



Interest Rates and Valuing Cash Flows

PART 2

Valuation Principle Connection. In this part of the text, we introduce the basic tools for making financial decisions. Chapter 3 presents the most important idea in this book, the *Valuation Principle*. The Valuation Principle states that we can use market prices to determine the value of an investment opportunity to the firm. As we progress through our study of corporate finance, we will demonstrate that the Valuation Principle is the one unifying principle that underlies all of finance and links all the ideas throughout this book.

Every day, managers in companies all over the world make financial decisions. These range from relatively minor decisions such as a local hardware store owner's determination of when to restock inventory, to major decisions such as Starbucks' 2008 closing of more than 600 stores, Apple's 2010 launch of the iPad, and Microsoft's 2012 software overhaul launching Windows 8. What do these diverse decisions have in common? They all were made by comparing the costs of the action against the value to the firm of the benefits. Specifically, a company's managers must determine what it is worth to the company today to receive the project's future cash inflows while paying its cash outflows.

In Chapter 3, we start to build the tools to undertake this analysis with a central concept in financial economics—the *time value of money*. In Chapter 4, we explain how to value any series of future cash flows and derive a few useful shortcuts for valuing various types of cash flow patterns. Chapter 5 discusses how interest rates are quoted in the market and how to handle interest rates that compound more frequently than once per year. In Chapter 6, we will apply what we have learned about interest rates and the present value of cash flows to the task of valuing bonds. In the last chapter of Part 2, Chapter 7, we discuss the features of common stocks and learn how to calculate an estimate of their value.

Chapter 3
Time Value of Money:
An Introduction

Chapter 4
Time Value of Money:
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Streams

Chapter 5
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3

Time Value of Money: An Introduction

LEARNING OBJECTIVES

- Identify the roles of financial managers and competitive markets in decision making
- Understand the Valuation Principle, and how it can be used to identify decisions that increase the value of the firm
- Assess the effect of interest rates on today's value of future cash flows
- Calculate the value of distant cash flows in the present and of current cash flows in the future

notation

r	interest rate	C	cash flow
PV	present value	n	number of periods
FV	future value		

In 2011, Amazon.com managers decided to more directly compete in the tablet market with the launch of the Kindle Fire, and they priced it at \$199, which by some estimates was either at or below the cost to build it. How did Amazon's managers decide this was the right decision for the company?

Every decision has future consequences that will affect the value of the firm. These consequences will generally include both benefits and costs. For example, in addition to the up-front cost of developing its own mobile phone and software, Amazon will also incur ongoing costs associated with future software and hardware development for the Fire, marketing efforts, and customer support. The benefits to Amazon include the revenues from the sales as well as additional content purchased through the device. This decision will increase Amazon's value if these benefits outweigh the costs.

More generally, a decision is good for the firm's investors if it increases the firm's value by providing benefits whose value exceeds the costs. But how do we compare costs and benefits that occur at different points in time, or are in different currencies, or have different risks associated with them? To make a valid comparison, we must use the tools of finance to express all costs and benefits in common terms. We convert all costs and benefits into a common currency and common point of time, such as dollars today. In this chapter, we learn (1) how to use market information to evaluate costs and benefits and (2) why market prices are so important. Then, we will start to build the critical tools relating to the time value of money. These tools will allow you to correctly compare the costs and benefits of a decision no matter when they occur.

3.1 Cost-Benefit Analysis

The first step in decision making is to identify the costs and benefits of a decision. In this section, we look at the role of financial managers in evaluating costs and benefits and the tools they use to quantify them.

Role of the Financial Manager

A financial manager's job is to make decisions on behalf of the firm's investors. Our objective in this book is to explain how to make decisions that increase the value of the firm to its investors. In principle, the idea is simple and intuitive: For good decisions, the benefits exceed the costs. Of course, real-world opportunities are usually complex and the costs and benefits are often difficult to quantify. Quantifying them often means using skills from other management disciplines, as in the following examples:

Marketing: to determine the increase in revenues resulting from an advertising campaign

Economics: to determine the increase in demand from lowering the price of a product

Organizational Behavior: to determine the effect of changes in management structure on productivity

Strategy: to determine a competitor's response to a price increase

Operations: to determine production costs after the modernization of a manufacturing plant

For the remainder of this text, we will assume we can rely on experts in these areas to provide this information so the costs and benefits associated with a decision have already been identified. With that task done, the financial manager's job is to compare the costs and benefits and determine the best decision for the value of the firm.

Quantifying Costs and Benefits

Any decision in which the value of the benefits exceeds the costs will increase the value of the firm. To evaluate the costs and benefits of a decision, we must value the options in the same terms—cash today. Let's make this concrete with a simple example.

Suppose a jewelry manufacturer has the opportunity to trade 200 ounces of silver for 10 ounces of gold today. An ounce of silver differs in value from an ounce of gold. Consequently, it is incorrect to compare 200 ounces to 10 ounces and conclude that the larger quantity is better. Instead, to compare the cost of the silver and the benefit of the gold, we first need to quantify their values in equivalent terms—cash today.

Consider the silver. What is its cash value today? Suppose silver can be bought and sold for a current market price of \$20 per ounce. Then the 200 ounces of silver we would give up has a cash value of:¹

$$(200 \text{ ounces of silver}) \times (\$20/\text{ounce of silver}) = \$4000$$

If the current market price for gold is \$1000 per ounce, then the 10 ounces of gold we would receive has a cash value of

$$(10 \text{ ounces of gold}) \times (\$1000/\text{ounce of gold}) = \$10,000$$

We have now quantified the decision. The jeweler's opportunity has a benefit of \$10,000 and a cost of \$4000. The net benefit of the decision is $\$10,000 - \$4000 = \$6000$ today.

¹You might wonder whether commissions and other transactions costs need to be included in this calculation. For now, we will ignore transactions costs, but we will discuss their effect in later chapters.

The net value of the decision is positive, so by accepting the trade, the jewelry firm will be richer by \$6000.

Role of Competitive Market Prices. Suppose the jeweler works exclusively on silver jewelry or thinks the price of silver should be higher. Should his decision change? The answer is no—he can always make the trade and then buy silver at the current market price. Even if he has no use for the gold, he can immediately sell it for \$10,000, buy back the 200 ounces of silver at the current market price of \$4000, and pocket the remaining \$6000. Thus, independent of his own views or preferences, the value of the silver to the jeweler is \$4000.

Because the jeweler can both buy and sell silver at its current market price, his personal preferences or use for silver and his opinion of the fair price are irrelevant in evaluating the value of this opportunity. This observation highlights an important general principle related to goods trading in a **competitive market**, a market in which a good can be bought *and* sold at the same price. Whenever a good trades in a competitive market, that price determines the value of the good. This point is one of the central and most powerful ideas in finance. It will underlie almost every concept we develop throughout the text.

competitive market A market in which a good can be bought *and* sold at the same price.

