Table of Contents

Section 2.1-2.1 Inductive Reasoning and Conditional Statements

Venn Diagrams

Section 2.3 Deductive Reasoning

Section 2.5-2.7 Properties and Proofs

2-1, 2-2: Inductive Reasoning and Conditional Statements

Introduction to Logic

Complete the sequence:

2, 4, 6, ____

123; 9 123; 18 123; 27 ________

Monday: pizza Tuesday: burger Wednesday: pizza Thursday: burger _______________

How did you know what came next? We used **inductive reasoning**, which is arriving at a conclusion (called a **conjecture**) based on a set of observations; looking for a pattern and applying it as a rule.

We can't use this type of reasoning to prove something to be true, but we can use it to ***disprove*** a conjecture.

Counterexample: ____

Examples: use a counterexample to disprove the statement.

1. All supplementary pairs of angles are linear pairs.

2. When I subtract one number from another, the difference is always smaller than the larger number.

3. If $x^2 = 4$, then $x = 2$

Symbols Used in Logic

Logical statements and expressions are often written using symbols to represent words. We will use the following symbols in this chapter:

p, q, r, s, t, ect	Symbols used to represent statements such as hypothesis and conclusions		
→		∨	
~		∴	
^		↔	

Example: let **p** represent "**Geometry is boring**" and **q** represent "**Geometry is difficult**".

Translate the following into symbolic form:

- Geometry is not boring ____________________________
- Geometry is boring and Geometry is difficult ____________________________
- Geometry is not boring or Geometry is difficult ____________________________

Example: let **r** represent "**I save my money**" and **s** represent "**I buy a car**".

Translate the following from symbolic form:

- $r \vee \sim s$ ____
- $r \rightarrow s$ ____
- $\sim r \rightarrow \sim s$ ____
- $s \leftrightarrow r$ ____
- $\therefore r$ ____

<u>Conditional Statements</u>

Conditional Statement: a logical statement with 2 parts, a _____________and a ___________

If – Then: "if" part starts the ________________and the "then" part introduces the _____________.

hypothesis *conclusion*

If an animal meows, then it is a cat.

True/False Conditional Statements

- Is the statement above true? Why or why not?
- **<u>True conditional statement</u>:**
- **<u>False conditional statement</u>:**

Examples: write 1 true conditional statement and 1 false conditional statement. Circle the hypothesis on each and <u>underline</u> the conclusion.

True Conditional	
False Conditional	

Translating Conditional Statements into "If, Then" Form

Some statements are conditional statements in disguise:

- All birds have feathers.

If, Then Form: ____

- I'm watching baseball if it's a Sunday afternoon.

If, Then Form: ____

- Linear pairs of angles are supplementary.

If, Then Form: ____

Forms of Conditional Statements

Name	Symbolic Form	Description
Conditional	$p \to q$	If, Then statement
Converse		the hypothesis and conclusion
Inverse		the hypothesis and conclusion
Contrapositive		the hypothesis and conclusion

Examples:

1. Right angles measure 90°.

	Statement	**True or False?**
Conditional		
Converse		
Inverse		
Contrapositive		

2. Basketball players are athletes.

	Statement	**True or False?**
Conditional		
Converse		
Inverse		
Contrapositive		

3. All math teachers teach Geometry.

	Statement	True or False?
Conditional		
Converse		
Inverse		
Contrapositive		

Conditional statement is equivalent to the contrapositive – both ________ or both ___________

Converse statement is equivalent to the converse – both ________ or both __________

Biconditional Statements

Biconditional Statement ($p \leftrightarrow q$): a statement that contains the phrase

________________________ : _________*typically definitions are biconditional statements.

Biconditional statement is true if 1.) the __________________ is __________

AND 2.) the __________________ is __________

Practice: Determine if the statements can be rewritten as a biconditional. If so, write in biconditional form.

- If x = 3, then $x^2 = 9$
  Conditional true or false? _______

  Converse true or false? ________

  Biconditional (if possible): ____

- If three points are collinear, then they are on the same line.

  Conditional true or false? _______

  Converse true or false? ________

  Biconditional (if possible): ____

Vocabulary Review

Term	Definition
Conditional Statement	Statement that can be written in "if…, then" form
Hypothesis	Part of a conditional statement that follows "if"
Conclusion	Part of a conditional statement that follows "then"
Negation	Denial of a statement formed by adding or removing the word *not* from a statement
Negate	To add or remove the word *not* from a statement to change its truth value from true to false or from false to true
Converse	Statement formed from a conditional statement by switching the hypothesis and conclusion
Inverse	Statement formed from a conditional statement by negating the hypothesis and conclusion
Contrapositive	Statement formed from a conditional statement by switching AND negating the hypothesis and conclusion
Biconditional	Statement combining a conditional statement and its converse, using the phrase "if and only if"

Part I: Underline the hypothesis, and circle the conclusion of each conditional statement.

1. If the anchor gets loose, then the boat will drift.

2. If you study hard, then you will make a good grade on the test.

3. If $3x - 10 = 23$, then $x = 11$.

4. If you eat breakfast, then you will feel better at school.

5. If two lines are perpendicular, then they form right angles.

6. If two angles are supplementary, then their sum is $180°$.

Part II: Write each statement in if-then form.

7. All music lovers buy cds.

8. An obtuse angle has a measure greater than $90°$.

9. All right angles measure $90°$.

10. Every dog has four legs.

11. All vertical angles are congruent.

12. All cats chase mice.

Part III: Write the converse, inverse, and contrapositive of each conditional statement.

13. If it is Saturday, then school is closed.

Converse:____.

Inverse:____.

Contrapositive:____.

14. If two angles are adjacent, then they have a common vertex.

Converse:____.

Inverse:____.

Contrapositive:____.

15. If a line bisects a segment, then the segment is divided into two congruent parts.

Converse:____.

Inverse:____.

Contrapositive:____.

16. If two angles form a linear pair, then they are supplementary.

Converse:____.

Inverse:____.

Contrapositive:____.

