Solution to
STAT 350 Exam 2 Review Questions (Spring 2015) **04/11//2015**

1. A random sample of 26 offshore oil workers took part in a simulated escape exercise, and their times (sec) to complete the escape are recorded. The sample mean is 370.69 sec and the sample standard deviation is 24.36 sec. Construct a 95% confidence interval on the true average escape time. Interpret your interval.

$n = 26$
$\bar{x} = 370.69$
$S = 24.36$
Since population standard deviation σ is not know, use T - procedure.

$DF = n - 1 = 25$
$t^*(95\%) = 2.060$
A 95% CI for μ is

$$\bar{x} \pm t^* \cdot s/\sqrt{n}$$

$$370.69 \pm 2.060 \times 24.36/\sqrt{26}$$
margin of error 9.84

$\rightarrow$ (360.9, 380.5)
(360.85, 380.53)

We are 95% confident that the true mean escape time is between 360.9 sec and 380.5 sec.

2. An investigator wishes to estimate the difference between population mean SAT-M scores of incoming freshmen in the College of Engineering and in the College of Science at Purdue University. The population standard deviations are both roughly 100 points and equal sample sizes are to be selected. What value of the common sample size n will be necessary to estimate the difference to within 10 points with 99% confidence?

Two-sample Z problem,

99% CI : $z^* = 2.576$

$$(\bar{x}_1 - \bar{x}_2) \pm \underbrace{z^* \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{10 = \text{margin of error}}$$

$\sigma_1^2 = \sigma_2^2 = 100^2$ (given)

$n_1 = n_2 = n$.

$$2.576 \times \sqrt{\frac{100^2}{n} + \frac{100^2}{n}} = 10$$

solve for $n = 1327.15 \nearrow$

≈ 1328

Computation detail

$$257.6 \times \sqrt{\frac{2}{n}} = 10$$

$$n = (25.76)^2 \times 2$$

$$= 1327.15 \nearrow \boxed{1328}$$

3. The life in hours of a battery is known to be approximately normally distributed. The manufacture claims that the average battery life exceeds 40 hours. A random sample of 10 batteries has a mean life of 40.5 hours and sample standard deviation s=1.25 hours. Carry out a test of significance for $H_0 : \mu = 40\ hrs$ vs $H_1\ \ \mu > 40\ hrs$. $\alpha = 0.05$

a) $H_0 : \mu = 40$
$H_a : \mu > 40$
choose $\alpha = 0.05$

b) 1- sample t test
test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{10}}$$

$$= \frac{40.5 - 40}{1.25/\sqrt{10}}$$

$$= 1.26$$

c) DF = 9

$0.1 < p\text{-value} < 0.15$

so, we fail to reject H_0

d) There is not enough evidence to support the claim that the average battery life exceeds 40 hours.

4. The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:

Brand 1: 275, 286, 287, 271, 283, 271, 279, 275, 263, 267
Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268

a) Which procedure is the most appropriate, matched pairs T or two sample T? Explain
b) Find a 95 % confidence interval for the difference of the mean.
c) Use the four-step procedure to carry out a hypothesis test to determine whether the mean overall distance for brand 1 and brand 2 are different?

P4 a) Two Independent Samples. i.e. two-sample T procedure, since there isn't any matching involved

b)

	n	$\bar{x}$	S
1	10	275.7	8.03
2	10	265.3	10.045

(use your calculator to obtain above information)

DF = 9, $C = 95\% \Rightarrow t^* = 2.262$

95% CI : $(\bar{x}_1 - \bar{x}_2) \pm t^* \cdot SE$,

$$SE = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$= \sqrt{\frac{8.03^2}{10} + \frac{10.045^2}{10}}$$

$$= 4.066$$

$$\bar{x}_1 - \bar{x}_2 = 10.4$$

So, a 95% CI is

$10.4 \pm 2.262 \times 4.066$

$(1.20, 19.60)$

c) ① $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 \neq \mu_2$

② $t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE} = \dfrac{10.4}{4.066} = 2.56$

DF = 11 ← DF = 9

③ $0.01 \times 2 < p\text{-value} < 0.02 \times 2 < \underset{\alpha}{0.05}$

reject H_0

The average distances traveled by different brands golf balls are different.

5. The Indiana State Police wish to estimate the average mph being traveled on the Interstate Highways, which cross the state. If the estimate is to be within ±5 mph of the true mean with 95% confidence and the estimated population standard deviation is 25 mph, how large a sample size must be taken?

