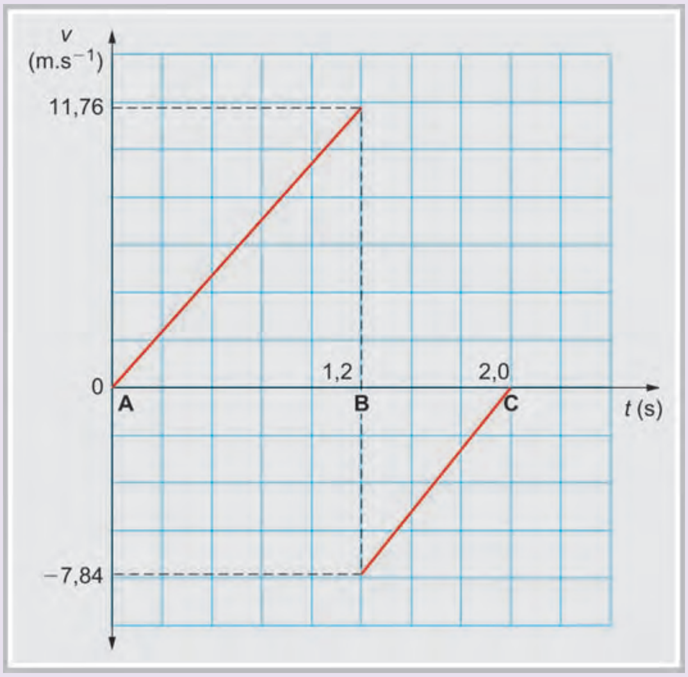


Fig 2.24b: Velocity vs time graph

a) Without the use of equations of motion, calculate:

   i) the distance travelled from A to B.

   ii) the distance travelled from B to C.

   iii) the distance x on Figure 2.24a.

   iv) the acceleration of the ball from A to B.

b) Sketch the graph of:

   i) *position vs time* for the motion of the ball from A to C. Include values on the x and y axis.

   ii) *acceleration vs time* for the motion of the ball from A to C. Include values on the x and y axis.

2. A ball is thrown upward from the top of a building. The path of the ball is shown in the sketch in Figure 2.25a. The graph of *velocity vs time* for the motion of the ball is given in Figure 2.25b.

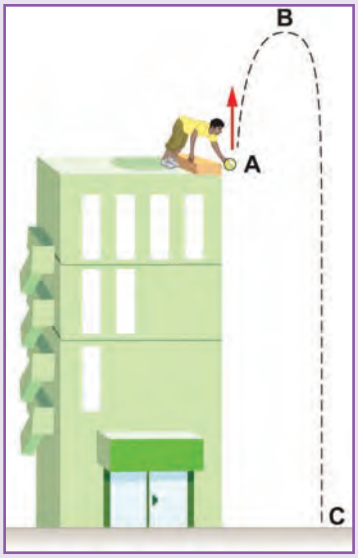


Fig 2.25a

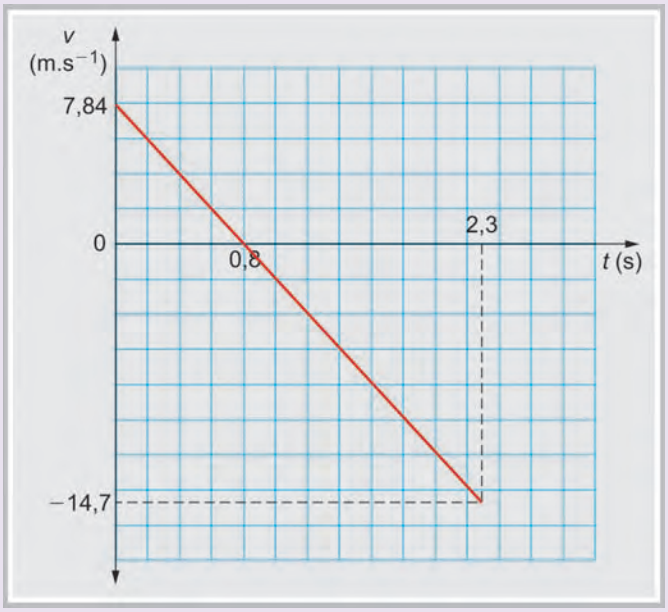


Fig 2.25b: Velocity vs time graph

   a) Without the use of equations of motion, calculate

      i) the distance travelled by the ball from A to B.

