

10

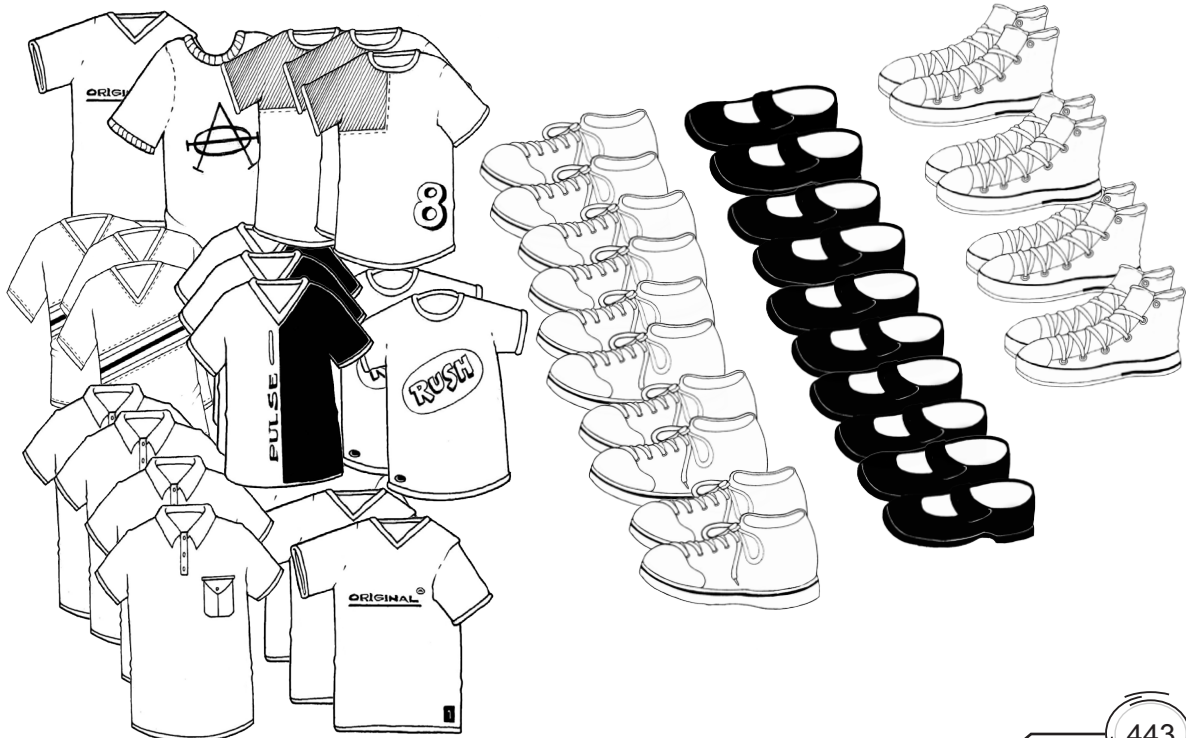
MEASURES OF CENTRAL TENDENCY AND MEASURES OF VARIABILITY

I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever wondered why a certain size of shoe or brand of shirt is made more available than other sizes?

Have you asked yourself why a certain basketball player gets more playing time than the rest of his team mates?

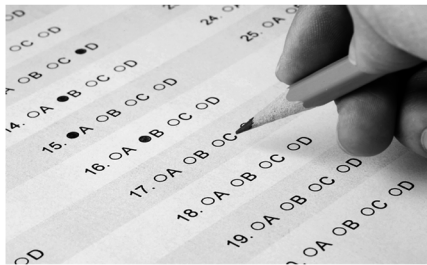
Have you thought of comparing your academic performance with your classmates? Have you wondered what score you need for each subject area to qualify for honors? Have you, at a certain time, asked yourself how norms and standards are made?



Purefoods TJ Giants | 2007-08 PBA Philippine Cup Stats

#	PLAYER	GP	MIN	FT		2PT		3PT		REBS			AST	BLK	STL	TO	FOU	PTS	AVE
				ATT	MD	ATT	MD	ATT	MD	TOT	OFF	DEF							
18	Yap, James	31	1136	182	144	343	149	246	88	113	39	74	44	9	19	71	80	706	22.77
21	Raymundo, Kerby	32	1164	169	135	428	166	78	17	302	100	202	148	10	25	##	97	518	16.19
8	Simon, Peter	31	697	117	88	198	109	119	37	67	21	46	22	1	13	63	66	417	13.45
	Villanueva, Enrico	20	537	92	52	195	95	4	0	148	55	93	28	19	9	39	63	242	12.1
1	Lanete, Chico	30	563	52	41	121	49	103	30	83	16	67	61	3	32	52	50	229	7.63
13	Yap, Roger	29	751	54	35	165	74	53	12	142	26	116	90	12	28	69	85	219	7.55
10	Adducul, Rommel	32	679	83	39	148	77	0	0	227	107	120	16	27	7	40	105	193	6.03
5	Salvador, Jondan	31	648	45	34	125	68	1	0	203	58	145	14	11	12	33	104	170	5.48
40	Castillo, Noy	14	243	9	4	11	9	30	10	18	7	11	15	0	0	9	30	52	3.71
24	Cablay, Brandon	11	154	8	4	15	7	24	7	20	4	16	9	1	3	7	10	39	3.55
7	Evangelista, Rey	17	192	14	6	32	17	6	2	57	23	34	8	1	8	12	10	46	2.71
28	Yee, Richard	23	200	28	20	40	20	0	0	36	9	27	11	0	4	14	32	60	2.61
9	Sanz, Robert	21	146	19	13	23	11	17	5	23	11	12	7	0	7	10	29	50	2.38
20	Larong, Ardy	17	121	9	5	16	6	9	2	7	4	3	3	3	2	11	35	23	1.35
6	Rodriguez, Omanzie	2	6	0	0	0	0	0	0	2	0	2	0	0	0	2	0	0	0

<http://basketball.exchange.ph/2008/03/28/giant-steps-for-purefoods/>



<http://www.whrhs.org/whrhs/site/default.asp>



http://www.launchingsuccess.com/store/index.php?main_page=product_info&products_id=13830

In this module you will find out the measures of central tendency and measures of variability. Remember to search for the answer to the following question(s):

- How can I make use of the representations and descriptions of a given set of data?
- What is the best way to measure a given set of data?

In this module, you will examine these questions when you study the following lessons.

II. LESSONS AND COVERAGE

Lesson 1: Measures of Central Tendency of Ungrouped Data

Lesson 2: Measures of Variability of Ungrouped Data

Lesson 3: Measures of Central Tendency of Grouped Data

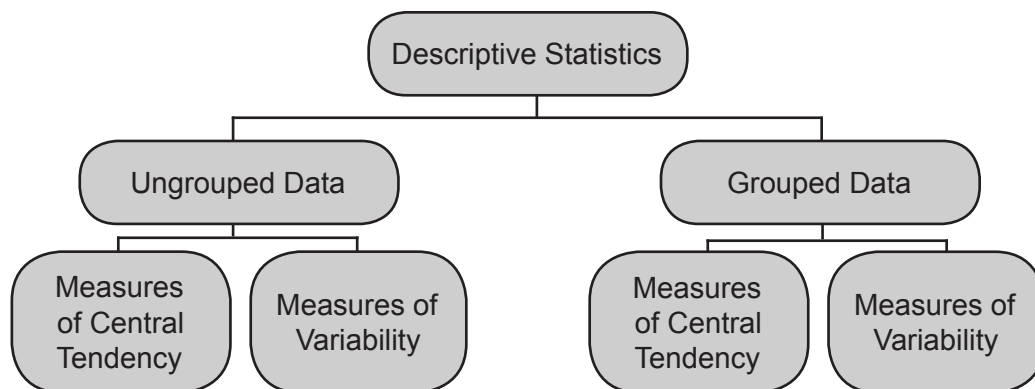
Lesson 4: Measures of Variability of Grouped Data

In these lessons, you will learn to:

Lesson	Objectives:
1	<ul style="list-style-type: none"> Find the mean, median, and mode of ungrouped data Describe and illustrate the mean, median, and mode of ungrouped data
2	<ul style="list-style-type: none"> Discuss the meaning of variability Calculate the different measures of variability of a given ungrouped data: range, standard deviation, and variance Describe and interpret data using measures of central tendency and measures of variability
3	<ul style="list-style-type: none"> Find the mean, median, and mode of grouped data Describe and illustrate the mean, median, and mode of grouped data
4	<ul style="list-style-type: none"> Discuss the meaning of variability Calculate the different measures of variability of a given grouped data: range, standard deviation, and variance Describe and interpret data using measures of central tendency and measures of variability

Module Map

Here is a simple map of the lessons that will be covered in this module.



To do well in this module, you will need to remember and do the following:

1. Study each part of the module carefully.
2. Take note of all the formulas given in each lesson.
3. Have your own scientific calculator. Make sure you are familiar with the keys and functions in your calculator.

III. PRE - ASSESSMENT

Find out how much you already know about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question.

- Which measure of central tendency is generally used in determining the size of the most saleable shoes in a department store?
 - mean
 - median
 - mode
 - range
- The most reliable measure of variability is _____ .
 - range
 - variance
 - average deviation
 - standard deviation
- For the set of data consisting of 8, 8, 9, 10, 10, which statement is true?
 - mean = mode
 - median = mode
 - mean = median
 - mean < median
- Which measure of central tendency is greatly affected by extreme scores?
 - mean
 - median
 - mode
 - none of the three
- Margie has grades 86, 68 and 79 in her first three tests in Algebra. What grade must she obtain on the fourth test to get an average of 78?
 - 76
 - 77
 - 78
 - 79
- What is the median age of a group of employees whose ages are 36, 38, 18, 10, 16 and 15 years?
 - 10
 - 15
 - 16
 - 17
- Nine people gave contributions in pesos 100, 200, 100, 300, 300, 200, 200, 150, 100, and 100 for a door prize. What is the median contribution?
 - Php 100
 - Php 150
 - Php 175
 - Php 200

14. What are the average sales of the two stores?
 a. 18 b. 18.4 c. 19 d. 19.5
15. Which store is consistently performing? Why?

For items 16 – 20 refer to the data below. Choose the letter that corresponds to the best answer:

Class	Frequency
46 – 50	1
41 – 45	2
36 – 40	3
31 – 35	10
26 – 30	6
21 – 25	9
16 – 20	5
11 – 15	6
6 – 10	4
1 – 5	2

16. What is the class size?
 a. 4 b. 3 c. 5 d. 6
17. What is the value of the median score?
 a. 24.10 b. 24.29 c. 24.15 d. 24.39
18. What is the range of the given set of data?
 a. 50 b. 50.5 c. 49.5 d. 99.5
19. What is the variance?
 a. 119.59 b. 119.49 c. 119.40 d. 119.50
20. What is the standard deviation?
 a. 10.90 b. 10.91 c. 10.92 d. 10.93

LEARNING GOALS AND TARGETS

After this lesson, you are expected to:

- demonstrate understanding of the key concepts of the different measures of tendency and measures of variability of a given data.
- compute and apply accurately the descriptive measures in statistics to data analysis and interpretation in solving problems related to research, business, education, technology, science, economics and others.

