

**REVENUE MANAGEMENT THROUGH
DYNAMIC CROSS-SELLING IN CALL CENTERS**

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Abstract

This paper models the cross-selling problem of a call center as a dynamic admission control problem. The key tradeoff between revenue generation and congestion in a call center is addressed in a dynamic framework. The question of when and to whom to cross-sell is explored using this model. The analysis shows that unlike current marketing practice which targets cross-sell attempts to entire customer segments, optimal dynamic policies may target selected customers from different segments. Structural properties of optimal policies are explored. Sufficient conditions are established for the existence of preferred calls and classes; i.e. calls that will always generate a cross-sell attempt. Numerical examples, that are motivated by a real call center, identify call center characteristics that increase the significance of considering dynamic policies rather than simple static cross-selling rules as currently observed. The value of these dynamic policies and static rules are compared. Finally, the structural properties lead to a heuristic that generates sophisticated static rules leading to near optimal performance.

1 Introduction

Many firms in mature industries, like the financial services industry, resort to growth by deepening customer relationships and making them more profitable rather than increasing market

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share. A significant part of this profitability comes from revenues generated by the sale of additional products and services to existing customers, in other words through tactics that improve customer life time value. Felvey (1982) states that existing customers are better sales prospects, compared to new customers without a relationship. Given the growing dislike among consumers for telemarketing, this type of selling is increasingly being performed via cross-selling and up-selling initiatives (Kresbach, 2002; Walker, 2003). According to Kamakura et al. (1991) cross-selling is emerging as one of the important customer relationship management (CRM) tools used to strengthen relationships. CRM refers to the whole strategy of building relationships and extracting more revenues from existing customers. The global market for CRM systems, service and technology is estimated to be around \$ 25 billion (Benjamin, 2001).

Inbound call centers are an important point of contact with the customer, where this type of selling takes place. According to a Tower Group estimate for 2003, in banking, 25 % of transactions are projected to take place in call centers. Given the increasing percentage of these centers that are organized as profit centers, focus is shifting to cross-selling. According to a Wells Fargo executive (The Economist, 2004) 80 % of the bank's growth is coming from selling additional products to existing customers. As the leader in cross-selling, this bank's customers hold an average of a little over four products per household. Given that an average American household has sixteen financial products, the opportunity for cross-selling growth in this industry is apparent.

A major concern for managers is identifying the right person and the right time to attempt a sale. While it is believed that cross-selling ensures that customers acquire multiple products of a firm, improves customer retention (Marple and Zimmerman, 1999) and reduces customer churn (Kamakura et al., 2003), excessive selling can motivate a customer to switch (Kamakura et al., 2003). Database marketing techniques that address this issue are being developed (Paas and Kuijlen, 2001; Kamakura et al. 1991, 2003), and software that helps insurance agents or bankers cross-sell more effectively is becoming more common (Insurance Advocate, 2003; American Banker, 2003) as companies embrace this tactic.

Cross-selling in a call center requires a customer service agent to transform an inbound service call into a sales call. According to an article in the Call Center Magazine (2003), call centers can use integrated predictive analysis and service automation software to make real-time recommendations to banking customers. However, in a review of existing products Chambers (2002) states that real-time automation is relatively immature and many products offer only the option of setting preset business rules that make promotion recommendations based on

previously captured and stored data. Common practice is to segment the customer base into groups based on their sales potential, and to target sales to high potential segments. In the absence of real-time automation, the customer service representative will use segment based estimates to determine whether it is appropriate to attempt a cross-sell to a particular call.

Irrespective of the type of automation in place, a cross-sell attempt in a call center implies additional talk time from the agent. One can expect the magnitude of this difference to be smaller in a call center that provides real-time automation since less time will be spent on information processing, however there will still be additional time during which the offer is made to the customer. Thus cross-selling will influence the load experienced by a call center, as documented in Akşin and Harker (1999). The biggest challenge of a call center manager is to manage the tension between costs and customer service (Dawson, 2003). While for the long-term this corresponds to determining the right number of service representatives to hire, in the shorter term it is resolved through capacity allocation. The primary role of such inbound call centers is service, and demand for service varies during the day, creating peaks in the system load. It may be the case that even for calls presenting high revenue potential, cross-selling during such peak times is not desirable due to its detrimental effect on capacity and service.

This basic description of cross-selling in a call center identifies a key challenge for managers: When should a cross-selling attempt be made such that revenue generation is maximized while congestion costs are kept as low as possible? Current practice identifies off-peak times during the day for cross-selling. However it is clear that a dynamic policy will utilize valuable capacity more effectively. It is this question of dynamic capacity allocation that motivates the research herein. In a more general setting, Güneş and Akşin (2004) consider this tradeoff between revenue generation and service costs, and analyze the interaction with a market segmentation decision and server incentives. The analysis in that paper optimizes steady-state performance metrics, and does not consider the dynamic nature of the problem. The only other paper that considers a dynamic cross-selling model in call centers is the one by Byers and So (2003). The authors model a call center as a single server Markovian queue, and compare the performance of cross-selling policies that consider queue state information as well as customer profile information. Their analysis extends part of the analysis in Güneş and Akşin (2004) to a dynamic setting. Netessine et al. (2004) analyze the dynamic cross-selling problem of an e-commerce retailer, focusing on the packaging of multiple products and their pricing. These aspects of the problem are not considered herein.

We model the cross-selling problem as a dynamic admission control problem in a multi-

server loss system. A customer's revenue potential is modeled as a random variable. For a call center with real-time automation, the realization of this random variable is observed before a cross-selling decision is made. Otherwise we consider a system where the decision is based on expected revenues. Both of these models are described in detail in the following section. In a similar admission control setting, random rewards have been considered by Ghoneim and Stidham (1985) and Örmeci et al. (2002). Both papers show the existence of optimal threshold policies. The paper by Örmeci et al. (2002) further characterizes conditions for the existence of preferred jobs, where preferred jobs are those which are always admitted to the system whenever there is at least one available server. Koole (1998), Altman et al. (2001), Örmeci et al. (2001), and Savin et al. (2003) have considered admission control problems with fixed rewards where results on the optimality of threshold type policies can be found. The latter two papers also characterize conditions for the existence of preferred classes. The fact that all calls have to be admitted for service and the admission control is performed only for the sales decision constitutes the key difference between earlier models and the one studied herein. In this paper we will characterize sufficient conditions for preferred classes and preferred calls.

Section 2 formulates the model with revenue realizations. Section 3 presents sufficient conditions for the existence of preferred classes and preferred calls. It is shown that unlike the prevailing practice of attempting a cross-sell on all customers in a segment, the optimal dynamic policy will sometimes dictate that only some customers in a segment, or in some cases even only some customers from each segment receive a cross-sell attempt. In Section 4 the model with expected revenues is analyzed. Taking data from a real retail banking call center as the basis, a set of numerical examples are developed in Section 5. Using these examples, it is first explored when dynamic cross-selling is valuable compared to segment-based simple static policies. We show that call centers with long service calls and long additional talk time for cross-selling, with customer profiles that are difficult to segment and that exhibit narrow ranges for high segment revenues, and centers that are designed to operate in a quality or quality-efficiency regime benefit more from the dynamic optimization of their cross-selling policies. A preliminary analysis that compares the actual policies to those suggested by our sufficient conditions in these numerical experiments, indicates a relatively good fit. Taking this as a starting point, a heuristic for cross-selling is proposed. The performance of this heuristic is analyzed numerically, where its average gain in 3888 examples is 97.6 % of the optimal average gain. The paper ends with concluding remarks.

2 A Dynamic Cross-Selling Model

2.1 Description of the system

In order to study the dynamic cross-selling problem, we model an inbound call center as a loss system with c identical parallel servers. We assume there is no waiting room. This assumption is made primarily for tractability purposes. However, it is also the case that one cannot sell to a customer who has been waiting for service for a long time so that the no waiting assumption constitutes a good approximation for the cross-selling problem. The inbound call center is a service center, so treats all call requests that are not blocked due to capacity limitations. Treated service calls generate a fixed revenue of r . The service time of a service call is distributed exponentially with rate μ .

Consider a setting where customers are divided into two types or segments $s = H, L$, based on their cross-sell potential. Customers of each type arrive to the system according to a Poisson process with rate λ_s . Customers of segment s generate a revenue ρ_s , which is a random variable following a probability distribution F_s with finite mean, if a cross-sell attempt is made. It is assumed that revenues of successive calls are independent. Each time a call arrives, there will be a decision to attempt a cross-sell or not. If the decision is to attempt a cross-sell, the call will generate a random revenue $r + \rho_s$ and the service time will be distributed exponentially with rate $\mu_1 = \mu - k$, where k is a constant that reflects the impact of the selling activity on the duration of the call. If the decision is not to cross-sell, then the call is treated as a service call with the earlier mentioned fixed revenue r and service rate μ . Note that the service rate does not depend on the customer type; i.e. all service calls have the same service time distribution, and all service calls where a cross-sell is attempted have another service time distribution with a slower rate. The objective of the call center is to maximize the total expected discounted revenues over an infinite time horizon and/or maximize long-run average revenue of the center.