      ii) the distance travelled by the ball from B to C.

      iii) the height of the building.

      iv) the acceleration of the ball from A to B.

b) What will the acceleration of the ball be from B to C?

c) Sketch the graph of:

   i) *position vs time* for the motion of the ball from A to C. Include values on the x and y axis.

   ii) *acceleration vs time* for the motion of the ball from A to C. Include values on the x and y axis.

3. A ball is thrown upward and eventually falls back down to the position from which it was thrown. The path of the ball is shown in the sketch in Figure 2.26a. The graph of *velocity vs time* for the motion of the ball is given in Figure 2.26b.

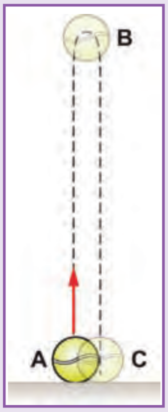


Fig 2.26a

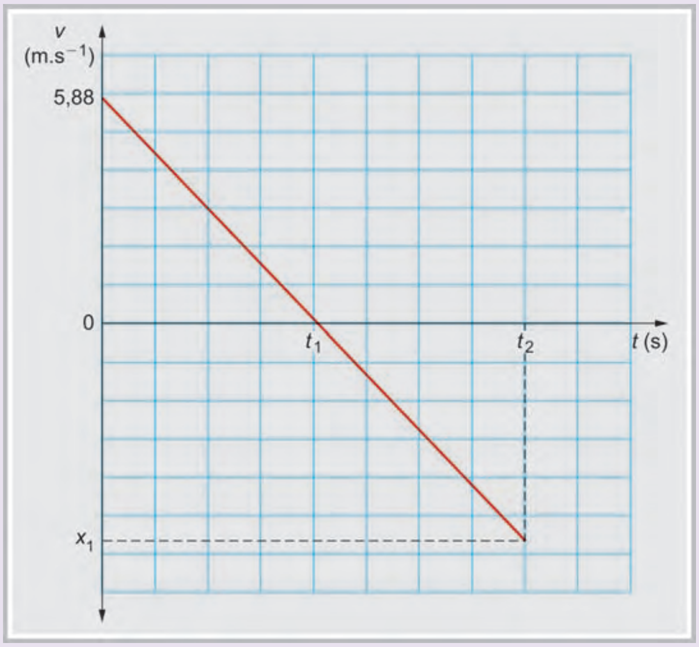


Fig 2.26b: Velocity vs time graph

a) What will be the value of the acceleration of the ball from A to B?

b) Without the use of equations of motion, calculate the value of t_1.

c) What is the value of t_2?

d) What is the value of x_1?

4. A ball is dropped from rest at point A and bounces twice at B and D, as shown in Figure 2.27a. The graph of *velocity vs time* is shown in Figure 2.27b.

