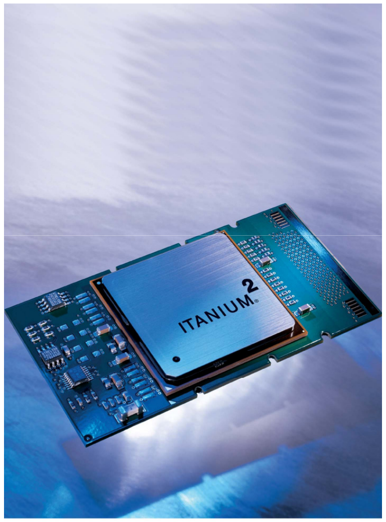
Chapter 32. Fundamentals of Circuits

Surprising as it may seem, the power of a computer is achieved simply by the controlled flow of charges through tiny wires and circuit elements.

Chapter Goal: To understand the fundamental physical principles that govern electric circuits.



Chapter 32. Fundamentals of Circuits

Topics:

- Circuit Elements and Diagrams
- Kirchhoff's Laws and the Basic Circuit
- Energy and Power
- Series Resistors
- Real Batteries
- Parallel Resistors
- Resistor Circuits
- Getting Grounded
- *RC* Circuits

Chapter 32. Reading Quizzes

How many laws are named after Kirchhoff?

A. 0
B. 1
C. 2
D. 3
E. 4

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A. 0
B. 1
✔ **C. 2**
D. 3
E. 4

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A. Short circuit
B. Chemical potential
C. Internal resistance
D. Effective capacitance
E. Inductive constant

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In an *RC* circuit, what is the name of the quantity represented by the symbol τ?

A. Period
B. Torque
C. Terminal voltage
D. Time constant
E. Coefficient of thermal expansion

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Which of the following are *ohmic* materials:

A. batteries.
B. wires.
C. resistors.
D. Materials a and b.
E. Materials b and c.

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✔ E. **Materials b and c.**

The equivalent resistance for a group of parallel resistors is

A. less than any resistor in the group.
B. equal to the smallest resistance in the group.
C. equal to the average resistance of the group.
D. equal to the largest resistance in the group.
E. larger than any resistor in the group.

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Chapter 32. Basic Content and Examples

FIGURE 32.1 An electric circuit.

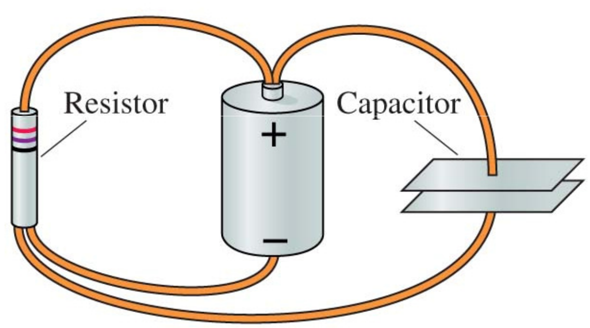


FIGURE 32.2 A library of basic symbols used for electric circuit drawings.

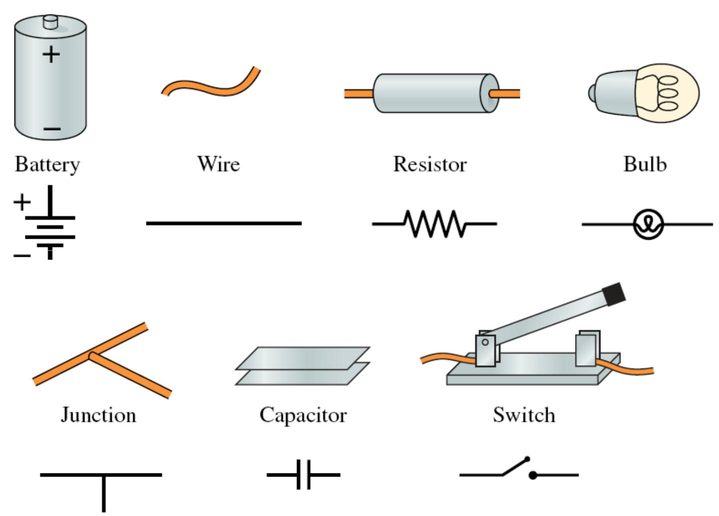
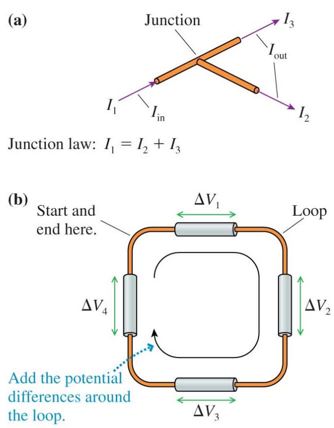


FIGURE 32.5 Kirchhoff's laws apply to junctions and loops.