EXAMPLE 3.1

Competitive Market Prices Determine Value

MyFinanceLab

PROBLEM

You have just won a radio contest and are disappointed to learn that the prize is four tickets to the Def Leppard reunion tour (face value \$40 each). Not being a fan of 1980s power rock, you have no intention of going to the show. However, the radio station offers you another option: two tickets to your favorite band's sold-out show (face value \$45 each). You notice that, on eBay, tickets to the Def Leppard show are being bought and sold for \$30 apiece and tickets to your favorite band's show are being bought and sold at \$50 each. What should you do?

SOLUTION

PLAN

Market prices, not your personal preferences (or the face value of the tickets), are relevant here:

- 4 Def Leppard tickets at \$30 apiece
- 2 of your favorite band's tickets at \$50 apiece

You need to compare the market value of each option and choose the one with the highest market value.

EXECUTE

The Def Leppard tickets have a total value of \$120 ($4 \times \30) versus the \$100 total value of the other 2 tickets ($2 \times \$50$). Instead of taking the tickets to your favorite band, you should accept the Def Leppard tickets, sell them on eBay, and use the proceeds to buy 2 tickets to your favorite band's show. You'll even have \$20 left over to buy a T-shirt.

EVALUATE

Even though you prefer your favorite band, you should still take the opportunity to get the Def Leppard tickets instead. As we emphasized earlier, whether this opportunity is attractive depends on its net value using market prices. Because the value of Def Leppard tickets is \$20 more than the value of your favorite band's tickets, the opportunity is appealing.

Concept Check

1. When costs and benefits are in different units or goods, how can we compare them?
2. If crude oil trades in a competitive market, would an oil refiner that has a use for the oil value it differently than another investor would?

3.2 Market Prices and the Valuation Principle

In the previous examples, the right decisions for the firms were clear because the costs and benefits were easy to evaluate and compare. They were easy to evaluate because we were able to use current market prices to convert them into equivalent cash values. Once we can express costs and benefits in terms of cash today, it is a straightforward process to compare them and determine whether the decision will increase the firm's value.

The Valuation Principle

Our discussion so far establishes competitive market prices as the way to evaluate the costs and benefits of a decision in terms of cash today. Once we do this, it is a simple matter to determine the best decision for the firm. The best decision makes the firm and its investors wealthier, because the value of its benefits exceeds the value of its costs. We call this idea the Valuation Principle:

The Valuation Principle:

The value of a commodity or an asset to the firm or its investors is determined by its competitive market price. The benefits and costs of a decision should be evaluated using those market prices. When the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm.

The Valuation Principle provides the basis for decision making throughout this text. In the remainder of this chapter, we apply it to decisions whose costs and benefits occur at different points in time.

EXAMPLE 3.2

Applying the Valuation Principle

MyFinanceLab

PROBLEM

You are the operations manager at your firm. Due to a pre-existing contract, you have the opportunity to acquire 200 barrels of oil and 3000 pounds of copper for a total of \$25,000. The current market price of oil is \$90 per barrel and for copper is \$3.50 per pound. You are not sure that you need all the oil and copper, so you are wondering whether you should take this opportunity. How valuable is it? Would your decision change if you believed the value of oil or copper would plummet over the next month?

SOLUTION

PLAN

We need to quantify the costs and benefits using market prices. We are comparing \$25,000 with:

200 barrels of oil at \$90 per barrel

3000 pounds of copper at \$3.50 per pound

EXECUTE

Using the competitive market prices we have:

$$\begin{aligned} (200 \text{ barrels of oil}) \times (\$90/\text{barrel today}) &= \$18,000 \text{ today} \\ (3000 \text{ pounds of copper}) \times (\$3.50/\text{pound today}) &= \$10,500 \text{ today} \end{aligned}$$

The value of the opportunity is the value of the oil plus the value of the copper less the cost of the opportunity, or $\$18,000 + \$10,500 - \$25,000 = \$3,500$ today. Because the value is positive, we should take it. This value depends only on the *current* market prices for oil and copper. If we do not need all of the oil and copper, we can sell the excess at current market prices. Even if we thought the value of oil or copper was about to plummet, the value of this investment would be unchanged. (We can always exchange them for dollars immediately at the current market prices.)

EVALUATE

Since we are transacting today, only the current prices in a competitive market matter. Our own use for or opinion about the future prospects of oil or copper do not alter the value of the decision today. This decision is good for the firm and will increase its value by \$3500.

Why There Can Be Only One Competitive Price for a Good

The Valuation Principle and finance in general rely on using a competitive market price to value a cost or benefit. We cannot have two different competitive market prices for the same good—otherwise we would arrive at two different values. Fortunately, powerful market forces keep competitive prices the same. To illustrate, imagine what you would do if you saw gold simultaneously trading for two different prices. You and everyone else who noticed the difference would buy at the low price and sell at the high price for as many ounces of gold as possible, making instant risk-free profits. The flood of buy and sell orders would push the two prices together until the profit was eliminated. These forces establish the **Law of One Price**, which states that in competitive markets, the same good or securities must have the same price. More generally, securities that produce exactly the same cash flows must have the same price.

Law of One Price In competitive markets, securities with the same cash flows must have the same price.

arbitrage The practice of buying and selling equivalent goods to take advantage of a price difference.

arbitrage opportunity Any situation in which it is possible to make a profit without taking any risk or making any investment.

In general, the practice of buying and selling equivalent goods in different markets to take advantage of a price difference is known as **arbitrage**. We refer to any situation in which it is possible to make a profit without taking any risk or making any investment as an **arbitrage opportunity**. Because an arbitrage opportunity's benefits are more valuable than its costs, whenever an arbitrage opportunity appears in financial markets, investors will race to take advantage of it and their trades will eliminate the opportunity.

Retail stores often quote different prices for the same item in different countries. *The Economist* magazine has long compared prices for a McDonald's Big Mac around the world. Here, we compare Big Mac prices from January 2013. The price in the local currency and converted to U.S. dollars is listed. Of course, these prices are not examples of competitive market prices, because you can only buy a Big Mac at these prices. Hence, they do not present an arbitrage opportunity. Even if shipping were free, you could buy as many Big Macs as you could get your hands on in India but you would not be able to sell those rotten Big Macs in Venezuela for a profit!