95% CI: $\bar{x} \pm z^* \cdot \sigma/\sqrt{n}$

margin of error $= z^* \cdot \sigma/\sqrt{n}$

$5 = 1.96 \times 25/\sqrt{n}$

$n = (1.96 \times 5)^2 = 96.04$

$\approx \underline{\underline{97}}$

6. A laboratory is testing the concentration level in mg/ml for the active ingredient found in a pharmaceutical product. In a random sample of 10 vials of the product, the mean and the sample standard deviation of the concentrations are 2.58 mg/ml and 0.09 mg/ml. Find a 95% confidence interval for the mean concentration level in mg/ml for the active ingredient found in this product.

$n = 10$, $\bar{x} = 2.58$, $S = 0.09$

use one-sample T, as σ unknown

$DF = 9 \Rightarrow t^* = 2.262$ $(C = 95\%)$

$2.58 \pm 2.262 \times 0.09/\sqrt{10}$

$(2.52, 2.64)$

7. An investigator wishes to estimate the difference between two population mean lifetimes of two different brands of batteries under specified conditions. If the population standard deviations are both roughly 2 hr and the sample size from the first brand will be twice the sample size from the second brand, what values of the sample sizes will be necessary to estimate the difference to within 0.5 hours with 99% confidence?

P7

$\sigma_1 = \sigma_2 = 2$

$n_1 = 2n_2$

$C = 99\% \Rightarrow z^* = 2.576$

two-sample means CI for $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm \underbrace{z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{\text{margin of error}}$$

$$2.576 \times \sqrt{\frac{2^2}{2n_2} + \frac{2^2}{n_2}} = 0.5$$

$$2.576 \times \sqrt{\frac{6}{n_2}} = 0.5$$

$$n_2 = 6 \times \left(\frac{2.576}{0.5}\right)^2$$

$$= 159.2 \nearrow = 160$$

$$n_1 = 2n_2 = 320$$

$$\boxed{n_1 = 320, \quad n_2 = 160}$$

8. The following summary data on proportional stress limits for two different type of woods, Red oak and Douglas fir.

Type of Wood	Sample Size	Sample Mean	s
Red oak	50	8.51	1.52
Douglas fir	62	7.69	3.25

a) Find a 90% confidence interval for the difference between true average proportional stress limits for the Red oak and that for the Douglas fir. Interpret your result.
b) A test of hypotheses is conducted at α=0.10 to determine if the stress limits are the same for the two type of woods.
c) Explain how you can use the confidence interval in part (a) to draw a conclusion in the test of hypotheses.

P8 a) $n_1 = 50, \quad n_2 = 62 \Rightarrow DF = 49$

$$\bar{x}_1 - \bar{x}_2 = 8.51 - 7.69 = 0.82$$

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \checkmark$$

$$= \sqrt{\frac{1.52^2}{50} + \frac{3.25^2}{62}}$$

$$= 0.465$$

Actual DF = 49
use DF = 40 $\Rightarrow t^* = 1.684$.
↕ 90%
DF = 50 ✓
calculator DF = 90 ✓ Full Credit for using DF=90 calculator #.
the 90% CI is

$$0.82 \pm 1.684 \times 0.465$$
$$(0.036, \ 1.604)$$

We are 90% confident that the difference of proportional stress limits for Red Oak and Douglas Fir is between (0.036, 1.604)

b) $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 \neq \mu_2$
test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{0.82}{0.465} = 1.76$$

use DF = 40,

$0.05 = 2 \times 0.025 < \text{p-value} < 2 \times 0.05 = 0.1 = \alpha$

reject H_0, conclude that the average proportional stress limit for Red Oak and Douglas Fir is different.

& from 90% CI, 0 is outside. $\Rightarrow$ reject H_0

9. The accompanying summary data on the ratio of strength to cross-sectional area for knee extensors is from the article "Knee Extensor and Knee Flexor Strength: Cross Sectional Area Ratios in Young and Elderly Men":

Group	Sample Size	Sample Mean	Sample Standard Deviation
Young Men	50	7.47	0.44
Elderly Men	45	6.71	0.56

Does the data suggest that the true average ratio for young men exceeds that for elderly men? Carry out a test of significance using $\alpha = 0.01$.