Junction law: $I_1 = I_2 + I_3$

Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

Tactics: Using Kirchhoff's loop law

TACTICS BOX 32.1 **Using Kirchhoff's loop law**

❶ **Draw a circuit diagram.** Label all known and unknown quantities.

❷ **Assign a direction to the current.** Draw and label a current arrow I to show your choice.

- If you know the actual current direction, choose that direction.
- If you don't know the actual current direction, make an arbitrary choice. All that will happen if you choose wrong is that your value for I will end up negative.

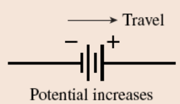
Tactics: Using Kirchhoff's loop law

❸ **"Travel" around the loop.** Start at any point in the circuit, then go all the way around the loop in the direction you assigned to the current in step 2. As you go through each circuit element, ΔV is interpreted to mean

$$\Delta V = V_{downstream} - V_{upstream}$$

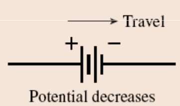
- For an ideal battery in the negative-to-positive direction:

$$\Delta V_{bat} = +\mathcal{E}$$

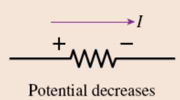


- For an ideal battery in the positive-to-negative direction:

$$\Delta V_{bat} = -\mathcal{E}$$



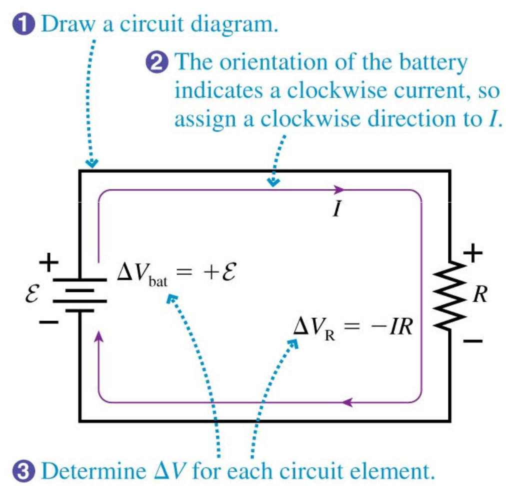
- For a resistor: $\Delta V_R = -IR$.



❹ **Apply the loop law:** $\sum (\Delta V)_i = 0$.

Exercises 4–7

FIGURE 32.7 Analysis of the basic circuit using Kirchhoff's loop law.



Energy and Power

The power supplied by a battery is

$$P_{bat} = I\mathcal{E} \qquad \text{(power delivered by an emf)}$$

The units of power are J/s, or W.
The power dissipated by a resistor is

$$P_R = \frac{dE_{th}}{dt} = \frac{dq}{dt}\Delta V_R = I\Delta V_R$$

Or, in terms of the potential drop across the resistor

$$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R} \qquad \text{(power dissipated by a resistor)}$$

EXAMPLE 32.4 The power of light

QUESTION:

EXAMPLE 32.4 The power of light
How much current is "drawn" by a 100 W lightbulb connected to a 120 V outlet?

EXAMPLE 32.4 The power of light

MODEL Most household appliances, such as a 100 W lightbulb or a 1500 W hair dryer, have a power rating. The rating does *not* mean that these appliances *always* dissipate that much power. These appliances are intended for use at a standard household voltage of 120 V, and their rating is the power they will dissipate *if* operated with a potential difference of 120 V. Their power consumption will differ from the rating if they are operated at any other potential difference.