Country	Local Cost	U.S. Dollar Cost
India	89.00 rupees	\$1.67
Hong Kong	17.00 HK dollars	\$2.19
Russia	72.88 rubles	\$2.43
China	16.00 yuan	\$2.57
Mexico	37.00 pesos	\$2.90
UAE	12.00 dirhams	\$3.27
Japan	320.00 yen	\$3.51
United States	4.37 US dollars	\$4.37
France	3.60 euros	\$4.89
Australia	4.70 Australian dollars	\$4.90
Germany	3.64 euros	\$4.94
Canada	5.41 Canadian dollars	\$5.39
Brazil	11.25 reais	\$5.64
Switzerland	6.50 Swiss francs	\$7.12
Sweden	48.40 Swedish krona	\$7.62
Venezuela	39.00 bolivares fuertes	\$9.08

Source: Economist.com.

Your Personal Financial Decisions

While the focus of this text is on the decisions a financial manager makes in a business setting, you will soon see that concepts and skills you will learn here apply to personal decisions as well. As a normal part of life we all make decisions that trade off benefits and costs across time. Going to college, purchasing

this book, saving for a new car or house down payment, taking out a car loan or home loan, buying shares of stock, and deciding between jobs are just a few examples of decisions you have faced or could face in the near future. As you read through this book, you will see that the Valuation Principle is the foundation of *all* financial decision making—whether in a business or in a personal context.

Concept Check

- How do investors' profit motives keep competitive market prices correct?
- How do we determine whether a decision increases the value of the firm?

3.3 The Time Value of Money and Interest Rates

Unlike the examples presented so far, most financial decisions have costs and benefits that occur at different points in time. For example, typical investment projects incur costs up front and provide benefits in the future. In this section, we show how to account for this time difference when using the Valuation Principle to make a decision.

The Time Value of Money

Your company has an investment opportunity with the following cash flows:

Cost: \$100,000 today
Benefit: \$105,000 in one year

Both are expressed in dollar terms, but are the cost and benefit directly comparable? No. Calculating the project's net value as $\$105,000 - \$100,000 = \$5,000$ is incorrect because it ignores the *timing* of the costs and benefits. That is, it treats money today as equivalent to money in one year. In general, a dollar received today is worth *more* than a dollar received in one year: If you have \$1 today, you can invest it now and have more money in the future. For example, if you deposit it in a bank account paying 10% interest, you will have \$1.10 at the end of one year. We call the difference in value between money today and money in the future the **time value of money**.

time value of money The difference in value between money received today and money received in the future; also, the observation that two cash flows at two different points in time have different values.

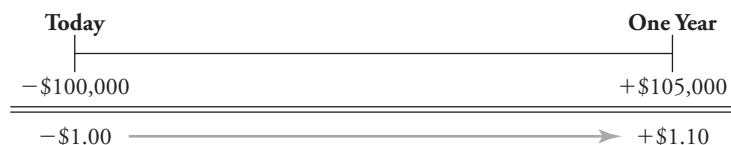


Figure 3.1 illustrates how we use competitive market prices and interest rates to convert between dollars today and other goods, or dollars in the future. Just like silver and gold, money today and money tomorrow are not the same thing. We compare them just like we did with silver and gold—using competitive market prices. But in the case of money, what is the price? It is the interest rate, the price for exchanging money today for money in a year. We can use the interest rate to determine values in the same way we used competitive market prices. Once we quantify all the costs and benefits of an investment

FIGURE 3.1
Converting Between Dollars Today and Gold or Dollars in the Future

We can convert dollars today to different goods or points in time by using the competitive market price or interest rate. Once values are in equivalent terms, we can use the Valuation Principle to make a decision.



in terms of dollars today, we can rely on the Valuation Principle to determine whether the investment will increase the firm's value.

The Interest Rate: Converting Cash Across Time

We now develop the tools needed to value our \$100,000 investment opportunity correctly. By depositing money into a savings account, we can convert money today into money in the future with no risk. Similarly, by borrowing money from the bank, we can exchange money in the future for money today. Suppose the current annual interest rate is 10%. By investing \$1 today we can convert this \$1 into \$1.10 in one year. Similarly, by borrowing at this rate, we can exchange \$1.10 in one year for \$1 today. More generally, we define the **interest rate**, r , for a given period as the interest rate at which money can be borrowed or lent over that period. In our example, the interest rate is 10% and we can exchange 1 dollar today for $(1 + .10)$ dollars in one year. In general, we can exchange 1 dollar today for $(1 + r)$ dollars in one year, and vice versa. We refer to $(1 + r)$ as the **interest rate factor** for cash flows; it defines how we convert cash flows across time, and has units of “\$ in one year/\$ today.”

Like other market prices, the interest rate ultimately depends on supply and demand. In particular, the interest rate equates the supply of savings to the demand for borrowing. But regardless of how it is determined, once we know the interest rate, we can apply the Valuation Principle and use it to evaluate other decisions in which costs and benefits are separated in time.

Value of \$100,000 Investment in One Year. Let's reevaluate the investment we considered earlier, this time taking into account the time value of money. If the interest rate is 10%, then your company faces a choice: use \$1 today, or deposit it and have \$1.10 in one year. That means we can think of \$1.10 as the cost of every dollar used today. So, we can express the cost of the investment as:

$$\begin{aligned} \text{Cost} &= (\$100,000 \text{ today}) \times \left(\frac{\$1.10 \text{ in one year}}{\$1 \text{ today}} \right) \\ &= \$110,000 \text{ in one year} \end{aligned}$$

interest rate The rate at which money can be borrowed or lent over a given period.

interest rate factor One plus the interest rate, it is the rate of exchange between dollars today and dollars in the future. It has units of “\$ in the future/\$ today.”

Think of this amount as the opportunity cost of spending \$100,000 today: The firm gives up the \$110,000 it would have had in one year if it had left the money in the bank. Alternatively, by borrowing the \$100,000 from the same bank, the firm would owe \$110,000 in one year.

	Today	One Year
Investment	-\$100,000	+\$105,000
Bank	-\$100,000	+\$110,000

We have used a market price, the interest rate, to put both the costs and benefits in terms of “dollars in one year,” so now we can use the Valuation Principle to compare them and compute the investment’s net value by subtracting the cost of the investment from the benefit in one year:

$$\$105,000 - \$110,000 = -\$5,000 \text{ in one year}$$

In other words, the firm could earn \$5,000 more in one year by putting the \$100,000 in the bank rather than making this investment. Because the net value is negative, we should reject the investment: If we took it, the firm would be \$5,000 poorer in one year than if we didn’t.