Lesson

1

Measures of Central Tendency of Ungrouped Data

What to Know



Let us begin with exploratory activities that will introduce you to the basic concepts of measures of central tendency and how these concepts are applied in real life.

Activity 1 contains familiar exercises provided to you in your Grade 7 modules

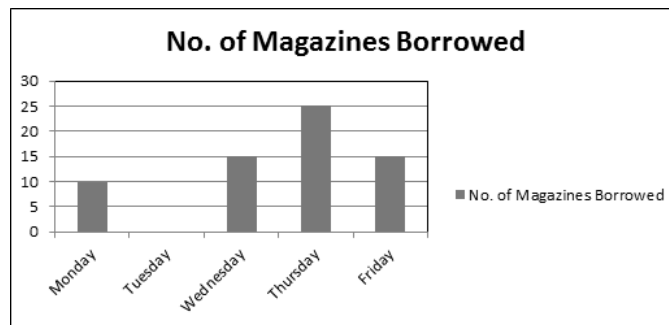
Activity 1

WHAT'S THE STORY BEHIND?

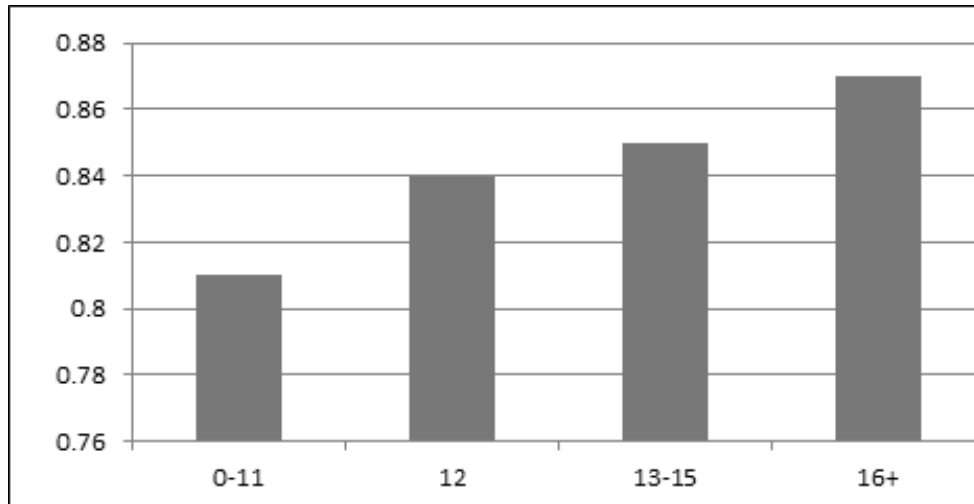
1. Daria bought T-shirts from a department store. She paid an average of Php 74 per shirt. Part of the torn receipt is shown below.

Official Receipt			
Quantity	Item	Unit Price	Subtot
5	Red Shirt	78.00	
3	Blue Shirt	76.00	
2	White Shirt		

- a. How much did she pay for each white shirt?
 - b. How much did she pay in all? Why?
2. The bar chart shows the number of magazines borrowed from the library last week.



- a. How many magazines were borrowed on Friday? Why?
 - b. What is the average number of magazines borrowed per day last week? What does this value tell you? Why?
 - c. On what day is the most number of magazines borrowed? Why?
 - d. Describe the number of magazines borrowed on a Tuesday. Why do you think so?
3. The graph below shows the percentage of survey respondents reporting that they are satisfied with their current job. The horizontal axis is the years of schooling for different respondents.



- a. What information can be obtained from the graph?
- b. What conclusion can be made? Why?
- c. What made you say that your conclusion was correct?
- d. What necessary adjustment could be made to provide accurate information based on the graph?

Activity 2 MEAL DEAL

To cater to five hundred (500) students having snacks all at the same time, your school canteen designed three meal packages for the students to choose from. The monitors of each section were tasked to collect the weekly orders of each student.



MEAL 1		MEAL 2		MEAL 3	
Item	Price	Item	Price	Item	Price
Hamburger	₱15.00	Baked Mac	₱15.00	Hotdog Sandwich	₱10.00
Spaghetti	₱10.00	Garlic Bread	₱5.00	Fruit Salad	₱7.00
French Fries	₱5.00	Veggie Salad	₱5.00	French Fries	₱5.00
Juice	₱5.00	Juice	₱5.00	Juice	₱5.00
Cost	₱35.00	Cost	₱30.00	Cost	₱27.00

Directions: Form yourselves into groups. Distribute to each member of the group the three meal packages. Make a week list of your preferred meal package. Record your group's order for the week on the sheet of paper below. Discuss with your group mate the answer to the questions below.

Meal Package	DAILY MEAL PACKAGE PREFERENCE						
	Monday	Tuesday	Wednesday	Thursday	Friday	Total	Sales
1							
2							
3							
Total							
Sales							



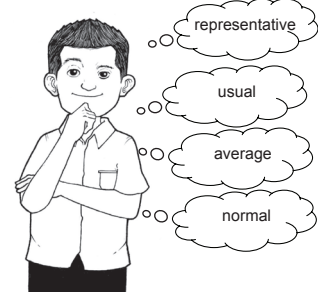
- A. In your group,
 1. what is the most preferred meal package?
 2. how much was the canteen's daily sales from each package? weekly sales?

- B. If all the groups will summarize their report,
 3. what might be the average weekly sales of the school canteen on each type of package?
 4. explain how these will help the canteen manager improve
 - 4.1 the sales of the school canteen.
 - 4.2 the combination of the food in each package.

- C. Make a combination of the food package of your choice.

The activities that you have just accomplished provided you situations where the basic concepts of statistics are applied. In this module, you will do activities that will help you in answering the question “How can I make use of the representations and descriptions of a given set of data?”.

Activity 3 WHICH IS TYPICAL?



Directions: Read the statements found at the right column in the table below. If you agree with the statement, place a checkmark (✓) in the Before-Lesson-Response column beside it. If you don't, mark it with (x).

Before Lesson Response	Statement
	24 is typical to the numbers 17, 25 and 30
	6 is the typical score in the set of data 3, 5, 8, 6, 9
	10 is a typical score in: 8, 7, 9, 10, and 6
	18 is typical age in workers' ages 17, 19, 20, 17, 46, 17, 18
	5 is typical in the numbers 3, 5, 4, 5, 7, and 5
	The mean is affected by the size of extreme values
	The median is affected by the size of extreme values
	The mode is affected by the size of extreme values
	The mean is affected by the number of measures
	The median is affected by number of measures
	The mode is affected by the number of measures

What to Process



Here are some enabling activities/experiences that you will perform to validate your understanding on averages in the What to Know phase.

After doing the activities in this section, it is expected that you will be able to answer the question, “**What is the best way to measure a given set of data?**”. The understanding gained would erase misconceptions about the different measures of central tendency that you have encountered before.

Activity 4 WATCH THIS!

4.1 A group of students obtained the following scores in a math quiz:

8, 7, 9, 10, 8, 6, 5, 4, 3

Arranging these scores in increasing order: 3, 4, 5, 6, 7, 8, 8, 9, 10,

the *mean* is 6.7.

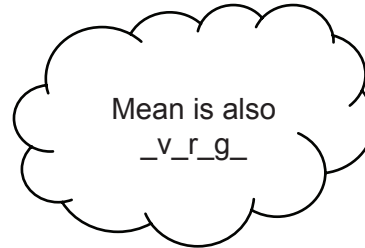
3, 4, 5, 6, 7, 8, 8, 9, 10

the *median* is 7.

3, 4, 5, 6, 7, 8, 8, 9, 10

the *mode* is 8.

3, 4, 5, 6, 7, 8, 8, 9, 10



Observe how the mean, median and mode of the scores were obtained. Make a guess and complete the statements below.

- The mean 6.7 was obtained by _____.
- The median 7 is the _____.
- The mode 8 is the _____.

If you have not discovered how the values were obtained proceed to Activity 4.2.

4.2 If the score 5 of another student is included in the list.

3, 4, 5, 5, 6, 7, 8, 8, 9, 10

The mean is 6.5.

3, 4, 5, 5, 6, 7, 8, 8, 9, 10

The median is 6.5

3, 4, 5, 5, 6, 7, 8, 8, 9, 10

The mode is 5 and 8.

3, 4, 5, 5, 6, 7, 8, 8, 9, 10

QUESTIONS ?

Discuss with your groupmates,

- your observation about the values of the mean, the median and the mode;
- how each value was obtained; and
- your generalizations based on your observations.

From these activities, you will see that the values are made to represent or describe a given set of data. You will know more about the characteristics of each type of measures of central tendency in the next activities and discussions.

Let's take a look at the mean.

The Mean

The **mean** (also known as the *arithmetic mean*) is the most commonly used measure of central position. It is used to describe a set of data where the measures cluster or concentrate at a point. As the measures cluster around each other, a single value appears to represent distinctively the typical value.

It is the sum of measures x divided by the number N of measures in a variable. It is symbolized as \bar{x} (read as x bar). To find the *mean* of an ungrouped data, use the formula

$$\bar{x} = \frac{\sum x}{N}$$

where $\sum x$ = the summation of x (sum of the measures) and N = number of values of x .

Example:

The grades in Geometry of 10 students are 87, 84, 85, 85, 86, 90, 79, 82, 78, 76. What is the average grade of the 10 students?

Solution:

$$\bar{x} = \frac{\sum x}{N}$$

$$\bar{x} = \frac{87 + 84 + 85 + 85 + 86 + 90 + 79 + 82 + 78 + 76}{10}$$

$$= \frac{832}{10}$$

$$\bar{x} = 83.2$$

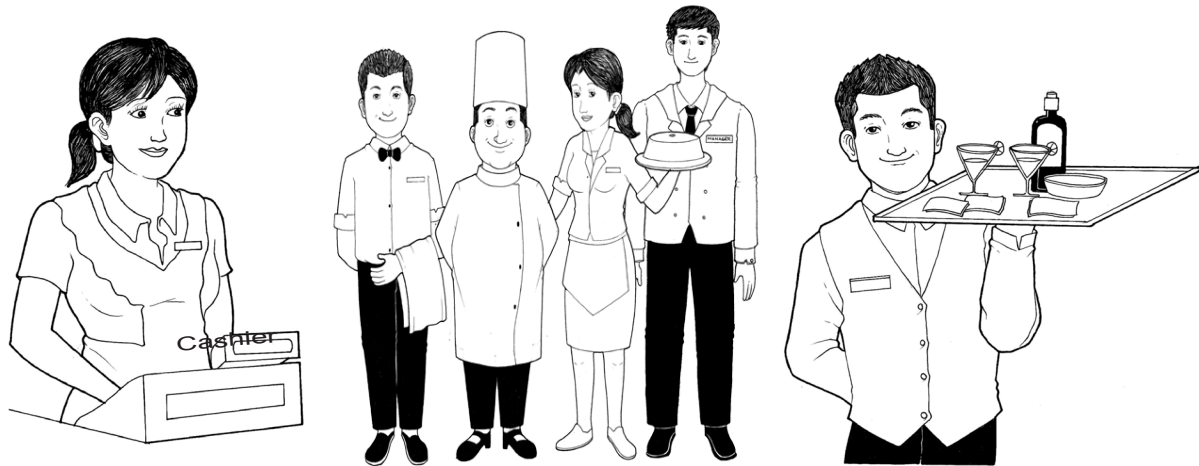
Hence, the average grade of the 10 students is 83.2.