Part IV:

*Let **p** represent **"Math is fun"**, and let **q** represent **"Math is difficult"**.*

Translate the following into symbolic form.

17. Math is not fun. ________________

18. Math is fun or math is difficult. ________________

19. Math is not fun and math is difficult. ________________

Translate the following from symbolic form to written form.

20. $\sim q \vee p$

____.

21.

____.

22.

____.

Part V: Write the converse of each of the following conditional statements, and then write the biconditional with symbolic form in parenthesis.

23. If an angle is acute, then its measure is less than $90°$.

converse:____(____________).

biconditional:____(____________).

24. If the measure of an angle is $180°$, then it is a straight angle.

converse:____(____________).

biconditional:____(____________).

Name____**Date** _________________________

Use the following conditional statement to answer the problems: "If I win, then you don't lose."

1. Write the hypothesis. ____
2. Write the conclusion. ____
3. Negate the hypothesis. ____
4. Negate the conclusion. ____
5. Write the converse. ____
6. Write the inverse. ____
7. Write the contrapositive. ____
8. Write the biconditional. ____

Use the following conditional statement to answer the problems: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, not symbols.

1. p is the hypothesis. Write p. ____
2. q is the conclusion. Write q. ____
3. $\sim p$ means "the negation of p." Write $\sim p$. ____
4. $\sim q$ means "the negation of q." Write $\sim q$. ____
5. (converse) $q \rightarrow p$ means "q implies p" or "If q, then p." Write $q \rightarrow p$.

   ____
6. (inverse) $\sim p \rightarrow \sim q$ means "Not p implies not q" or "If not p, then not q." Write $\sim p \rightarrow \sim q$.

   ____
7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not q implies not p" or "If not q, then not p." Write $\sim q \rightarrow \sim p$. ____
8. $p \wedge q$ means "p and q." Write $p \wedge q$. ____
9. $p \vee q$ means "p or q." Write $p \vee q$. ____
10. $\therefore p$ means "therefore p." Write $\therefore p$. ____
11. $p \leftrightarrow q$ means "p if and only if q." Write $p \leftrightarrow q$. ____________________________________

Activity Sheet 3: Logic and Conditional Statements

Name______________________________________**Date** ________________________

1. Write each of the following statements as a conditional statement. Then, circle the hypothesis, and underline the conclusion.
   a. Mark Twain wrote, “If you tell the truth, you don’t have to remember anything.”

      ____

   b. Helen Keller wrote, “One can never consent to creep when one feels the impulse to soar.”

      ____

   c. Mahatma Ghandi wrote, “Freedom is not worth having if it does not include the freedom to make mistakes.”

      ____

   d. Benjamin Franklin wrote, “Early to bed and early to rise makes a man healthy, wealthy, and wise.”

      ____

2. Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine whether each is true or false.
   a. “If I win, then you don’t lose.”

      Converse:____

      Inverse:____

      Contrapositive:____

      True orfalse: ____

   b. “If two segments are congruent, then they have the same length.”

      Converse:____

      Inverse:____

      Contrapositive:____

      True orfalse: ____

3. Use the Law of Detachment to reach a logical conclusion about the following statement: “If it is raining, then Sam and Sarah will not go to the football game.” This is a true conditional, and it is raining.

   ____

4. Statement 1: "If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°."
   Statement 2: "If the sum of the measures of two angles is 180°, then the angles are supplementary."
   By the Law of Syllogism, which statement below follows from Statements 1 and 2?__________
   a. If the sum of the measures of two angles is 180°, then the angles form a linear pair.
   b. If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°.
   c. If two adjacent angles form a linear pair, then the angles are supplementary.
   d. If two angles are supplementary, then the sum of the measures of the angles is 180°.

5. Let *p*: you see lightning and *q*: you hear thunder. Write each of the following statements in symbolic notation:
   a. If you see lightning, then you hear thunder. ____
   b. If you hear thunder, then you see lightning. ____
   c. If you don't see lightning, then you don't hear thunder. ______________________________
   d. If you don't hear thunder, then you don't see lightning. ______________________________

6. Let *p*: two planes intersect and *q*: the intersection is a line. Write each of the following statements in symbolic notation:
   a. If two planes don't intersect, then the intersection is a line. __________________________
   b. If the intersection is not a line, then two planes do not intersect. ______________________

7. Draw a Venn Diagram below for each of the following statements:
   a. All squares are rhombi.
   b. Some rectangles aresquares.
   c. No trapezoids are parallelograms.

HW: 2.1-2.2 Inductive Reasoning & Conditional Statements

Choose the best multiple choice answer for numbers 1-3.

1.) Consider the following statements.

$\boldsymbol{p}$: the sum of two angles is 90°

$\boldsymbol{q}$: The two angles are complements.

Which of the following is a symbolic representation of the statement:

If two angles are not complements, then the sum of the two angles is not 90°?

a. $\sim q \to \sim p$

b. $\sim p \to \sim q$

c. $q \to p$

d. $p \to q$

2.) Let p represent $\sqrt{11} = z$ and q represent z *is a rational number*. Which is a representation of the statement below?

If $\sqrt{11} = z$, *then z is not a rational number.*

a. $\sim p \to \sim q$

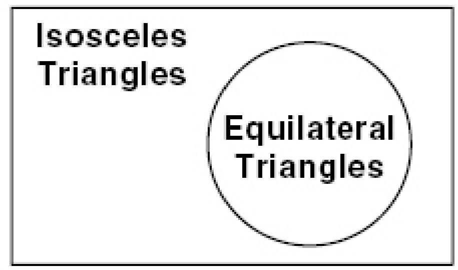
b. $p \to q$

c. $p \to \sim q$

d. $\sim q \to \sim p$

3.) According to the Venn diagram, which statement is true?

a. All isosceles triangles are also equilateral triangles.
b. All equilateral triangles are also isosceles triangles.
c. Some equilateral triangles are also isosceles triangles.
d. No isosceles triangles are equilateral triangles.