The revenue generated by a class- s call, ρ_s , is assumed to follow the probability distribution F_s . We first define

$$\bar{\rho}_s = \inf\{t : P\{\rho_s \leq t\} = 1\},$$

where we set $\inf \emptyset = \infty$. In other words, $\bar{\rho}_s$ is the supremum of the reward that can be received from a class- s call. Note that $\bar{\rho}_s$ is well defined because $P\{\rho_s \leq t\}$ is right continuous in t , therefore, the set $\{t : P\{\rho_s \leq t\} = 1\}$ either is empty or it has an infimum. It is straightforward to show that when $\bar{\rho}_s = \infty$, there are preferred calls of class s . Moreover, the random reward

to be received from a cross-sell is bounded. Hence, we assume without loss of generality that $\bar{\rho}_s < \infty$ for both $s = H, L$. We can also define a lower bound on the cross-sell revenues:

$$\underline{\rho}_s = \sup\{t : P\{\rho_s \leq t\} = 0\}.$$

We assume that $0 \leq \rho_s$, so $\underline{\rho}_s$ always exists. This assumption is realistic, since the server will never attempt to sell if the random reward is negative. Lemma 1 will provide mathematical justification, so that this assumption does not create any restriction on our model. Finally, we also assume $\bar{\rho}_L \leq \bar{\rho}_H$, to reflect the expectation that class H brings higher rewards. Hence, this assumption is without loss of generality. Then, there are three possible scenarios:

- **Scenario 1:** $\underline{\rho}_L \leq \bar{\rho}_L \leq \underline{\rho}_H \leq \bar{\rho}_H$
- **Scenario 2:** $\underline{\rho}_L \leq \underline{\rho}_H < \bar{\rho}_L \leq \bar{\rho}_H$
- **Scenario 3:** $\underline{\rho}_H < \underline{\rho}_L \leq \bar{\rho}_L \leq \bar{\rho}_H$.

Each scenario represents a segmentation scheme, whereby individual customers are aggregated into homogeneous groups according to their cross-sell revenue generation potentials. For example based on demographic, past purchase, and psychographics a probability of purchase is estimated for each customer. This is then coupled with likely purchase volume and profit margin or revenue information to lead to a customer profitability or revenue potential estimate. Scenario 1 represents discrete segments for two types of customers. According to Lilien and Rangaswamy (1998), this type of segmentation is easier to understand and apply but sacrifices some information. Scenario 2 is known as overlapping segments, and represents a more realistic and theoretically accurate segmentation scheme. Note that Scenario 2 includes the case with $\underline{\rho}_L = \underline{\rho}_H$. Scenario 3, on the other hand, is an example for an *irrelevant segmentation*, and so will not be considered further.

The system we have described so far will be referred to as the *model with revenue realizations* due to the underlying assumption about the observability of the revenue potential of a customer at the time of the decision: In this case, it is assumed that a server can observe the realization of the random demand before taking the decision to cross-sell or not. However, it is also possible that the server takes a decision and the realization is observed at service completion. We will consider this model as well, which will be labeled as the model with expected revenues. The model with revenue realizations represents the case of a call center where marketing and technology support is such that as soon as a customer call arrives, the system

is capable of displaying its revenue potential. This represents a setting with software that has real-time automation capability as described in Section 1. The expected revenue case represents a setting where technology only enables historical analysis, and hence the server only has distribution information. How much revenue is eventually realized from a particular customer will be determined at call completion.

The model with expected revenues is a special case of the model with random revenues when we set $\underline{\rho}_L = \bar{\rho}_L = r_L$ and $\underline{\rho}_H = \bar{\rho}_H = r_H$ in Scenario 1. Since the server has to make a decision before s/he actually observes the random revenue, his/her decision can be based only on the expectation of the rewards. Hence, we can take $r_s = E(\rho_s)$.

2.2 The discrete time model of the system

In this section, we build a discrete time Markov decision process (MDP) for the system described above with the objective of maximizing total expected discounted returns over a finite time horizon with β as the discount rate. The states of the system are x_1 and x_2 representing the number of cross-sell calls and the number of service calls in the system respectively. The states of the system change at service completions and at arrivals to a system with idle server(s). Because the decision to attempt a cross-sell depends on the customer revenue potential, the state at arrival instants is defined as $(x; s, \rho_s) = (x_1, x_2; s, \rho_s)$. At all other times the state information is described by $x = (x_1, x_2)$. Note that in both definitions x is restricted to the set $\bar{\mathcal{S}} = \{x \in \mathcal{Z}^2 : e_0 \leq x, x_1 + x_2 \leq c\}$, where \mathcal{Z} is the set of integers, and $e_0 = (0, 0)$. Now let \mathcal{R} be the set of real numbers, and $\rho_0 = 0$. Then the state space can be expressed by the set $\mathcal{S} = \{(x; s, \rho_s) : x \in \bar{\mathcal{S}}, s \in \{0, L, H\}, \rho_s \in \mathcal{R}\}$, where states of type x are denoted by $(x; 0, 0)$ so that \mathcal{S} contains all possible states.

Cross-sell decisions are made at arrival instants. The corresponding action sets for systems with at least one idle server are $A(x; s, \rho_s) = \{1, 2\}$, with a one denoting the decision to attempt a cross-sell and a two to treat the customer request as a pure service call. When all the servers in the system are occupied, $A(x; s, \rho_s) = \{0\}$ showing that all the incoming calls have to be rejected. We assume that rewards by successive customers from class s are i.i.d. random variables with probability distribution function F_s .

We interpret discounting as exponential failures, i.e., the system closes down in an exponentially distributed time with rate β (for the equivalence of the processes with discounting

and without discounting but with an exponential deadline, see e.g., Walrand, 1988). Then, the maximum possible rate out of any state is $\lambda_H + \lambda_L + c\mu + \beta$. Since the time between transitions is always exponentially distributed and the maximum rate of transitions is finite, we can use uniformization and normalization to build a discrete time equivalent of the original system. We assume, using the appropriate time scale, $\lambda_H + \lambda_L + c\mu + \beta = 1$ so that the system will be observed at exponentially distributed intervals with mean 1. There will be an arrival with probability $\lambda_H + \lambda_L$ and a potential service completion with probability $c\mu$ so that a real service completion due to a standard service call occurs with probability $x_2\mu$ and due to a cross-sell call with probability $x_1\mu_1$, while a fictitious service completion occurs with probability $c\mu - x_2\mu - x_1\mu_1$. Upon an arrival, if all servers are busy, then the state of the system does not change. Otherwise, the server either decides to cross-sell in state $(x; s, \rho_s)$, so that the state is changed instantaneously to $(x + e_1)$, or s/he takes the call as a standard-service call which moves the system to state $(x + e_2)$, where e_j is the two-dimensional unit vector with the j^{th} component equal to 1.

We now develop the optimality equations for the transformed system in finite horizon. Let $u_n(x)$ and $v_n(x; s, \rho_s)$ be the maximal expected reward, starting in state x and $(x; s, \rho_s)$, respectively, until n transitions occur. When the system has available server(s), computing $v_n(x; s, \rho_s)$ requires comparison of two actions: cross-selling the incoming class- s call which implies moving to state $x + e_1$ with a reward of $\rho_s + r$, and giving the standard service so that the system moves to state $x + e_2$ with a reward of r . Define $a_n(x; s, \rho_s)$ to be the optimal action in state $(x; s, \rho_s)$ with n remaining transitions.

The optimality equations of this model are as follows. For $x_1 + x_2 < c$:

$$v_n(x; s, \rho_s) = \max\{\rho_s + r + u_n(x + e_1), r + u_n(x + e_2)\} \quad (1)$$

$$\begin{aligned} u_{n+1}(x) &= \lambda_H E[v_n(x; H, \rho_H)] + \lambda_L E[v_n(x; L, \rho_L)] + \\ &\quad x_2\mu u_n(x - e_2) + x_1\mu_1 u_n(x - e_1) + \\ &\quad (c\mu - x_2\mu - x_1\mu_1)u_n(x), \end{aligned} \quad (2)$$

where we set $u_n(-1, x_2) = u_n(0, x_2)$, $u_n(x_1, -1) = u_n(x_1, 0)$, and $E[h(x; s, \rho_s)]$ denotes expectation with respect to the probability distribution F_s . For $x_1 + x_2 = c$, no calls can be accepted so that $a_n(x; s, \rho_s) = 0$, and thus $v_n(x; s, \rho_s) = u_n(x)$. We assume that ties in equation 1 are broken by selecting pure service. This assumption guarantees the validity of the results for general (continuous, discrete and mixed) probability distributions of rewards.

We prove certain properties of an optimal policy under the objective of maximizing total

expected β -discounted reward for a finite number of transitions, n . The state space of the model is compact, while the action space in each state is finite. Also, we have $E(\rho_s) < \infty$, so that the optimal value functions are bounded. These properties, used with the corresponding results of Hernandez-Lerma and Lasserre (1999), ensure that optimal policies under the expected β -discounted reward over an infinite horizon and/or under the long-run average criterion (when $\beta \rightarrow 0$) inherit the structural properties of optimal policies operating with finite n . We refer to the technical report Örmeci and Akşin (2004) for the details.