Consider another activity.

Activity 5 WHO'S REPRESENTING?

Sonya's Kitchen received an invitation for one person from food exposition. The service crew seven numbers is very eager to go. To be fair to all, Sonya decided to choose a person whose age falls within the mean age of her seven members.

Sonya's Kitchen Crew



She made a list such as below:

Service Crew	Age
Manager	47
Cook A	21
Cook B	20
Cashier	19
Waiter A	18
Waiter B	18
Waitress	18

QUESTIONS ?

- What is the mean age of the service crew?
- Is there someone in this group who has this age?
- How many persons are older than the mean age? How many are younger?
- Do you think this is the best measure of central tendency to use? Explain.

Take note of how the mean is affected by extreme values. Very high or very low values can easily change the value of the mean.

Do the next activity to solve problems encountered.

Activity 6 WHO'S IN THE MIDDLE?

From our previous example, the ages of the crew are given as 18, 20, 18, 19, 21, 18 and 47. Follow the steps below.



- Arrange the ages in numerical order.
- What is the middle value?
- Is there a crew with this representative age?
- How many crew are younger than this age? Older than this age?
- Who is now the representative of Sonya's Kitchen in the Food Fair?
- Compare the results from the previous discussion (how the mean is affected by the set of data). Explain.

The middle value here or term in a set of data arranged according to size/magnitude (either increasing or decreasing) is called the median.

Consider another situation in the next activity.

Activity 7 THE NEWLY-HIRED CREW

If at the end of the month, Sonya's Kitchen hired another crew members whose age is 22, the data now consists of eight ages: 18, 20, 18, 19, 21, 18, 47 and 22, an even number. How many middle entries are there?

Sonya's Kitchen Crew



Let us find out by following these simple steps:



- a. Arrange the crew's ages in numerical order.
- b. Find the two middle values (ages).
- c. Get the average of the two middle values.
- d. What is now the median age?
- e. How many are below this age? above this age?

Here are more examples for you to develop your skills in finding the median of a set of data.

Example 1:

The library logbook shows that 58, 60, 54, 35, and 97 books, respectively, were borrowed from Monday to Friday last week. Find the median.

Solution:

Arrange the data in increasing order.

35, 54, 58, 60, 97

We can see from the arranged numbers that the middle value is 58.

Since the middle value is the median, then the *median* is 58.

Example 2:

Andrea's scores in 10 quizzes during the first quarter are 8, 7, 6, 10, 9, 5, 9, 6, 10, and 7. Find the median.

Solution:

Arrange the scores in increasing order.

5, 6, 6, 7, 7, 8, 9, 9, 10, 10

Since the number of measures is even, then the median is the average of the two middle scores.

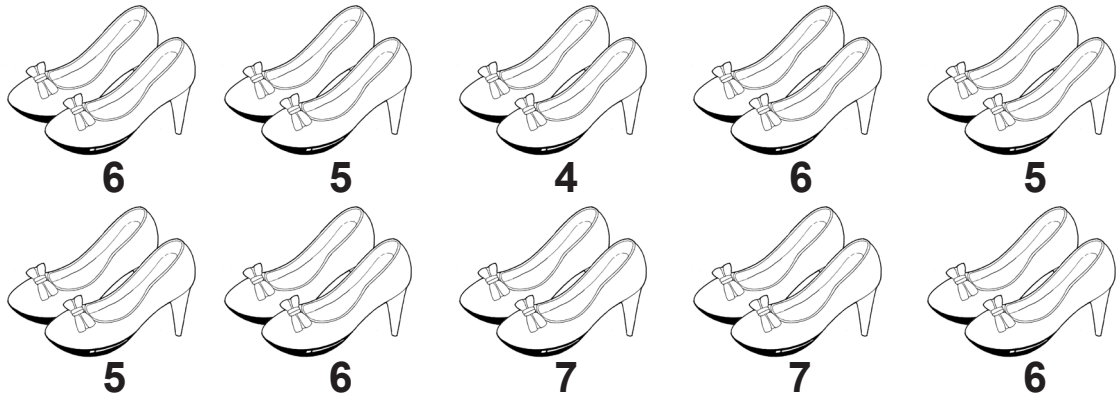
$$Md = \frac{7 + 8}{2} = 7.5$$

Hence, the median of the set of scores is 7.5

The next activity is another measure of central tendency. Try and discover for yourself the typical value we are looking for.

Activity 8 THE MOST POPULAR SIZE

1. A shoe store was able to sell 10 black pair of shoes in one day. Which shoe size is saleable? How many of this size were sold for the day?



2. The principal of a school had the number of students posted at the door of each section. What section(s) has the same number of students? What is that number?

I-Camia 50 Students	I-Rosal 52 Students	I-Lily 50 Students	I-Adelfa 53 Students
I-Santan 51 Students	I-Tulip 53 Students	I-Rose 52 Students	I-Iris 53 Students
I-Ilang-Ilang 50 Students			

3. The scores of five students in a ten-item test. How many got same score?



Loida	<u>5</u>	Jackie	<u>8</u>	Jen	<u>7</u>	Julie	<u>3</u>	Fe	<u>9</u>
1. b	6. d	1. a	6. c	1. b	6. a	1. b	6. d	1. a	6. a
2. b	7. b	2. b	7. b	2. b	7. b	2. b	7. b	2. b	7. b
3. b	8. b	3. b	8. a	3. b	8. a	3. c	8. b	3. a	8. a
4. c	9. d	4. c	9. d	4. c	9. a	4. d	9. a	4. c	9. d
5. b	10. c	5. b	10. a	5. b	10. a	5. b	10. c	5. b	10. c

From this activity, what is the characteristic of this value that we are looking for? This typical value is what we call the mode.

The next discussion will give you a clearer idea about the *mode*.

The Mode

The *mode* is the measure or value which occurs most frequently in a set of data. It is the value with the greatest frequency.

To find the *mode* for a set of data:

1. select the measure that appears most often in the set;
2. if two or more measures appear the same number of times, then each of these values is a mode; and
3. if every measure appears the same number of times, then the set of data has no mode.

Try answering these items.

Find the mode in the given sets of scores.

1. {10, 12, 9, 10, 13, 11, 10}
2. {15, 20, 18, 19, 18, 16, 20, 18}
3. {5, 8, 7, 9, 6, 8, 5}
4. {7, 10, 8, 5, 9, 6, 4}
5. {12, 16, 14, 15, 16, 13, 14}

Activity 9 WHO'S THE CONTENDER

The Mathematics Department of Juan Sumulong High School is sending a contestant in a quiz bee competition. The teachers decided to select the contestant from among the top two performing students of Section 1. With very limited data, they considered only the scores of each student in 10 quizzes.

The scores are tabulated below.

Quiz Number	Zeny	Richard
1	11	10
2	13	10
3	5	12
4	13	15
5	7	15
6	10	9
7	35	13
8	13	13
9	9	12
Total	117	108

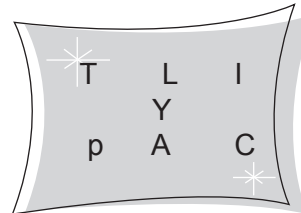
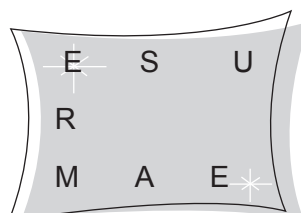
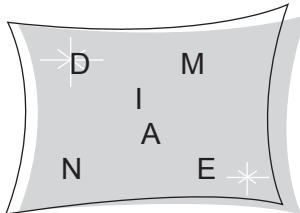
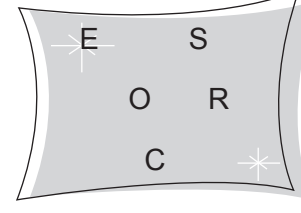
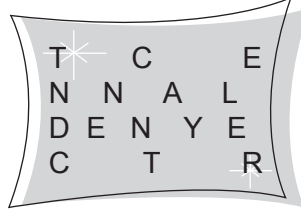
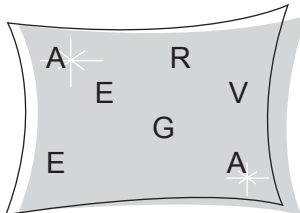
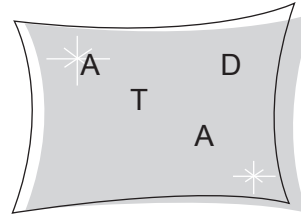
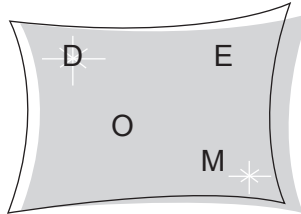
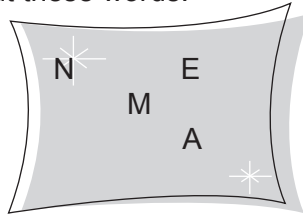
- What is the mean of the scores of both students?
- How many scores are above and below the mean of these scores?
- Check once more the distribution of scores. Which of the two has a more consistent performance?
- Which of the two students do you think should be sent to represent the school in the competition?
- Try getting the median of these scores and compare with their mean.
- Which do you think is the best measure to use to assess their performance? Explain.

Activity 10 JOURNAL WRITING

Write your reflection about where you have heard or encountered averages (e.g. business, sports, weather). How did this help you analyze a situation in the activities discussed?

Activity 11 WHAT A WORD!

Rearrange the letters to name the important words you have learned. Tell something about these words.



Practice Exercise:

- Find the mean, median, and mode/modes of each of the following sets of data.
 - 29, 34, 37, 22, 15, 38, 40
 - 5, 6, 7, 7, 9, 9, 9, 10, 14, 16, 20
 - 82, 61, 93, 56, 34, 57, 92, 53, 57
 - 26, 32, 12, 18, 11, 12, 15, 18, 21
 - The scores of 20 students in a Biology quiz are as follows:

25	33	35	45	34
26	29	35	38	40
45	38	28	29	25
39	32	37	47	45
- Athena got the following scores in the first quarter quizzes: 12, 10, 16, x , 13, and 9. What must be her score on the 4th quiz to get an average of 12?
- The mean of 12 scores is 68. If two scores, 70 and 63 are removed, what is the mean of the remaining scores?