4.) **For the given statement, write the converse, the invers and the symbolic representation.**

___ → ___ **If-then**: If angles add to 90°, then they are complementary.

___ → ___ **Converse**:

____

___ → ___ **Inverse**:

____

___ → ___ **Contrapositive**:

____

Decide whether the statement is true or false. If false provide a counterexample.

5.) The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$. true or false

6.) If $m\angle A$ is 85°, then the measure of the complement of $\angle A$ is 5°. true or false

7.) Supplementary angles are always linear pairs. true or false

8.) If a number is an integer, then it is rational. true or false

Rewrite the definition as a biconditional statement. (iff)

9.) An angle with a measure between 90° and 180° is called obtuse.

Conditional: ____

Converse: ____

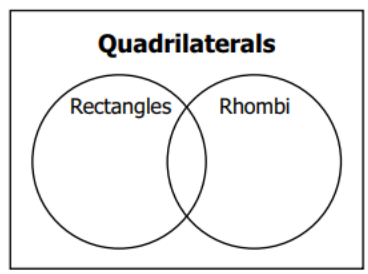
Biconditional: ____

10.) Which statement has the same meaning as the given statement?

Given: You can go to the movie after you do your homework.

- **a.** If you do your homework, then you can go to the movie afterwards.
- **b.** If you do not do your homework, then you can go to the movie afterwards.
- **c.** If you cannot go to the movie afterwards, then do your homework.
- **d.** If you are going to the movie afterwards, then do not do your homework.

11.) Which of the following statements must be true about the Venn diagram?



a. All rectangles are rhombi.

b. Some rhombi are rectangles.

c. Some rhombi are quadrilaterals.

d. Rectangles are never rhombi.

Review – Sketch and label the following:

12.) Point B lies between points A and C.

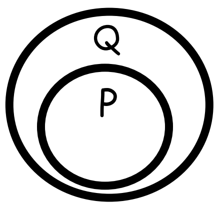
13.) $\overrightarrow{CD}$ is an angle bisector of $\angle ACB$.

Notes: Venn Diagrams

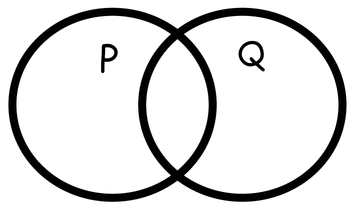
A VENN DIAGRAM is a drawing that uses shapes to represent sets of objects and the relationship between various sets. **Venn Diagrams** are useful in explaining conditional statements and showing logic visually.

Venn diagrams are useful to show different relationships between sets, including when all, some or none of the elements overlap.

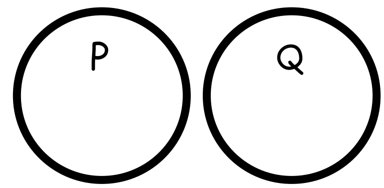
All elements in P are also in Q, and some elements of Q are in P.



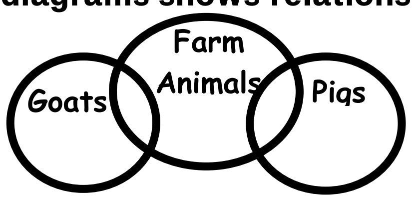
Some elements in P are also in Q, and some elements of Q are in P.



Sets P and Q have no elements in common.

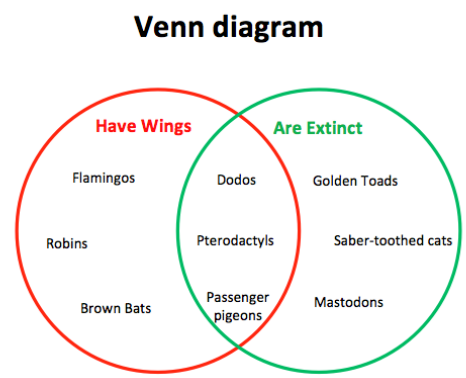


Combining diagrams shows relationships between three or more sets.



The Venn diagram illustrates the relationship between goats, pigs, and farm animals.

It shows that some goats are farm animals, and some pigs are farm animals, but no goats are pigs.



Create a Venn Diagram to show the relationship between the following sets.

5.) Dogs, Puppies, Animals

6.) Basketball players, Musicians, Girls

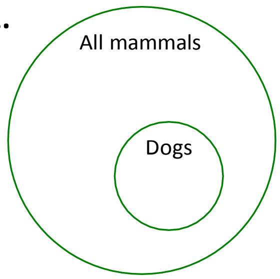
7.) Rabbits, Bunnies, Wolves, Mammals

8.) Sons, Fathers, Grandfathers

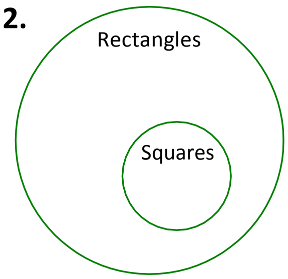
Classwork 2-2 Logic

Use each venn diagram to write an if-then statement

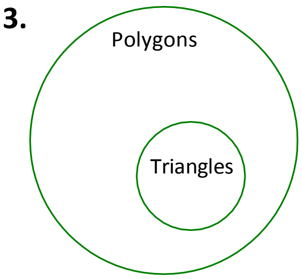
1.



2.



3.



1. ______________________________ **2.** ______________________________ **3.** ______________________________

______________________________ ______________________________ ______________________________

______________________________ ______________________________ ______________________________

Use each conditional statement to draw a venn diagram.

*4. **All milk contains calcium***

*5. **If you live in Richmond, then you live in Virginia.***

6. Every ant is an insect.

4. ______________________________ **5.** ______________________________ **6.**

Create a venn diagram in order to solve the following problem.

7. Thirty-six people were polled on telephone usage. One-third of the people owned cell phones. One-fourth of the people owned pagers. Five people owned both pagers and cell phones. The rest of the people did not own a pager or a cell phone. How many people did not own a pager or a cell phone.