We conclude this section by introducing some notation that will be useful in the subsequent analysis of the optimality equations. Let

$$D_n(ij)(x) = u_n(x + e_i) - u_n(x + e_j), \quad i = 0, 1, 2, \quad j = 1, 2.$$

The quantity $D_n(ij)(x)$ is equal to the relative benefit of starting in state $x + e_i$ vs. $x + e_j$, with a horizon of n transitions. It is now easy to see from equation (1) that

$$a_n(x; s, \rho_s) = 1 \iff D_n(21)(x) < \rho_s,$$

since we assume that we choose “pure service”, if both actions are optimal. Notice that the decision to cross-sell or not depends only on the observed potential revenue, not on the type of the call. However, we prefer to keep the state as $(x; s, \rho_s)$ to prevent any misunderstandings with regard to the expectation $E[v_n(x; s, \rho_s)]$. Now, it is optimal to attempt cross-sell in state $(x; s, \rho_s)$ if and only if the reward it brings exceeds a threshold equal to $D_n(21)(x) = u_n(x + e_2) - u_n(x + e_1)$, which represents the loss in future rewards because of the increased load due to the slow service. Similarly, the difference $D_n(0j)(x)$ represents the expected additional burden, that an additional cross-sell ($j = 1$) or pure ($j = 2$) call brings to the system in state x when there are n remaining transitions.

3 Preferred Classes and Preferred Calls

Preferred calls are those that always generate a cross-sell attempt. More precisely, a call with a reward of ρ is said to be preferred if $D_n(21)(x) < \rho$ for all $x + e_1 \in \bar{\mathcal{S}}$. If all calls of class s are preferred, i.e., if $D_n(21)(x) < \rho_s$ for all possible ρ_s and for all $x + e_1 \in \bar{\mathcal{S}}$, then class s is called preferred.

Scenarios 1 and 2 described in subsection 2.1 and the thresholds on the potential revenues, $D_n(21)(x)$, described in subsection 2.2 together lead to six different policies in terms of preferred

calls and classes. Given our interest in preferred calls and classes, we will not consider Policy O any further.

- **Policy O** Cross-sell dynamically (no preferred calls)
- **Policy I** Cross-sell attempt to chosen calls of segment H only
- **Policy II** Cross-sell attempt to segment H only
- **Policy III** Cross-sell attempt to segment H and chosen calls of segment L
- **Policy IV** Cross-sell attempt to chosen calls of segment H and L
- **Policy V** Cross-sell attempt to everyone

In this section, we show that under certain conditions there are preferred classes and/or preferred calls from either of the classes. To derive these conditions, we first define:

$$D(21) = \max\{D_n(21)(x) : x \in \bar{\mathcal{S}}, n \geq 0\}, \quad (3)$$

$$D(0j) = \max\{D_n(0j)(x) : x \in \bar{\mathcal{S}}, n \geq 0\}. \quad (4)$$

The value of $D(21)$ with respect to $\underline{\rho}_s$ and $\bar{\rho}_s$ determines the existence of preferred calls/classes: If $\underline{\rho}_s \leq D(21) \leq \bar{\rho}_s$, there are preferred class- s calls, and if $D(21) \leq \underline{\rho}_s$, class s is preferred. Without solving the problem completely, we cannot compute the exact value of $D(21)$. Moreover, it is not possible to find lower bounds on $D(21)$ (except for 0 as established Lemma 1). Hence, we will derive sufficient conditions for $\underline{\rho}_s$ and $\bar{\rho}_s$ to be upper bounds on $D(21)$.

The main duty of the call center we consider is to answer regular service calls to obtain a fixed reward of r . The call center cannot fulfill this duty if all servers are busy. On the other hand, cross-sell decisions increase the total load of the call center, which, in turn, increases the probability of having all servers busy. Hence, in cross-sell decisions comparison of the fixed reward r with the random reward ρ_s plays an important role. However, this is not the only aspect: A cross-sell decision for the current call affects the potential cross-sell decision for a future call, which requires a comparison of the random reward offered now with the random reward to be offered in the future. These two comparisons together will determine the sufficient conditions for the existence of preferred calls/classes. Since the existence of preferred calls/classes involves a comparison of random rewards for different customer segments, we need to consider the relations between them.

3.1 Preliminary results on $D(21)$ and $D(0j)$

This subsection provides a lower bound on $D(21)$ and $D(0j)$, as well as a relationship between $D(21)$ and $D(01)$, which will allow us to concentrate on the quantity $D(01)$ in our subsequent analysis. The proof technique that will be used in this result, called coupling, is common in many of our proofs, so we describe it here in detail: Coupling is a widely used method in Markov decision models. When comparing two systems, we explicitly couple all the random variables for the two systems. Specifically, both systems will have the same arrival stream. The service times of calls in the two systems are coupled as follows: If the coupled calls are of the same class, they depart at the same time, otherwise we use the assumption that $\mu_1 \leq \mu$: We let ξ be a uniformly distributed random variable in $(0, 1)$, and we generate the service time of the class-1 call, say call d_1 , and the class-2 call, say call d_2 , using the same ξ , so call d_2 has a shorter service time than call d_1 with probability 1. In terms of discrete time, this translates to the following: Both calls are completed with probability μ_1 , and a regular service call departs from the system with probability $\mu - \mu_1$ leaving the coupled cross-sell call in the system. Thus, coupling does not allow a coupled cross-sell call to leave the system while the coupled service call is still there. Now we are ready to present our result:

Lemma 1 *For $j = 1, 2$, for all $x \in \bar{\mathcal{S}}$ and $n \geq 0$:*

- (a) *For $j = 1, 2$, $0 \leq D(0j)$.*
- (b) $0 \leq D(21) \leq \frac{\mu - \mu_1}{\mu + \beta} D(01)$.

The proof of Lemma 1 can be found in the Appendix. We have three remarks: (1) The proofs of these two statements have no reference to the assumptions on the rewards, i.e., on $\bar{\rho}_s$, and $\underline{\rho}_s$ so that Lemma 1 holds for all possible F_s . Hence, this lemma assures that a class- s call has to be taken as a regular service call whenever $\rho_s < 0$, justifying our assumption $\underline{\rho}_s \geq 0$. (2) The system collects the rewards upon arrival, so that the calls already in the system bring no benefit, but only the burden of preventing us to accept new arrivals. So, $u_n(x)$ contains only rewards obtained from future calls. Hence, it is always preferable to be in a state where there are less or faster calls, which is just what is stated in Lemma 1. (3) Our previous discussion has identified $D(21)$ as the quantity which determines the existence of preferred calls/classes. However, now by Lemma 1 deriving an upper bound on $D(01)$ gives an upper bound on $D(21)$.

Hence, in the rest of this section we concentrate on finding sufficient conditions to establish upper bounds on $D(01)$.

3.2 Sufficient conditions for preferred calls/classes

In this subsection, we derive sufficient conditions to establish upper bounds on $D(01)$, which immediately translates to sufficient conditions for the existence of preferred calls/classes by Lemma 1. The proof technique to derive these conditions is similar to the one used in Lemma 1, i.e., induction on the number transitions combined with a coupling argument. Here, we present this technique, but note that the specific conditions depend on the actual scenarios on $\underline{\rho}_s$ and $\bar{\rho}_s$. Thus, the exact statements of the sufficient conditions will be given for scenarios 1 and 2 below.

All the subsequent statements will claim an upper bound on $D(01)$, hence our aim is to prove the following statement: “Under certain conditions, $D(01) \leq U$, where $U \geq 0$.” The proof is by induction on the number of transitions, n , combined with a coupling argument: The statement is satisfied for $u_0(x) = 0$ for all $x \in \bar{\mathcal{S}}$, so that we can assume that $D_n(01)(x) \leq U$ for all $x \in \bar{\mathcal{S}}$ for n . Now consider period $n + 1$: Assume that system A is in state x and system B is in $x + e_1$ in period $n + 1$, and we couple the two systems. We allow system A to follow the optimal policy, whereas system B imitates system A whenever possible. Note that it cannot imitate system A only if all of its servers are busy so that it has to reject all incoming calls. Hence, if system B has at least one idle server, the difference between the two systems due to the additional cross-sell call remains the same. Otherwise, system A either accepts the incoming class- s call as a regular call so that the systems move to two different states $x + e_2$ and $x + e_1$ with a difference of r , or it accepts the call as cross-sell which couples the two systems with a difference of $r + \rho_s$ in reward. With the departure of the additional class-1 call in system B, the systems again enter the same state but with no return, whereas all other service completions keep the difference between the two systems the same. Then, letting $u_{n+1}^B(x + e_1)$ be the total expected discounted reward of system B in period $n + 1$:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq u_{n+1}(x) - u_{n+1}^B(x + e_1) \\
&\leq \lambda_H \max\{D(01), D(21) + r, \rho_H + r\} \\
&\quad + \lambda_L \max\{D(01), D(21) + r, \rho_L + r\} + (c\mu - \mu_1)D(01) \\
&\leq \lambda_H \max\left\{U, \frac{\mu - \mu_1}{\mu + \beta}U + r, \bar{\rho}_H + r\right\} + \lambda_L \max\left\{U, \frac{\mu - \mu_1}{\mu + \beta}U + r, \bar{\rho}_L + r\right\}
\end{aligned}$$

$$+(1 - \lambda_H - \lambda_L - \mu_1 - \beta)U \quad (5)$$

where the first inequality is due to the optimality of u_n 's, the second due to coupling and the definitions of $D(01)$ and $D(21)$, the third due to the definition of $\bar{\rho}_s$, the induction hypothesis, Lemma 1 and uniformization. Inequality (5) is valid for all possible scenarios, and so it will be used directly in all the subsequent proofs, which are placed in the Appendix.