EXAMPLE 32.4 The power of light

SOLVE Because the lightbulb is operating as intended, it will dissipate 100 W of power. Thus

$$I = \frac{P_R}{\Delta V_R} = \frac{100\text{ W}}{120\text{ V}} = 0.833\text{ A}$$

EXAMPLE 32.4 The power of light

ASSESS A current of 0.833 A in this lightbulb transfers 100 J/s to the thermal energy of the filament, which, in turn, dissipates 100 J/s as heat and light to its surroundings.

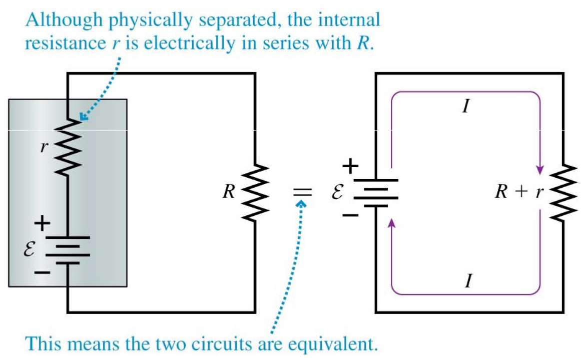
Series Resistors

- Resistors that are aligned end to end, *with no junctions between them,* are called **series resistors** or, sometimes, resistors "in series."
- The current I is the same through all resistors placed in series.
- If we have N resistors in series, their **equivalent resistance** is

$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad \text{(series resistors)}$$

The behavior of the circuit will be unchanged if the N series resistors are replaced by the single resistor R_{eq}.

FIGURE 32.20 A single resistor connected to a real battery is in series with the battery's internal resistance, giving $R_{eq} = R + r$.



EXAMPLE 32.7 Lighting up a flashlight

QUESTION:

EXAMPLE 32.7 Lighting up a flashlight

A 6 Ω flashlight bulb is powered by a 3 V battery with an internal resistance of 1 Ω. What are the power dissipation of the bulb and the terminal voltage of the battery?

EXAMPLE 32.7 Lighting up a flashlight

MODEL Assume ideal connecting wires but not an ideal battery.

EXAMPLE 32.7 Lighting up a flashlight

VISUALIZE The circuit diagram looks like Figure 32.20. R is the resistance of the bulb's filament.

EXAMPLE 32.7 Lighting up a flashlight

SOLVE Equation 32.19 gives us the current:

$$I = \frac{\mathcal{E}}{R + r} = \frac{3\text{ V}}{6\ \Omega + 1\ \Omega} = 0.43\text{ A}$$

This is 15% less than the 0.5 A an ideal battery would supply. The potential difference across the resistor is $\Delta V_R = IR = 2.6$ V, thus the power dissipation is

$$P_R = I\Delta V = 1.1\text{ W}$$

The battery's terminal voltage is

$$\Delta V_{bat} = \frac{R}{R + r}\mathcal{E} = \frac{6\ \Omega}{6\ \Omega + 1\ \Omega}3\text{ V} = 2.6\text{ V}$$

EXAMPLE 32.7 Lighting up a flashlight

ASSESS 1 Ω is a typical internal resistance for a flashlight battery. The internal resistance causes the battery's terminal voltage to be 0.4 V less than its emf in this circuit.

Parallel Resistors

- Resistors connected *at both ends* are called **parallel resistors** or, sometimes, resistors "in parallel."
- The left ends of all the resistors connected in parallel are held at the same potential V_1, and the right ends are all held at the same potential V_2.
- The potential differences ΔV are the *same* across all resistors placed in parallel.
- If we have N resistors in parallel, their **equivalent resistance** is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \qquad \text{(parallel resistors)}$$

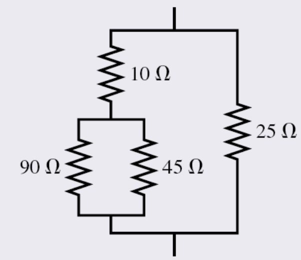
The behavior of the circuit will be unchanged if the N parallel resistors are replaced by the single resistor R_{eq}.

EXAMPLE 32.10 A combination of resistors

QUESTION:

EXAMPLE 32.10 A combination of resistors
What is the equivalent resistance of the group of resistors shown in FIGURE 32.26?

FIGURE 32.26 A combination of resistors.