2-3: Deductive Reasoning

<u>Deductive Reasoning</u>: uses facts, definitions, accepted properties and **laws of logic** to form a logical argument

Inductive Reasoning: uses specific examples and patterns to form a conjecture

*Recall: In logic, we use symbols to represent statements. In the following examples, the letters **p, q,** and **r** are intended to represent specific statements.*

<u>Two Laws of Deductive Reasoning:</u>

Law of Detachment –

If the hypothesis of a specific conditional is true, then the conclusion is also true conclusion

1. If ____________________
2. and ____________________
3. then ____________________

*you state the conclusion at the end

Example:
If an angle measures more than 90°, then it is not acute.

$m\angle ABC = 120°$.

$\therefore$ ________________________

Law of Syllogism –

If the hypothesis implies conclusion and the conclusion implies a second conclusion, then the hypothesis implies the second

1. If ____________________
2. and ____________________
3. then ____________________

*you make a new "if-then" statement at the end

Example:
If you wear the school colors, then you have school spirit

If you have school spirit, then the team feels great.

$\therefore$ ____________________________________

____

<u>Practice</u>: Find a conclusion that will make the arguments valid, if possible. State the Law used. If the argument is invalid, state INVALID.

1.) If I drive to work, then I will not be late.
If I am not late, then I do not lose any pay.

Conclusion: ____

Law used: ____

2.) If a quadrilateral is a square, then it has four right angles.
Quadrilateral ABCD is a square.

Conclusion: ____

Law used: ____

3.) If it is Tuesday, then I pack a turkey sandwich for lunch.
Today is Friday.

Conclusion: ____

Law used: ____

4.) If I go to college, then I graduated high school.
This fall, I attend Georgetown University.

Conclusion: ____

Law used: ____

5.) If I spend too much time on my phone, then I don't sleep well.
If I spend too much time on my phone, then my grades suffer.

Conclusion: ____

Law used: ____

6.) If I do well on the test, then my parents are pleased.
If do all my homework correctly, then I do well on the test.

Conclusion: ____

Law used: ____

Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write INVALID.

1. (1) All dogs are mammals
(hint: rewrite in If-then form)

(2) Joe is a llama

(3) Joe is a mammal

____

2. (1) If $\angle 2$ is acute, then $\angle 3$ is obtuse.

(2) If $\angle 3$ is obtuse, then $\angle 4$ is acute.

(3) If $\angle 2$ is acute, then $\angle 4$ is acute.

____

3. (1) If two lines are perpendicular, then they intersect to form a right angle

(2) Line l is perpendicular to line m

(3) Line l and m intersect to form a right angle. ____________________________

*4. (1) $r \rightarrow s$

(2) $s \rightarrow t$

(3) $\sim t \rightarrow \sim r$ ________________

5. (1) $h \rightarrow k$

(2) k

(3) h ________________

*Recall what we learned about true conditional statements and their contrapositive…

YOU DO: LAW OF DETACHMENT

True Conditional Statement: If it is Friday, then I wear jeans. $(p \rightarrow q)$

____. (p)

$\therefore$ ____. (q)

Now build your own:

____. $(p \rightarrow q)$

____. (p)

$\therefore$ ____. (q)

YOU DO: LAW OF SYLLOGISM

True Conditional Statement: If I am tired, then I drink coffee. $(p \rightarrow q)$

____. $(q \rightarrow r)$

$\therefore$ ____. $(p \rightarrow r)$

Now build your own:

____. $(p \rightarrow q)$

____. $(q \rightarrow r)$

$\therefore$ ____. $(p \rightarrow r)$

Geometry Practice on Law of Detachment and Law of Syllogism

Assume the following conditionals are true. A) What conclusion can you make, if any? B) Which law of logic are you using?

1. If I go to the movie, then I'll eat popcorn.
   If I eat popcorn, then I'll enjoy the movie.
   A. Conclusion? ____
   B. Law of logic? ____

2. If a figure is a square, then it is a rectangle.
   If a figure is a rectangle, then it has 4 right angles.
   A. Conclusion? ____
   B. Law of logic? ____

3. If I miss my bus, then I'll be late for school.
   I miss my bus.
   A. Conclusion? ____
   B. Law of logic? ____

4. If this wind keeps up, then we will lose some trees.
   We lose some trees.
   A. Conclusion? ____
   B. Law of logic? ____

5. All accountants enjoy mathematics.
   People who enjoy mathematics are intelligent.
   A. Conclusion? ____
   B. Law of logic? ____

6. If a person is a librarian, then she reads books.
   If a person is a friend of Dana's, then she reads books.
   A. Conclusion? ____
   B. Law of logic? ____

7. If an animal is a pig, then it is a mammal.
   Leon is a pig.
   A. Conclusion? ____
   B. Law of logic? ____

8. If a quadrilateral is a square, then it has four congruent sides.
   Quadrilateral ABCD has four congruent sides.
   A. Conclusion? ____
   B. Law of logic? ____

9. If my mother lets me go to a movie, then I will be happy.
   My mom lets me go to a movie.
   A. Conclusion? ____
   B. Law of logic? ____

10. If a man is wealthy, then he buys a mansion.
    If a man can buy a mansion, then he can hire a housekeeper.
    A. Conclusion? ____
    B. Law of logic? ____

Laws of Logic Worksheet

Use the Law of Syllogism (Transitive Property) and the Law of Detachment to determine the logical conclusion for each of the following. If the statements do not follow either pattern, write "No Valid Conclusion".