Proposition 1

If $\frac{\lambda_H + \lambda_L}{\lambda_H + \lambda_L + \mu + \beta} \frac{\mu - \mu_1}{\mu_1 + \beta} \leq \frac{\bar{\rho}_H}{r}$, then there are preferred class – H calls.

Simply stated, this Proposition provides a condition that compares the value of a lost service call to the maximum revenue that a class- H customer can bring. For this policy of selective sales to calls from the H -segment, the condition is easier to satisfy for higher upper bounds on the revenue potential relative to r , low call volumes and/or low k values, and faster calls. Given that the first ratio will typically take values that are very close to one, and that one can expect $\bar{\rho}_H$ to be much larger than r , only very slow service calls and (or) very high k values will prevent this condition from being true in most settings.

Proposition 2 *Let*

$$\begin{aligned} \alpha_1 &= \frac{\mu - \mu_1}{\mu + \beta} (\bar{\rho}_H + r) \\ \alpha_2 &= \frac{\lambda_H}{\lambda_H + \mu_1 + \beta} \alpha_1 \\ \alpha_3 &= \left(\frac{\mu - \mu_1}{\mu_1 + \beta} + \frac{\mu + \beta}{\lambda_H} \right) r \\ \alpha_4 &= \frac{(\lambda_H \bar{\rho}_H + (\lambda_H + \lambda_L)r)(\mu - \mu_1)}{(\lambda_H + \mu_1 + \beta)(\mu + \beta) + \lambda_L(\mu_1 + \beta)} \\ \alpha_5 &= \frac{\mu - \mu_1}{\mu_1 + \beta} r \end{aligned}$$

Then:

- (i) *If $\alpha_1 \leq R$, $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1} R$.*
- (ii) *If $\bar{\rho}_H < \alpha_3$, then having $\alpha_5 \leq R < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1} R$.*
- (iii) *If $\bar{\rho}_H \geq \alpha_3$, then having $\alpha_2 \leq R < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1} R$.*

(iv) If $\bar{\rho}_H < \alpha_3$, then having $\alpha_4 \leq R < \alpha_5$ implies $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}R$,

where R with its corresponding implication on the optimal policy is specified as follows:

- (a) If $R = \underline{\rho}_H$ in Scenario 1, class-H is preferred.
- (b) If $R = \bar{\rho}_L$ in Scenario 1, class-H is preferred, and there are preferred class-L calls.
- (c) If $R = \bar{\rho}_L$ in Scenario 2, there are preferred class-H and preferred class-L calls.

First note that α_1 and α_5 do not contain λ terms. Thus, condition (i) only depends on call characteristics in terms of revenue and service rates. All other conditions have terms that depend on call volume, so in some way depend on “congestion”. Looking more closely, the condition in (i) can hold if μ and μ_1 are sufficiently close (i.e. low k) while μ and β have high values. In other words in environments where both service calls and cross-sell calls are fast and relatively similar. Another situation when this condition may apply is if the upper and lower bound values of the random revenues, specifically $\bar{\rho}_H$ and $\underline{\rho}_H$ for part (a), and $\bar{\rho}_H$ and $\bar{\rho}_L$ for parts (b) and (c), is relatively close and the base service revenue r is small. In other words if there is a significant cross-sell revenue component that is furthermore coming from a tight distribution. The other three conditions (ii)-(iv) are relatively weak. One can check that conditions that imply the first part make the second half of the statements more difficult to satisfy. One observation that is worth noting is that higher values of λ_H relative to the sum of the service rate and the discount rate (i.e. the probability of an arrival of type H before the previous service call has been completed) render it more difficult to satisfy these conditions. In other words if the high segment is generating a lot of calls, cross-selling to all customers from this segment becomes more difficult, as one would also expect intuitively.

In parts (b) and (c), since $\bar{\rho}_H + r > \bar{\rho}_L$ the only way for condition (i) to hold is for fast service and cross-sell, a high discount rate, and a relatively low service revenue r . This is similar to what was stated for part (a) above, but the described effects need to be stronger here. Like before, this condition does not depend on call arrival rates. Conditions (ii) and (iii) depend on λ_H but are independent of λ_L . For (ii) one would normally expect $\bar{\rho}_H > r$. So in order to satisfy the first half of this condition low λ_H , high μ and low μ_1 would be required. However these conditions make the second half more difficult to satisfy, so that this condition is relatively weak. The first half of condition (iii) would be satisfied as long as the term multiplying r is small enough. One possibility would be a high value for λ_H , however this tightens the interval

stated in the subsequent condition. If on the other hand λ_H is small, then once again relatively fast service and cross-sell calls would be required. Finally condition (iv) provides an alternative to the second half of the condition in (ii). Note that this condition depends on both arrival rates.

We note that Scenario 1 assumes $\underline{\rho}_L \leq \bar{\rho}_L \leq \underline{\rho}_H \leq \bar{\rho}_H$, so there are no calls which will incur a reward of ρ with $\bar{\rho}_L < \rho < \underline{\rho}_H$. Hence, it is not surprising to have identical conditions on $\bar{\rho}_L$ to have $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}\bar{\rho}_L$, with the conditions on $\underline{\rho}_H$ to guarantee $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}\underline{\rho}_H$. For Scenario 2, the corresponding condition will be different due to the presence of calls that incur a reward of ρ with $\bar{\rho}_L < \rho < \underline{\rho}_H$, as stated below.

Proposition 3 *Let*

$$\begin{aligned}\alpha_1 &= \frac{\mu - \mu_1}{\mu + \beta}(\bar{\rho}_H + r) \\ \alpha_2 &= \frac{\lambda_H}{\lambda_H + \mu_1 + \beta}\alpha_1 \\ \alpha_6 &= (\lambda_H + \mu_1 + \beta)\bar{\rho}_L + (\mu_1 + \beta)r \\ \alpha_7 &= \frac{(\lambda_H\bar{\rho}_H + \lambda_L\bar{\rho}_L + (\lambda_H + \lambda_L)r)(\mu - \mu_1)}{(\lambda_H + \lambda_L + \mu_1 + \beta)(\mu + \beta)} \\ \alpha_8 &= \frac{\mu - \mu_1}{\mu + \beta}(\bar{\rho}_L + r).\end{aligned}$$

Then:

- (i) *If $\alpha_1 \leq R$, $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}R$.*
- (ii) *If $\alpha_6 > \lambda_H\bar{\rho}_H$, then having $\alpha_8 \leq R < \alpha_1$ implies $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}R$.*
- (iii) *If $\alpha_6 \leq \lambda_H\bar{\rho}_H$, then having $\alpha_2 \leq R < \alpha_1$ implies $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}R$.*
- (iv) *If $\alpha_6 > \lambda_H\bar{\rho}_H$, then having $\alpha_7 \leq R \leq \alpha_8$ implies $D(01) \leq \frac{\mu+\beta}{\mu-\mu_1}R$,*

where R with its corresponding implication on the optimal policy is specified as follows:

- (a) *If $R = \underline{\rho}_H$ in Scenario 2, class-H is preferred, and there are preferred class-L calls.*
- (b) *If $R = \underline{\rho}_L$ in Scenario 2, both class-H and class-L are preferred.*
- (c) *If $R = \underline{\rho}_L$ in Scenario 1, both class-H and class-L are preferred.*

The condition stated in (i) is similar to the same one in Proposition 2, but now the R values on the right hand side of the inequality are lower in all parts (a), (b) and (c). This implies that having sufficient conditions for cross-selling to all customers based on call characteristics only, becomes more difficult in this case. The first half of condition (ii) and (iv) will be true if $(\mu_1 + \beta)(\bar{\rho}_L + r)$ is sufficiently large. In other words if cross-selling is fast, the discount rate is high, the upper bound on the low segment revenue is high, and the service revenue is high. These conditions narrow the subsequent interval in (ii). Satisfaction of the second half of (iv) depends on the magnitude of λ_L and λ_H . The first half of (iii) is the reverse of that in (ii) and (iv). In addition to the opposite of the above stated conditions, it will be true for high values of λ_H or sufficiently high upper bound on revenue from the high segment. However for high values of λ_H the subsequent part of the condition will be difficult to satisfy. So once again, we can state the conditions for a “cross-sell to all policy” in terms of call characteristics only, however this is harder to do than for the other policies. Otherwise, sufficient conditions for this policy can be obtained if cross-selling is fast, the service revenue is high, or the relative values of λ_H and λ_L are appropriately chosen.