4. The average weight of Loida, Jackie and Jen is 55 kilograms.
- What is the total weight of these girls?
 - Julie weighs 60 kilograms. What is the average weight of the four girls?
5. The data below show the score of 40 students in the 2010 Division Achievement Test (DAT).

35	16	28	43	21	17	15	16
20	18	25	22	33	18	32	38
23	32	18	25	35	18	20	22
36	22	17	22	16	23	24	15
15	23	22	20	14	39	22	38

- What score is typical to the group of the students? Why?
- What score appears to be the median? How many students fall below that score?
- Which score frequently appears?
- Find the Mean, Median and Mode.
- Describe the data in terms of the mean, median, and mode.

What to Understand



Reflect and analyze how you were able to develop a concept out of the activities you have studied. The knowledge gained here will further help you understand and answer the next activities.

Activity 12

WORK IN PAIRS

Analyze the following situations and answer the questions that follow. Make the necessary justifications if possible.

1. The first three test scores of each of the four students are shown. Each student hopes to maintain an average of 85. Find the score needed by each student on the fourth test to have an average of 85, or explain why such average is not possible.

- a. Lisa: 78, 80, 100 c. Lina: 79, 80, 81
b. Mary: 90, 92, 95 d. Willie: 65, 80, 80

2. The weekly salaries in pesos of 6 workers of a construction firm are 2400, 2450, 2450, 2500, 2500 and 4200.

- a. Compute for the mean, the median, and the mode
b. If negotiations for new salaries are to be proposed, and you represent the management, which measure of central tendency will you use in the negotiation? Explain your answer.
c. If you represent the labor union, which measure of central tendency will you use in the negotiation? Explain your answer.



3. The monthly salaries of the employees of ABC Corporation are as follows:

Manager:	Php 100 000
Cashier:	Php 20 000
Clerk (9):	Php 15 000
Utility Workers (2):	Php 8 500



In the manager's yearly report, the average salary of the employees is Php 20 923.08. The accountant claimed that the average monthly salary is Php 15 000.

Both employees are correct since the average indicates the typical value of the data.

Which of the two salaries is the average salary of the employees? Justify your answer.

Activity 13 WHICH IS TYPICAL?

Direction: Read the statements found at the right column in the table below. If you agree with the statement, place a checkmark (✓) in the After-Lesson-Response column beside it. If you don't, mark it with (x).

Statement	After-Lesson Response
24 is typical to the numbers 17, 25 and 30	
6 is the typical score in the set of data 3, 5, 8, 6, 9	
10 is a typical score in: 8, 7, 9, 10, and 6	
18 is typical age in workers' ages 17, 19, 20, 17, 46, 17, 18	
5 is typical in the numbers 3, 5, 4, 5, 7, and 5	
The mean is affected by the size of extreme values	
The median is affected by the size of extreme values	
The mode is affected by the size of extreme values	
The mean is affected by the number of measures	
The median is affected by number of measures	
The mode is affected by the number of measures	

Activity 14 LET'S SUMMARIZE!



The three measures of central tendency that you have learned in the previous module do not give an adequate description of the data. We need to know how the observations spread out from the average or mean.

Lesson

2

Measures of Variability

What to Know



Let's begin with interesting and exploratory activities that would lead to the basic concepts of measures of variability. You will learn to interpret, draw conclusions and make recommendations.

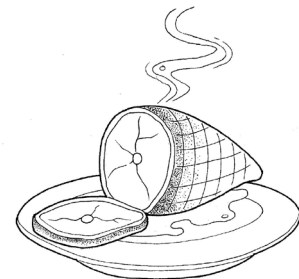
After these activities, the learners shall be able to answer the question, “**How can I make use of the representations and descriptions of a given set of data in real-life situations?**”.

The lesson on measures of variability will tell you how the values are scattered or clustered about the typical value.

It is quite possible to have two sets of observations with the same mean or median that differs in the amount of spread about the mean. Do the following activity.

Activity 1 WHICH TASTES BETTER?

A housewife surveyed canned ham for a special family affair. She picked 5 cans each from two boxes packed by company A and company B. Both boxes have the same weight. Consider the following weights in kilograms of the canned Ham packed by the two companies (sample A and sample B).



Sample A: 0.97, 1.00, 0.94, 1.03, 1.11

Sample B: 1.06, 1.01, 0.88, 0.90, 1.14

Help the housewife choose the best sample by doing the following procedure.

QUESTIONS

- Arrange the weights in numerical order.
- Find the mean weight of each sample.
- Analyze the spread of the weights of each sample from the mean.
- Which sample has weights closer to the mean?
- If you are to choose from these two samples, which would you prefer? Why?
- Was your choice affected by the weight or the taste? Explain.

Measures other than the mean may provide additional information about the same data. These are the *measures of dispersion*.

Measures of dispersion or variability refer to the spread of the values about the mean. These are important quantities used by statisticians in evaluation. Smaller dispersion of scores arising from the comparison often indicates more consistency and more reliability.

The most commonly used measures of dispersion are the range, the average deviation, the standard deviation, and the variance.

What to Process



Here you will be provided with enabling activities that you have to go through to validate your understanding on measures of variability after the activities in the What to Know phase. These would answer the question **“How can I make use of the representations and descriptions of given set of data in real-life situations?”**.

The Range

The range is the simplest measure of variability. It is the difference between the largest value and the smallest value.

$$R = H - L$$

where R = Range, H = Highest value, L = Lowest value

Test scores of 10, 8, 9, 7, 5, and 3, will give us a range of 7 from $10 - 3 = 7$.

Let us consider this situation.

The following are the daily wages of 8 factory workers of two garment factories. Factory A and factory B. Find the range of salaries in peso (Php).

Factory A: 400, 450, 520, 380, 482, 495, 575, 450.

Factory B: 450, 400, 450, 480, 450, 450, 400, 672

Workers of both factories have mean wage = 469



Finding the range of wages: $\text{Range} = \text{Highest wage} - \text{Lowest wage}$

$$\text{Range A} = 575 - 380 = 195$$

$$\text{Range B} = 672 - 350 = 322$$

Comparing the two wages, you will note that wages of workers of factory B have a higher range than wages of workers of factory A. These ranges tell us that the wages of workers of factory B are more scattered than the wages of workers of factory A.

Look closely at wages of workers of factory B. You will see that except for 672 the highest wage, the wages of the workers are more consistent than the wages in A. Without the highest wage of 672 the range would be 80 from $480 - 400 = 80$. Whereas, if you exclude the highest wage 575 in A, the range would be 140 from $520 - 380 = 140$.

Can you now say that the wages of workers of factory B are more scattered or variable than the wages of workers of factory A?

The range tells us that it is not a stable measure of variability because its value can fluctuate greatly even with a change in just a single value, either the highest or lowest.

Activity 2

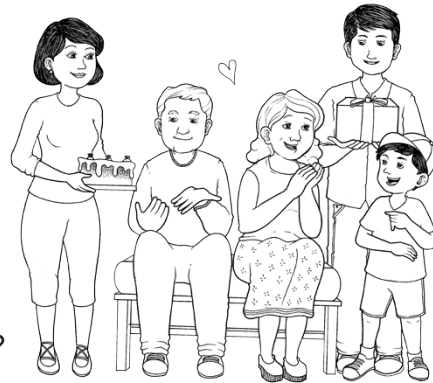
WHO IS SMARTER?

1. The IQs of 5 members of 2 families A and B are:

Family A: 108, 112, 127, 118 and 113

Family B: 120, 110, 118, 120 and 110

- Find the mean IQ of the two families.
- Find the range of the IQ of both families.
- Which of the two families has consistent IQ?



2. The range of each of the set of scores of the three students is as follows:

Ana	$H = 98, L = 92, R = 98 - 92 = 6$
Josie	$H = 97, L = 90, R = 97 - 90 = 7$
Lina	$H = 98, L = 89, R = 98 - 89 = 7$

- What have you observed about the range of the scores of the three students?
- What does it tell you?

3. Consider the following sets of scores: Find the range and the median.

Set A	Set B
3	3
4	7
5	7
6	7
8	8
9	8
10	8
12	9
15	15

Activity 3
TRY THIS!

- A. Compute the range for each set of numbers.

1. {12, 13, 17, 22, 22, 23, 25, 26}
2. {12, 13, 14, 15, 16, 17, 18}
3. {12, 12, 13, 13, 13, 13, 13, 15, 19, 20, 20}
4. {7, 7, 8, 12, 14, 14, 14, 14, 15, 15}
5. {23, 25, 27, 27, 32, 32, 36, 38}

- B. Solve the following:

1. If the range of the set of scores is 29 and the lowest score is 18, what is the highest score?
2. If the range of the set of scores is 14, and the highest score is 31, what is the lowest score?
3. The reaction times for a random sample of 9 subjects to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1 and 3.4 seconds. Calculate range.
4. Two students have the following grades in six math tests. Compute the mean and the range. Tell something about the two sets of scores.

Pete	Ricky
82	88
98	94
86	89
80	87
100	92
94	90

The Average Deviation

The dispersion of a set of data about the average of these data is *the average deviation or mean deviation*.

To compute the average deviation of an ungrouped data, we use the formula:

$$\text{A.D.} = \frac{\sum|x-\bar{x}|}{N}$$

where A.D. is the average deviation;

x is the individual score;

\bar{x} is the mean; and

N is the number of scores.

$|x-\bar{x}|$ is the absolute value of the deviation from the mean.

Procedure in computing the average deviation:

1. Find the mean for all the cases.
2. Find the absolute difference between each score and the mean.
3. Find the sum of the difference and divide by N .

Example:

Find the average deviation of the following data: 12, 17, 13, 18, 18, 15, 14, 17, 11

1. Find the mean (\bar{x}).

$$\bar{x} = \frac{\sum x}{N} = \frac{12 + 17 + 13 + 18 + 18 + 15 + 14 + 17 + 11}{9}$$

$$\bar{x} = \frac{135}{9} = 15$$

2. Find the absolute difference between each score and the mean.

$$\begin{aligned}|x-\bar{x}| &= |12 - 15| = 3 \\ &= |17 - 15| = 2 \\ &= |13 - 15| = 2 \\ &= |18 - 15| = 3 \\ &= |18 - 15| = 3 \\ &= |15 - 15| = 0 \\ &= |14 - 15| = 1 \\ &= |17 - 15| = 2 \\ &= |11 - 15| = 4\end{aligned}$$

3. Find the sum of the absolute difference $\sum|x-\bar{x}|$.

$$\begin{aligned}|x-\bar{x}| &= |12 - 15| = 3 \\ &= |17 - 15| = 2 \\ &= |13 - 15| = 2 \\ &= |18 - 15| = 3\end{aligned}$$

$$\begin{aligned}
&= |18 - 15| = 3 \\
&= |15 - 15| = 0 \\
&= |14 - 15| = 1 \\
&= |17 - 15| = 2 \\
&= |11 - 15| = 4
\end{aligned}$$

$$\Sigma|x-\bar{x}| = 20$$

This can be represented in tabular form as shown below.

x	\bar{x}	$ x-\bar{x} $
12	15	3
17	15	2
13	15	2
18	15	3
18	15	3
15	15	0
14	15	1
17	15	2
11	15	4
		$\Sigma x-\bar{x} = 20$

4. Solve for the average deviation by dividing the result in step 3 by N.

$$\text{A.D.} = \frac{\Sigma|x-\bar{x}|}{N} = \frac{20}{9} = 2.22$$

Activity 4 **TRY THIS!**

Solve the average deviation of the following:

- Science achievement test scores: 60, 75, 80, 85, 90, 95
- The weights in kilogram of 10 students: 52, 55, 50, 55, 43, 45, 40, 48, 45, 47.
- The diameter (in cm) of balls: 12, 13, 15, 15, 15, 16, 18.
- Prices of books (in pesos): 85, 99, 99, 99, 105, 105, 120, 150, 200, 200.
- Cholesterol level of middle-aged persons: 147, 154, 172, 195, 195, 209, 218, 241, 283, 336.

