CBSE NCERT Solutions for Class 10 Mathematics Chapter 8

Back of Chapter Questions

1. In Δ ABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine:

(i) $\sin A, \cos A$

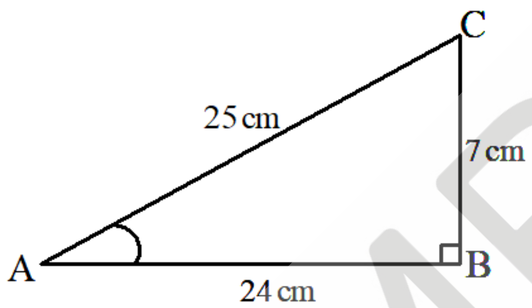
(ii) $\sin C, \cos C$

Solution:

In ΔABC, apply Pythagoras theorem

$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2 = 625$

$AC = \sqrt{625} = 25$ cm



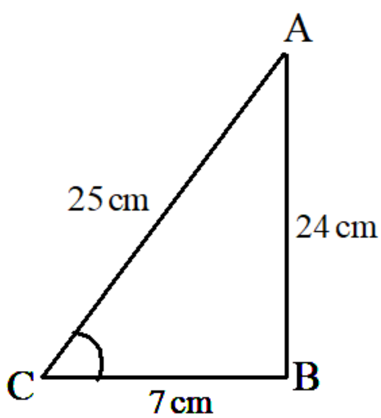
(i) $\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$

$= \frac{24}{25}$

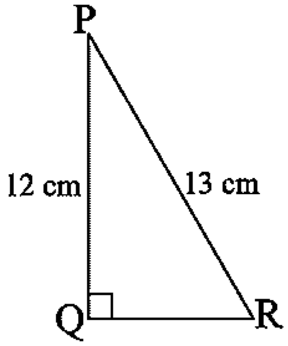
(ii)



$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

2. In Fig., find $\tan P - \cot R$.



Solution:

Apply Pythagoras theorem in ΔPQR

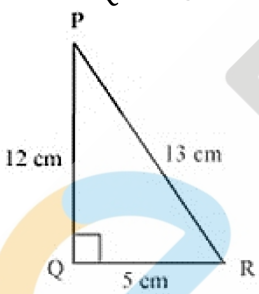
$PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 = 144 + QR^2$

$25 = QR^2$

$QR = 5$



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

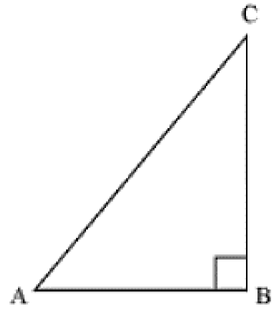
$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

3. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution:

Let us assume that Δ ABC is a right triangle, right angled at vertex B.



Given that

$$\sin A = \frac{3}{4}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}}$$

$$\Rightarrow \frac{BC}{AC} = \frac{3}{4}$$

Let BC and AC be 3R and 4R respectively, where R is any positive number.

In ΔABC, apply Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (4\text{ R})^2 = AB^2 + (3\text{ R})^2$$

$$\Rightarrow 16\text{ R}^2 - 9\text{ R}^2 = AB^2$$

$$\Rightarrow 7\text{ R}^2 = AB^2$$

$$\Rightarrow AB = \sqrt{7}R$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}R}{4R} = \frac{\sqrt{7}}{4}$$

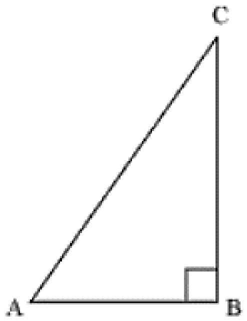
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{3R}{\sqrt{7}R} = \frac{3}{\sqrt{7}}$$

4. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Solution:

Let us assume that ΔABC is a right triangle, right angled at vertex B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

$\cot A = \frac{8}{15}$ (given)

$$\Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB and BC be 8R and 15R respectively, where R is a positive number.

Now applying Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

$$= (8R)^2 + (15R)^2$$

$$= 64\ R^2 + 225\ R^2$$

$$= 289\ R^2$$

$$\Rightarrow AC = 17\ R$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15\ R}{17\ R} = \frac{15}{17}$$

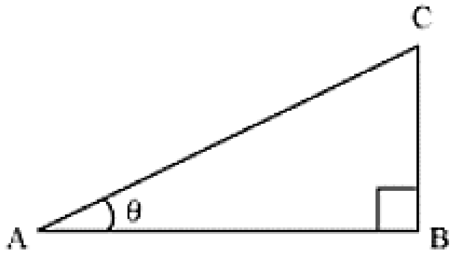
$$\sec A = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A}$$

$$= \frac{AC}{AB} = \frac{17}{8}$$

5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution:

Let us assume that ΔABC is a right angled triangle, right angled at vertex B.