Note that the sufficient conditions stated in Propositions 2 and 3 are independent of the revenue distribution and only depend on the lower and upper bounds of each segment’s revenues. Policies like Policy II and V correspond to common practices: sell to everyone in a particular segment or to all customers. Typically, discrete segments as in Scenario 1 lead to the former and overlapping segments as in Scenario 2 lead to the latter policy in practice. On the other hand Policies I, III and IV suggest that just knowing the segment will not be sufficient to operationalize the cross-sell rule: who will be cross-sold within a segment will depend on who is being cross-sold and when. Policies I and III where only part of a segment is cross-sold are possible even in the case when discrete segments can be formed. If the initial segmentation is overlapping, then sufficient conditions for a rather surprising policy of cross-selling to chosen calls from both segments can be stated. This is an important result that suggests that when the dynamic nature of the cross-selling problem is explicitly accounted for, revenue based segmentation will not necessarily overlap with the optimal cross-selling policy. In our numerical examples, we will explore the extent to which the optimal policies deviate from standard marketing practice.

4 The model with expected revenues

As mentioned earlier, the model with expected revenues is a special case of the model with revenue realizations when we set $\underline{\rho}_L = \bar{\rho}_L = r_L$ and $\underline{\rho}_H = \bar{\rho}_H = r_H$ in Scenario 1. Moreover, we can have only three possible policies in this model, since “preferred calls” do not exist:

- **Policy O** Cross-sell dynamically (there are no preferred calls)
- **Policy II** Cross-sell attempt to segment H only
- **Policy V** Cross-sell attempt to everyone

Once again, we are not going to consider Policy 0 any further. Now we easily see that Proposition 2 provides sufficient conditions for observing policy II.

Corollary 1 *Let*

$$\begin{aligned}\alpha_1 &= \frac{\mu - \mu_1}{\mu + \beta}(r_H + r) \\ \alpha_2 &= \frac{\lambda_H}{\lambda_H + \mu_1 + \beta}\alpha_1 \\ \alpha_3 &= \left(\frac{\mu - \mu_1}{\mu_1 + \beta} + \frac{\mu + \beta}{\lambda_H} \right) r \\ \alpha_4 &= \frac{(\lambda_H r_H + (\lambda_H + \lambda_L)r)(\mu - \mu_1)}{(\lambda_H + \mu_1 + \beta)(\mu + \beta) + \lambda_L(\mu_1 + \beta)} \\ \alpha_5 &= \frac{\mu - \mu_1}{\mu_1 + \beta}r\end{aligned}$$

Then:

- (i) *If $\alpha_1 \leq r_H$, $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_H$.*
- (ii) *If $r_H < \alpha_3$, then having $\alpha_5 \leq r_H < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_H$.*
- (iii) *If $r_H \geq \alpha_3$, then having $\alpha_2 \leq r_H < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_H$.*
- (iv) *If $r_H < \alpha_3$, then having $\alpha_4 \leq r_H < \alpha_5$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_H$,*

so that class- H is preferred.

Proposition 3 gives sufficient conditions for both classes, in other words all calls, to be preferred:

Corollary 2 *Let*

$$\begin{aligned}\alpha_1 &= \frac{\mu - \mu_1}{\mu + \beta}(r_H + r) \\ \alpha_2 &= \frac{\lambda_H}{\lambda_H + \mu_1 + \beta}\alpha_1 \\ \alpha_6 &= (\lambda_H + \mu_1 + \beta)r_L + (\mu_1 + \beta)r \\ \alpha_7 &= \frac{(\lambda_H r_H + \lambda_L r_L + (\lambda_H + \lambda_L)r)(\mu - \mu_1)}{(\lambda_H + \lambda_L + \mu_1 + \beta)(\mu + \beta)} \\ \alpha_8 &= \frac{\mu - \mu_1}{\mu + \beta}(r_L + r).\end{aligned}$$

Then:

- (i) *If $\alpha_1 \leq r_L$, $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_L$.*
- (ii) *If $\alpha_6 > \lambda_H r_H$, then having $\alpha_8 \leq r_L < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_L$.*
- (iii) *If $\alpha_6 \leq \lambda_H r_H$, then having $\alpha_2 \leq r_L < \alpha_1$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_L$.*
- (iv) *If $\alpha_6 > \lambda_H r_H$, then having $\alpha_7 \leq r_L \leq \alpha_8$ implies $D(01) \leq \frac{\mu + \beta}{\mu - \mu_1}r_L$,*

so that both class-H and class-L are preferred.

5 Numerical Analysis

In this Section, our objective is two-fold. First, we would like to understand numerically the difference between the model with expected revenues and the model with revenue realizations. This comparison will provide an assesment of the value of real-time marketing automation. Similarly the numerical analysis will illustrate when dynamic cross-selling will be valuable, compared to simple static policies. Subsequently, we will explore the effectiveness of a heuristic, that makes explicit use of the structural properties developed before in order to suggest more sophisticated static policies that are easy to implement. The effectiveness of this heuristic will be assessed numerically. To this end, we have developed a set of test problems. In these problems, we vary base service call lengths (μ), cross-sell durations (k or μ_1), basic service

revenues (r), the upper and lower bounds on random revenues ($\underline{\rho}_L, \bar{\rho}_L, \underline{\rho}_H, \bar{\rho}_H$), the size of the call center in terms of number of servers (c), and the load ($\lambda/c\mu$) of the call center. For the numerical analysis, we assume that both high segment and low segment revenues have a uniform distribution.

Case 1 (C1) is motivated by a real retail banking call center. Using data estimated for this center, the average length of a service call is taken as 2.7 minutes. Two organizational designs are possible for cross-selling: either the service representative attempts a cross-sell and forwards to a sales department if successful, or the service representative attempts a cross-sell and closes the sale if successful. The latter is expected to take longer in terms of additional talk time. We label these two options as *attempt and forward* (f) and *attempt and close* (c). Increase in talk times will be 27 % for *attempt and forward* and 220 % for *attempt and close*, again based on estimates from this call center. Average revenue from a call with cross-selling is estimated as 75 units. We take this as the lowest value of the upper limit of high segment revenues and consider three values $\bar{\rho}_H \in \{75, 125, 175\}$. Revenues from basic service calls are taken as $r = 1$ to reflect the situation that service calls generate very low revenue compared to sales in this call center. For all our examples $\underline{\rho}_L = 0$, $\bar{\rho}_L \in \{0.3\bar{\rho}_H, 0.6\bar{\rho}_H, 0.9\bar{\rho}_H\}$ and $\underline{\rho}_H \in \{0.3\bar{\rho}_H, 0.6\bar{\rho}_H, 0.9\bar{\rho}_H\}$. These values will result in instances with discrete and overlapping segments. We consider call centers with $c \in \{100, 150, 200\}$ servers. Call volumes are obtained to ensure four different loads ($\lambda/c\mu$), characterizing quality driven (0.75), quality-efficiency driven (0.9), and efficiency driven (1.05, 1.2) centers (for precise definitions of these terms see Gans et al. (2003)). The total call volume is split in three different ways, such that $\lambda_H/(\lambda_H + \lambda_L) \in \{0.1, 0.25, 0.4\}$.

Case 2 (C2) is constructed taking Case 1 as a basis. It is assumed to represent the setting of an insurance call center or an investment bank, where the basic service length is longer. We assume an average service call length of 5.5 minutes, taking the instance of a major insurance call center in the U.S. The percentage increase for *attempt and forward* and *attempt and close* are taken to be the same as in Case 1. The only other difference from Case 1 is the assumption that basic service calls generate more revenue, so that $r = 20$. The two cases result in 3888 problem instances.

5.1 The Value of Real-Time Marketing Automation

In this subsection, we compare the performance of the model with expected revenues and the one with revenue realizations. Different assumptions about the marketing automation in place

at a call center led to the formulation of these two versions of the model. A comparison of optimal revenues will quantify the value of having real-time automation, as is assumed for the revenue realizations case. In the numerical examples, we set r_H and r_L to the mean revenue of the corresponding revenue distributions. Table 1 tabulates the averages for each call volume split and overall range of the ratio of the optimal gain from the model with expected revenues to the optimal gain in the model with revenue realization.

		Scenario 1		Scenario 2	
		f: 0.1,0.25,0.4	c: 0.1,0.25,0.4	f: 0.1,0.25,0.4	c: 0.1,0.25,0.4
C1	avg. %	99,99,99	85,93,97	99,99,99	87,88,90
	range %	98-100	81-100	99-100	86-90
C2	avg. %	98,98,99	95,98,99	99,99,99	86,90,93
	range %	97-100	81-100	98-100	81-98

Table 1: Ratio of the optimal gain with expected revenues to the optimal gain with revenue realizations

We observe that the difference ranges from less than one percent to nineteen percent. The biggest value from real-time automation is observed in the C1:c and C2:c cases, i.e. the attempt and close sales organization increases this value. For these cases, the optimal policy under the model with expected revenues is mostly to sell to everyone, or to high segment calls only whereas the model with random revenues sells more selectively. Similarly, in Scenario 2, r_H and r_L take closer values, thus making it harder for the model with expected revenues to be selective. On the other hand, for the cases with attempt and forward type sales organizations, the optimal policy under revenue realizations sells to all high segment customers and a big proportion of the low segment customers, thus approaching the selling to all policy observed in the expected revenues model. Thus, in these cases the value of revenue realizations is minimal. Finally, we note that the volume split between high segment and low segment customers has an impact, as demonstrated by the three averages in each case. The model with expected revenues performs better for higher proportions of high segment calls. As the two segment's volumes approach each other there is less need for selective selling, which once again helps the model with expected revenues. In summary, we can state that the value of real-time automation is more pronounced in settings where the relative additional load of cross-selling is higher, where segmentation is more difficult from a revenue potential standpoint, and where the H -segment generates fewer calls relative to the total call volume. In the remaining parts of the paper, we restrict our

attention to the more general model with revenue realizations.