2 ① $H_0: \mu_1 = \mu_2$ 1 – young
$H_a: \mu_1 > \mu_2$ 2 – elderly

② $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{7.47 - 6.71}{\sqrt{\frac{0.44^2}{50} + \frac{0.56^2}{45}}} = 7.3$$

DF = 44 ⇒ use DF = 40

③ p-value < 0.0005
reject H_0

④ Yes, the data suggest the true average ratio for young men exceeds that for elderly men

10. Coronary heart disease (CHD) begins in young adulthood and is the fifth leading cause of death among adults aged 20 to 24 years. Studies of serum cholesterol levels among college students, however, are very limited. A 1999 study looked at a large sample of students from a large southeastern university and reported that the mean serum cholesterol level among women is 168 mg/dl with a standard deviation of 27 mg/dl.[15] A more recent study at a southern university investigated the lipid levels in a cohort of sedentary university students.[16] The mean total cholesterol level among *n* = 71 females was $\bar{x} = 173.7$. Is there evidence that the mean cholesterol level among sedentary students differs from this average over all students? Use the four-step procedure to carry out a test of significance. Use α = 0.05.

P 10

1. $H_0: \mu = 168$
$H_a: \mu \neq 168$ $\quad \alpha = 0.05$

$\sigma = 27 \rightarrow$ use z-test

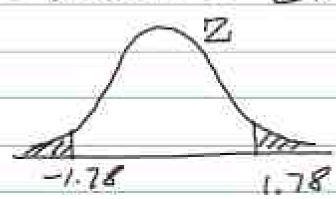
2. $z = \dfrac{\bar{x} - 168}{\sigma / \sqrt{n}}$

$= \dfrac{173.7 - 168}{27/\sqrt{71}}$

$= 1.78$

$P(Z > 1.78) = 1 - 0.9625$
$= 0.0375$

3. p-value $= 2 \times 0.0375 = 0.075 > \alpha$



4. fail to reject Ho.
there is not strong enough evidence that the mean cholesterol level among sedentary students differs from that of overall student.

11. Fifteen adult males between the ages 35 and 45 participated in a study to evaluate the effect of diet and exercise on blood cholesterol levels. The total cholesterol was measured in each subject initially, and then three months after participating in an aerobic exercise program and switching to a low-fat diet.The data are shown in the accompanying table.

Table I: Blood Cholesterol Levels for 15 Adult Males

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Before	265	240	258	296	251	245	287	314	260	279	283	240	238	225	247
After	229	231	227	240	238	241	234	256	247	239	246	218	219	226	233

	N	**Mean**	**StDev**	**SE Mean**
Before	**15**	**261.80**	**24.96**	**6.45**
After	**15**	**234.93**	**10.48**	**2.71**
Diff (Before - After)	**15**	**26.87**	**19.04**	**4.92**

a) Find a 90% confidence interval for the true mean reduction of the cholesterol reduction.
b) carry out a test of hypotheses to determine if the data support the claim that the low-fat diet and aerobic exercise are of value in producing a mean reduction in blood cholesterol levels? Use α=0.05.

Please see next page for solution.

P11 Before & After ⇒ Classical Matched Pairs

a) a 90% CI:

$$\bar{x}_d \pm t^* \cdot s_d/\sqrt{n}$$

$\bar{x}_d = 26.87$

$s_d = 19.04$

$n = 15 \Rightarrow DF = 14$
$\downarrow$
$t^* = 1.761$

$26.87 \pm 1.761 \times 19.04/\sqrt{15}$
26.87 ± 8.66
$(18.21, 35.53)$

b). ① $H_0: \mu_d = 0$ $\mu_d = \mu_{before - after}$
$H_a: \mu_d > 0$

② $t = \dfrac{\bar{x}_d - 0}{s_d/\sqrt{n}}$

$= \dfrac{26.87}{19.04/\sqrt{15}} = 5.47$

③ $DF = 14$
p-value < 0.0005
reject H_0

④ Yes, data support the claim that the lowe fat diet and aerobic exercise are of value in reducing cholesterol level.

12. True of False Questions (explain why):

A. ANOVA tests the null hypothesis that the sample means are all equal.
FALSE
ANOVA tests the null hypothesis that the **population** means are all equal.

B. A strong case for causation is best made in an observational study.
FALSE
A strong case for causation is best made in an **experiment.**

C. You use ANOVA to compare the variances of the populations.
FALSE
You use ANOVA to compare the **means** of the populations.

D. A multiple-comparisons procedure is used to compare a relation among means that was specified prior to looking at the data.