EXAMPLE 32.10 A combination of resistors

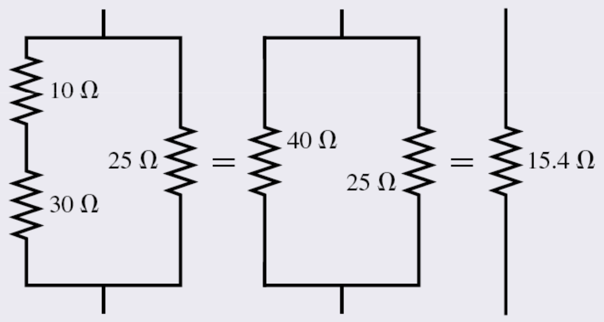
MODEL This circuit contains both series and parallel resistors.

EXAMPLE 32.10 A combination of resistors

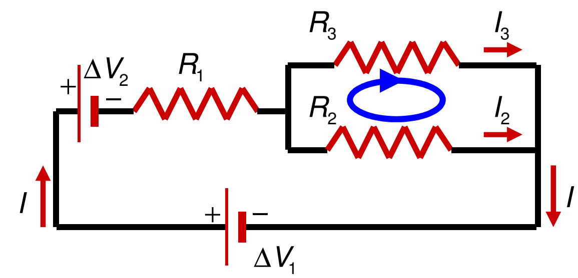
SOLVE Reduction to a single equivalent resistance is best done in a series of steps, with the circuit being redrawn after each step. The procedure is shown in FIGURE 32.27. Note that the 10 Ω and 25 Ω resistors are *not* in parallel. They are connected at their top ends but not at their bottom ends. Resistors must be connected at *both* ends to be in parallel. Similarly, the 10 Ω and 45 Ω resistors are *not* in series because of the junction between them. If the original group of four resistors occurred within a larger circuit, they could be replaced with a single 15.4 Ω resistor without having any effect on the rest of the circuit.

EXAMPLE 32.10 A combination of resistors

FIGURE 32.27 The combination is reduced to a single equivalent resistor.



Example: Kirchoff's Rules

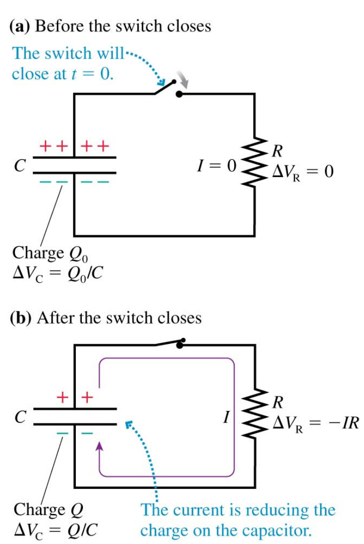


$$I = I_2 + I_3$$

$$-I_3 R_3 + I_2 R_2 = 0$$

$$\Delta V_1 - \Delta V_2 - I_2 R_2 - I R_1 = 0$$

FIGURE 32.35 An *RC* circuit.



RC Circuits

- Consider a charged capacitor, an open switch, and a resistor all hooked in series. This is an *RC* Circuit.
- The capacitor has charge Q_0 and potential difference $\Delta V_C = Q_0/C$.
- There is no current, so the potential difference across the resistor is zero.
- At $t = 0$ the switch closes and the capacitor begins to discharge through the resistor.
- The capacitor charge as a function of time is

$$Q = Q_0 e^{t/\tau}$$

where the time constant τ is

$$\tau = RC$$

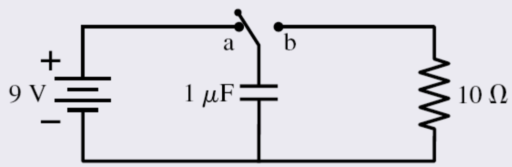
EXAMPLE 32.14 Exponential decay in an RC circuit

QUESTION:

EXAMPLE 32.14 Exponential decay in an *RC* circuit

The switch in FIGURE 32.37 has been in position a for a long time. It is changed to position b at $t = 0$ s. What are the charge on the capacitor and the current through the resistor at $t = 5.0\ \mu s$?

FIGURE 32.37 An *RC* circuit.



