Examples CH 4

1. Consider the discrete probability distribution when answering the following question.

X	2	4	5	10
P(X)	.2	.3	?	.2

a. Find the probability that **X is large than 2.**

b. Calculate the **mean** and **variance** for this distribution.

c. Find the probability that **X is at most 4.**

d. Find the probability that **$\mu - \sigma < X < \mu + \sigma$.**

Answer:

$P(x > 2) = [P(x=4) + P(x=5) + P(x=10)] = (.3 + .3 + .2) = .8$

$\mu = \Sigma x p(x) = 2(.2) + 4(.3) + 5(.3) + 10(.2) = 5.1$

$\sigma^2 = \Sigma x^2 p(x) - \mu^2 = 4(.2) + 16(.3) + 25(.3) + 100(.2) - (5.1)^2 = 33.1 - 26.01 = 7.09$

$\sigma = 2.66$

$P(X \leq 4) = P(x=2) + P(x=4) = .5$

$\mu - \sigma = 5.1 - 2.66 = 2.44$ **$\mu + \sigma = 5.1 + 2.66 = 7.76$**

$P(2.44 < X < 7.76) = P(x=4) + P(x=5) = .6$

2. *Ladies Home Journal* polled its readers to determine the proportion of wives who, if given a second chance, would marry their husband. According to the responses from the survey, 80% of wives would marry their husbands again. Suppose a random and independent sample of $n = 20$ wives is collected. Define the random variable X = the number of the 20 wives who would marry their husband again. Then we know X is a binomial random variable. Find the probability that more than 15 of the wives responded that they would marry their husbands again.

Answer: Let a success be a wife who would marry her husband again.

Then x is a binomial random variable with $n = 20$ and $p = .80$.(Use Table)

$P(X > 15) = 1 - P(X \leq 15) = 1 - .370 = .630$

3. It has been stated that 20% of the general population donate their time and energy to working on community projects. Suppose 20 people have been randomly sampled from a community. Find the probability that more than seven of the 20 do donate their time and energy to community projects.

Answer: Let X is a binomial random variable with $n = 20$ and $p = .20$. (Use Table)

$$P(x > 7) = 1 - P(x \leq 7) = 1 - .968 = .032$$

4. An automobile manufacturer has determined that 30% of all gas tanks that were installed on its 2013 compact model are defective. If 15 of these cars are independently sampled, what is the probability that more than half need new gas tanks?

Answer: Let x = the number of the 15 cars with defective gas tanks. Then x is a binomial random variable with $n = 15$ and $p = .30$. (Use Table)

$$P(\text{more than half}) = P(x > 7.5) = P(x \geq 8) = 1 - P(x \leq 7) = 1 - .950 = .050$$

5. A local newspaper claims that 60% of the items advertised in its classified advertisement section are sold within 1 week of the first appearance of the ads. To check the validity of the claim, the newspaper randomly selected 20 advertisements. They found that 13 of the 20 items sold within a week. Based on this claim, is it likely to observe ***at most*** **13** who were not sold their item within a week?

Answer: Let x = the number of the 20 ads that resulted in the item being sold within a week. Then x is a binomial random variable with $n = 25$ and $p = 1 - .60. = 0.40$

$$P(x \leq 13) = 0.994$$

6. The chance that a Montesuma rose will blossom during the first year after planting is 80%. Let say, you bought 15 bushes of Montesuma rose and planted them in your garden. What is the probability that less than 7 of your roses will not blossom the first year?

$$P(x < 7) = P(x \leq 6) = 0.982 \qquad p = 1 - .80. = 0.20$$

7. Ann bakes six pies a day that cost $2 each to produce. On 21% of the days she sells only 3 pies. On 19% of the days, she sells 5 pies, and on the remaining 60% of the days, she sells all six pies. If Ann sells her pies for $6 each, what is her expected profit for a day's worth of pies?

X	**P(X)**
3	0.21
5	0.19
6	0.60

E(x) = 3*.21+ 5*.19 + 6*.60 = 5.18

Profit = $5.18*6 - 6*2 = $31.08 - 12 = $19.08

7. Current estimates suggest that in west Africa only 45% of the households have computers with access to on-line services. Suppose 15 people with home-based computers were randomly and independently sampled.

a) Find the probability that **more than two** of those sampled currently have access to on-line services.

n = 15 [the total number of people with computers]

p = .45 [the probability that they have access]

q = .55 [the probability that they do not have access]

Formula: $\mathbf{P(x > 2) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]}$

$$P(x = 0) = \frac{15!}{0!(15)!}(.45^0)(.55^{15}) = 0.00013$$

$$P(x = 1) = \frac{15!}{1!(15-1)!}(.45^1)(.55^{15-1}) = 0.00156$$

$$P(x = 2) = \frac{15!}{2!(15-2)!}(.45^2)(.55^{15-2}) = 0.00896$$

Calculations: $\mathbf{P(x > 2) = 1 - 0.016065 = 0.989349}$

b) Find the probability that **fewer than two** of those sampled currently have access to on-line services.

n = 15 [the total number of people with computers]

p = .45 [the probability that they have access]

q = .55 [the probability that they do not have access]

Formula: $\mathbf{P(x < 2) = [P(x = 0) + P(x = 1)]}$

$$P_{(1)} = \frac{15!}{1!(15-1)!}(.45^1)(.55^{15-1}) = 0.00156$$

$$P_{(0)} = \frac{15!}{0!(15)!}(.45^0)(.55^{15}) = 0.00013$$

Calculations: P(x < 2) = [P(x = 0) + P(x = 1)] = 0.00169

8. **Current estimates suggest that in east Africa almost 60% of the households have a computers with access to on-line services. Suppose 20 people with home-based PC were randomly and independently sampled.**

a) Find the probability that **at least 10 but at most 14** currently have access to on-line services.

$P(10 \leq x \leq 14) = P(x \leq 14) - P(x \leq 9)$ **Use n=20, p= .6**

b) Find the probability that **fewer than quarter** of those sampled have **no access** to on-line services.

$P(x < 5) = P(x \leq 4)$ **Use n=20, p= 1 - .6 =.4**

c) Find the probability that **at least 18** currently have access to on-line services.

$P(x \geq 18) = 1 - P(x \leq 17)$ **Use n=20, p= .6**

d) How many people **do you expect** currently do not have access to on-line services?

E(x) = **μ = np = 20 x .6 = 12**

9. According to a recent study, 1 in every 4 women has been a victim of domestic abuse at some point in their lives. Suppose we have randomly and independently sampled twenty women. Find the probability **that at least 2** of the sampled women have been the victim of domestic abuse at some point in their lives.

n = 20	[the total number of women]
k = 2	[the number of women you are looking for]
p = .25	[the probability that they are victims]
q = .75	[the probability that they are not victims]

P(x ≥ 2) = 1 – P(x < 2) = 1 – [P(x = 0) + P(x = 1)] = 0.976

$$P_{(1)} = \frac{20!}{1!(20-1)!}(.25^1)(.75^{20-1}) = 0.021$$

$$P_{(0)} = \frac{20!}{1!(20)!}(.25^0)(.75^{20}) = 0.003$$