$\sec\theta = \dfrac{\text{hypotenuse}}{\text{Side adjacent to } \angle B}$

$\Rightarrow \dfrac{13}{12} = \dfrac{AC}{AB}$

Let AC and AB be 13R and 12R, where R is a positive number.

Now applying Pythagoras theorem in ΔABC

$(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow (13\,R)^2 = (12\,R)^2 + BC^2$

$\Rightarrow 169\,R^2 = 144\,R^2 + BC^2$

$\Rightarrow 25\,R^2 = BC^2$

$\Rightarrow BC = 5R$

$\sin\theta = \dfrac{\text{Side opposite to } \angle\theta}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{5R}{13R} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{Side adjacent to } \angle\theta}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{12R}{13R} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \dfrac{BC}{AB} = \dfrac{5R}{12R} = \dfrac{5}{12}$

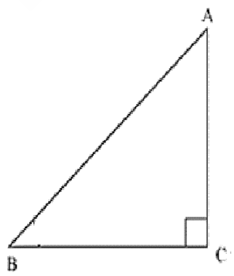
$\cot\theta = \dfrac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \dfrac{AB}{BC} = \dfrac{12R}{5R} = \dfrac{12}{5}$

$\text{coses}\,\theta = \dfrac{\text{hypotenuse}}{\text{Side opposite to } \angle\theta} = \dfrac{AC}{BC} = \dfrac{13R}{5R} = \dfrac{13}{5}$

6. If ∠A and ∠B are acute angles such that $\cos A = \cos B$, then show that ∠A = ∠B.

Solution:

Let us assume a right triangle, right angled at vertex C.



$cosA = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}}$

$= \frac{AC}{AB}$

$cosB = \frac{\text{Side adjacent to } \angle \mathbf{B}}{\text{hypotenuse}}$

$= \frac{\text{BC}}{\text{AB}}$

Since $\cos \text{A} = \cos \text{B}$

So $\frac{AC}{AB} = \frac{\text{BC}}{\text{AB}}$

$\Rightarrow \text{AC} = \text{BC}$

So $\angle A = \angle B$ (Angle opposite to equal sides are equal in length)

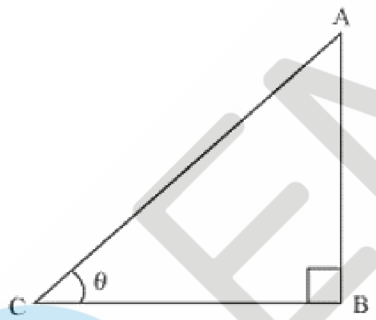
7. If $\cot \theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$,

(ii) $\cot^2 \theta$

Solution:

Let us assume that Δ ABC is a right triangle, right angled at vertex B.



$$\cot \theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{BC}}{\text{AB}}$$

$$= \frac{7}{8}$$

Let BC and AB be 7R and 8R respectively, where R is a positive number.

In ΔABC, apply Pythagoras theorem

$\text{AC}^2 = \text{AB}^2 + \text{BC}^2$

$= (8\text{ R})^2 + (7\text{ R})^2$

$= 64\text{ R}^2 + 49\text{ R}^2$

$= 113\text{ R}^2$

$\text{AC} = \sqrt{113}\text{ R}$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8\,R}{\sqrt{113}\,R} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacen to } \angle\theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{7\,R}{\sqrt{113}\,R} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-sin^2\theta\ )}{(1-\cos^2\theta\ )}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta$

$$= (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

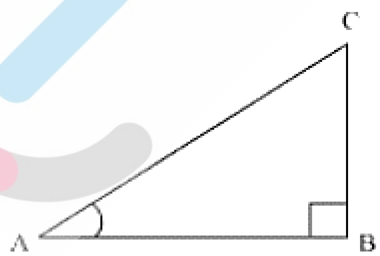
8. If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution:

Given that $3\cot A = 4$

or $\cot A = \frac{4}{3}$

Let us assume that Δ ABC is a right triangle, right angled at vertex B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Let AB and BC be 4R and 3R respectively, where R is a positive number.