Value of \$100,000 Investment Today. The preceding calculation expressed the value of the costs and benefits in terms of dollars in one year. Alternatively, we can use the interest rate factor to convert to dollars today. Consider the benefit of \$105,000 in one year. What is the equivalent amount in terms of dollars today? That is, how much would we need to have in the bank today so we end up with \$105,000 in the bank in one year? We find this amount by dividing \$105,000 by the interest rate factor:

$$\begin{aligned} \text{Benefit} &= (\$105,000 \text{ in one year}) \div \left(\frac{\$1.10 \text{ in one year}}{\$1 \text{ today}} \right) \\ &= \$95,454.55 \text{ today} \end{aligned}$$

This is also the amount the bank would lend to us today if we promised to repay \$105,000 in one year.² Thus, it is the competitive market price at which we can “buy” or “sell” today an amount of \$105,000 in one year.

	Today	One Year
Value of Cost Today	-\$100,000	+\$105,000
Value of Benefit Today	+\$95,454.55	← $\frac{105,000}{1.10}$ ←

Now we are ready to compute the net value of the investment today (as opposed to its net value in one year) by subtracting the cost from the benefit:

$$\$95,454.55 - \$100,000 = -\$4,545.45 \text{ today}$$

Because this net value is calculated in terms of dollars today (in the present), it is typically called the *net present value*. We will formally introduce this concept in Chapter 8. Once again, the negative result indicates that we should reject the investment. Taking the investment would make the firm \$4,545.45 poorer today because it gave up \$100,000 for something worth only \$95,454.55.

²We are assuming the bank is willing to lend at the same 10% interest rate, which would be the case if there were no risk associated with the cash flow.

Present Versus Future Value. This calculation demonstrates that our decision is the same whether we express the value of the investment in terms of dollars in one year or dollars today: We should reject the investment. Indeed, if we convert from dollars today to dollars in one year,

$$(-\$4545.45 \text{ today}) \times (\$1.10 \text{ in one year}/\$1 \text{ today}) = -\$5000 \text{ in one year}$$

we see that the two results are equivalent, but expressed as values at different points in time. When we express the value in terms of dollars today, we call it the **present value (PV)** of the investment. If we express it in terms of dollars in the future, we call it the **future value (FV)** of the investment.

present value (PV) The value of a cost or benefit computed in terms of cash today.

future value (FV) The value of a cash flow that is moved forward in time.

discount factor The value today of a dollar received in the future.

discount rate The appropriate rate to discount a cash flow to determine its value at an earlier time.

Discount Factors and Rates. In the preceding calculation, we can interpret

$$\frac{1}{1+r} = \frac{1}{1.10} = 0.90909$$

as the *price* today of \$1 in one year. In other words, for just under 91 cents, you can “buy” \$1 to be delivered in one year. Note that the value is less than \$1—money in the future is worth less today, so its price reflects a discount. Because it provides the discount at which we can purchase money in the future, the amount $1/(1+r)$ is called the one-year **discount factor**. The interest rate is also referred to as the **discount rate** for an investment.

EXAMPLE 3.3

Comparing Revenues at Different Points in Time

MyFinanceLab

PROBLEM

The launch of Sony's PlayStation 3 was delayed until November 2006, giving Microsoft's Xbox 360 a full year on the market without competition. Sony did not repeat this mistake in 2013 when PS4 launched at the same time as Xbox One. Imagine that it is November 2005 and you are the marketing manager for the PlayStation. You estimate that if PlayStation 3 were ready to be launched immediately, you could sell \$2 billion worth of the console in its first year. However, if your launch is delayed a year, you believe that Microsoft's head start will reduce your first-year sales by 20% to \$1.6 billion. If the interest rate is 8%, what is the cost of a delay of the first year's revenues in terms of dollars in 2005?

SOLUTION

PLAN

Revenues if released today: \$2 billion Revenue if delayed: \$1.6 billion Interest rate: 8%

We need to compute the revenues if the launch is delayed and compare them to the revenues from launching today. However, in order to make a fair comparison, we need to convert the future revenues of the PlayStation if they are delayed into an equivalent present value of those revenues today.

EXECUTE

If the launch is delayed to 2006, revenues will drop by 20% of \$2 billion, or \$400 million, to \$1.6 billion. To compare this amount to revenues of \$2 billion if launched in 2005, we must convert it using the interest rate of 8%:

$$\$1.6 \text{ billion in 2006} \div (\$1.08 \text{ in 2006}/\$1 \text{ in 2005}) = \frac{\$1.6 \text{ billion}}{(1.08)} = \$1.481 \text{ billion in 2005}$$

Therefore, the cost of a delay of one year is

$$\$2 \text{ billion} - \$1.481 \text{ billion} = \$0.519 \text{ billion } (\$519 \text{ million}).$$

(Continued)

EVALUATE

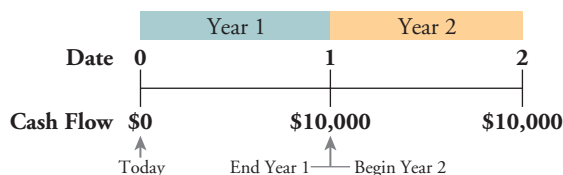
Delaying the project for one year was equivalent to giving up \$519 million in cash. In this example, we focused only on the effect on the first year's revenues. However, delaying the launch delays the entire revenue stream by one year, so the total cost would be calculated in the same way by summing the cost of delay for each year of revenues.

Timelines

timeline A linear representation of the timing of (potential) cash flows.

Our visual representation of the cost and benefit of the \$100,000 investment in this section is an example of a **timeline**, a linear representation of the timing of the expected cash flows. Timelines are an important first step in organizing and then solving a financial problem. We use them throughout this text.

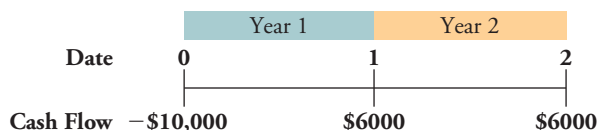
Constructing a Timeline. To understand how to construct a timeline, assume a friend owes you money. He has agreed to repay the loan by making two payments of \$10,000 at the end of each of the next two years. We represent this information on a timeline as follows:



Identifying Dates on a Timeline. To track cash flows, we interpret each point on the timeline as a specific date. The space between date 0 and date 1 represents the first year of the loan. Date 0 is today, the beginning of the first year, and date 1 is the end of the first year. The \$10,000 cash flow below date 1 is the payment you will receive at the end of the first year. Similarly, date 1 is the beginning of the second year, date 2 is the end of the second year, and the \$10,000 cash flow below date 2 is the payment you will receive at the end of the second year. Note that date 1 signifies *both* the end of year 1 *and* the beginning of year 2, which makes sense since those dates are effectively the same point in time.³

Distinguishing Cash Inflows from Outflows. In this example, both cash flows are inflows. In many cases, however, a financial decision will include inflows and outflows. To differentiate between the two types of cash flows, we assign a different sign to each: Inflows (cash flows received) are positive cash flows, whereas outflows (cash flows paid out) are negative cash flows.