Law of Syllogism (Transitive Property)	Law of Detachment
$p \rightarrow q$	$p \rightarrow q$
$q \rightarrow r$	p is true
$r \rightarrow s$	therefore, q is true

1. If 2x + 3 = 17, then x = 7
   2x + 3 = 17
   Conclusion: ____

2. If Joan goes out with David, then she will have a good time Saturday night.
   Joan had a good time Saturday night.
   Conclusion: ____

3. If I go on vacation, I'll spend money.
   If I spend money, I'll be broke.
   If I'm broke, I'll have to get another job.
   Conclusion: ____

4. If Paul divorces Veronica, then he will not receive his inheritance.
   Paul divorces Veronica.
   Conclusion: ____

5. If I pass geometry, I won't have to go to summer school.
   If I don't go to summer school, I'll get a job.
   If I get a job, I'll make money.
   Conclusion: ____

6. I ABCD is a rectangle, then the diagonals are congruent.
   The diagonals are congruent.
   Conclusion: ____

7. If it's sunny Saturday, then I'll go to the beach.
   If I go to the beach, then I'll lay in the sun.
   If I lay in the sun, I'll get a sunburn.
   Conclusion: ____

8. If you use Shining Smile toothpaste, then you will be popular.
You are popular.
Conclusion: ______________________________________

9. If Chris is a sophomore, he takes English II.
Chris is a sophomore.
Conclusion: ______________________________________

10. If the treasure is discovered, then pirate Ruffian will walk the plank.
The treasure is discovered.
Conclusion: ______________________________________

11. If I don’t wear a sweater, I’ll get a cold.
If I get a cold, I’ll get sick.
If I get sick, I’ll miss the dance.
Conclusion: ______________________________________

12. If <1 and <2 are vertical angles, then they are equal.
<1 and <2 are equal.
Conclusion: ______________________________________

13. If 2 angles are linear, then they are supplementary.
If 2 angles are supplementary, then their sum is 180°.
Conclusion: ______________________________________

14. If Linda takes the bus, she’ll be on time for work.
Linda take the bus.
Conclusion: ______________________________________

15. If Hemlock Bones decodes the secret message, then the agent will live.
The agent lives.
Conclusion: ______________________________________

16. If Mr. Jones gets home on time, then he and Mrs. Jones will attend a meeting.
If Mr. and Mrs. Jones attend a meeting, then Lisa will visit grandma.
If Lisa visits grandma, then she will practice the piano.
If Lisa practices the piano, then she will go to bed late.
Conclusion: ______________________________________

Classwork 2-2 Logic

***Use the Law of Detachment or the Law of Syllogism to find a valid conclusion. If no valid* conclusion exists write "no valid conclusion".**

8. If it is snowing, then school is closed. It is snowing today.

____

9. If a student does well in geometry, then the student will do well on the SAT.
If a student does well on the SAT, then the student will get into a good college.

____

10. If it rains, then the soil gets wet.
If the soil gets wet, then the grass grows.

____

11. If it is cold, then you wear a jacket.
If you go snowboarding, then you wear a jacket.

____

Use the law of syllogism to solve the following.

12. If $x \to \sim w$, $y \to w$, $z \to y$, then $x \to$ _____.

13. If $a \to b$, $d \to f$, $\sim c \to d$, $b \to \sim f$, then $a \to$ _____.

14. If $r \to \sim m$, $s \to r$, $\sim c \to m$, then $\sim c \to$ _______.

Worksheet 4: Laws of Logic

Use the Law of Detachment to draw a conclusion.

1. If a student gets an A on a final exam, then the student will pass the course.
   The student gets an A on the final exam.

2. If a student wants to go to college, then the student must study hard.
   Rashid wants to go to the University of North Carolina.

3. If two lines intersect, then they intersect at exactly at one point.
   Two lines intersect.

4. If there is lightning, then it is not safe to be out in the open.
   Marla sees lightning from the soccer field.

5. If Galileo High School wins the championship, then the school will celebrate.
   Galileo High School wins the championship.

6. If x has a value of 4, then 2x has a value of 8. The
   value of x is 4.

Use the Law of Syllogism to draw a conclusion.

7. If an animal is a red wolf, then its scientific name is Canis rufus. If an
   animal is named Canis rufus, then it is endangered.

8. If two planes intersect, then they intersect in a line. If
   two planes are not parallel, then they intersect.

9. If you read a good book, then you enjoy yourself. If
   you enjoy yourself, then your time is well spent.

10. If you are studying botany, then you are studying biology. If you
    are studying biology, then you are studying a science.

11. If the sun is shining, then it is a beautiful day. If it is
    a beautiful day, then we will go out.

12. If the stereo is on, then the volume is loud.
    If the volume is loud, then the neighbors will complain.

Geometry HW - 2.3 Deductive Reasoning

Logic

[ID:0ZT7V4]

Directions: Go through and write what you think it should be, then choose the answer that fits with what you were thinking.

1 **What is the *converse* of the following statement?**

If Joe goes fishing, then he needs bait.

A *If Joe goes fishing, then he does not need bait.*
B *If he does not need bait, then Joe does not go fishing.*
C *If he needs bait, then Joe goes fishing.*
D *If Joe does not go fishing, then he does not need bait.*

2 **What is the contrapositive of the following statement?**

If two segments are congruent, then they have the same length.

F *If two segments do not have the same length, then they are not congruent.*
G *Two segments are congruent if and only if they have the same length*
H *If two segments have the same length, then they are congruent.*
J *If two segments are not congruent, then they do not have the same length.*

3 **Statement:** ***If a polygon is a pentagon, then it has 5 sides.***

What is the contrapositive of the statement?

A *If a polygon does not have 5 sides, then it is not a pentagon.*
B *If a polygon has 5 sides, then it is a pentagon.*
C *If a polygon is a pentagon, then it does not have 5 sides.*
D *If a polygon is not a pentagon, then it does not have 5 sides.*

4 **Statement:** ***If $3x - 1 = 8$, then $x = 3$.***

What is the inverse of the following statement?

F If $3x - 1 = 8$, then $x \neq 3$.
G If $3x - 1 \neq 8$, then $x \neq 3$.
H If $x \neq 3$, then $3x - 1 \neq 8$.
J If $x = 3$, then $3x - 1 = 8$.

5 **Let p represent $x^2 = 21$,**

and let q represent x is *not* a whole number.
Which is a representation of the statement below?

If x is a whole number, then $x^2 \neq 21$

A $p \rightarrow \sim q$

B $\sim q \rightarrow \sim p$
C $\sim p \rightarrow \sim q$
D $\sim p \rightarrow q$

6 **Let p represent**

Two angles are vertical angles.

Let q represent

The angles are congruent.