5.2 The Value of Dynamic Cross-selling

In order to understand what type of environments lead to more dynamic policies, we compare the difference between the maximum and minimum values that $D_{21}(x)$ takes. Problem instances where this difference, or *threshold range*, is larger, are labeled as more dynamic. The first set of comparisons are made between C1: attempt and forward (f), attempt and close (c), and C2: attempt and forward (f), attempt and close (c). Comparisons are made such that all other parameters are the same. As expected, all test instances demonstrate that longer talk times and longer cross-selling content lead to more dynamic policies. Ordered from least to most dynamic, one has (C1:f, C1:c, C2:c) and (C1:f, C2:f, C2:c). Thus the retail banking call center using the organizational design of attempt and forward has the least dynamic policies, whereas the insurance call center which uses attempt and close has the most dynamic policies, *ceteris paribus*.

We next explore the role call center load and size play on the dynamic nature of optimal policies. All else being equal, we compare the threshold range for four different loads and three call center sizes. Figure 1 shows the most dynamic instances displayed by call center type. Both for C1:f and C2:f the most dynamic policies are observed for the quality-efficiency regime represented by a load of 90%, and by a large-sized call center represented by 200 servers. In these examples, if the load is set to 75 %, then within the policies with this load those for medium-sized centers with 150 servers are slightly more dynamic than those with 200 servers. It seems the extra slack created by a low load coupled with a large number of servers makes policies less dynamic in this case. For C1:c and C2:c the pattern shifts such that a quality regime represented by a load of 75 % and the largest size with 200 servers lead to the most dynamic policies. The examples demonstrate how the organizational design for cross-selling, and capacity choice impact the dynamic nature of the underlying problem.

The effect of the different cross-selling revenue bounds ($\bar{\rho}_L, \underline{\rho}_H, \bar{\rho}_H$) on the resulting policies are established via pairwise comparisons of the threshold range within each segmentation scheme (Scenario 1: discrete, Scenario 2: overlapping). The pairwise comparisons are made such that either the high segment or the low segment revenue bounds are the same in each pair and the effects of changes in the other segment are explored. The impacts of narrower segment- H revenues, and wider segment- L revenues (more overlapping segment- L revenues in the case of

C1:f, C2:f				C1:c, C2:c			
Size	100	150	200	Size	100	150	200
load				load			
120				120			
105				105			
90				90			
75				75			

Figure 1: Highest threshold ranges observed in problems with shading

Scenario 2) are analyzed. Table 2 tabulates the percentage of comparisons in each case where the stated segment properties lead to more dynamic policies. Thus it is observed that for most problem instances, narrower H -segment revenues and wider or more overlapping L -segment revenues lead to more dynamic policies. This effect is less pronounced in C2:c (and to some extent C1:c), which represents call center environments with long base talk times and long cross-sell durations. The volume difference between the two segments also makes an impact: we observe that as the H -segment volume proportion decreases from 0.4 to 0.1, the stated properties in Table 2 hold for a higher percentage of instances. In other words, as the H -segment becomes more distinct, both in revenue terms and in volume, observed policies tend to be more dynamic.

		Scenario 1		Scenario 2	
		f	c	f	c
C1	narrow H	100 %	100 %	100 %	100 %
	wide L	100 %	78 %	100 %	100 %
C2	narrow H	100 %	96 %	100 %	82 %
	wide L	100 %	54 %	100 %	83 %

Table 2: Percentages where stated segment properties lead to more dynamic policies

So far we have characterized instances that lead to more dynamic policies. To summarize, these are call centers where talk times are long and the cross-selling portion of calls is significant; capacity is designed such that the number of servers is large and the centers operate in quality or quality-efficiency regimes; and there is a premium high segment that is difficult to distinguish

from the remaining calls. Interactions between these different effects were also illustrated. Our next objective is to establish the value of dynamic policies compared to simple static policies used in practice. Tables 3 and 4 tabulate the differences in gain between the optimal dynamic policy, a static policy of targeting all segment- H calls, and a static policy of cross-selling to all calls. In particular, the averages (for each volume split), and overall range of the ratio of the gain from each static policy to the optimal gain are tabulated for the different settings.

		Scenario 1		Scenario 2	
		f: 0.1,0.25,0.4	c: 0.1,0.25,0.4	f: 0.1,0.25,0.4	c: 0.1,0.25,0.4
C1	avg. %	36,63,78	66,91,96	21,43,60	39,69,72
	range %	20-91	40-100	15-68	30-84
C2	avg. %	63,76,85	94,98,99	47,61,83	80,89,92
	range %	35-97	66-100	32-90	60-97

Table 3: Ratio of the gain from H -segment only policy to gain from optimal policy

The H -segment only policy, which is a selective segment based policy performs better in settings where calls are long and cross-selling content is significant, i.e. in settings where the congestion effects are important. The strong effect that the volume proportion of H -segment calls has on the performance of this policy can be seen from the big differences in the averages reported in Table 3. In line with our intuition, relatively higher H -segment call volumes imply a better performance of this static policy. Also, this policy, which will never be optimal under Scenario 2, performs better when discrete segments can be formed. It is interesting to note that the difference between this widely used policy and the optimal dynamic policy can be quite large in some instances, as large as 85 % in the worst case.

		Scenario 1		Scenario 2	
		f: 0.1,0.25,0.4	c: 0.1,0.25,0.4	f: 0.1,0.25,0.4	c: 0.1,0.25,0.4
C1	avg. %	99,99,99	83,80,81	99,99,99	87,88,89
	range %	98-100	70-89	99-100	86-90
C2	avg. %	98,98,99	69,71,75	99,99,99	79,81,83
	range %	97-100	53-87	98-100	68-88

Table 4: Ratio of the gain from cross-sell to all policy to gain from optimal policy

For these examples, overall, the static policy of selling to all seems to perform better than

selling to H -segment customers only. In some instances, the cross-sell to all policy performs very close to the optimal dynamic policy, though even for this case differences can be significant (with a worst case of 47 % below the optimal) for attempt and close type sales organizations. When the best performing cases of each static policy are combined, we note that these simple policies, if carefully selected to reflect the operating environment, do quite well compared to dynamic optimal policies. In the subsequent section, we develop a heuristic that generates more sophisticated static policies that can improve performance vis-a-vis these basic static policies.

5.3 Heuristics for Cross-selling

The sufficient conditions developed in Section 3 are used to construct two heuristics. We have already mentioned that conditions (*ii* – *iv*) of Proposition 1 and 2 are rather weak. Hence, we concentrate on the quantity α_1 , which can be viewed as a function of $\bar{\rho}_H$:

$$\alpha_1(\bar{\rho}_H) = \frac{\mu - \mu_1}{\mu + \beta}(\bar{\rho}_H + r).$$

The first heuristic we consider cross-sells to a customer only if his/her revenue, R , is larger than $\alpha_1(\bar{\rho}_H)$. In general, the sufficient conditions tend to be more conservative than the optimal policies, by targeting a narrower segment of customers. Without reporting the detailed performance, we note that this static policy performs consistently well across all cases, however the best of one of the simple static policies from the previous subsection always outperforms it. Encouraged by this consistency, we propose the second heuristic.

Our first heuristic compares the current revenue, R , with $\alpha_1(\bar{\rho}_H)$, a function of the highest possible revenue, $\bar{\rho}_H$, which is a rather conservative approach. In our second heuristic, we replace $\bar{\rho}_H$ with $E(R) = E(\rho|\rho > R)$: $E(R)$ is the expected reward that we can gain from cross-selling a future customer who offers a revenue of more than R , the current revenue. The current revenue, R , is compared with $\alpha_1(E(R))$, the “expected future gain” when we choose to provide only service to the current customer, and choose to cross-sell a future customer only if s/he offers a revenue of R' with $R' > R$. Then, the heuristic decides to cross-sell the current customer only if $R > \alpha_1(E(R))$. When the revenue distributions for both classes are uniform, the equation $R = \alpha_1(E(R))$ has a unique solution, R^* , such that for all $R < R^*$, $R < \alpha_1(E(R))$, and for all $R > R^*$, $R > \alpha_1(E(R))$. Hence, the heuristic cross-sells only if $R \geq R^*$.