To illustrate, suppose you have agreed to lend your brother \$10,000 today. Your brother has agreed to repay this loan with interest by making payments of \$6000 at the end of each of the next two years. The timeline is:



³ That is, there is no real time difference between a cash flow paid at 11:59 P.M. on December 31 and one paid at 12:01 A.M. on January 1, although there may be some other differences such as taxation, which we will overlook for now.

Notice that the first cash flow at date 0 (today) is represented as $-\$10,000$ because it is an outflow. The subsequent cash flows of $\$6000$ are positive because they are inflows.

Representing Various Time Periods. So far, we have used timelines to show the cash flows that occur at the end of each year. Actually, timelines can represent cash flows that take place at any point in time. For example, if you pay rent each month, you could use a timeline such as the one in our first example to represent two rental payments, but you would replace the “year” label with “month.”

Many of the timelines included in this chapter are simple. Consequently, you may feel that it is not worth the time or trouble to construct them. As you progress to more difficult problems, however, you will find that timelines identify events in a transaction or investment that are easy to overlook. If you fail to recognize these cash flows, you will make flawed financial decisions. Therefore, approach *every* problem by drawing the timeline as we do in this chapter and the next.

Concept Check

- How is an interest rate like a price?
- Is the value today of money to be received in one year higher when interest rates are high or when interest rates are low?

3.4 Valuing Cash Flows at Different Points in Time

The example of the $\$100,000$ investment in the previous section laid the groundwork for how we will compare cash flows that happen at different points in time. In this section, we will generalize from the example by introducing three important rules central to financial decision making that allow us to compare or combine values across time.

Rule 1: Comparing and Combining Values

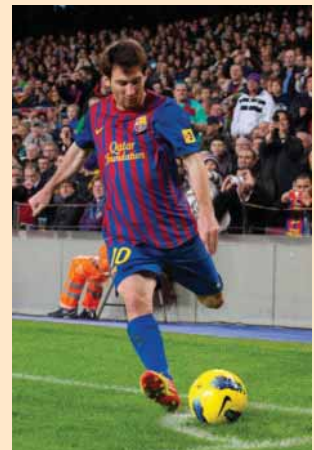
Our first rule is that it is only possible to compare or combine values at the same point in time. This rule restates a conclusion from the last section: Only cash flows in the same units can be compared or combined. A dollar today and a dollar in one year are not equivalent. Having money now is more valuable than having money in the future; if you have the money today you can invest it and earn interest.

To compare or combine cash flows that occur at different points in time, you first need to convert the cash flows into the same units by moving them to the same point in time. The next two rules show how to move the cash flows on the timeline.

COMMON MISTAKE

Summing Cash Flows Across Time

Once you understand the time value of money, our first rule may seem straightforward. However, it is very common, especially for those who have not studied finance, to violate this rule, simply treating all cash flows as comparable regardless of when they are received. One example is in sports contracts. In 2011, Albert Pujols signed a contract with the Los Angeles Angels that was repeatedly referred to as a “ $\$240$ million” contract. The $\$240$ million comes from simply adding up all the payments Pujols would receive over the 10 years of the contract—treating dollars received in 10 years the same as dollars received today. The same thing occurred when Lionel Messi signed a contract extension with FC Barcelona in 2013, giving him a “ $\$150$ million” contract through 2018.



Rule 2: Compounding

Suppose we have \$1000 today, and we wish to determine the equivalent amount in one year's time. If the current market interest rate is 10%, we saw in the last section that we can use that rate as an exchange rate, meaning the rate at which we exchange money today for money in one year, to move the cash flow forward in time. That is:

$$(\$1000 \text{ today}) \times (\$1.10 \text{ in one year}/\$1 \text{ today}) = \$1100 \text{ in one year}$$

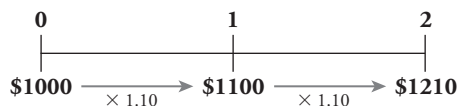
In general, if the market interest rate for the year is r , then we multiply by the interest rate factor, $(1 + r)$ to move the cash flow from the beginning to the end of the year. We multiply by $(1 + r)$ because at the end of the year you will have $(1 \times \text{your original investment})$ plus interest in the amount of $(r \times \text{your original investment})$. This process of moving forward along the timeline to determine a cash flow's value in the future (its future value) is known as **compounding**. *Our second rule stipulates that to calculate a cash flow's future value, you must compound it.*

compounding Computing the return on an investment over a long horizon by multiplying the return factors associated with each intervening period.

We can apply this rule repeatedly. Suppose we want to know how much the \$1000 is worth in two years' time. If the interest rate for year 2 is also 10%, then we convert as we just did:

$$(\$1100 \text{ in one year}) \times (\$1.10 \text{ in two years}/\$1 \text{ in one year}) = \$1210 \text{ in two years}$$

Let's represent this calculation on a timeline:



Given a 10% interest rate, all of the cash flows—\$1000 at date 0, \$1100 at date 1, and \$1210 at date 2—are equivalent. They have the same value but are expressed in different units (different points in time). An arrow that points to the right indicates that the value is being moved forward in time—that is, compounded.

In the preceding example, \$1210 is the future value of \$1000 two years from today. Note that the value grows as we move the cash flow further in the future. In the last section, we defined the time value of money as the difference in value between money today and money in the future. Here, we can say that \$1210 in two years is equivalent to \$1000 today. The reason money is more valuable to you today is that you have opportunities to invest it. As in this example, by having money sooner, you can invest it (here at a 10% return) so that it will grow to a larger amount in the future. Note also that the equivalent amount grows by \$100 the first year, but by \$110 the second year. In the second year, we earn interest on our original \$1000, plus we earn interest on the \$100 interest we received in the first year. This effect of earning interest on both the original principal plus the accumulated interest, so that you are earning “interest on interest,” is known as **compound interest**. Figure 3.2 shows how over time the amount of money you earn from interest on interest grows so that it will eventually exceed the amount of money that you earn as interest on your original deposit.