What is the symbolic representation of the following statement?

If two angles are congruent, then the angles are vertical angles.

F $q \rightarrow p$
G $p \rightarrow q$
H $\sim q \rightarrow \sim p$
J $\sim p \rightarrow \sim q$

7 **Consider the following statements.**

p: The sum of two angles is 90°.
q: The two angles are complements.

Which of the following is a symbolic representation of the statement:

If two angles are not complements, then
the sum of the two angles is not 90°?

A $\sim q \rightarrow \sim p$
B $\sim p \rightarrow \sim q$
C $q \rightarrow p$

D $p \rightarrow q$

8 **If the conditional statement**

"If you have a laptop, then you have a computer"

is represented by $\boldsymbol{p \rightarrow q}$ what is the symbolic representation of "If you have a computer, then you do not have a laptop" ?

F $q \rightarrow \sim p$

G $p \rightarrow \sim q$

H $\sim q \rightarrow p$

J $\sim q \rightarrow \sim p$

9 **Which of the following statements represents a valid argument?**

A If $a < b$ and $a < c$, then $c < b$.

B If $a > b$ and $a > c$, then $b > c$.

C If $a > b$ and $b > c$, then $a > c$.

D If $a > b$ and $a > c$, then $a > b + c$.

10 **Which is a valid conclusion that can be drawn from these statements?**

If a quadrilateral is a rhombus, then it is a parallelogram.

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

F *Every quadrilateral is a rhombus.*

G *Every parallelogram is a rhombus.*

H *Opposite angles of a rhombus are congruent.*

J *Opposite angles of a quadrilateral are congruent.*

11 **If $\boldsymbol{p \rightarrow q}$, and $\boldsymbol{q \rightarrow r}$, then —**

A $p \rightarrow r$

B $r \rightarrow \sim p$

C $r \rightarrow p$

D $\sim r \rightarrow p$

12 **Consider the following arguments. If the first two statements are true, in which argument is the 3rd statement an *incorrect* conclusion?**

F ***1. If it rains, then we will stay inside.***
2. If we stay inside, then we will play games.
3. If it rains, then we will play games.

1. If Susan eats her broccoli, then she will get ice cream.
G ***2. If Susan gets ice cream, then she will stay up late.***
3. If Susan eats her broccoli, then she will stay up late.

H ***1. If we win the championship, then we will get a trophy.***
2. If we win the game, then we will win the championship.
3. If we win the game, then we will get a trophy.

J ***1. If John studies, then he will pass the test.***
2. If John passes the test, then he will not be grounded.
3. If John is grounded, then he will study.

2.5-2.7 Properties & Proofs

Algebraic Properties of Equality

Let *a*, *b*, and *c*, be real numbers

Addition Property: If $a = b$, then ___________________

Multiplication Property: If $a = b$, then ___________________

Subtraction Property: If $a = b$, *then* _______________________

Division Property: If $a = b$ and $c \neq 0$, then __________________

Substitution Property: If a = b, then __________________ and ________________

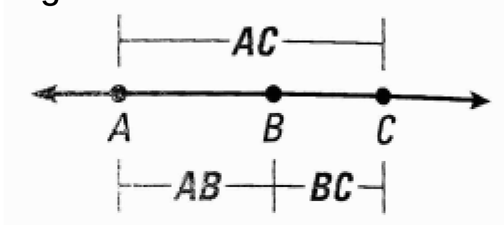
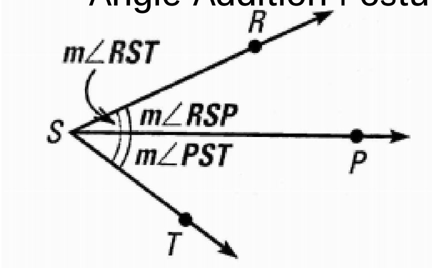
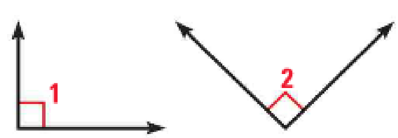
Distributive Property: ab + ac = ________________________

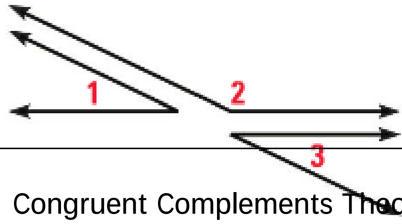
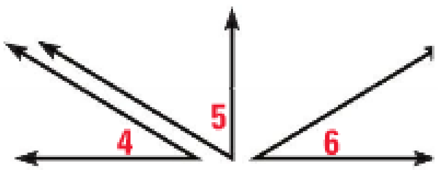
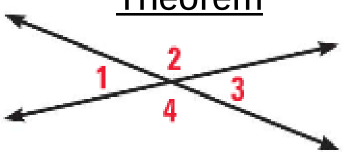
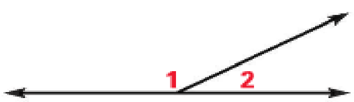
Solve the equations and write a reason for each step:

Solve 2x + 3 = 9 – x. Write a reason for each step

Solve –4(6x + 2) = 64. Write a reason for each step.

Chapter 1 & 2 Theorems and Postulates

Segment Addition Postulate 	If B is between A and C, then AB + BC = AC. If AB + BC = AC, then B is between A and C.
Angle Addition Postulate 	**Words:** If *P* is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measure of $\angle RSP$ *and* $\angle PST$ **Symbols:** If *P* is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$
Definition of Midpoint	A point that divides a segment into 2 congruent segments.
Definition of Segment Bisector	A point, ray, line, line segment, or plane that intersects a segment at its midpoint.
Definition of Angle Bisector	A ray that divides an angle into 2 angles that are congruent.
Definition of a Right Angle	An angle that measures $90°$.
Definition of a Linear Pair	2 adjacent angles whose noncommon sides are opposite rays. Their measures sum to 180°.
Definition of Vertical Angles	2 angles whose sides form 2 pairs of opposite rays. Their measures are equal.
Definition of Complementary Angles	2 angles whose measures sum to $90°$.
Definition of Supplementary Angles	2 angles whose measures sum to $180°$.
Definition of Congruence	2 angles/segments are congruent if they have the same measure.
Right Angle Congruence Theorem 	All right angles are congruent. If $\angle 1$ and $\angle 2$ are right angles then $\angle 1 \cong \angle 2$.