EXAMPLE 32.14 Exponential decay in an RC circuit

MODEL The battery charges the capacitor to 9.0 V. Then, when the switch is changed to position b, the capacitor discharges through the 10 Ω resistor. Assume ideal wires.

EXAMPLE 32.14 Exponential decay in an RC circuit

SOLVE The time constant of the *RC* circuit is

$$\tau = RC = (10\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10 \times 10^{-6}\ \text{s} = 10\ \mu\text{s}$$

The capacitor is initially charged to 9.0 V, giving $Q_0 = C\Delta V_C = 9.0\ \mu C$. The capacitor charge at $t = 5.0\ \mu s$ is

$$Q = Q_0 e^{-t/RC} = (9.0\ \mu\text{C})e^{-(5.0\ \mu\text{s})/(10\ \mu\text{s})}$$
$$= (9.0\ \mu\text{C})e^{-0.5} = 5.5\ \mu\text{C}$$

The initial current, immediately after the switch is closed, is $I_0 = Q_0/\tau = 0.90$ A. The resistor current at $t = 5.0\ \mu s$ is

$$I = I_0 e^{-t/RC} = (0.90\ \text{A})e^{-0.5} = 0.55\ \text{A}$$

EXAMPLE 32.14 Exponential decay in an RC circuit

ASSESS This capacitor will be almost entirely discharged $5\tau = 50\ \mu s$ after the switch is closed.

Chapter 32. Summary Slides

General Strategy

MODEL Assume that wires and, where appropriate, batteries are ideal.

VISUALIZE Draw a circuit diagram. Label all known and unknown quantities.

SOLVE Base the solution on Kirchhoff's laws.

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Write one loop equation for each independent loop.
- Find the current and the potential difference.
- Rebuild the circuit to find I and ΔV for each resistor.

ASSESS Verify that

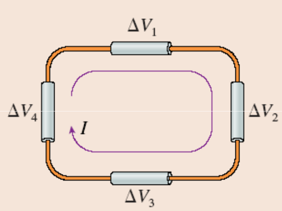
- The sum of potential differences across series resistors matches ΔV for the equivalent resistor.
- The sum of the currents through parallel resistors matches I for the equivalent resistor.

General Strategy

Kirchhoff's loop law

For a closed loop:

- Assign a direction to the current I.
- $\sum_i (\Delta V)_i = 0$

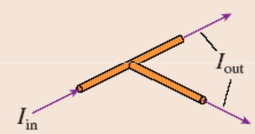


General Strategy

Kirchhoff's junction law

For a junction:

- $\sum I_{in} = \sum I_{out}$



Important Concepts

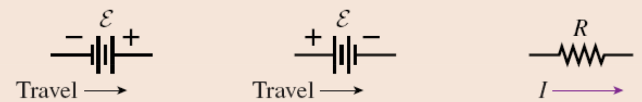
Ohm's Law

A potential difference ΔV between the ends of a conductor with resistance R creates a current

$$I = \frac{\Delta V}{R}$$

Important Concepts

Signs of ΔV



$\Delta V_{bat} = +\mathcal{E}$ $\quad$ $\Delta V_{bat} = -\mathcal{E}$ $\quad$ $\Delta V_R = -IR$

Important Concepts

The **energy used by a circuit** is supplied by the emf $\mathcal{E}$ of the battery through the energy transformations

$$E_{chem} \rightarrow U \rightarrow K \rightarrow E_{th}$$

The battery *supplies* energy at the rate

$$P_{bat} = I\mathcal{E}$$

The resistors *dissipate* energy at the rate

$$P_R = I\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R}$$

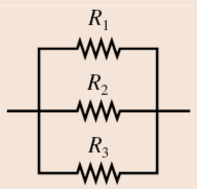
Applications

Series resistors

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Parallel resistors

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$$



Applications

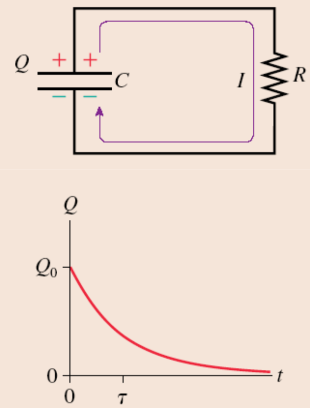
RC circuits

The discharge of a capacitor through a resistor satisfies:

$$Q = Q_0 e^{-t/\tau}$$

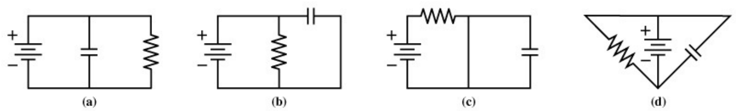
$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}$$

where $\tau = RC$ is the **time constant.**



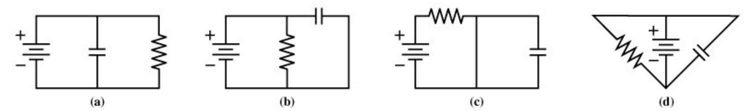
Chapter 32. Questions

Which of these diagrams represent the same circuit?



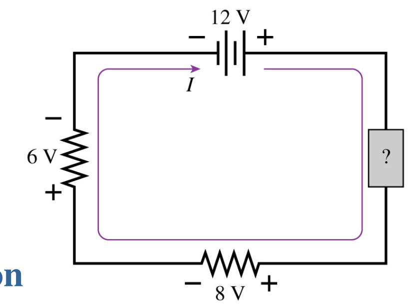
A. a and b
B. b and c
C. a and c
D. a, b, and d
E. a, b, and c

Which of these diagrams represent the same circuit?



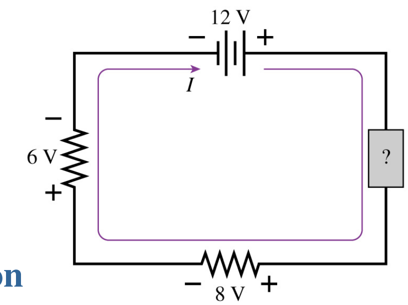
A. a and b
B. b and c
C. a and c
✔ **D. a, b, and d**
E. a, b, and c

What is ΔV across the unspecified circuit element? Does the potential increase or decrease when traveling through this element in the direction assigned to I?



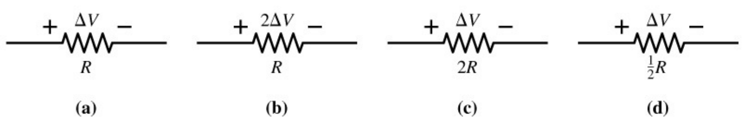
A. ΔV decreases by 2 V in the direction of I.
B. ΔV increases by 2 V in the direction of I.
C. ΔV decreases by 10 V in the direction of I.
D. ΔV increases by 10 V in the direction of I.
E. ΔV increases by 26 V in the direction of I.

What is ΔV across the unspecified circuit element? Does the potential increase or decrease when traveling through this element in the direction assigned to I?



A. ΔV decreases by 2 V in the direction of I.
✔ **B. ΔV increases by 2 V in the direction of I.**
C. ΔV decreases by 10 V in the direction of I.
D. ΔV increases by 10 V in the direction of I.
E. ΔV increases by 26 V in the direction of I.

Rank in order, from largest to smallest, the powers P_a to P_d dissipated in resistors a to d.



A. $P_b > P_a = P_c = P_d$
B. $P_b = P_d > P_a > P_c$
C. $P_b = P_c > P_a > P_c$
D. $P_b > P_d > P_a > P_c$
E. $P_b > P_c > P_a > P_d$

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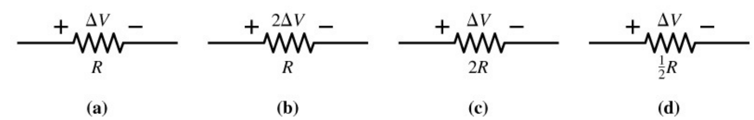
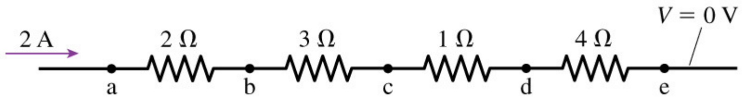


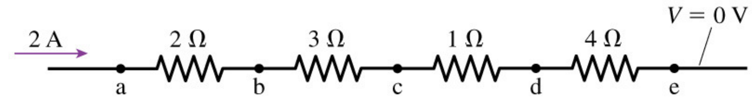
A. $P_b > P_a = P_c = P_d$
B. $P_b = P_d > P_a > P_c$
C. $P_b = P_c > P_a > P_c$
✔ **D. $P_b > P_d > P_a > P_c$**
E. $P_b > P_c > P_a > P_d$

What is the potential at points a to e ?