Table 5 reports averages and the overall range of the ratio of the gain obtained with this second heuristic to the optimal gain. The heuristic approaches optimal performance on the

average consistently across all operating environments considered, and its average performance is always superior to that of the best of the simple static policies. Still we need to address its worst case behavior in C1:c under Scenario 1, as it is worse than that of the best simple static policies. In this case, all systems with performances less than 70% (the worst case of cross-sell to all policy) have a wide range of $(\underline{\rho}_H, \bar{\rho}_H)$ with $\bar{\rho}_L = \underline{\rho}_H$, and $\lambda_H/(\lambda_H + \lambda_L) = 0.1$. We have a total of 36 such incidences out of 216 in C1:c under Scenario 1. Hence, when the call volume of H -segment is low, and the segments are close to each other with a large range, our heuristic becomes very conservative with a high threshold, and so does not perform as well as the best of the simple static heuristics. However, these incidences occur rarely, 36 out of 3888, so we can conclude that this heuristic outperforms the simple static policies in all cases especially when their average performances are compared, and approaches optimal performance consistently across all operating environments considered.

		Scenario 1		Scenario 2	
		f: 0.1,0.25,0.4	c: 0.1,0.25,0.4	f: 0.1,0.25,0.4	c: 0.1,0.25,0.4
C1	avg. %	100,100,100	85,95,98	100,100,100	97,98,98
	range %	98-100	55-100	99-100	96-100
C2	avg. %	100,99,99	95,98,98	100,100,100	95,96,97
	range %	96-100	76-100	99-100	75-99

Table 5: Ratio of the gain with second heuristic to the optimal gain

Our heuristic explicitly uses the probability distributions of the random revenues along with the service rates, while completely ignoring the current state of the system. On the other hand, the model with expected revenues uses the information about the current state of the system, while it has no reference to the random revenue distributions. Hence, comparing their performances may show the value of these two kinds of information. The average performance of our heuristic is almost always superior, where the most significant improvements are observed under Scenario 2 in C1:c and C2:c. In terms of the worst case behavior, the expected revenue model generally performs better, with an exception in C1:c under Scenario 2, and with the most notable case in C1:c under Scenario 1, since 56 out of 216 incidences are below 81% (the worst case of the expected revenue model). As a result, there is no clear winner, and we can conclude that as long as one of the valuable information kinds, the current state of the system or the probability distributions of the revenues, is used efficiently, the system will perform very closely to an optimal one.

6 Concluding Remarks

This paper formulates and analyzes the first dynamic model of cross-selling in a call center with multiple servers and random revenues. The existence of preferred calls and classes in this type of dynamic admission control problem is demonstrated. The resulting sufficient conditions enable structural comparisons of optimal dynamic policies and prevailing marketing practice. It is shown that unlike current practice, which targets entire segments of customers, optimal policies can target only some calls from a segment. Even policies where some calls from each segment, with both high and low revenue potential customers, are possible. This suggests that considering the dynamic capacity effects together with the revenue potential of cross-selling leads to more complex cross-selling rules, that do not always overlap with revenue-based policies. The sufficient conditions are also useful as a building block for a heuristic, which is shown to generate sophisticated static policies, that result in near optimal performance. These static policies are furthermore very easy to understand and implement, making them more valuable in practice.

Two versions of the model are analyzed: the model with random realizations and with expected revenues. These represent the two extremes in terms of marketing automation available for cross-selling in call centers, namely real-time automation and historical data analysis. As such, our value comparisons between the optimal dynamic policies under these two versions of the model also characterize the value of real-time automation vis-a-vis historical data analysis. It is shown that in certain operating environments, the difference can be as high as nineteen percent. However, the settings with an attempt and forward type sales organization are shown to lead to a small difference, thus underlining the importance of understanding the operating environment of a call center before investing in expensive CRM products. Considering the fact that customer reactions to badly targeted cross-sell attempts are not included in our modeling framework, this difference should be viewed as a lower bound on the real value.

Using the case of a real retail banking call center, a set of numerical examples are developed. These are used to characterize environments where consideration of the dynamic nature of the problem is more important. It is shown that large call centers that operate under a quality-efficiency or quality regime, with call and cross-selling durations that are relatively long, having smaller differences between base service and cross-sell revenues, and where there is a premium high segment of customers who are difficult to segment from the remaining calls, will benefit more from dynamic optimization of their cross-selling policies. As a caveat, it should be men-

tioned that the value of dynamic optimization demonstrated through our numerical examples should be viewed as an upper bound on the real value today, since exact observation of revenue realizations at the beginning of a call represents an idealized view of the current technology landscape.

Finally, it is possible to show the monotonicity of optimal admission policies and hence the optimality of threshold-type policies for this cross-selling problem, when concavity of the value functions u_n in x_1 are assumed. This result can be found in the technical report Örmeci and Akşin (2004). Concavity of value functions even in simpler systems cannot be shown for all possible parameters when the state space is finite, see Altman, Jimenez and Koole (2001), Örmeci, Burnetas and van der Wal (2002), and Örmeci, Burnetas and Emmons (2001), although no example of such systems is reported that violated concavity. In the 3888 example test suite that we consider herein, we have similarly not observed a single value function which is not concave.

7 Appendix

Proof of Lemma 1 Proof. We prove only the second inequality of part (b) in detail. The other statements can be proved similarly.

Let $D(01)$ be as defined by (4), so that $D_n(01)(x) \leq D(01)$ for all $x \in \bar{\mathcal{S}}$, and for all $n \geq 0$. We will show that $D_n(21)(x) \leq \frac{\mu - \mu_1}{\mu + \beta} D(01)$ for all $x \in \bar{\mathcal{S}}$, and for all $n \geq 0$, which proves the statement. The proof is by induction on the number of transitions. The initial condition $u_0(x) = 0$ for all $x \in \bar{\mathcal{S}}$ clearly satisfies the statement. Assume that the statement is also true for period n , hence $D_n(21)(x) \leq \frac{\mu - \mu_1}{\mu + \beta} D(01)$ for all $x \in \bar{\mathcal{S}}$. Now, consider period $n + 1$.

We use the coupling argument described above: Assume that system A starts in state $x + e_2$ and system B starts in $x + e_1$. We couple the two systems via the service and interarrival times, so that all the departure and arrival times are the same in both systems except for the additional calls. Moreover, the additional regular call, say call d_2 , in system A is coupled with the additional cross-sell call, say call d_1 , in system B, to ensure that d_2 leaves the system whenever d_1 finishes service. Then, we can let system A follow the optimal policy and system B imitate all the decisions of system A. Defining $u_n^B(x + e_2)$ as the expected discounted return

of system B, we have:

$$\begin{aligned}
D_{n+1}(21)(x) &= u_{n+1}(x + e_2) - u_{n+1}(x + e_1) \leq u_{n+1}(x + e_2) - u_{n+1}^B(x + e_1) \\
&\leq (\lambda_H + \lambda_L)D(21) + \mu_1 \times 0 + (\mu - \mu_1)D(01) + (c - 1)\mu D(21) \\
&\leq (1 - \mu - \beta)\frac{\mu - \mu_1}{\mu + \beta}D(01) + (\mu - \mu_1)D(01) = \frac{\mu - \mu_1}{\mu + \beta}D(01),
\end{aligned}$$

where the first inequality follows from the description of the policies for systems A and B, the second inequality is due to the coupling and definitions of $D(21)$ and $D(01)$, and the last inequality follows by uniformization and the induction hypothesis. This proves the lemma. \square

Proof of Proposition 1: In inequality (5), we have $U = \frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H$:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq \lambda_H \max\left\{\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H, \bar{\rho}_H + r\right\} + \lambda_L \max\left\{\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H, \bar{\rho}_H + r, \bar{\rho}_L + r\right\} \\
&\quad + (1 - \lambda_H - \lambda_L - \mu_1 - \beta)\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H \\
&\leq (\lambda_H + \lambda_L) \max\left\{\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H, \bar{\rho}_H + r\right\} + (1 - \lambda_H - \lambda_L - \mu_1 - \beta)\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H
\end{aligned}$$

where the first inequality is due to the definition of $\bar{\rho}_s$ and the induction hypothesis, and the second is due to the assumption that $\bar{\rho}_L \leq \bar{\rho}_H$. If $\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H \geq \bar{\rho}_H + r$, the statement is proven; otherwise:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq (\lambda_H + \lambda_L)(\bar{\rho}_H + r) + (1 - \lambda_H - \lambda_L - \mu_1 - \beta)\frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H \\
&\leq \bar{\rho}_H \left(\lambda_H + \lambda_L + (1 - \lambda_H - \lambda_L - \mu_1 - \beta)\frac{\mu + \beta}{\mu - \mu_1} \right. \\
&\quad \left. + (\lambda_H + \lambda_L + \mu + \beta)\frac{\mu_1 + \beta}{\mu - \mu_1} \right) \\
&= \bar{\rho}_H \left(\frac{\mu + \beta}{\mu - \mu_1} + (\lambda_H + \lambda_L) \left(1 - \frac{\mu + \beta}{\mu - \mu_1} + \frac{\mu_1 + \beta}{\mu - \mu_1} \right) \right. \\
&\quad \left. + \frac{(\mu_1 + \beta)(\mu + \beta) - (\mu_1 + \beta)(\mu + \beta)}{\mu - \mu_1} \right) \\
&= \frac{\mu + \beta}{\mu - \mu_1}\bar{\rho}_H
\end{aligned}$$

where the second inequality is due to the assumption of the theorem. Thus, the statement is true for all $x \in \bar{\mathcal{S}}$ and for all $n \geq 0$. \square

Proof of Proposition 2: We can use inequality (5) with $U = \frac{\mu + \beta}{\mu - \mu_1}R$:

$$D_{n+1}(01)(x) \leq \lambda_H \max\left\{\frac{\mu + \beta}{\mu - \mu_1}R, R + r, \bar{\rho}_H + r\right\} + \lambda_L \max\left\{\frac{\mu + \beta}{\mu - \mu_1}R, R + r, \bar{\rho}_L + r\right\}$$

$$\begin{aligned}
& +(1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R \\
\leq & \lambda_H \max\left\{\frac{\mu + \beta}{\mu - \mu_1} R, \bar{\rho}_H + r\right\} + \lambda_L \max\left\{\frac{\mu + \beta}{\mu - \mu_1} R, R + r\right\} \\
& +(1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R
\end{aligned} \tag{6}$$

where the inequalities are due to the definition of $\bar{\rho}_s$, the possible values of R , explicitly $\bar{\rho}_L \leq R \leq \bar{\rho}_H$ in all statements, and the induction hypothesis. Now we differentiate the cases:

(i) $\alpha_1 \leq R$: In this case, we have $\frac{\mu + \beta}{\mu - \mu_1} R \geq \bar{\rho}_H + r$, which immediately proves the statement.