<u>Congruent Supplements Theorem</u> 	If two angles are supplementary to the same angle, then they are congruent. If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$
Congruent Complements Theorem 	If two angles are complementary to the same angle, then they are congruent If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$
<u>Vertical Angles Congruence Theorem</u> 	Vertical angles are congruent. $\angle 1$ and $\angle 3$ are vertical angles, then $\angle 1 \cong \angle 3$ $\angle 4$ and $\angle 2$ are vertical angles, then $\angle 4 \cong \angle 2$
<u>Linear Pair Postulate</u> 	If two angles form a linear pair, then they are supplementary. $\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and add up to $180°$

KEY CONCEPT *For Your Notebook*

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Property Practice

Name the property that justifies the 2nd statement:

statement 1: $a = c$

statement 2: $a + 3 = c + 3$

Name the property that justifies the 2nd statement:

statement 1: $ST = MN$

statement 2: $ST - AB = MN - AB$

Name the property that justifies the 2nd statement:

statement 1: $AB = 5$
$AB + BC = AC$

statement 2: $5 + BC = AC$

Name the property that justifies the statement:

$$m\angle ABC = m\angle CBA$$

Name the property that justifies the second statement:

statement 1: $AB = CD$

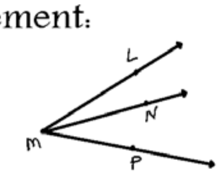
statement 2: $CD = AB$

Name the property that justifies the second statement:

statement 1: $m\angle A = m\angle B$
$m\angle B = m\angle C$

statement 2: $m\angle A = m\angle C$

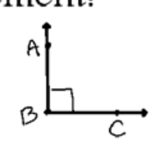
Name the DEFINITION that justifies the second statement:

statement 1: 

$\overrightarrow{MN}$ bisects $\angle LMP$

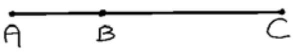
statement 2: $m\angle LMN = m\angle NMP$

Name the DEFINITION that justifies the second statement:

statement 1: 

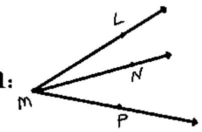
statement 2: $m\angle ABC = 90^\circ$

Name the POSTULATE that justifies the second statement:

Given Diagram: 

statement 2: $AB + BC = AC$

Name the POSTULATE that justifies the second statement:

Given Diagram: 

statement 2: $m\angle LMN + m\angle NMP = m\angle LMP$

Name the DEFINITION that justifies the second statement:

statement 1: B is the midpoint of $\overline{QR}$

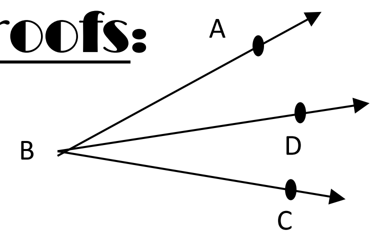
statement 2: $QB = BR$

Name the DEFINITION that justifies the second statement:

statement 1: $\angle 1 \cong \angle 2$

statement 2: $m\angle 1 = m\angle 2$

Practice Geometric Proofs:



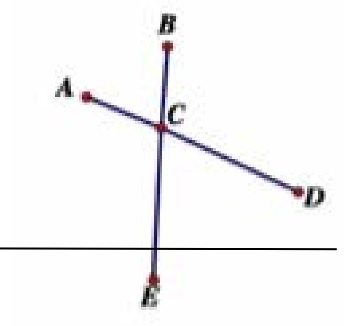
1. **Given:** $\overrightarrow{BD}$ bisects $\angle ABC$

Prove: $2(m\angle ABD) = m\angle ABC$

1. $\overrightarrow{BD}$ bisects $\angle$ ABC	1. ________________
2. ________________	2. Definition of angle bisector
3. ________________	3. Definition of congruent angles
4. $m\angle ABD + m\angle DBC = m\angle ABC$	4. ________________
5. $m\angle ABD + m\angle ABD = m\angle ABC$	5. ________________
6. ________________	6. Simplify/Distributive Property

2. **Given:** $AC = BC$; $CE = CD$

Prove: $AD = BE$



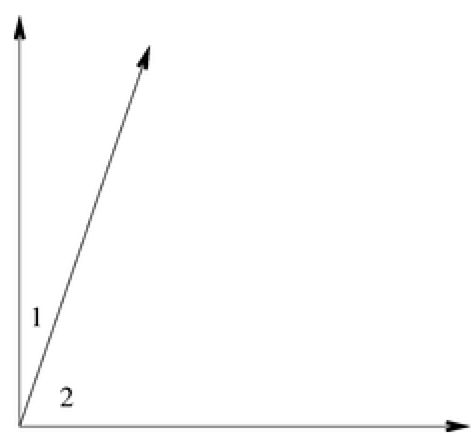
Statements	Reasons
1. ________________ ________________	1. Given
2. $AC + CD = AD$	2. ______________________________
3. __________________	3. Segment Addition Postulate
4. $AC + CE = BE$	4. Substitution Property of Equality
5. $AC + CD = BE$	5. ______________________________
6. _________________	6. ______________________________

3. **Given :**

- $\angle 1$ & $\angle 2$ are complementary
- $m\angle 2 = 74^{\circ}$

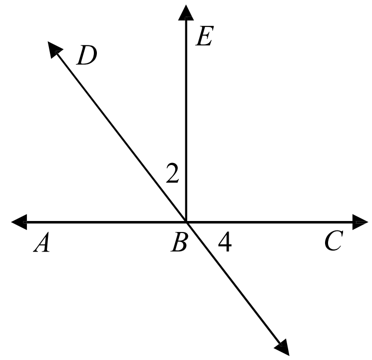
Prove :

$m\angle 1 = 16^{\circ}$



Statements	Reasons
1. ________________________	1. Given
2. ________________________	2. Given
3. ________________________	3. Definition of Complementary Angles
4. $m\angle + 74° = 90$	4. ______________________________
5. ________________________	5. ______________________________

Complete the two-column proof.