Now in ΔABC

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4\,R)^2 + (3\,R)^2$

$= 16\,R^2 + 9\,R^2$

$= 25\,R^2$

$AC = 5\,R$

$\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{htpotenuse}} = \dfrac{AB}{AC}$

$= \dfrac{4R}{5R} = \dfrac{4}{5}$

$\sin A = \dfrac{\text{Side adjacent to } \angle A}{\text{htpotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{3R}{5R} = \dfrac{3}{5}$

$\tan A = \dfrac{\text{Side opposite to } \angle A}{\text{htpotenuse}} = \dfrac{BC}{AB}$

$= \dfrac{3R}{4R} = \dfrac{3}{4}$

$$LHS = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$RHS = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Since, LHS = RHS

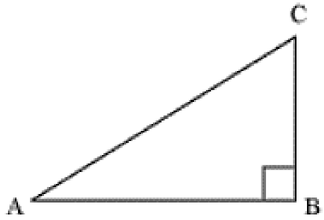
Hence proved.

9. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Solution:



$\tan A = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let BC and AB be R and $\sqrt{3}$ R, Where R is a positive number.

In ΔABC, apply Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}\, R\right)^2 + (R)^2$

$= 3\, R^2 + R^2 = 4\, R^2$

$\Rightarrow AC = 2R$

$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{R}{2\, R} = \frac{1}{2}$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}R}{2\, R} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}R}{2\, R} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{R}{2\, R} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

10. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

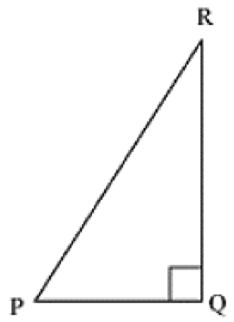
Solution:

Given that $PR + QR = 25$

$PQ = 5$

Let PR be a

So, $QR = 25 - a$



In ΔPQR, apply Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$a^2 = (5)^2 + (25 - a)^2$

$a^2 = 25 + 625 + a^2 - 50a$

$50a = 650$

$a = 13$

So, $PR = 13$ cm

$QR = 25 - 13 = 12$ cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of cot and A.

(iv) $\sin\theta = \frac{4}{3}$ for some angle θ.

Solution:

(i) If $0^o \le A \le 45^o$

$0 \le \tan A \le 1$

If $45^o \le A \le 90^o$

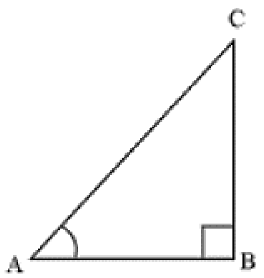
$1 \leq \tan A < \infty$

Clearly value of tan A is from 0 to ∞ and not always less than 1.

Hence, the given statement is false.

(ii) Let us assume that Δ ABC is a right-angled triangle, right angled at vertex B.

$\sec A = \frac{12}{5}$



$\frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$

Let AC and AB be 12R and 5R respectively, where R is a positive number.

In ΔABC, apply Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$(12\ R)^2 = (5\ R)^2 + BC^2$

$144\ R^2 = 25\ R^2 + BC^2$

$BC^2 = 119\ R^2$

$BC = 10.9\ R$

We may observe that for given two sides AC = 12 R and AB = 5 R

BC should be such that –

$AC - AB < BC < AC + AB$ (sum of any two sides of a triangle is greater than third side and difference of any two sides of a triangle is smaller than the third side)

so, $12\ R - 5\ R < BC < 12\ R + 5\ R$

$7\ R < BC < 17\ R$

But BC = 10.9 R. Clearly such a triangle is possible and hence such value of sec A is possible. Hence, the given statement is true.

(iii) cosecA is the abbreviation used for cosecant of angle A, and cos A is the abbreviation used for cosine of angle A. Hence, the given statement is false.

(iv) cot A is the abbreviation used for cotangent of angle A and not the product of cot and A. So, given statement is false.

(v) $\sin\theta = \frac{4}{3}$

In a right-angle triangle, we know that

$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{4}{3}$

So, side opposite to $\angle\theta = \frac{4}{3}\,hypotenuse$

But in a right-angle triangle hypotenuse is always greater than the remaining two sides.

Hence such value of $\sin\theta$ is not possible.

Hence the given statement is false.