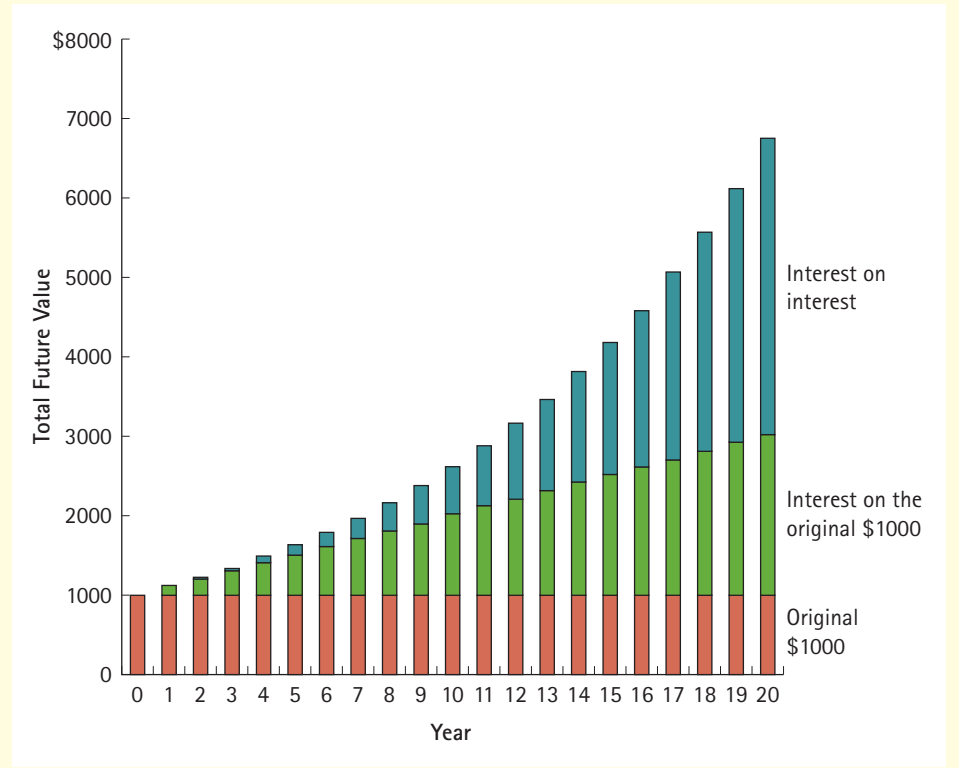
compound interest The effect of earning “interest on interest.”

How does the future value change in the third year? Continuing to use the same approach, we compound the cash flow a third time. Assuming the competitive market interest rate is fixed at 10%, we get:

$$\$1000 \times (1.10) \times (1.10) \times (1.10) = \$1000 \times (1.10)^3 = \$1331$$

FIGURE 3.2
The Composition of Interest over Time

This bar graph shows how the account balance and the composition of the interest changes over time when an investor starts with an original deposit of \$1000, represented by the red area, in an account earning 10% interest over a 20-year period. Note that the turquoise area representing interest on interest grows, and by year 15 has become larger than the interest on the original deposit, shown in green. By year 20, the interest on interest the investor earned is \$3727.50, while the interest earned on the original \$1000 is \$2000.



In general, to compute a cash flow C 's value n periods into the future, we must compound it by the n intervening interest rate factors. If the interest rate r is constant, this calculation yields:

Future Value of a Cash Flow

$$FV_n = C \times \underbrace{(1 + r) \times (1 + r) \times \cdots \times (1 + r)}_{n \text{ times}} = C \times (1 + r)^n \quad (3.1)$$

Rule of 72

Another way to think about the effect of compounding is to consider how long it will take your money to double given different interest rates. Suppose you want to know how many years it will take for \$1 to grow to a future value of \$2. You want the number of years, n , to solve:

$$FV_n = \$1 \times (1 + r)^n = \$2$$

If you solve this formula for different interest rates, you will find the following approximation:

$$\text{Years to double} \approx 72 \div (\text{interest rate in percent})$$

This simple "Rule of 72" is fairly accurate (that is, within one year of the exact doubling time) for interest rates higher than 2%. For example, if the interest rate is 9%, the doubling time should be about $72 \div 9 = 8$ years. Indeed, $1.09^8 = 1.99!$ So, given a 9% interest rate, your money will approximately double every 8 years.

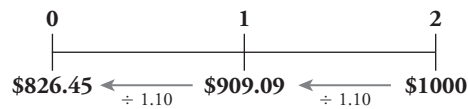
Rule 3: Discounting

The third rule describes how to put a value today on a cash flow that comes in the future. Suppose you would like to compute the value today of \$1000 that you anticipate receiving in one year. If the current market interest rate is 10%, you can compute this value by converting units as we did in the last section:

$$(\$1000 \text{ in one year}) \div (\$1.10 \text{ in one year}/\$1 \text{ today}) = \$909.09 \text{ today}$$

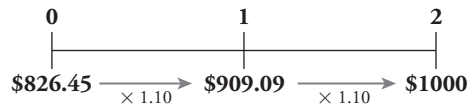
That is, to move the cash flow back along the timeline, we divide it by the interest rate factor, $(1 + r)$, where r is the interest rate. This process of finding the equivalent value today of a future cash flow is known as **discounting**. *Our third rule stipulates that to calculate the value of a future cash flow at an earlier point in time, we must discount it.*

Suppose that you anticipate receiving the \$1000 two years from today rather than in one year. If the interest rate for both years is 10%, you can prepare the following timeline:

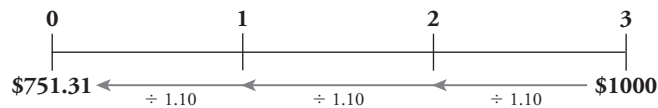


When the interest rate is 10%, all of the cash flows—\$826.45 at date 0, \$909.09 at date 1, and \$1000 at date 2—are equivalent. They represent the same value in different units (different points in time). The arrow points to the left to indicate that the value is being moved backward in time or discounted. Note that the value decreases the further in the future is the original cash flow.

The value of a future cash flow at an earlier point on the timeline is its present value at the earlier point in time. That is, \$826.45 is the present value at date 0 of \$1000 in two years. Recall that the present value is the “do-it-yourself” price to produce a future cash flow. Thus, if we invested \$826.45 today for two years at 10% interest, we would have a future value of \$1000, using the second rule of valuing cash flows:



Suppose the \$1000 were three years away and you wanted to compute the present value. Again, if the interest rate is 10%, we have:



That is, the present value today of a cash flow of \$1000 in three years is given by:

$$\$1000 \div (1.10) \div (1.10) \div (1.10) = \$1000 \div (1.10)^3 = \$751.31$$

In general, to compute the present value of a cash flow C that comes n periods from now, we must discount it by the n intervening interest rate factors. If the interest rate r is constant, this yields:

Present Value of a Cash Flow

$$PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n} \tag{3.2}$$

discounting Finding the equivalent value today of a future cash flow by multiplying by a discount factor, or equivalently, dividing by 1 plus the discount rate.