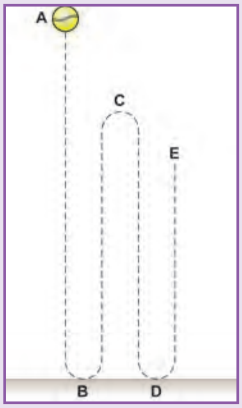


Fig 2.27a

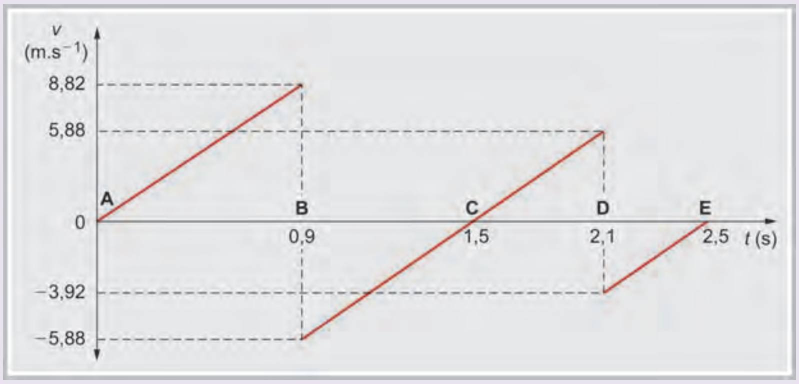


Fig 2.27b: Velocity vs time graph

a) Without the use of equations of motion, calculate:
   i) the distance from A to B.
   ii) the acceleration from A to B.
   iii) the distance from B to C.
   iv) the distance from point A to point C.
   v) the total distance travelled by the ball from A to E.

b) Draw the corresponding *displacement vs time* graph for the motion, including values on the *x* and *y* axes.

5. The graph in Figure 2.28 shows the motion of a parachutist from the moment she jumps out of the plane until she hits the ground.

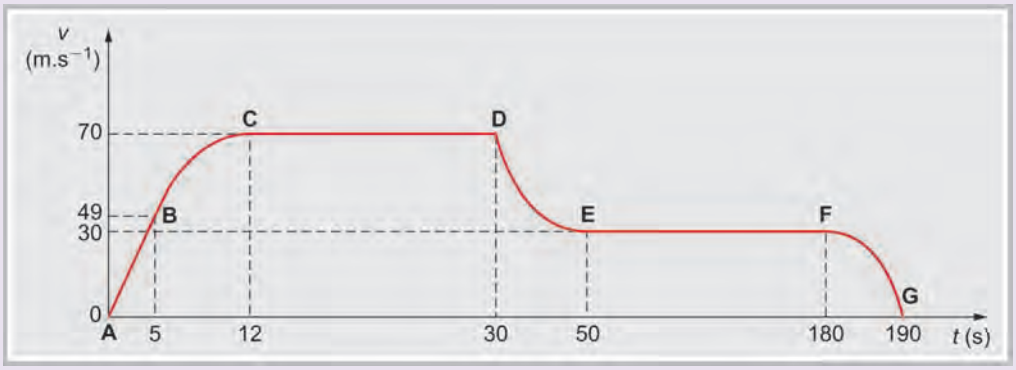


Fig 2.28

a) Describe her motion from 0 – 30 seconds.

b) What happens at 30 seconds that changes the shape of the graph so significantly?

c) From the graph, calculate:

i) the acceleration of the parachutist from 0 – 5 seconds.

ii) the distance that the parachutist falls from 0 – 5 seconds.

iii) the acceleration of the parachutist from 12 – 30 seconds.

iv) the distance that the parachutist falls from 12 – 30 seconds.

d) Why is the velocity of the parachutist so much slower from 50 – 180 s than from 12 – 30 s? Explain.

6. A ball is thrown upward at 25 $m.s^{-1}$ from the roof of a building (point A). It hits the ground at point C, as shown in Figure 2.29a. The corresponding *displacement vs time* graph for the motion is given in Figure 2.29b.