(ii) $\bar{\rho}_H < \alpha_3$ and $\alpha_5 \leq R < \alpha_1$: Having $R < \alpha_1$ implies that $\frac{\mu + \beta}{\mu - \mu_1} R < \bar{\rho}_H + r$, and $\alpha_5 \leq R$ implies $\frac{\mu + \beta}{\mu - \mu_1} R \geq R + r$. Moreover, whenever $\bar{\rho}_H < \alpha_3$, $\alpha_2 < \alpha_5$, so that $\alpha_2 < R$. Hence, we have:

$$\begin{aligned}
D_{n+1}(01)(x) & \leq \lambda_H(\bar{\rho}_H + r) + \lambda_L \frac{\mu + \beta}{\mu - \mu_1} R + (1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R \\
& \leq \frac{\mu + \beta}{\mu - \mu_1} R \left(\lambda_H \frac{\lambda_H + \mu_1 + \beta}{\lambda_H} + (1 - \lambda_H - \mu_1 - \beta) \right) = \frac{\mu + \beta}{\mu - \mu_1} R,
\end{aligned}$$

where the second inequality follows since $\alpha_2 < R$.

(iii) $\bar{\rho}_H \geq \alpha_3$ and $\alpha_2 \leq R < \alpha_1$: By $R < \alpha_1$, $\frac{\mu + \beta}{\mu - \mu_1} R < \bar{\rho}_H + r$. Moreover, if $\bar{\rho}_H \geq \alpha_3$, $\alpha_5 \leq \alpha_2$, so that $\alpha_5 \leq R$. Hence, $\frac{\mu + \beta}{\mu - \mu_1} R \geq R + r$. Then, we have:

$$\begin{aligned}
D_{n+1}(01)(x) & \leq \lambda_H(\bar{\rho}_H + r) + \lambda_L \frac{\mu + \beta}{\mu - \mu_1} R + (1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R \\
& \leq \frac{\mu + \beta}{\mu - \mu_1} R,
\end{aligned}$$

where the second inequality follows since $\alpha_2 \leq R$.

(iv) $\bar{\rho}_H < \alpha_3$, $\alpha_4 \leq R < \alpha_5$: We first note that $\alpha_4 < \alpha_5$ if and only if $\bar{\rho}_H < \alpha_3$. Now, since $R < \alpha_5$, $R + r \geq \frac{\mu + \beta}{\mu - \mu_1} R$. Then, we have:

$$\begin{aligned}
D_{n+1}(01)(x) & \leq \lambda_H(\bar{\rho}_H + r) + \lambda_L(R + r) + (1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R \\
& = \lambda_H \bar{\rho}_H + (\lambda_H + \lambda_L)r + R(\lambda_L + (1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1}) \\
& \leq \frac{R}{\mu - \mu_1} ((\lambda_H + \mu_1 + \beta)(\mu + \beta) + \lambda_L(\mu_1 + \beta)) \\
& \quad + \lambda_L(\mu - \mu_1) + (1 - \lambda_H - \lambda_L - \mu_1 - \beta)(\mu + \beta) \\
& = \frac{\mu + \beta}{\mu - \mu_1} R
\end{aligned}$$

where the second inequality follows since $\alpha_4 \leq R$.

Thus, all three statements are true for all $x \in \bar{S}$ and for all $n \geq 0$. \square

Proof of Proposition 3: In this case, inequality (5) becomes with $U = \frac{\mu+\beta}{\mu-\mu_1}R$:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq \lambda_H \max\left\{\frac{\mu+\beta}{\mu-\mu_1}R, R+r, \bar{\rho}_H+r\right\} + \lambda_L \max\left\{\frac{\mu+\beta}{\mu-\mu_1}R, R+r, \bar{\rho}_L+r\right\} \\
&\quad + (1-\lambda_H-\lambda_L-\mu_1-\beta)\frac{\mu+\beta}{\mu-\mu_1}R \\
&\leq \lambda_H \max\left\{\frac{\mu+\beta}{\mu-\mu_1}R, \bar{\rho}_H+r\right\} + \lambda_L \max\left\{\frac{\mu+\beta}{\mu-\mu_1}R, \bar{\rho}_L+r\right\} \\
&\quad + (1-\lambda_H-\lambda_L-\mu_1-\beta)\frac{\mu+\beta}{\mu-\mu_1}R
\end{aligned} \tag{7}$$

where the inequalities are due to the definition of $\bar{\rho}_s$, possible values R since $R \leq \bar{\rho}_L$ in all statements, and the induction hypothesis. Now we differentiate the cases:

(i) $\alpha_1 \leq R$: In this case, we have $\frac{\mu+\beta}{\mu-\mu_1}R \geq \bar{\rho}_H+r$, which immediately proves the statement.

(ii) $\lambda_H \bar{\rho}_H < \alpha_6$ and $\alpha_8 \leq R < \alpha_1$: Having $R < \alpha_1$ implies that $\frac{\mu+\beta}{\mu-\mu_1}R < \bar{\rho}_H+r$, and $\alpha_8 \leq R$ implies $\frac{\mu+\beta}{\mu-\mu_1}R \geq \bar{\rho}_L+r$. Moreover, whenever $\lambda_H \bar{\rho}_H < \alpha_6$, $\alpha_2 < \alpha_8$, so that $\alpha_2 < R$. Hence, we have:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq \lambda_H(\bar{\rho}_H+r) + \lambda_L \frac{\mu+\beta}{\mu-\mu_1}R + (1-\lambda_H-\lambda_L-\mu_1-\beta)\frac{\mu+\beta}{\mu-\mu_1}R \\
&\leq \frac{\mu+\beta}{\mu-\mu_1}R
\end{aligned}$$

where the second inequality follows from $\alpha_2 < R$.

(iii) $\alpha_6 \leq \lambda_H \bar{\rho}_H$ and $\alpha_2 \leq R < \alpha_1$: We first note that whenever $\alpha_6 \leq \lambda_H \bar{\rho}_H$, $\alpha_8 \leq \alpha_2$ so that $\frac{\mu+\beta}{\mu-\mu_1}R \geq \bar{\rho}_L+r$. On the other hand, $\frac{\mu+\beta}{\mu-\mu_1}R < \bar{\rho}_H+r$ because $R < \alpha_1$. Hence, we have:

$$\begin{aligned}
D_{n+1}(01)(x) &\leq \lambda_H(\bar{\rho}_H+r) + \lambda_L \frac{\mu+\beta}{\mu-\mu_1}R + (1-\lambda_H-\lambda_L-\mu_1-\beta)\frac{\mu+\beta}{\mu-\mu_1}R \\
&\leq \frac{\mu+\beta}{\mu-\mu_1}R
\end{aligned}$$

where the second inequality follows from $\alpha_2 \leq R$.

(iv) $\alpha_6 > \lambda_H \bar{\rho}_H$ and $\alpha_7 \leq R \leq \alpha_8$: We first note that $\alpha_7 < \alpha_8$ if and only if $\alpha_6 > \lambda_H \bar{\rho}_L$. Now, since $R < \alpha_8$, $R + r \geq \frac{\mu + \beta}{\mu - \mu_1} R$. Then, we have:

$$\begin{aligned} D_{n+1}(01)(x) &\leq \lambda_H(\bar{\rho}_H + r) + \lambda_L(\bar{\rho}_L + r) + (1 - \lambda_H - \lambda_L - \mu_1 - \beta) \frac{\mu + \beta}{\mu - \mu_1} R \\ &\leq \frac{\mu + \beta}{\mu - \mu_1} R (1 - \lambda_H - \lambda_L - \mu_1 - \beta + \lambda_H + \lambda_L + \mu_1 + \beta) \\ &= \frac{\mu + \beta}{\mu - \mu_1} R \end{aligned}$$

where the second inequality is due to the assumption $\alpha_7 \leq R$.

Thus, all three statements are true for all $x \in \bar{\mathcal{S}}$ and for all $n \geq 0$. □

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