PERSONAL FINANCE

EXAMPLE 3.4

Present Value
of a Single Future
Cash Flow

MyFinanceLab

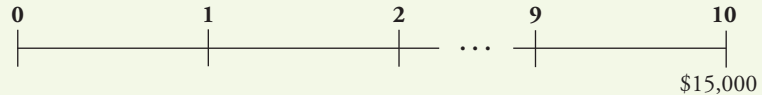
PROBLEM

You are considering investing in a savings bond that will pay \$15,000 in 10 years. If the competitive market interest rate is fixed at 6% per year, what is the bond worth today?

SOLUTION

PLAN

First, set up your timeline. The cash flows for this bond are represented by the following timeline:



Thus, the bond is worth \$15,000 in 10 years. To determine the value today, we compute the present value using Equation 3.2 and our interest rate of 6%.

EXECUTE

$$PV = \frac{15,000}{1.06^{10}} = \$8375.92 \text{ today}$$

EVALUATE

The bond is worth much less today than its final payoff because of the time value of money.

As we've seen in this section, we can compare cash flows at different points in time as long as we follow the Three Rules of Valuing Cash Flows, summarized in Table 3.1. Armed with these three rules, a financial manager can compare an investment's costs and benefits that are spread out over time and apply the Valuation Principle to make the right decision. In Chapter 4, we will show you how to apply these rules to situations involving multiple cash flows at different points in time.

TABLE 3.1
The Three Rules of
Valuing Cash Flows

Rule	Formula
1: Only values at the same point in time can be compared or combined.	None
2: To calculate a cash flow's future value, we must compound it.	Future value of a cash flow: $FV_n = C \times (1 + r)^n$
3: To calculate the present value of a future cash flow, we must discount it.	Present value of a cash flow: $PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n}$

Concept
Check

- Can you compare or combine cash flows at different times?
- What do you need to know to compute a cash flow's present or future value?

Using a Financial Calculator

Financial calculators are programmed to perform most present and future value calculations. However, we recommend that you develop an understanding of the formulas before using the shortcuts. We provide a more extensive discussion of financial calculators on page 95 and in the appendix to Chapter 4, but we'll cover the relevant functions for this chapter here. To use financial calculator functions, you always enter the known values first and then the calculator solves for the unknown.

To answer Example 3.4 with a financial calculator, do the following:

Concept	Number of Periods	Interest Rate per Period	Recurring Payments	Future Value
Calculator Key	N	I/Y	PMT	FV
Enter	10	6	0	15000

Because you are solving for the present value (PV), press the **PV** key last (on an HP calculator), or press **CPT** then the **PV** key on a TI calculator. The calculator will return -8375.92 . Note that the calculator balances inflows with outflows, so because the FV is positive (an inflow), it returns the PV as a negative (an outflow).

If you were solving for the future value instead, you would enter:

N	I/Y	PV	PMT
10	6	-8375.92	0

And finally, on an HP, press the **FV** key or on a TI, press **CPT** and then the **FV** key.

MyFinanceLab Here is what you should know after reading this chapter. MyFinanceLab will help you identify what you know, and where to go when you need to practice.

Key Points and Equations

3.1 Cost-Benefit Analysis

- To evaluate a decision, we must value the incremental costs and benefits associated with that decision. A good decision is one for which the value of the benefits exceeds the value of the costs.
- To compare costs and benefits that occur at different points in time we must put all costs and benefits in common terms. Typically, we convert costs and benefits into cash today.
- A competitive market is one in which a good can be bought and sold at the same price. We use prices from competitive markets to determine the cash value of a good.

Key Terms

competitive market, p. 73

Online Practice

MyFinanceLab Study Plan 3.1

3.2 Market Prices and the Valuation Principle

- Arbitrage is the process of trading to take advantage of equivalent goods that have different prices in different competitive markets.
- If equivalent goods or securities trade simultaneously in different competitive markets, they will trade for the same price in each market. This is equivalent to saying that no arbitrage opportunities should exist.
- The Valuation Principle states that the value of a commodity or an asset to the firm or its investors is determined by its competitive market price. The benefits and costs of a decision should be evaluated using those market prices. When the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm.

arbitrage, p. 75
 arbitrage opportunity,
 p. 75
 Law of One Price, p. 75

MyFinanceLab
 Study Plan 3.2

3.3 The Time Value of Money and Interest Rates

- The time value of money is the difference in value between money today and money in the future.
- The rate at which we can exchange money today for money in the future by borrowing or investing is the current market interest rate.
- The present value (PV) of a cash flow is its value in terms of cash today.
- Timelines are a critical first step in organizing the cash flows in a financial problem.

discount factor, p. 79
 discount rate, p. 79
 future value (*FV*), p. 79
 interest rate, p. 77
 interest rate factor, p. 77
 present value (*PV*), p. 79
 time value of money,
 p. 76
 timeline, p. 80

MyFinanceLab
 Study Plan 3.3

3.4 Valuing Cash Flows at Different Points in Time

- There are three rules of valuing cash flows:
 - Only cash flows that occur at the same point in time can be compared or combined.
 - To calculate a cash flow's future value, you must compound it.
 - To calculate a cash flow's present value, you must discount it.
- The future value in n years of a cash flow C today is:
- The present value today of a cash flow C received in n years is:

$$C \times (1 + r)^n \quad (3.1)$$

$$C \div (1 + r)^n \quad (3.2)$$

compounding, p. 82
 compound interest,
 p. 82
 discounting, p. 84

MyFinanceLab
 Study Plan 3.4
 Financial
 Calculator
 Tutorials:
 Calculating the
 Future Value of a
 Lump Sum and
 Solving for the
 Present Value of
 a Single Future
 Cash Flow

**CRITICAL
THINKING**

1. What makes an investment decision a good one?
2. How important are our personal preferences in valuing an investment decision?
3. Why are market prices useful to a financial manager?
4. Why is arbitrage important to competitive market prices?
5. How does the Valuation Principle help a financial manager make decisions?
6. Can we directly compare dollar amounts received at different points in time?

7. Why is a cash flow in the future worth less than the same amount today?
8. What is a discount rate?
9. What is compound interest?
10. What is the intuition behind the geometric growth in interest?

PROBLEMS

All problems are available in [MyFinanceLab](#). An asterisk * indicates problems with a higher level of difficulty.