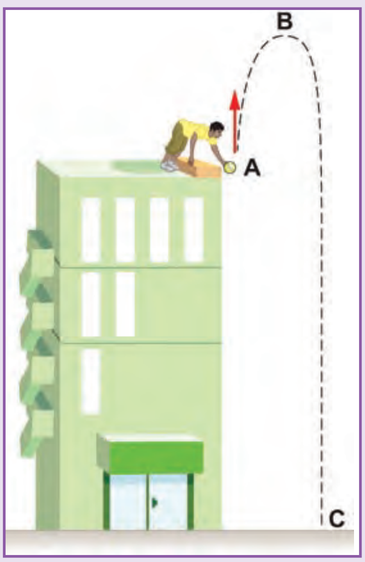


Fig 2.29a

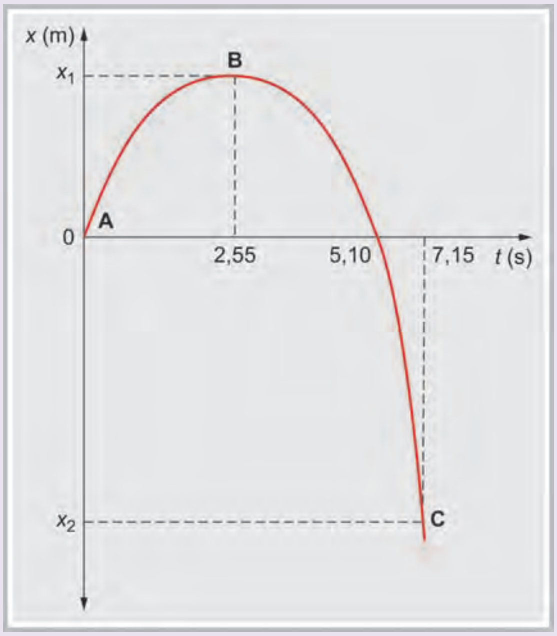


Fig 2.29b: Displacement vs time graph

a) Use equations of motion to find:

i) the distance from A to B (fill this value in on the graph)

ii) the velocity at C

b) What is the value of:

i) x_1 on the graph?

ii) x_2 on the graph?

c) Sketch the *velocity vs time* graph for the motion, including values on the x and y axes.

d) Sketch the *acceleration vs time* graph for the motion, including values on the x and y axes.

7. A ball is thrown upward from A to its highest point B. The path of the ball is shown in the sketch in figure 2.30a. The graph of *position vs time* for the motion of the ball is given in Figure 2.30b.

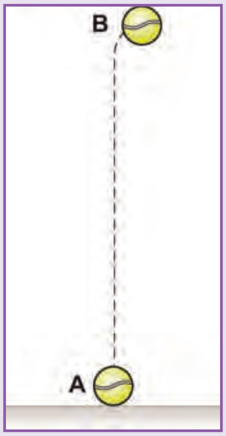


Fig 2.30a

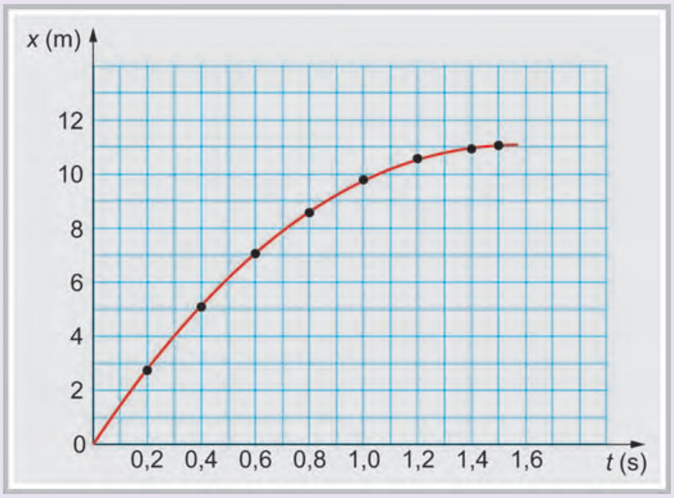


Fig 2.30b: Position vs time graph

Without the use of equations of motion, calculate the velocity of the ball at 0,8 s.

Extend yourself

1. A group of hikers come to a krans (a sheer cliff). The first hiker uses a chain ladder to climb from the soft dry river bed below to the top of the 20 m krans. The hikers below ask him to send some oranges down to them before they climb the ladder. The oranges strike the soft river bed below. Assume that air resistance is negligible.