The *average deviation* gives a better approximation than the *range*. However, it does not lend itself readily to mathematical treatment for deeper analysis.

Let us do another activity to discover another measure of dispersion, the *standard deviation*.

The Standard Deviation

Activity 5 WORKING IN PAIRS

Compute the standard deviation of the set of test scores: {39, 10, 24, 16, 19, 26, 29, 30, 5}.



- Find the mean.
- Find the deviation from the mean $(x-\bar{x})$.
- Square the deviations $(x-\bar{x})^2$.
- Add all the squared deviations. $\sum(x-\bar{x})^2$
- Tabulate the results obtained:

x	$x-\bar{x}$	$(x-\bar{x})^2$
5		
10		
16		
19		
24		
26		
29		
30		
39		
		$\sum(x-\bar{x})^2$

- Compute the standard deviation (SD) using the formula

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$$

- Summarize the procedure in computing the standard deviation.

From the activity, you have learned how to compute for the standard deviation.

Like the average deviation, standard deviation differentiates sets of scores with equal averages. But the advantage of standard deviation over mean deviation is that it has several applications in inferential statistics

To compute for the standard deviation of an ungrouped data, we use the formula:

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$$

Where SD is the standard deviation;
 x is the individual score;
 \bar{x} is the mean; and
 N is the number of scores.

In the next discussion, you will learn more about the importance of using the standard deviation.

Let us consider this example.

Compare the standard deviation of the scores of the three students in their Mathematics quizzes.

Student A	97, 92, 96, 95, 90
Student B	94, 94, 92, 94, 96
Students C	95, 94, 93, 96, 92

Solution:

Student A:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{92 + 92 + 96 + 95 + 90}{5} = 94$$

Step 2. Complete the table below.

x	$x-\bar{x}$	$(x-\bar{x})^2$
97	3	9
92	-2	4
96	2	4
95	1	1
90	4	16
		$\sum(x-\bar{x})^2 = 34$

Step 3. Compute the standard deviation.

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{34}{5}} = \sqrt{6.8} = 2.6$$

Student B:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{92 + 92 + 96 + 95 + 90}{5} = 94$$

Step 2. Complete the table below.

x	$x - \bar{x}$	$(x - \bar{x})^2$
94	0	0
94	0	0
92	-2	4
94	0	0
96	2	4
		$\sum(x - \bar{x})^2 = 8$

Step 3. Compute the standard deviation.

$$SD = \sqrt{\frac{\sum(x - \bar{x})^2}{N}} = \sqrt{\frac{8}{5}} = \sqrt{1.6} = 1.3$$

Student C:

Step 1. Compute the mean score.

$$\bar{x} = \frac{\sum x}{N} = \frac{95 + 94 + 93 + 96 + 92}{5} = 94$$

Step 2. Complete the table below.

x	$x - \bar{x}$	$(x - \bar{x})^2$
95	1	1
94	0	0
93	-1	1
96	2	4
92	-2	4
		$\sum(x - \bar{x})^2 = 10$

Step 3. Compute the standard deviation.

$$SD = \sqrt{\frac{\sum(x - \bar{x})^2}{N}} = \sqrt{\frac{10}{5}} = \sqrt{2} = 1.4$$

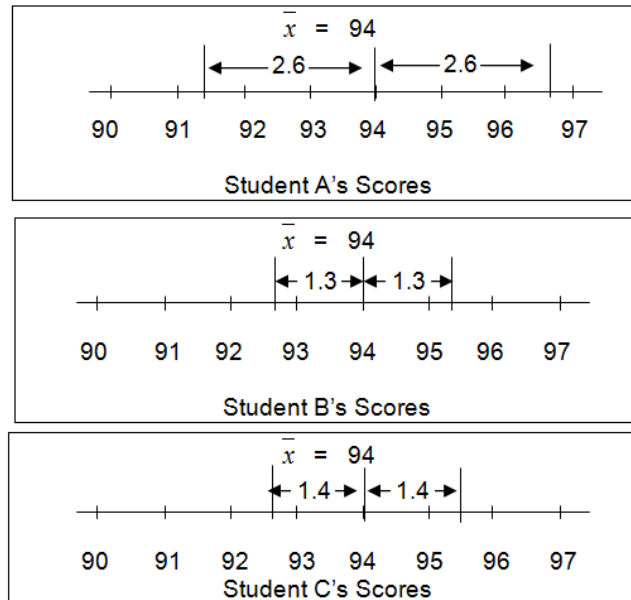
The result of the computation of the standard deviation of the scores of the three students can be summarized as:

$$\text{SD (A)} = 2.6$$

$$\text{SD (B)} = 1.3$$

$$\text{SD (C)} = 1.4$$

The standard deviation of the scores can be illustrated below by plotting the scores on the number line.



Graphically, a standard deviation of 2.6 means most of the scores are within 2.6 units from the *mean*. A Standard deviation of 1.3 and 1.4 suggest that most of the scores are within 1.3 and 1.4 units from the *mean*.

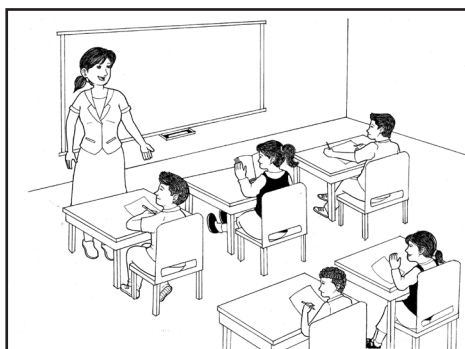
The scores of Student B is clustered closer to the mean. This shows that the score of Student B is the most consistent among the three sets of scores.

The concept of standard deviation is especially valuable because it enables us to compare data points from different sets of data. When two groups are compared, the group having a smaller standard deviation is less varied.

Activity 6

WORKING IN PAIRS

- A. Compute the standard deviation for each set of numbers.
- (12, 13, 14, 15, 16, 17, 18)
 - (7, 7, 8, 12, 14, 14, 14, 14, 15, 15)
 - (12, 12, 13, 13, 13, 13, 13, 15, 19, 20, 20)
 - (12, 13, 17, 22, 22, 23, 25, 26)
 - (23, 25, 27, 27, 32, 32, 36, 38)
- B. The reaction times for a random sample of nine subjects to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1 and 3.4 seconds. Calculate the range and standard deviation.
- C. Suppose two classes achieved the following grades on a math test, find the range and the standard deviation.



Class 1: 64, 70, 73, 77, 85, 90, 94

Class 2: 74, 75, 75, 76, 79, 80, 94

You may use a scientific calculator to solve for the standard deviation.

Activity 7

WORKING IN PAIRS

The grades of a student in nine quizzes: 78, 80, 80, 82, 85, 85, 85, 88, 90. Calculate for the mean and standard deviation using a scientific calculator.

Procedure

Press the following keys:

Shift

Mode (Setup)



4 (Stat)

1 (ON)

Mode

3 (Stat)

1 (1-var)

	x	$f(x)$
1		
2		
3		

Is displayed. Input values of x .

78 [=] 80 [=] 82 [=] 85 [=] 88 [=] 90 [=]

(↓) (⇒) 1 [=] 2 [=] 1 [=] 3 [=] 1 [=] 1

	x	$f(x)$
1	78	1
2	80	2
3	82	1
4	85	3
5	88	1
6	90	1

The displayed output.

AC Shift 1(Stat) 4 (var) $2(\bar{x})$ [=]

Answer: Mean ≈ 83.67

AC Shift 1(Stat) 4 (var) $2(x\sigma n)$ [=]

Answer: SD ≈ 3.74

In the next discussion, you will learn about another measure of variability.

The Variance

The variance (∂^2) of a data is equal to $\frac{1}{N}$. The sum of their squares minus the square of their mean. It is virtually the square of the standard deviation.

$$\partial^2 = \frac{\sum(x-\bar{x})^2}{N}$$

where ∂^2 is the variance;

N is the total number of observations;

x is the raw score; and

\bar{x} is the mean of the data.

Variance is not only useful, it can be computed with ease and it can also be broken into two or more component sums of squares that yield useful information.

Activity 8

ANSWER THE FOLLOWING.

The table shows the daily sales in peso of two sari-sari stores near a school.

Store A	Store B
300	300
310	120
290	500
301	100
299	490
295	110
305	300
300	480

Compute for the Variance and interpret.

What to Understand



You will be provided with activities that will allow you to reflect, revisit, revise and rethink about a variety of experiences in life. Moreover, you will be able to express your understanding on the concept of *measures of variability* that would engage you in multidirectional self-assessment.

Activity 9

ANSWER THE FOLLOWING.

- Find the range for each set of data.
 - scores on quizzes: 10, 9, 6, 6, 7, 8, 8, 8, 8, 9
 - Number of points per game: 16, 18, 10, 20, 15, 7, 16, 24
 - Number of VCR's sold per week: 8, 10, 12, 13, 15, 7, 6, 14, 18, 20
- Given the scores of two students in a series of test
Student A: 60, 55, 40, 48, 52, 36, 52, 50
Student B: 62, 48, 50, 46, 38, 48, 43, 39
 - Find the mean score of each student?
 - Compute the range.
 - Interpret the result.

3. The minimum distances (in feet) a batter has to hit the ball down the center of the field to get a home run in 8 different stadiums is 410, 420, 406, 400, 440, 421, 402 and 425 ft. Compute for the standard deviation.
4. The scores received by Jean and Jack in ten math quizzes are as follows:
Jean: 4, 5, 3, 2, 2, 5, 5, 3, 5, 0
Jack: 5, 4, 4, 3, 3, 1, 4, 0, 5, 5
- Compute for the standard deviation.
 - Which student had the better grade point average?
 - Which student has the most consistent score?