Cost-Benefit Analysis

1. Honda Motor Company is considering offering a \$2000 rebate on its minivan, lowering the vehicle's price from \$30,000 to \$28,000. The marketing group estimates that this rebate will increase sales over the next year from 40,000 to 55,000 vehicles. Suppose Honda's profit margin with the rebate is \$6000 per vehicle. If the change in sales is the only consequence of this decision, what are its costs and benefits? Is it a good idea?
2. You are an international shrimp trader. A food producer in the Czech Republic offers to pay you 2 million Czech koruna today in exchange for a year's supply of frozen shrimp. Your Thai supplier will provide you with the same supply for 3 million Thai baht today. If the current competitive market exchange rates are 25.50 koruna per dollar and 41.25 baht per dollar, what is the value of this exchange to you?
3. Suppose your employer offers you a choice between a \$5000 bonus and 100 shares of the company's stock. Whichever one you choose will be awarded today. The stock is currently trading at \$63 per share.
 - a. Suppose that if you receive the stock bonus, you are free to trade it. Which form of the bonus should you choose? What is its value?
 - b. Suppose that if you receive the stock bonus, you are required to hold it for at least one year. What can you say about the value of the stock bonus now? What will your decision depend on?
4. Suppose Big Bank offers an interest rate of 5.5% on both savings and loans, and Bank Enn offers an interest rate of 6% on both savings and loans.
 - a. What profit opportunity is available?
 - b. Which bank would experience a surge in the demand for loans? Which bank would receive a surge in deposits?
 - c. What would you expect to happen to the interest rates the two banks are offering?
5. If the cost of buying a CD and ripping the tracks to your iPod (including your time) is \$25, what is the most Apple could charge on iTunes for a whole 15-track CD?
6. Some companies cross-list their shares, meaning that their stock trades on more than one stock exchange. For example, Research In Motion, the maker of BlackBerry mobile devices, trades on both the Toronto Stock Exchange and NASDAQ. If its price in Toronto is 50 Canadian dollars per share and anyone can exchange Canadian dollars for U.S. dollars at the rate of US\$0.95 per C\$1.00, what must RIM's price be on NASDAQ?

Market Prices and the Valuation Principle

7. Bubba is a shrimp farmer. In an ironic twist, Bubba is allergic to shellfish, so he cannot eat any shrimp. Each day he has a one-ton supply of shrimp. The market price of shrimp is \$10,000 per ton.

- a. What is the value of a ton of shrimp to him?
 - b. Would this value change if he were not allergic to shrimp? Why or why not?
8. Brett has almond orchards, but he is sick of almonds and prefers to eat walnuts instead. The owner of the walnut orchard next door has offered to swap this year's crop with him. Assume he produces 1000 tons of almonds and his neighbor produces 800 tons of walnuts. If the market price of almonds is \$100 per ton and the market price of walnuts is \$110 per ton:
- a. Should he make the exchange?
 - b. Does it matter whether he prefers almonds or walnuts? Why or why not?

The Time Value of Money and Interest Rates

9. You have \$100 and a bank is offering 5% interest on deposits. If you deposit the money in the bank, how much will you have in one year?
10. You expect to have \$1000 in one year. A bank is offering loans at 6% interest per year. How much can you borrow today?
11. A friend asks to borrow \$55 from you and in return will pay you \$58 in one year. If your bank is offering a 6% interest rate on deposits and loans:
 - a. How much would you have in one year if you deposited the \$55 instead?
 - b. How much money could you borrow today if you pay the bank \$58 in one year?
 - c. Should you loan the money to your friend or deposit it in the bank?
12. You plan to borrow \$1000 from a bank. In exchange for \$1000 today, you promise to pay \$1080 in one year. What does the cash flow timeline look like from your perspective? What does it look like from the bank's perspective?
13. The local electronics store is offering a promotion "1-year: same as cash," meaning that you can buy a TV now, and wait a year to pay (with no interest). So, if you take home a \$1000 TV today, you will owe them \$1000 in one year. If your bank is offering 4% interest, what is the true cost of the TV to you today?

Valuing Cash Flows at Different Points in Time

14. Suppose the interest rate is 4%.
 - a. Having \$200 today is equivalent to having what amount in one year?
 - b. Having \$200 in one year is equivalent to having what amount today?
 - c. Which would you prefer, \$200 today or \$200 in one year? Does your answer depend on when you need the money? Why or why not?
15. You are considering a savings bond that will pay \$100 in 10 years. If the interest rate is 2%, what should you pay today for the bond?
16. If your bank pays you 1.5% interest and you deposit \$500 today, what will your balance be in 5 years?
17. Consider the following alternatives:
 - i. \$100 received in one year
 - ii. \$200 received in 5 years
 - iii. \$300 received in 10 years
 - a. Rank the alternatives from most valuable to least valuable if the interest rate is 10% per year.
 - b. What is your ranking if the interest rate is only 5% per year?
 - c. What is your ranking if the interest rate is 20% per year?

- *18.** Suppose you invest \$1000 in an account paying 8% interest per year.
- What is the balance in the account after 3 years? How much of this balance corresponds to “interest on interest”?
 - What is the balance in the account after 25 years? How much of this balance corresponds to “interest on interest”?
- 19.** Calculate the future value of \$2000 in
- 5 years at an interest rate of 5% per year.
 - 10 years at an interest rate of 5% per year.
 - 5 years at an interest rate of 10% per year.
- *d.** Why is the amount of interest earned in part (a) less than half the amount of interest earned in part (b)?
- 20.** What is the present value of \$10,000 received
- 12 years from today when the interest rate is 4% per year?
 - 20 years from today when the interest rate is 8% per year?
 - 6 years from today when the interest rate is 2% per year?
- 21.** Your brother has offered to give you either \$5000 today or \$10,000 in 10 years. If the interest rate is 7% per year, which option is preferable?
- 22.** Your bank pays 2.5% interest per year. You put \$1000 in the bank today and \$500 more in the bank in one year. How much will you have in the bank in 2 years?
- 23.** Your cousin is currently 12 years old. She will be going to college in 6 years. Your aunt and uncle would like to have \$100,000 in a savings account to fund her education at that time. If the account promises to pay a fixed interest rate of 4% per year, how much money do they need to put into the account today to ensure that they will have \$100,000 in 6 years?
- 24.** Your mom is thinking of retiring. Her retirement plan will pay her either \$250,000 immediately on retirement or \$350,000 five years after the date of her retirement. Which alternative should she choose if the interest rate is
- 0% per year?
 - 8% per year?
 - 20% per year?
- 25.** You are planning to invest \$5000 in an account earning 9% per year for retirement.
- If you put the \$5000 in an account at age 23, and withdraw it 42 years later, how much will you have?
 - If you wait 10 years before making the deposit, so that it stays in the account for only 32 years, how much will you have at the end?
- *26.** Your grandfather put some money in an account for you on the day you were born. You are now 18 years old and are allowed to withdraw the money for the first time. The account currently has \$3996 in it and pays an 8% interest rate.
- How much money would be in the account if you left the money there until your 25th birthday?
 - What if you left the money until your 65th birthday?
 - How much money did your grandfather originally put in the account?