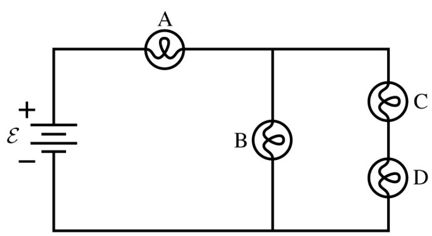
A. $V_a = 0$ V, $V_b = -4$ V, $V_c = -10$ V, $V_d = -12$ V, $V_e = -20$ V
B. $V_a = -20$ V, $V_b = -16$ V, $V_c = -10$ V, $V_d = -8$ V, $V_e = 0$ V
C. $V_a = -4$ V, $V_b = -6$ V, $V_c = -2$ V, $V_d = -8$ V, $V_e = 0$ V
D. $V_a = 4$ V, $V_b = 6$ V, $V_c = 2$ V, $V_d = 8$ V, $V_e = 0$ V
E. $V_a = 20$ V, $V_b = 16$ V, $V_c = 10$ V, $V_d = 8$ V, $V_e = 0$ V

What is the potential at points a to e ?



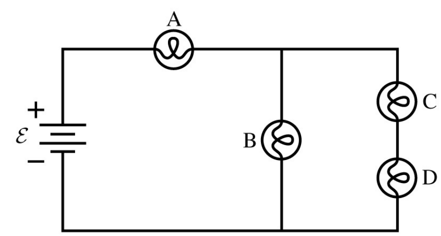
A. $V_a = 0$ V, $V_b = -4$ V, $V_c = -10$ V, $V_d = -12$ V, $V_e = -20$ V
B. $V_a = -20$ V, $V_b = -16$ V, $V_c = -10$ V, $V_d = -8$ V, $V_e = 0$ V
C. $V_a = -4$ V, $V_b = -6$ V, $V_c = -2$ V, $V_d = -8$ V, $V_e = 0$ V
D. $V_a = 4$ V, $V_b = 6$ V, $V_c = 2$ V, $V_d = 8$ V, $V_e = 0$ V
✔ **E. $V_a = 20$ V, $V_b = 16$ V, $V_c = 10$ V, $V_d = 8$ V, $V_e = 0$ V**

Rank in order, from brightest to dimmest, the identical bulbs A to D.



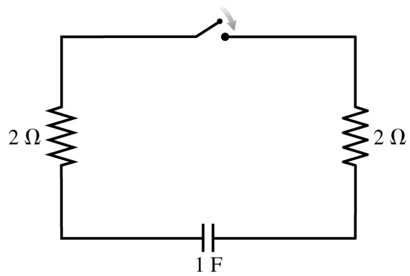
A. C = D > B > A
B. A > C = D > B
C. A = B = C = D
D. A > B > C = D
E. A > C > B > D

Rank in order, from brightest to dimmest, the identical bulbs A to D.



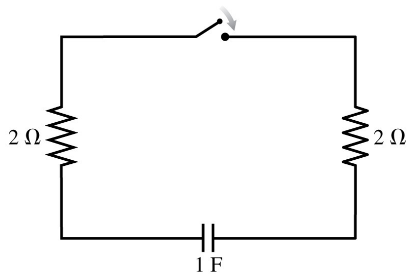
A. C = D > B > A
B. A > C = D > B
C. A = B = C = D
✔ **D. A > B > C = D**
E. A > C > B > D

The time constant for the discharge of this capacitor is



A. 5 s.
B. 1 s.
C. 2 s.
D. 4 s.
E. The capacitor doesn't discharge because the resistors cancel each other.

The time constant for the discharge of this capacitor is



A. 5 s.
B. 1 s.
C. 2 s.
✔ **D. 4 s.**
E. The capacitor doesn't discharge because the resistors cancel each other.