   - He drops the first orange (A) from a height of 20 m.
   - He sends the second orange (B) down to them, throwing it with a velocity of 10 m.s^{-1} downward.
   - He throws a third orange (C) upward into the air with velocity of 10 m.s^{-1}.

   a) Calculate the time taken for orange A to reach the river bed below. (3)

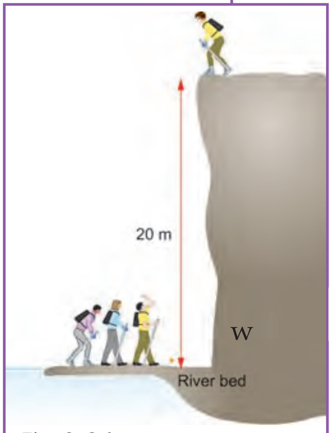


Fig 2.31

b) Calculate the magnitude of the impact velocity of each of the oranges when it reaches the river bed 20 m below him. Do this for:

i) Orange A (4)

ii) Orange B (3)

iii) Orange C (3)

c) Draw *velocity vs time* graphs (on the same set of axes) for the flight of each of the three oranges. Label them graph A, B and C to correspond with each of the oranges A, B and C. (12)

d) In which way are the flights of the three oranges similar? (1)

e) In which way do the flights of the three oranges differ? (1)

One of the hikers is curious about how the depth of the crater formed by a falling orange varies with the impact velocity of the orange. She asks you to design an experiment to answer his question.

f) Write a hypothesis for his question. (2)

g) Design an experiment to test your hypothesis. (5)

2. John designed and constructed a model rocket. He placed it on top of a high platform to allow his friends to get the best possible view.

He recorded the launch on a video tape. Using the video tape, John was able to plot the *velocity vs time* graph of the motion of the rocket. Refer to Figure 2.32.

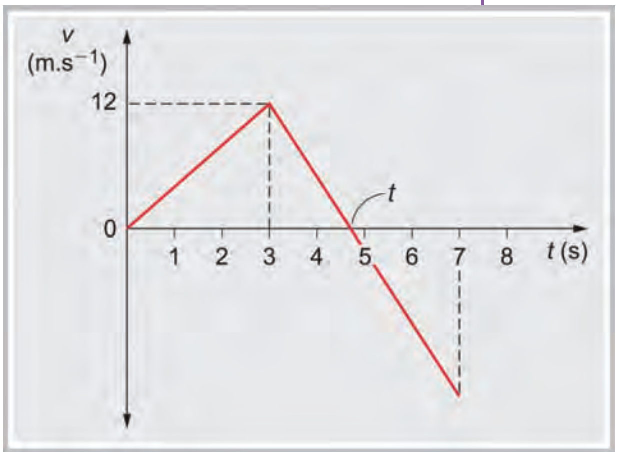


Fig 2.32

a) For the first 3 seconds:

i) describe the motion of the rocket. (4)

ii) draw and label a free-body diagram of the forces acting on the rocket. (4)

b) After 3 seconds, all the rocket fuel is burnt up and the rocket is now in free-fall.

i) What is the gradient of the graph between $t = 3$ s and $t = 7$ s? (1)

ii) Determine the time at t. (6)

c) Sketch a *displacement vs time* graph for the motion of the rocket. (5)

3. Neil and Susan are interested in the bouncing vertical motion of a ball. They set up an experiment to do this. A ball of mass 250 g is dropped from a fixed height of 2 m. Neil times the time taken from the moment the ball leaves Susan's hand until it touches the floor. He uses a stopwatch. They record the height reached after the bounce by taking photographs of the ball bouncing back to maximum height in front of a 2 m ruler. They repeat these measurements three times and record the following results. The times taken to reach the floor are 0,61 s, 0,65 s and 0,64 s respectively. The corresponding maximum heights of the bounces are 1,54 m, 1,60 m and 1,58 m respectively.