EXERCISE 8.2

1. Evaluate the following:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Solution:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}=\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}=\frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$=\frac{\sqrt{3}(2\sqrt{6}-2\sqrt{2})}{(2\sqrt{6}+2\sqrt{2})(2\sqrt{6}-2\sqrt{2})}$$

$$=\frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{(2\sqrt{6})^2-(2\sqrt{2})^2}=\frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{24-8}=\frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{16}$$

$$=\frac{\sqrt{18}-\sqrt{6}}{8}=\frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\frac{\sin 30^o+\tan 45^o-\text{coses } 60^o}{\sec 30^o+\cos 60^o+\cot 45^o}$

$$=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$=\frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}=\frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$$

On multiplying and dividing by $3\sqrt{3}-4$, we get

$$=\frac{(3\sqrt{3}-4)^2}{(3\sqrt{3}+4)(3\sqrt{3}-4)}$$

$$=\frac{27+16-24\sqrt{3}}{27-16}=\frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60^o+4\sec^2 30^o-\tan^2 45^o}{\sin^2 30^o+\cos^2 30^o}$

$$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\frac{5\left(\frac{1}{4}\right)+\left(\frac{16}{3}\right)-1}{\frac{1}{4}+\frac{3}{4}}$$

$$=\frac{\frac{15+64-12}{12}}{\frac{4}{4}}=\frac{67}{12}$$

2. Choose the correct option and justify your choice:

(i) $\frac{2\tan 30^o}{1+\tan^2 30^o} =$

(A) $\sin 60^o$

(B) $\cos 60^o$

(C) $\tan 60^o$

(D) $\sin 30^o$

(ii) $\frac{1-\tan^2 45^o}{1+\tan^2 45^o}$

(A) $\tan 90^o$

(B) 1

(C) $\sin 45^o$

(D) 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(A) 0^o

(B) 30^o

(C) 45^o

(D) 60^o

(iv) $\frac{2\tan 30^o}{1-\tan^2 30^o} =$

(A) $\cos 60^o$

(B) $\sin 60^o$

(C) $\tan 60^o$

(D) $\sin 30^o$

Solution:

(i) $\frac{2\tan 30^o}{1+\tan^2 30^o}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Also, $\sin 60^o = \frac{\sqrt{3}}{2}$

So option (A) is correct.

(ii) $\frac{1-\tan^2 45^o}{1+\tan^2 45^o}$

$$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Hence (D) is correct.

(iii) Out of given options only option (A) is correct.

As $\sin 2A = \text{Sin}0^o = 0$

$2 \sin A = 2 \sin 0^o = 2(0) = 0$

(iv) $\frac{2 \tan 30^o}{1-\tan^2 30^o} =$

$$= \frac{2(\frac{1}{\sqrt{3}})}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$= \sqrt{3}$

Also, $\tan 60^o = \sqrt{3}$

So option (C) is correct.

3. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^o < A+B \le 90^o; A > B$, find A and B.

Solution:

$\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^o$

$\Rightarrow A+B = 60^o$ ...(i)

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^o$

$\Rightarrow A-B = 30^o$ ...(ii)

Adding both equations

$2A = 90^o$

$\Rightarrow A = 45^o$

From equation (i)

$45^o + B = 60^o$

$B = 15^o$

So, $\angle A = 45^o$ and $\angle B = 15^o$

4. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0^o$.

Solution:

(i) $\sin(A + B) = \sin A + \sin B$

Let $A = 45^o$ and $B = 45^o$

$LHS = \sin(A + B) = \sin(45^o + 45^o)$

$= \sin 90^o$

$= 1$

$\text{RHS} = \sin A + \sin B = \sin 45^o + \sin 45^o$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$=\sqrt{2}$

Clearly LHS $\neq$ RHS.

Hence the given statement is false.

(ii) We know that,

$\sin 0^o = 0$

$\sin 30^o = \frac{1}{2} = 0.5$

$\sin 45^o = \frac{1}{\sqrt{2}} = 0.707$

$\sin 60^o = \frac{\sqrt{3}}{2} = 0.866$

$\sin 90^o = 1$

Clearly, we can see that value of $\sin\theta$ increases as θ increases in the interval of $0^o < \theta < 90^o$

Hence the given statement is true.

(iii) $\cos 0^o = 1$

$\cos 30^o = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45^o = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60^o = \frac{1}{2} = 0.5$

$\cos 90^o = 0$

Clearly, we can see that value of $\cos\theta$ decreases as θ increases from 0^o *to* 90^o.

Hence the given statement is false.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

when $\theta = 45^o$

$\sin 45^o = \frac{1}{\sqrt{2}}$

$\cos 45^o = \frac{1}{\sqrt{2}}$

This is true for $\theta = 45^o$. But not true for all other values of θ.