Lesson

3

Measures of Central Tendency of Grouped Data

What to Know



Start the lesson by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding Measures of Central Tendency for Grouped Data. As you go through this lesson, think of the following important question: **How is the measures of central tendency for grouped data used in solving real-life problems and in making decisions?** To find out the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier.

Activity 1

DO YOU STILL REMEMBER THIS?

Directions:

A. Write the following expressions in expanded form:

1. $\sum_{i=1}^4 4x_1$

2. $\sum_{i=2}^5 (Y_i - 5)$

3. $\sum_{i=1}^4 (X_i + 2Y_i)$

B. Evaluate the following expressions using the given data:

$x_1 = 5$	$x_2 = -2$	$x_3 = -1$	$x_4 = 7$	$x_5 = 2$
$y_1 = 1$	$y_2 = 6$	$y_3 = -4$	$y_4 = -3$	$y_5 = -5$

1. $\sum_{i=2}^5 (5 - X_i)$

3. $\sum_{i=2}^4 \frac{2}{X_i - Y_i}$

2. $\sum_{i=1}^4 5Y_i$

4. $\sum_{i=1}^4 (3Y_i - X_i^2)$

QUESTIONS

1. How did you find the given activity?
2. Have you applied your previous knowledge about summation notation?

Activity 2 **TRY THIS!**

Directions: Complete the frequency distribution table by finding the unknown values. Write your complete solutions and answers on a piece of paper.

A. Scores of Grade 8 Section Avocado Students in the 4th Periodic Test in Mathematics

Score	Frequency (f)	Class Mark (X)	fX	Less Than Cumulative Frequency ($<cf$)	Lower Class Boundary (lb)
46 – 50					
41 – 45					
36 – 40					
31 – 35					
26 – 30					
21 – 25					
$i =$	$\Sigma f =$		$\Sigma(fX) =$		

B. Ages of San Pedro Jose High School Teachers

Age	f	X	fX	$<cf$	lb
21 – 25					
26 – 30					
31 – 35					
36 – 40					
41 – 45					
46 – 50					
51 – 55					
56 – 60					
61 – 65					
$i =$	$\Sigma f =$		$\Sigma(fX) =$		

Questions	A	B
<ol style="list-style-type: none"> 1. How did you determine the unknown values in the frequency distribution table? 2. What is the class size? 3. What is the class mark of the class with the highest frequency? 4. In each frequency distribution table, determine the following: <ol style="list-style-type: none"> a. Median class b. Cumulative frequency of the median class c. Modal class d. Lower boundary of the modal class 5. Find the following measures in each data set: <ol style="list-style-type: none"> a. Mean b. Median c. Mode 		

Were you able to complete the frequency distribution table? Were you able to find the unknown values in the frequency distribution table? In the next activity, you will calculate the mean, median, and mode of a given set of data.

Activity 3 NEXT ROUND...

Directions: The frequency distribution below shows the height (in cm) of 50 students in Buslo High School. Use the table to answer the questions that follow. Write your complete solutions and answers in a piece of paper.

Height (in cm) of 50 Students in Buslo High School

Height (in cm)	Frequency	X
170-174	8	
165-169	18	
160-164	13	
155-159	7	
150-154	4	

QUESTIONS ?

1. What is the total frequency of the given data set?
2. Complete the frequency distribution table. What is $\sum fX$?
3. How would you find the mean of the given data set?
4. Find the mean of the set of data.
5. Determine the following. Explain your answer.
 - a. Median class
 - b. Modal class
 - c. Lower boundary of the median class
 - d. Lower boundary of the modal class
6. Find the median and the mean of the set of data?
7. How do the mean, median, and the mode of the set of data compare?

What to Process



How did you find the previous activity? Were you able to find the unknown measures/values? Are you ready to perform the next activity? Will you be able to find the mean, median and the mode of a set of data such as the ages, grades, or test scores of your classmates? Before proceeding to these activities, read first some important notes on how to calculate the mean, median and mode for grouped data.

Before we proceed in finding the mean, median and mode of grouped data, let us recall the concepts about Summation Notation:

Summation Notation

It is denoted by the symbol using the Greek letter Σ (a capital sigma) which means “the summation of”.

The summation notation can be expressed as:

$$\sum_{i=1}^n X_i = X_1 + X_2 + X_3 + \dots + X_n$$

and it can be read as “the summation of X sub i where i starts from 1 to n .”

Illustrative Example:

1. Write the expression in expanded form:

- a.
$$\sum_{i=1}^5 2X_i = 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5$$
$$= 2(X_1 + X_2 + X_3 + X_4 + X_5)$$

- b.
$$\sum_{i=2}^4 (2X_i - Y_i) = (2X_2 - Y_2) + (2X_3 - Y_3) + (2X_4 - Y_4)$$

To find the mean, median and mode for grouped data, take note of the following:

1. Mean for Grouped Data

When the number of items in a set of data is too big, items are grouped for convenience. To find the mean of grouped data using class marks, the following formula can be used:

$$\text{Mean} = \frac{\sum(fX)}{\sum f}$$

where: f is the frequency of each class
 X is the class mark of class

Illustrative Example:

Directions: Calculate the mean of the Mid-year Test scores of Students in Filipino.

Mid-year Test Scores of Students in Filipino

Score	Frequency
41 – 45	1
36 – 40	8
31 – 35	8
26 – 30	14
21 – 25	7
16 – 20	2

Solutions:

Score	Frequency (f)	Class Mark (X)	fX
41 – 45	1	43	43
36 – 40	8	38	304
31 – 35	8	33	264
26 – 30	14	28	392
21 – 25	7	23	161
16 – 20	2	18	36
$i = 5$	$\sum f = 40$		$\sum(fX) = 1,200$

$$\text{Mean} = \frac{\sum(fX)}{\sum f} = \frac{1,200}{40} = 30$$

Therefore, the mean of mid-year test is **30**.

There is an alternative formula for computing the mean of grouped data and this makes use of coded deviation

$$\text{Mean} = A.M + \left[\frac{\sum(fd)}{\sum f} \right] i$$

where: A.M. is the assumed mean;
 f is the frequency of each class;
 d is the coded deviation from A.M.; and
 i is the class interval

Any class mark can be considered as the assumed mean. But it is convenient to choose the class mark with the highest frequency to facilitate computation. The class chosen to contain as the A.M. has no deviation from itself and so 0 is assigned to it.

Subsequently, similar on a number line or Cartesian coordinate system, consecutive positive integers are assigned to the classes upward and negative integers to the classes downward.

Let us find the mean of the given illustrative example about the Mid-year test scores of Students in Filipino using coded deviation.

Illustrative Example:

Mid-year Test Scores of Students in Filipino

Score	Frequency
41 – 45	1
36 – 40	8
31 – 35	8
26 – 30	14
21 – 25	7
16 – 20	2

Solutions:

Score	f	X	d	fd
41 – 45	1	43	3	3
36 – 40	8	38	2	16
31 – 35	8	33	1	8
26 – 30	14	28	0	0
21 – 25	7	23	-1	-7
16 – 20	2	18	-2	-4
$i = 5$	$\sum f = 40$			$\sum fd = 16$

$A.M. = 28$	$\Sigma f = 40$
$\Sigma fd = 16$	$i = 5$

$$\text{Mean} = A.M + \left[\frac{\Sigma(fd)}{\Sigma f} \right] i$$

$$\text{Mean} = 28 + \left[\frac{16}{40} \right] 5$$

$$\text{Mean} = 28 + \frac{16(5)}{40}$$

$$\text{Mean} = 28 + \frac{80}{40}$$

$$\text{Mean} = 28 + 2$$

$$\text{Mean} = 30$$

Therefore, the mean of mid-year test is **30**.

What have you observed? It implies that even you use class marks or coded deviation the results that you will get are the same.

2. Median for Grouped Data

The median is the middle value in a set of quantities. It separates an ordered set of data into two equal parts. Half of the quantities is located above the median and the other half is found below it, whenever the quantities are arranged according to magnitude (from highest to lowest.)

In computing for the median of grouped data, the following formula is used:

$$\text{Median} = lb_{mc} + \left[\frac{\frac{\Sigma f}{2} - <cf}{f_{mc}} \right] i$$

where: lb_{mc} is the lower boundary of the median class;

f is the frequency of each class;

$<cf$ is the cumulative frequency of the lower class next to the median class;

f_{mc} is the frequency of the median class; and

i is the class interval.

The median class is the class that contains the $\frac{\Sigma f}{2}$ th quantity. The computed median must be within the median class.

Illustrative Example:**Directions:** Calculate the median of the Mid-year Test Scores of students in Filipino.**Mid-year Test Scores of Students in Filipino**

Score	Frequency
41 – 45	1
36 – 40	8
31 – 35	8
26 – 30	14
21 – 25	7
16 – 20	2

Solutions:

Score	Frequency	lb	$<cf$	
41 – 45	1	40.5	40	
36 – 40	8	35.5	39	
31 – 35	8	30.5	31	
26 – 30	14	25.5	23	Median Class
21 – 25	7	20.5	9	
16 – 20	2	15.5	2	
$i = 5$	$\Sigma f = 40$			

$$\text{Median} = lb_{mc} + \left[\frac{\frac{\Sigma f}{2} - <cf}{f_{mc}} \right] i$$

a. $\frac{\Sigma f}{2} = \frac{40}{2} = 20$

The 20th score is contained in the class 26-30. This means that the median falls within the class boundaries of 26-30. That is, 25.5-30.5

b. $<cf = 9$

c. $f_{mc} = 14$

d. $lb_{mc} = 25.5$

e. $i = 5$

Solutions:

$$\text{Median} = lb_{mc} + \left[\frac{\frac{\Sigma f}{2} - <cf}{f_{mc}} \right] i$$

$$\text{Median} = 25.5 + \left[\frac{\frac{20}{2} - 9}{14} \right] 5$$

$$\text{Median} = 25.5 + \left[\frac{10 - 9}{14} \right] 5$$

$$\text{Median} = 25.5 + \left[\frac{1}{4} \right] 5$$

$$\text{Median} = 25.5 + \left[\frac{5}{14} \right] 5$$

$$\text{Median} = 25.5 + 0.38$$

$$\text{Median} = 25.88$$

Therefore, the median of the mid-year test scores is 25.88.

(Note: The median 25.88 falls within the class boundaries of 26-30 which is 25.5-30.5)

3. Mode for Grouped Data

The mode of grouped data can be approximated using the following formula:

$$\text{Mode} = lb_{mo} + \left[\frac{D_1}{D_1 + D_2} \right] i$$

where: lb_{mo} is the lower boundary of the modal class;

D_1 is the difference between the frequencies of the modal class and the next upper class;

D_2 is the difference between the frequencies of the modal class and the next lower class; and

i is the class interval.

The modal class is the class with the highest frequency.

Illustrative Example:

Directions: Calculate the mode of the Mid-year Test Scores of students in Filipino.