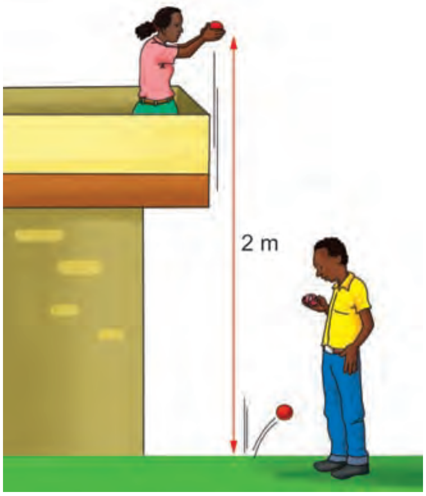


Fig 2.33

a) Draw up a table of the results of this experiment and calculate the average values of time taken and maximum height reached. Include these average values in the table. Make sure that your columns have appropriate headings and SI units. (4)

b) What is the magnitude of the acceleration of the ball while it falls to the floor? Ignore the effects of air resistance. (1)

c) Use the values from the table to calculate the magnitude of the ball's velocity when it reaches the floor. (3)

d) Use results from the table to calculate the magnitude of the ball's velocity when it leaves the floor. (3)

[54]

Definitions

free fall the motion of a body in which the only force acting on it is gravity

terminal velocity the constant speed a free falling object eventually reaches when the air resistance prevents further acceleration

Summary

- An object is in freefall when the only force acting on it is the force of gravity.
- Air resistance causes the acceleration of an object falling vertically to decrease, until eventually the acceleration is zero and the object has reached terminal velocity. Terminal velocity is a constant velocity and acceleration is zero.
- These equations can be used to solve for unknown values regarding linear motion with constant acceleration:

In symbols:	Where:
$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\Delta t$	$\boldsymbol{v}_i$ = initial velocity ($m.s^{-1}$)
$\boldsymbol{v}_f{}^2 = \boldsymbol{v}_i{}^2 + 2\boldsymbol{a}\Delta x$	$\boldsymbol{v}_f$ = final velocity ($m.s^{-1}$)
$\Delta \boldsymbol{x} = \boldsymbol{v}_i\Delta t + \frac{1}{2}\boldsymbol{a}\Delta t^2$	$\Delta \boldsymbol{x}$ = distance/displacement (m)
$\Delta x = \left(\frac{\boldsymbol{v}_i + \boldsymbol{v}_f}{2}\right)\Delta t$	Δt = time (s)
	$\boldsymbol{a}$ = acceleration ($m.s^{-2}$)

- When doing calculations from a *velocity vs time* graph:
  - Area under the graph = displacement or distance
  - Gradient of the graph = acceleration
- When doing calculations from a *displacement vs time* graph:
  - Gradient of the graph = velocity

Topic 3

Organic chemistry

What you will learn about in this topic

- Organic molecular structures
- Structure and physical property relationships
- Applications of organic chemistry
- Types of reactions of organic compounds
- Plastics and polymers

Let's talk about this topic

The photo shows an oil drill. Oil is a vital resource in the world today. From it we are able to produce fuels such as petrol, as well as other products such as road tar and plastics. The world simply cannot function without oil and its derivatives. However, it is believed that there is only enough oil reserves left in the world for another 35 to 40 years, if we continue consuming it at the current rate that we are. That is why there is so much interest in alternative energy resources. Oil is an example of an organic compound. You will learn more about organic compounds in this topic.

Organic molecular structures

What you know already

In Grade 11, in the topic 'Exploiting the lithosphere', you learnt about the use of coal and oil as fossil fuels. When these fossil fuels burn, a chemical reaction occurs that produces greenhouse gases that result in pollution and global warming. The combustion of fuels is just one type of reaction that we will investigate in this topic.

Check myself

1. What gases are formed when coal and oil are burned?
2. Are coal and oil renewable or non-renewable resources? Explain.