FALSE
A multiple-comparisons procedure is used to compare a relation among means that was specified prior to looking at the data.

E. In rejecting the null hypothesis, one can conclude that all the means are different from one another.

FALSE
In rejecting the null hypothesis, one can conclude that **at least one mean is different from others**.

F. A one-way ANOVA can be used only when there are two means to be compared.
FALSE
A one-way ANOVA can be used when there are **three or more means** to be compared.

G. The ANOVA *F* statistic will be large when the within-group variation is much larger than the between-group variation.
FALSE
The ANOVA *F* statistic will be **small** when the within-group variation is much larger than the between-group variation.

13 (12.13, 12.15) For each of the following situations, identify the response variable and the populations to be compared, and give I, N and (a) Degrees of freedom for group, for error, and for the total (b) Null and alternative hypotheses (c) Numerator and denominator degrees of freedom for the F statistic

A. A poultry farmer is interested in reducing the cholesterol level in his marketable eggs. He wants to compare two different cholesterol-lowering drugs added to the hens' standard diet as well as an all-vegetarian diet. He assigns 25 of his hens to each of the three treatments.

Response: egg cholesterol level
Population: chickens with different diets or drugs
I = 3, N = 75, n1=n2=n3=25

B. A researcher is interested in students' opinions regarding an additional annual fee to support non-income-producing varsity sports. Students were asked to rate their acceptance of this fee on a seven-point scale. She received 94 responses, of which 31 were from students who attend varsity football or basketball games only, 18 were from students who also attend other varsity competitions, and 45 were from students who did not attend any varsity games

Response: rating on 7 points scale
Population: students from three different groups
I = 3, N = 94, n1=31, n2=18, n3=45

C. A professor wants to evaluate the effectiveness of his teaching assistants. In one class period, the 42 students were randomly divided into three equal-sized groups, and each group was taught power calculations from one of the assistants. At the beginning of the next class, each student took a quiz on power calculations, and these scores were compared.

Response: quiz score
Population: students in each TA group
I = 3, N = 42, n1=n2=n3=14

For all three:
H0: u1 = u2 = u3, vs
Ha: at least one mean is different from others.

Degrees of freedom for group, for error, and for the total, along with the numerator and denominator degrees of freedom for the F statistic are listed in the table below for each question:

Situation	I	N	DFG	DFE	DFT	df for F statistic
(a) Egg cholesterol level	3	75	2	72	74	$F(2, 72)$
(b) Student opinions	3	94	2	91	93	$F(2, 91)$
(c) Teaching assistants	3	42	2	39	41	$F(2, 39)$

14. (12.12) An experiment was run to compare three groups. The sample sizes were 27, 31, and 122, and the corresponding estimated standard deviations were 37, 28, and 46.

A. Is it reasonable to use the assumption of equal standard deviations when we analyze these data? Give a reason for your answer.

   Yes, since the standard deviation ratio between max and min is 46/28 < 2.

B. Give the values of the variances for the three groups.

   1369, 784, and 2116

C. Find the pooled variance.

   N = 27+31+122 = 180, I = 3

   pooled variance =(26*1369+30*784+ 121*2116)/177 = 1780

D. What is the value of the pooled standard deviation?

   pooled standard deviation is sqrt(1780) = 42.20

E. Explain why your answer in part (d) is much closer to the standard deviation for the third group than to either of the other two standard deviations.

   The pooled variance is the weighted average of the variances of the three groups.

   The sample size for the third group is much larger than that of the other two groups.

15. (12.26) Various studies have shown the benefits of massage to manage pain. In one study, 125 adults suffering from osteoarthritis of the knees were randomly assigned to one of five 8-week regimens.[9] The primary outcome was the change in the Western Ontario and McMaster Universities Arthritis Index (WOMAC-Global). This index is used extensively to assess pain and functioning in those suffering from arthritis. Negative values indicate improvement. The following table summarizes the results of those completing the study.