Mid-year Test Scores of Students in Filipino

Score	Frequency
41 – 45	1
36 – 40	8
31 – 35	8
26 – 30	14
21 – 25	7
16 – 20	2

Solutions:

Score	Frequency	lb
41 – 45	1	40.5
36 – 40	8	35.5
31 – 35	8	30.5
26 – 30	14	25.5
21 – 25	7	20.5
16 – 20	2	15.5

Modal Class

Since class 26-30 has the highest frequency, therefore the modal class is 26-30.

$$\begin{aligned}lb_{mo} &= 25.5 \\D_1 &= 14 - 8 = 6 \\D_2 &= 14 - 7 = 7 \\i &= 5\end{aligned}$$

$$\text{Mode} = 25.5 + \left[\frac{D_1}{D_1 + D_2} \right] i$$

$$\text{Mode} = 25.5 + \left[\frac{7}{6 + 7} \right] 5$$

$$\text{Mode} = 25.5 + \left[\frac{7}{13} \right] 5$$

$$\text{Mode} = 25.5 + \frac{35}{13}$$

$$\text{Mode} = 25.5 + 2.69$$

$$\text{Mode} = \mathbf{28.19}$$

Therefore, the mode of the mid-year test is **28.19**.

If there are two or more classes having the same highest frequency, the formula to be used is:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean})$$

Illustrative Example:

Height of Nursing Students in Our Lady of Piat College

Height (cm)	Frequency
170-174	7
165-169	10
160-164	11
155-159	11
150-154	10

(Note: The given data has two classes with the highest frequency; therefore, the first formula in solving the mode is not applicable.)

Solutions:

a. Mean = $\frac{\sum(fX)}{\sum f} = \frac{8,075}{50}$

Mean = 161.5

b. Median
 $\frac{\sum f}{2} = \frac{50}{2} = 25$

The 25th score is contained in the class 160-164. This means that the median falls within the class boundaries of 160-164. That is, 159.5-164.5

$$<cf = 21$$

$$f_{mc} = 11$$

$$lb_{mc} = 159.5$$

$$i = 5$$

$$\text{Median} = lb_{mc} + \left[\frac{\frac{\sum f}{2} - <cf}{f_{mc}} \right] i$$

$$\text{Median} = 159.5 + \left[\frac{25 - 21}{11} \right] i$$

$$\text{Median} = 159.5 + \left[\frac{4}{11} \right] 5$$

$$\text{Median} = 159.5 + \left[\frac{4(5)}{11} \right]$$

$$\text{Median} = 159.5 + \left[\frac{20}{11} \right]$$

$$\text{Median} = 159.5 + 1.82$$

$$\text{Median} = 161.32$$

- c. Mode
 Mode = $3(\text{Median}) - 2(\text{Mean})$
 Mode = $3(161.32) - 2(161.5)$
 Mode = $483.96 - 323$
 Mode = 160.36

Therefore, the mode of the given data is **160.36**.

Were you able to learn different formulas in solving the mean, median and mode of grouped data? In the next activity, try to apply those important notes in getting the value of mean, median and mode of grouped data.

Activity 4 **LETS SOLVE IT...**

Directions: Calculate the mean, median and mode of the weight of IV-2 Students. Write your complete solutions and answers in a sheet of paper.

Weight of IV-2 Students

Weight in kg	Frequency
75 – 79	1
70 – 74	4
65 – 69	10
60 – 64	14
55 – 59	21
50 – 54	15
45 – 49	14
40 – 44	1

Mean = _____
 Median = _____
 Mode = _____



1. How did you find the mean, median, and mode of the data set?
2. What comparisons can you make about the three measures obtained?
3. What have you learned and realized while doing the activity?

Have you solved the mean, median, and mode easily with your partner? Were you able to apply the notes on how to calculate the mean, median and mode? Do the next activity by yourself.

Activity 5 **ONE MORE TRY**

Directions: Calculate the mean, median and mode of the given grouped data.

Pledges for the Victims of Typhoon Pablo

Pledges in Pesos	Frequency
9,000 – 9,999	4
8,000 – 8,999	12
7,000 – 7,999	13
6,000 – 6,999	15
5,000 – 5,999	19
4,000 – 4,999	30
3,000 – 3,999	21
2,000 – 2,999	41
1,000 – 1,999	31
0 – 999	14



1. What is the class interval of the given frequency distribution table?
2. How many pledges are there for the victims of typhoon?
3. Determine the following:
 - a. Class mark of the pledges having the highest number of donors
 - b. Median class
 - c. Modal class
4. How did you determine the mean, median, and the mode of the given data set?
How about the lower boundary of the median class of the pledges?
5. What is the lower boundary of the median class of the pledges in pesos?
6. What is the lower boundary of the modal class?
7. What is the modal score of the pledges in pesos?

What to Understand



Reflect how you were able to develop a concept out of the activities you have studied. The knowledge gained here will further help you understand and answer the next activities. After doing the following activities, you should be able to answer the following question: **How is the measures of central tendency for grouped data used in solving real-life problems and in making decisions?**

Activity 6

WE CAN DO IT...

1. Below are the scores of 65 students in Mathematics Test

Score	f	X	d	fd	$<cf$
55 – 58	2				
51 – 54	4				
47 – 50	5				
43 – 46	6				
39 – 42	10				
35 – 38	13				
31 – 34	8				
27 – 30	6				
23 – 26	6				
19 – 22	2				
15 – 18	2				
11 – 14	1				

- Complete the table by filling in the values of X (the class marks or midpoints), d (deviation), fd and $<cf$ (cumulative frequency). Explain how you arrived at your answer.
 - Find the mean, median, and the mode of the set of data.
 - How would you compare the mean, median, and the mode of the set of data?
 - Which measure best represents the average of the set of data? Why?
2. Is the median the most appropriate measure of averages (central tendency) for grouped data? Why? How about the mean? mode? Explain your answer.
3. Is it always necessary to group a set of data when finding its mean, median, or mode? Why?

What new insights do you have about solving measures of central tendency of grouped data? What do you realize after learning and doing different activities?

Let's extend your understanding. This time, apply what you have learned in real life by doing the tasks in the next section.

What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding of solving measures of central tendency of grouped data.

Activity 7 **LET'S APPLY IT...**

Prepare some power saving measures. Gather data from your classmates or peers which may include the following: electric bills, electric appliances and the estimated time of usage. Use the data and different statistical measures obtained for analysis and coming up with the power-saving measures.

RUBRIC ON GROUP TASK

	4	3	2	1
Understanding of Task	I/we demonstrated an in-depth understanding of the content, processes, and demands of the task.	I/we demonstrated substantial understanding of the content and task, even though some supporting ideas or details may be overlooked or misunderstood.	I/we demonstrated gaps in our understanding of the content and task.	I/we demonstrated minimal understanding of the content.

Completion of Task	I/we fully achieved the purpose of the task, including thoughtful, insightful interpretations and conjectures.	I/we accomplished the task.	I/we completed most of the assignment.	I/we attempted to accomplish the task, but with little or no success.
Communication of Findings	I/we communicated our ideas and findings effectively, raised interesting and provocative questions, and went beyond what was expected.	I/we communicated our findings effectively.	I/we communicated our ideas and findings.	I/we did not finish the investigation and/or were not able to communicate our ideas very well.
Group Process	We used all of our time productively. Everyone was involved and contributed to the group process and product.	We worked well together most of the time. We usually listened to each other and used each other's ideas.	We worked together some of the time. Not everyone contributed equal efforts to the task.	We really did not pull together or work very productively as a group. Not everyone contributed to the group effort.
Problem Solving	Problems did not deter us. We were proactive and worked together to solve problems.	We worked together to overcome problems we encountered.	We might have worked more productively as a group.	Some people did more work than others. OR Nobody worked very well in the group.

Adopted from Intel Teach Elements (Assessment on 21st Century Classroom)

In this section, your tasks were to cite real-life situations and formulate and solve problems involving measures of central tendency of grouped data

How did you find the performance task? How did the task help you see the real world application of measures of central tendency of grouped data?

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about measures of central tendency of grouped data. The lesson provided you opportunities to describe on how to solve mean, median and mode of the given grouped data. Moreover, you were given the chance to apply the given important notes on how to solve the mean, median and mode of the given grouped data and to demonstrate your understanding of the lesson by doing a practical task.

Lesson

4

Measures of Variability of Grouped Data

What to Know



Start the lesson by assessing your knowledge of the different mathematics concepts previously studied and your skills in performing mathematical operations. These knowledge and skills may help you in understanding Measures of Variability of Grouped Data. As you go through this lesson, think of the following important question: **How are the measures of variability of grouped data used in solving real-life problems and in making decisions?** To find out the answer, perform each activity. If you find any difficulty in answering the exercises, seek the assistance of your teacher or peers or refer to the modules you have gone over earlier.

Activity 1

LET'S TRY THIS!

Directions: Complete the frequency distribution table by finding the unknown values. Write your complete solutions and answers on a piece of paper.

Scores of Grade 8 Avocado students in the 4th Periodic Test in Mathematics

Score	Frequency (f)	Class Mark (X)	fX	$(X - \bar{x})$	$(X - \bar{x})^2$	$f(X - \bar{x})^2$
46 – 50	2					
41 – 45	9					
36 – 40	13					
31 – 35	11					
26 – 30	10					
21 – 25	5					
$i =$	$\Sigma f =$		$\Sigma fX =$			$\Sigma f(X - \bar{x})^2 =$

QUESTIONS
?

1. How did you determine the unknown values in the frequency distribution table?
2. What is the class size?
3. What is the $\sum fX$?
4. What is the value of the mean in the given distribution table?
5. What is the upper class boundary of the top interval? What about the lower class boundary of the bottom interval?
6. What is the range?
7. What is the variance of the given distribution table?
8. How would you find the variance?
9. What is the standard deviation?
10. How would you solve for the standard deviation?

Were you able to complete the frequency distribution table? Were you able to find the unknown values in the frequency distribution table? In the next activity, you will calculate the range, variance and standard deviation of a given data set.

Activity 2
GO FOR IT...

Directions: The frequency distribution below shows the number of mistakes of 50 students made in factoring 20 quadratic equations. Use the table to answer the questions that follow. Write your complete solutions and answers in a piece of paper.

Number of Mistakes Made by 50 Students in Factoring 20 Quadratic Equations

Number of Mistakes	Frequency	X
18 – 20	2	
15 – 17	5	
12 – 14	6	
9 – 11	10	
6 – 8	15	
3 – 5	8	
0 – 2	4	

QUESTIONS

1. What is the total frequency of the given data set?
2. Complete the frequency distribution table. What is $\sum fX$?
3. How would you find the mean of the given data set?
4. What is the mean of the set of data?
5. What is the upper class boundary of the top interval?
6. What is the lower class boundary of the bottom interval?
7. What is the range?
8. Find the variance and standard deviation of the set of data
9. How do the range, variance and standard deviation used in interpreting the set of data?