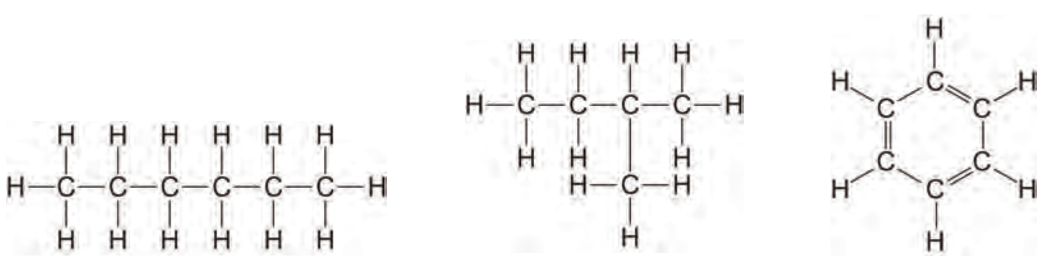
Introduction to organic molecular structures

Organic chemistry can be considered to be the chemistry of carbon compounds. Together with carbon, hydrogen is present in most organic compounds – these substances are called **hydrocarbons**. Organic compounds can also contain other elements along with carbon, such as oxygen, nitrogen, chlorine and bromine.

Carbon is the basic building block of organic compounds. Carbon recycles through the Earth's atmosphere, water, soil and living organisms, including human beings through the carbon cycle. Carbon is present in our bodies in the form of proteins, carbohydrates and fatty acids, whilst carbon is also present in other organic substances such as petrol and plastic.

Fig 3.1: Carbon is present in all these items.

There are millions of different organic materials and substances on Earth. The reason for this is because of carbon's unique bonding capabilities. Carbon has the unique characteristic among all elements to form long chains of its own atoms, a property called **catenation**. It is able to form very long chains of carbon atoms, as well as branches and rings, as shown in Figure 3.2:



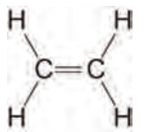
a) A straight-chain hydrocarbon

b) A branched-chain hydrocarbon

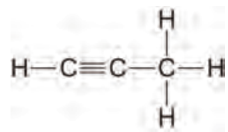
c) A hydrocarbon ring

Fig 3.2: Carboon atoms can form chains, branches and rings

Carbon can also form double and triple bonds between carbon atoms, as shown in Figure 3.3:



a) *A hydrocarbon compound containing a double bond*



b) *A hydrocarbon compound containing a triple bond*

Fig 3.3: Carbon atoms can form double and triple bonds.

Some factors which make carbon – carbon bonds unique include:

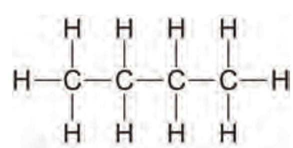
- The fact that the covalent bond between two carbon atoms is quite strong.
- Carbon compounds are not extremely reactive under ordinary conditions.
- A wide variety of carbon compounds are possible since carbon can form up to four single covalent bonds.
- The ability of carbon to make bonds with itself – a process known as catenation.
- The ability of carbon to make multiple bonds with itself.

Organic compounds can be represented in different ways. You need to know the following methods of representation:

- Molecular formula, e.g. C_4H_{10}

  This is the simplest method of representing an organic substance. It only shows how many atoms of each element there are within the molecule, but it gives us no indication as to where the atoms are found in relation to one another.

- Structural formula, e.g.

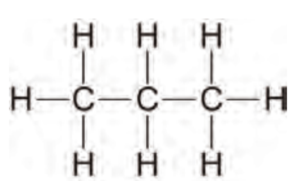
  This method shows us exactly where each atom within the molecule is found.

- Condensed structural formula, e.g. $CH_3CH_2CH_2CH_3$

  This method lists the carbon atoms in the molecule individually and shows how many hydrogen atoms are connected to each carbon atom without showing the bonds.

Checkpoint 1

Write down whether the following examples are showing the molecular formula, the condensed structural formula or the structural formula for the organic compound.