13)

Given: BD bisects $\angle ABE$

Prove: $\angle 2 \cong \angle 4$

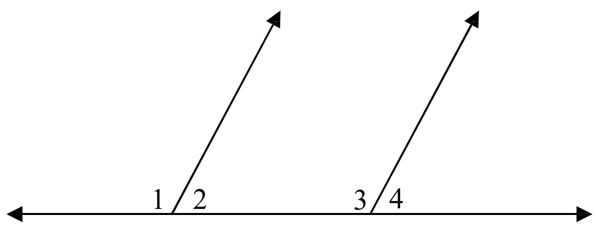
Statements:	Reasons:
BD bisects $\angle ABE$	
$\angle 2 \cong \angle 1$	
$\angle 1 \cong \angle 4$	
$\angle 2 \cong \angle 4$	

Complete the two-column proof.

14)

Given: $\angle 3$ is supplementary to $\angle 2$

Prove: $\angle 1 \cong \angle 3$



Statements:	Reasons:
	Given
$m\angle 1 + m\angle 2 = 180$	
$\angle 2$ is supplementary to $\angle 1$	
$\angle 3 \cong \angle 1$	

HW: 2.5 - 2.7 Properties and Proofs

Name: ___________________________ Block: ________ Date :____________

Write a reason for each step.

1.)

Statements	Reasons
1. $3x - 12 = 7x + 8$	1. Given
2. $-4x - 12 = 8$	2.
3. $-4x = 20$	3.
4. $x = -5$	4.

2.)

Statements	Reasons
1. $5(x-1) = 4x + 13$	1. Given
2. $5x - 5 = 4x + 13$	2.
3. $x - 5 = 13$	3.
4. $x = 18$	4.

Multiple Choice

____3.) Name the property of equality the statement illustrates: If XY=AB and AB =GH, then XY=GH.

A. Substitution **B**. Reflexive
C. Symmetric **D**. Transitive

Solve the equation. Write a reason for each step. (two-column proof: like # 1 and #2)

4.)

Statements	Reasons
1. $3(2x + 11) = 9$	1. Given

5.)

Statements	Reasons
1. $4(5x-9) = -2(x + 7)$	1. Given

Name the property illustrated by the statement.

6. $\angle ABC \cong \angle CBA$ ______________________

7. If $\angle RST \cong \angle 5$, then $\angle 5 \cong \angle RST$. ______________________

8. If $\overline{QS} \cong \overline{XR}$ and $\overline{RX} \cong \overline{SX}$, then $\overline{QS} \cong \overline{SX}$. ______________________

Use the property to complete the statement for 8 - 12.

9.) Substitution Property of Equality: If $AB = 20$, then $AB + CD =$ _____________.

10.) Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then _______________.

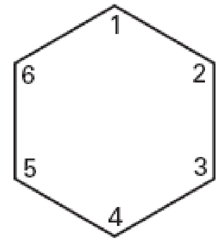
11.) Addition Property of Equality: If $AB = CD$, then ________ $+ EF =$ ________ $+ EF$.

12.) Distributive Property: If $5(x+8) = 2$, then _________x + ________ = 2.

13.) Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then _________________.

14. GIVEN: $m\angle 4 = 120°$, $\angle 2 \cong \angle 5$, $\angle 4 \cong \angle 5$

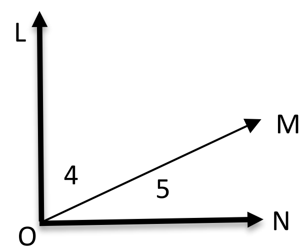
PROVE: $m\angle 2 = 120°$



Statements	**Reasons**
1. $m\angle 4 = 120°$, $\angle 2 \cong \angle 5$, $\angle 4 \cong \angle 5$	**1.** ______________________________
2. $\angle 2 \cong \angle 4$	**2.** ______________________________
3. ________________________	**3.** Definition of congruent angles
4. $m\angle 2 = 120°$	**4.** ______________________________

15. GIVEN: $\angle LON$ is a right angle

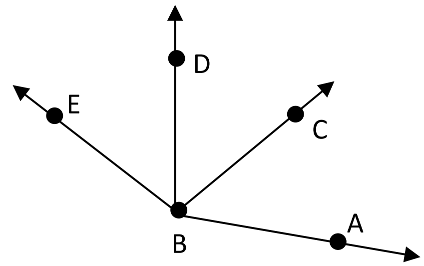
PROVE: $\angle 4 \; and \; \angle 5$ are Complementary



Statements	**Reasons**
1. $\angle LON$ is a right angle	**1.** ______________________________
2. $m\angle LON = 90°$	**2.** ______________________________
3. $m\angle \_\_\_ + m\angle \_\_\_ = m\angle LON$	**3.** Angle Addition Postulate
4. $m\angle \_\_\_ + m\angle \_\_\_ = 90°$	**4.** Substitution Property
5. ________________________________	**5.** ______________________________

6. GIVEN: $\angle ABC \cong \angle CBD$, $m\angle CBD = 50°$, $m\angle CBE = 100°$

PROVE: $m\angle ABC \cong \angle DBE$



Statements	Reasons
1. $\angle ABC \cong \angle CBD$, $m\angle CBD = 50°$, $m\angle CBE = 100°$	**1.** ______________________________
2. ___________________ $= m\angle CBE$	**2.** Angle Addition Postulate
3. $50° + m\angle DBE = 100°$	**3.** ______________________________
4. $m\angle DBE = 50°$	**4.** ______________________________
5. $m\angle CBD =$ _______	**5.** Substitution Property of Equality
6. _______________	**6.** Definition of congruent angles
7. $\angle ABC \cong \angle DBE$	**7.** ______________________________