What to Process



How did you find the previous activity? Were you able to find the unknown measures/values? Are you ready to perform the next activity? Will you be able to find the mean, range, variance and standard deviation of a set of data such as the grades, or test scores? Before proceeding to these activities, read first some important notes on how to calculate the range, variance and standard deviation of grouped data.

To find the range, variance and standard deviation of grouped data, take note of the following:

1. Range of Grouped Data

The range is the simplest measure of variability. The range of a frequency distribution is simply the difference between the upper class boundary of the top interval and the lower class boundary of the bottom interval.

$$\text{Range} = \text{Upper Class Boundary of the Highest Interval} - \text{Lower Class Boundary of the Lowest Interval}$$

Illustrative Example:

Solve for range:

Scores in Second Periodical Test of I – Faith in Mathematics I

Scores	Frequency
46 – 50	1
41 – 45	10
36 – 40	10
31 – 35	16
26 – 30	9
21 – 25	4

Solutions:

Upper Class Limit of the Highest Interval = 50

Upper Class Boundary of the Highest Interval = 50 + 0.5 = 50.5

Lower Class Limit of the Lowest Interval = 21

Lower Class Boundary of the Lowest Interval = 21 - 0.5 = 20.5

Range = Upper Class Boundary of the Highest Interval – Lower Class Boundary of the Lowest Interval

Range = 50.5 – 20.5

Range = 30

Therefore, the range of the given data set is **30**.

2. Variance of Grouped Data (σ^2)

Variance is the average of the square deviation from the mean. For large quantities, the variance is computed using frequency distribution with columns for the midpoint value, the product of the frequency and midpoint value for each interval; the deviation and its square; and the product of the frequency and the squared deviation.

To find variance of a grouped data, use the formula:

$$\sigma^2 = \frac{\sum f(X - \bar{x})^2}{\sum f - 1}$$

where; f = class frequency

X = class mark

\bar{x} = class mean

$\sum f$ = total number of frequency

In calculating the variance, do the following steps:

1. Prepare a frequency distribution with appropriate class intervals and write the corresponding frequency (f).
2. Get the midpoint (X) of each class interval in column 2.
3. Multiply frequency (f) and midpoint (X) of each class interval to get fX .
4. Add fX of each interval to get $\sum fX$.
5. Compute the mean using $\bar{x} = \frac{\sum fX}{\sum f}$.
6. Calculate the deviation ($X - \bar{x}$) by subtracting the mean from each midpoint.
7. Square the deviation of each interval to get $(X - \bar{x})^2$.
8. Multiply frequency (f) and $(X - \bar{x})^2$. Find the sum of each product to get $\sum fX(X - \bar{x})^2$.
9. Calculate the standard deviation using the formula

$$\sigma^2 = \frac{\sum f(X - \bar{x})^2}{\sum f - 1}$$

Illustrative Example:

Find the variance of the given data set:

Scores in Second Periodical Test of
I – Faith in Mathematics I

Scores	Frequency
46 – 50	1
41 – 45	10
36 – 40	10
31 – 35	16
26 – 30	9
21 – 25	4

Solutions:

Scores	Frequency (f)	Class Mark (X)	fX	$(X - \bar{x})$	$(X - \bar{x})^2$	$f(X - \bar{x})^2$
46 – 50	1	48	48	13.4	179.56	179.56
41 – 45	10	43	430	8.4	70.56	705.6
36 – 40	10	38	380	3.4	11.56	115.6
31 – 35	16	33	528	-1.6	2.56	40.96
26 – 30	9	28	252	-6.6	43.56	392.04
21 – 25	4	23	92	-11.6	134.56	538.24
$i = 5$	$\sum f = 50$		$\sum fX = 1,730$			$\sum f(X - \bar{x})^2 = 1,972$

$$\text{Mean } (\bar{x}) = \frac{\sum fX}{\sum f} = \frac{1,730}{50} = 34.60$$

$$\sigma^2 = \frac{\sum f(X - \bar{x})^2}{\sum f - 1}$$

$$\sigma^2 = \frac{1,972}{50 - 1}$$

$$\sigma^2 = \frac{1,972}{49} = 40.2448 \sim 40.24$$

Therefore, the variance(σ^2) is **40.24**.

3. Standard Deviation (s)

The standard deviation is considered the best indicator of the degree of dispersion among the measures of variability because it represents an average variability of the distribution. Given the set of data, the smaller the range, the smaller the standard deviation, the less spread is the distribution.

To get the value of the standard deviation (s), just simply get the square root of the variance (σ^2):

$$s = \sqrt{\sigma^2}$$

Illustrative Example:

Refer to the given previous example. Get the square root of the given value of variance:

$$\begin{aligned} s &= \sqrt{\sigma^2} \\ s &= \sqrt{40.24} \\ s &= 6.34 \end{aligned}$$

Therefore, the standard deviation of the Scores in Second Periodical Test of I – Faith in Mathematics I is **6.34**.

Were you able to learn different formulas in solving the range, variance, and standard deviation of grouped data? In the next activity, try to apply those important notes in getting the value of range, variance, and standard deviation of grouped data.

Activity 3 **LET'S APPLY IT...**

Directions: Calculate the range, variance and standard deviation of the Weekly Allowance of Students in Binago School of Fisheries. Write your complete solutions and answers on a sheet of paper.

Weekly Allowance of Students in Binago School of Fisheries

Weekly Allowance (in Pesos)	Frequency
500-549	2
450-499	3
400-449	1
350-399	3
300-349	4
250-299	14
200-249	12
150-199	21
100-149	10

Range = _____
Variance (σ^2) = _____
Standard Deviation (s) = _____

QUESTIONS

1. How did you find the range, variance and standard deviation?
2. What you can say about the value of range and variance?
3. What you can say about the standard deviation?
4. What have you learned and realized while doing the activity?

Were you able to solve the range, variance and standard deviation easily with your seatmate? Were you able to apply the notes on how to calculate the range, variance and standard deviation? Do the next activity by yourself.

Activity 4
CHALLENGE PART

Directions: Calculate the range, variance and standard deviation of the given grouped data.

Pledges for the Victims of Typhoon Pablo

Pledges in Pesos	Frequency
9,000 – 9,999	4
8,000 – 8,999	12
7,000 – 7,999	13
6,000 – 6,999	15
5,000 – 5,999	19
4,000 – 4,999	30
3,000 – 3,999	21
2,000 – 2,999	41
1,000 – 1,999	31
0 – 999	14

QUESTIONS

1. What is the $\sum fX$?
2. What is the value of the mean in the given distribution table?
3. What is the upper class boundary of the top interval? What about the lower class boundary of the bottom interval?
4. What is the range?
5. What is the variance of the given distribution table?
6. How would you find the variance?
7. What is the standard deviation?
8. How would you solve for the standard deviation?
9. What have you learned from the given activity?

What to Understand



Reflect how you were able to develop a concept out of the activities you have studied. The knowledge gained here will further help you understand and answer the next activities. After doing the following activities, you should be able to answer the following question: **How are the measures of variability of grouped data used in solving real-life problems and in making decisions?**

Activity 5

LET'S CHECK YOUR UNDERSTANDING...

1. Below are the scores of 65 students in a Mathematics test

Score	f	X	fX	$(X - \bar{x})$	$(X - \bar{x})^2$	$f(X - \bar{x})^2$
55 – 58	2					
51 – 54	4					
47 – 50	5					
43 – 46	6					
39 – 42	10					
35 – 38	13					
31 – 34	8					
27 – 30	6					
23 – 26	6					
19 – 22	2					
15 – 18	2					
11 – 14	1					

- Complete the table by filling in the values of X (the class marks or midpoints), $(X - \bar{x})$, $(X - \bar{x})^2$ and $f(X - \bar{x})^2$. Explain how you arrived at your answer.
 - Find the range, variance and standard deviation of the set of data.
 - What you can say about the standard deviation?
 - Which measure is considered unreliable? Why?
- Is the range the most appropriate measure of dispersion for grouped data? Why? How about the variance? standard deviation? Explain your answer.
 - Is it always necessary to group a set of data when finding its range, variance and standard deviation? Why?

What new insights do you have about solving measures of variability of grouped data? What do you realize after learning and doing different activities?

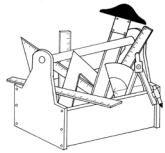
Now, you can extend your understanding by doing the tasks in the next section.

What to Transfer



Demonstrate your understanding on measures of central tendency and measures of variability through products that reflect meaningful and relevant problems/situations.

Create a scenario of the task in paragraph form incorporating GRASP: Goal, Role, Audience, Situation, Product/Performance, Standards.



- G: Make a criteria for a scholarship grant based on monthly family income and scholastic performance.
- R: Barangay Social Worker
- A: Local NGO
- S: An NGO in the locality will grant scholarship to qualified and deserving scholars
- P: Criteria
- S: Justification, Accuracy of data, Clarity of Presentation

SUMMARY/SYNTHESIS/GENERALIZATION:

This lesson was about measures of variability of grouped data. The lesson provided you opportunities to describe on how to solve range, variance and standard deviation of the given grouped data. Moreover, you were given the chance to apply the given important notes on how to solve the range, variance and standard deviation of the given grouped data and to demonstrate your understanding of the lesson by doing a practical task.

Glossary

Measure of Central Tendency - The score or value is where all the other values in a distribution tend to cluster.

Mean - The sum of measures x divided by the number n of measures in a variable. It is symbolized as \bar{x} (read as x bar).

Median - The middle entry or term in a set of data arranged in numerical order (either increasing or decreasing).

Mode - The measure or value which occurs most frequently in a set of data. It is the value with the greatest frequency.

Measure of Dispersion – The measure of spread of a data about the average of these data.

Range - The simplest measure of variability. It is the difference between the largest value and the smallest value.

Average Deviation or Mean Deviation - The dispersion of a set of data about the average of these data.

Standard Deviation - The most important measure of dispersion. It differentiates sets of scores with equal averages.

The variance (σ^2) of a data is equal to $\frac{1}{N}$ the sum of their squares minus the square of their mean. It is virtually the square of the standard deviation.

References:

Canonigo, A. (2012) Statistics, K – 12 Facilitators Guide
Math IV, Project EASE
Amsco_Integrated Algebra, Statistics
Javier, S. CASIO FX – 991ES PLUS Handbook
Pogoso, C., Montana, R, Introductory Statistics

http://www.picturesof.net/search_term_pages/meat.html
<http://www.stockfresh.com/image/289009/cartoon-kids>
<http://www.fotosearch.com/photos-images/garment-factory.html>
<http://www.equinoxlab.com/>
http://www.themall.ph/thumbs/images/2012/10/25/get1017125146/1_13511564701025145x217.JPG
http://media3.picsearch.com/is?tqgS6MH-ZoNLolkQZGJu2Qhkc_04U6Jd88KruCCxpz4
<http://cdn7.fotosearch.com/bthumb/CSP/CSP428/k4285733.jpg>