1. $CH_3CH_2CH_3$
2. C_3H_8
3. 

We will study organic compounds that are grouped together, as they have similar characteristics and properties. It is necessary to classify the millions of organic compounds that exist in some way. We do this by identifying the functional group and homologous series of the compound.

The **functional group** of a compound is an atom or a group of atoms that form the centre of chemical activity in the molecule.

A **homologous series** is a series of similar compounds which have the same functional group and whose consecutive members differ by $-CH_2$ in their molecular formula. All compounds in the same homologous series obey the same general formula.

The functional group of a compound assists us in identifying to which homologous series an organic molecule belongs.

Science around us

Rock paintings

Charcoal, which consists mostly of carbon, was used by the San people to mix black paints used in rock paintings, as shown in Figure 3.4.

Fig 3.4: Rock paintings made using charcoal.

Alkanes $-\overset{|}{\underset{|}{C}}-\overset{|}{\underset{|}{C}}-$

The alkanes are a homologous series of hydrocarbons where the molecules are characterised by single bonds between their carbon atoms. Alkanes are said to be saturated compounds. A **saturated** organic compound is one that only contains single bonds attached to carbon atoms that make up the compound. The functional group of all alkanes is $-\overset{|}{\underset{|}{C}}-\overset{|}{\underset{|}{C}}-$, as this indicates that all the carbon atoms within the molecule are connected by single bonds.

Alkanes have the general formula C_nH_{2n+2}. The general formula can be used to determine how many hydrogen atoms an alkane molecule will contain, if the number of carbon atoms within the molecule is known. For example, if the alkane contains 3 carbon atoms, then n=3. If n = 3, then the number of hydrogen atoms within the molecule will be 2(3) + 2 = 8. Therefore the molecular formula of the compound will be C_3H_8.

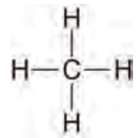
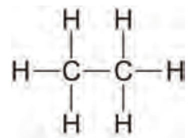
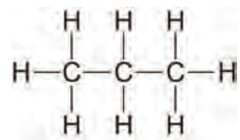
The name of the alkane is determined by the number of carbon atoms it contains in the molecule. The number of carbon atoms in the molecule determines the prefix of the name. Alkane molecules' names will always end with the suffix '-ane', indicating that they are alkanes.

Number of carbons in main carbon chain and prefix:

1-	meth	5-	pent
2-	eth	6-	hex
3-	prop	7-	hept
4-	but	8-	oct

Table 3A shows some alkanes.

Table 3A

Name	Structural Formula	Condensed Structural Formula	Molecular Formula
methane		CH_4	CH_4
ethane		CH_3CH_3	C_2H_6
propane		$CH_3CH_2CH_3$	C_3H_8

The structural formula of alkanes do not necessarily have to be drawn in a straight line. For example, octane can also be drawn as shown in Figure 3.5.

However, it is easiest to draw the carbon atoms in a straight line when drawing structural formulae.

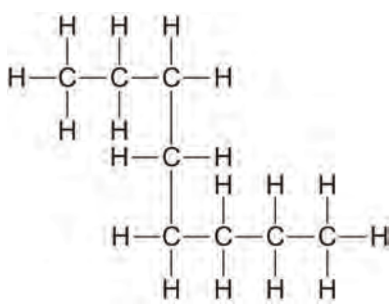


Fig 3.5: Octane

Checkpoint 2

1. Draw the structural formula for:
   a) butane
   b) hexane.
2. Use the general formula for alkanes to determine the molecular formula for:
   a) pentane
   b) octane.

Branched alkanes

We already know that organic molecules may have a branched-chain structure. We will now study branched alkanes.

To name the branches, we will need to indicate in the name the number of carbon atoms in each branch. Table 3B indicates the name of a branch, determined by how many carbon atoms the branch contains. These are known as alkyl groups, hence the name of the branch will always end in –yl.