Regimen	n	$\bar{x}$	s
30 min massage 1 × /wk	22	−17.4	17.9
30 min massage 2 × /wk	24	−18.4	20.7
60 min massage 1 × /wk	24	−24.0	18.4
60 min massage 2 × /wk	25	−24.0	19.8
Usual care, no massage	24	−6.3	14.6

1. What proportion of adults dropped out of the study before completion?
   N = 22+24+24+25+24 = 119; dropout rate = 6/125 = 4.8%
2. Is it reasonable to use the assumption of equal standard deviations when we analyze these data? Give a reason for your answer.
   Yes, the ratio of the max to min standard deviations is 20.7/14.6 < 2
3. Find the pooled standard deviation.
   N = 119, I = 5 pooled variance = **339.32**; so pooled SD = 18.42
4. The SS(Regimen) = 5060.346. Test the null hypothesis that the mean change in WOMAC-Global score is the same for all regimens.
   STEP1: H0: $\mu 1 = \mu 2 = \mu 3 = \mu 4 = \mu 5$
   Ha: Not all population means are the same
   STEP 2:SSG = 5060.346 (GIVEN), MSG = 5060/4 = 1265
   F = MSG/MSE = 1265/**339.32** = 3.73, DF1 = 4, DF2 = 114
   STEP3: p-value = 0.0069, reject H0
   STEP4: the mean change in WOMAC-Global score is **not** the same for all regimens.
5. There are 10 pairs of means to compare. For the Bonferroni multiple-comparisons method, the critical *t*-value is 2.863. Which pairs of means are found to be significantly different? Write a short summary of your analysis.

Please practice comparing one pair, say control vs another treatment.

(c) $s_p^2 = \dfrac{(22-1)17.9^2+(24-1)20.7^2+(24-1)18.4^2+(25-1)19.8^2+(24-1)14.6^2}{22+24+24+25+24-5} = 339.3193.$ $s_p = \sqrt{s_p^2} =$ 18.421. **(d)** Because $s_p^2 = \text{MSE}$, we have $F = \dfrac{5060.346/4}{339.3193} = 3.728.$ With df 4 and 114, $P = 0.0068$. This is extremely strong evidence that there is a difference in knee pain with the different regimens. **(e)** For the Bonferroni procedure, means that differ by more than $(2.863)(18.421)\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}$ are deemed significantly different. Comparing the 30-minute massage once a week ($n = 22$) with any of the three treatments with $n = 24$, their means would have to differ by more than 15.567 to be considered statistically different. None of those differences are significant. Comparing the 30-minute massage once a week ($n = 22$) to the 60-minute massage twice a week ($n = 25$), the means would have to differ by more than 15.417. They do not. In comparing the three treatments that have $n = 24$, their means would have to differ by at least 15.225; we see the 60-minute massage once a week is different from the control. 60-minute massage twice a week had $n = 25$. It will have to differ from a group with $n = 24$ by at least 15.072; that group is also different from the control. Summaries will differ.

16. (12.40) There have been numerous studies investigating the effects of restaurant ambiance on consumer behavior. One study investigated the effects of musical genre on consumer spending.[15] At a single high-end restaurant in England over a 3-week period, there were a total of 141 participants; 49 of them were subjected to background pop music (for example, Britney Spears, Culture Club, and Ricky Martin) while dining, 44 to background classical music (for example, Vivaldi, Handel, and Strauss), and 48 to no background music. For each participant, the total food bill, adjusted for time spent dining, was recorded. The following table summarizes the means and standard deviations (in British pounds):

Background music	Mean bill	n	s
Classical	24.130	44	2.243
Pop	21.912	49	2.627
None	21.697	48	3.332
Total	22.531	141	2.969

A. Plot the means versus the type of background music. Does there appear to be a difference in spending?
B. Is it reasonable to assume that the variances are equal? Explain.
C. The F statistic is 10.62. Give the degrees of freedom and either an approximate (from a table) or an exact (from software) P-value. What do you conclude?
D. Refer back to part (a). Without doing any formal analysis, describe the pattern in the means that is likely responsible for your conclusion in part (c).

SOLUTION:

12.40 (a) The plot of means suggests that spending is higher for classical music, while pop and no music appear to have the same effect. **(b)** Yes: The guidelines for pooling standard deviations say that the ratio of largest to smallest should be less than 2; we have $3.332/2.243 = 1.49 < 2$. **(c)** The degrees of freedom are $DFG = I - 1 = 2$ and $DFE = N - I = 138$. Comparing to an $F(2, 100)$ distribution in Table E, we see that $P < 0.001$; software gives $P = 0.00005$. We have strong evidence that the means are not all the same. **(d)** The higher average bill for classical music led to this conclusion; the difference between pop music and no background music is negligible. **(e)** The setting of this experiment ("a single high-end restaurant in England") might limit how much this conclusion can be generalized. It *might* extend to other high-end restaurants, but perhaps not to "family-style" restaurants, and almost certainly not to fast-food restaurants.