As $\sin 30^o = \frac{1}{2}$ and $\cos 30^o = \frac{\sqrt{3}}{2}$

Hence the given statement is false.

(v) $\cot A$ is not defined for $A = 0^o$

We know that,

$\cot A = \frac{\cos A}{\sin A}$

$\cot 0^o = \frac{\cos 0^o}{\sin 0^o} = \frac{1}{0}$ =undefined.

Hence the given statement is true.

EXERCISE 8.3

1. Evaluate:

(i) $\frac{\sin 18^o}{\cos 72^o}$

(ii) $\frac{\tan 26^o}{\cot 64^o}$

(iii) $\cos 48^o - \sin 42^o$

(iv) $\operatorname{cosec} 31^o - \sec 59^o$

Solution:

(i) $\frac{\sin 18^o}{\cos 72^o} = \frac{\sin(90^o - 72^o)}{\cos 72^o}$

$= \frac{\cos 72^o}{\cos 72^o} = 1$

(ii) $\frac{\tan 26^o}{\cot 64^o} = \frac{\tan(90^o - 64^o)}{\cot 64^o}$

$= \frac{\cot 64^o}{\cot 64^o} = 1$

(iii) $\cos 48^o - \sin 42^o = \cos(90^o - 42^o) - \sin 42^o$

$= \sin 42^o - \sin 42^o$

$= 0$

(v) $\text{coses } 31^o - \sec 59^o = \text{cosec}(90^o - 59^o) - \sec 59^o$

$= \sec 59^o - \sec 59^o$

$= 0$

2. Show that:

(i) $\tan 48^o \tan 23^o \tan 42^o \tan 67^o = 1$

(ii) $\cos 38^o \cos 52^o - \sin 38^o \sin 52^o = 0$

Solution:

(i) $\text{LHS} = \tan 48^o \ \tan 23^o \tan 42^o \ \tan 67^o$

$= \tan(90^o - 42^o) \ \tan(90^o - 67^o) \tan 42^o \tan 67^o$

$= \cot 42^o \cot 67^o \tan 42^o \tan 67^o$

$= (\cot 42^o \tan 42^o)(\cot 67^o \tan 67^o)$

$= (1)(1)$

$= 1 = \text{RHS}$

(ii) $\text{LHS} = \cos 38^o \cos 52^o - \sin 38^o \ \sin 52^o$

$= \cos(90^o - 52^o) \ \cos(90^o - 38^o) - \sin 38^o \sin 52^o$

$= \sin 52^o \sin 38^o - \sin 38^o \sin 52^o = 0 = \text{RHS}$

3. If $\tan 2A = \cot(A - 18^o)$, where $2A$ is an acute angle, find the value of A.

Solution:

Given that

$\tan 2A = \cot(A - 18^o)$

$\Rightarrow \cot(90^o - 2A) = \cot(A - 18^o)$

$\Rightarrow 90^o - 2A = A - 18^o$

$\Rightarrow 108^o = 3A$

$\Rightarrow A = 36^o$

4. If $\tan A = \cot B$, prove that $A + B = 90^o$.

Solution:

Given that

$\tan A = \cot B$

$\Rightarrow \tan A = \tan(90^o - B)$

$\Rightarrow A = 90^o - B$

$\Rightarrow A + B = 90^o$

Hence Proved

5. If $\sec 4A = \mathrm{cosec}(A - 20^o)$, where 4A is an acute angle, find the value of A.

Solution:

Given that

$\sec 4A = \mathrm{cosec}\ (A - 20^o)$

$\Rightarrow \mathrm{cosec}(90^o - 4A) = \mathrm{cosec}\ (A - 20^o)$

$\Rightarrow 90^o - 4A = A - 20^o$

$\Rightarrow 110^o = 5A$

$\Rightarrow A = 22^o$

6. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Solution:

We know that, sum of all angles of a triangle is 180^o

So, for a triangle Δ ABC

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle B + \angle C = 180^o - \angle A$

$$\Rightarrow \frac{\angle B + \angle C}{2} = 90^o - \frac{\angle A}{2}$$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^o - \frac{A}{2}\right)$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

7. Express $\sin 67^o + \cos 75^o$ in terms of trigonometric ratios of angles between 0^o and 45^o.

Solution:

$\sin 67^o + \cos 75^o$

$= \sin(90^o - 23^o) + \cos(90^o - 15^o)$

$= \cos 23^o + \sin 15^o$

EXERCISE 8.4

1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Solution:

We know that

$\text{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$\sin^2 A = \frac{1}{1+\cot^2 A}$ (Since, $cosecA = \frac{1}{sinA}$)

So, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

As we know, $\tan A = \frac{\sin A}{\cos A}$

Also, $\cot A = \frac{\cos A}{\sin A}$

So, we can easily see that $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$= 1 + \frac{1}{\cot^2 A}$ $(\tan A = \frac{1}{\cot A})$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Solution:

As we know, $\cos A = \frac{1}{\sec A}$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\Rightarrow \sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \tan^2 A = \sec^2 A - 1$

$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Solution:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1} \qquad (\text{As } \sin^2 A + \cos^2 A = 1)$$

$= 1$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= (\sin 25^\circ)\{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ\{\sin(90^\circ - 25^\circ)\}$

$= (\sin 25^\circ)(\sin 25^\circ) + (\cos 25^\circ)(\cos 25^\circ)$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1 \qquad (\text{As, } \sin^2 A + \cos^2 A = 1)$

4. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=$

(A) 0

(B) 1

(C) 2

(D) -1

(iii) $(\sec A+\tan A)(1-\sin A)=$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}=$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

Solution:

(i) $9\sec^2 A-9\tan^2 A$

$=9(\sec^2 A-\tan^2 A)$

$=9(1)$ $[\text{as}, \sec^2 A=\tan^2 A+1]$

$=9$

Hence option (B) is correct.

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Hence option (C) is correct.

(iii) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

So, option (D) is correct.

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence option (D) is correct.

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

[**Hint:** Write the expression in terms of $\sin\theta$ and $\cos\theta$]

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

[**Hint:** Simplify LHS and RHS separately]

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \operatorname{cosec} A + \cot A$, using the identity

$\operatorname{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[**Hint:** Simplify LHS and RHS separately]

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Solution:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS $= (\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

RHS $= \frac{1-\cos\theta}{1+\cos\theta}$

So, LHS = RHS

Hence Proved.

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$$\text{LHS} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

RHS $= 2\sec A$

Clearly, we can see LHS $=$ RHS

Hence proved.

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ \operatorname{cosec}\theta$

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1)}{(\sin\theta\cos\theta)} + \frac{(\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$= \sec\theta\operatorname{cosec}\theta + 1$

RHS $= \sec\theta\operatorname{cosec}\theta + 1$

Clearly, we can see that LHS $=$ RHS.

Hence Proved

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{LHS} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = (\cos A + 1)$$

$$= \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A}$$

$\text{RHS} = \frac{\sin^2 A}{1 - \cos A}$

Clearly, LHS = RHS

Hence Proved

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

On dividing the numerator and denominator by $\sin A$, we get

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$= \operatorname{cosec} A + \cot A$

$\text{RHS} = \operatorname{cosec} A + \cot A$

Clearly, LHS = RHS

Hence Proved

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \sec A + \tan A$$

$\text{RHS} = \sec A + \tan A$

Clearly, LHS = RHS

Hence Proved

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{LHS} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times \{2(1-\sin^2\theta) - 1\}}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times (1-2\sin^2\theta)}$$

$= \tan\theta$

$\text{RHS} = \tan\theta$

Clearly, LHS = RHS

Hence Proved

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$\text{LHS} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 + 2\ \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A\ \sec A$

$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\ \sin A\left(\frac{1}{\sin A}\right) +$
$2\cos A\left(\frac{1}{\cos A}\right)$ (Since, cosecA=$\frac{1}{\sin A}$ and secA=$\frac{1}{\cos A}$)

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A$

$=$ RHS

Hence Proved

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$ (Since, cosecA $= \frac{1}{\sin A}$ and secA $= \frac{1}{\cos A}$)

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$

$= \sin A \cos A$

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Since, LHS $=$ RHS

Hence Proved

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A}$$

$= \tan^2 A$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\frac{1+\tan^2 A-2\tan A}{1+\cot^2 A-2\cot A}$$

$$=\frac{\sec^2 A-2\tan A}{\operatorname{cosec}^2 A-2\cot A}$$

$$=\frac{\frac{1}{\cos^2 A}-\frac{2\sin\ A}{\cos A}}{\frac{1}{\sin^2 A}-\frac{2\cos A}{\sin A}}=\frac{\frac{1-2\sin A\cos A}{\cos^2 A}}{\frac{1-2\sin A\cos A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence Proved

♦ ♦ ♦