Basic Math Formulas for Crash Investigators

To convert a speed to a velocity

$$V = S\left(1.466\right)$$

V = Velocity in feet per seconds (ft/sec)
S = Speed in miles per hour (mph)
1.466 = Mathematical Constant

Example

Your driver just had a rear-end accident and says "I was traveling at 50 mph and had plenty of following distance – at least 4 bus lengths".

How many seconds was the bus actually following?

4 bus lengths = ~180 feet

50 mph x 1.466 = 73 ft/sec

180 ft ÷ 73 ft/sec = 2.5 seconds

Exercise

A driver was traveling at 55 mph and has a perception/reaction time of 2.5 seconds (slightly fatigued). How far will he/she travel before anything changes?

$$V = S(1.466) \qquad D = V(t)$$

$$V = 55(1.466) \qquad D = 80.63(2.5)$$

$$V = 80.63\ fps \qquad D = 201.6\ feet$$

Constant Velocity

The **time** required to travel a given distance at a constant velocity may be determined.

$$t = \frac{D}{V}$$

Examples:

5 miles ÷ 60 mph = .08 hours (5 mins)

30 miles ÷ 60 mph = .5 hours (30 mins)

110 miles ÷ 60 mph = 1.83 hours (1hr 50 mins)

Average Speed for Trip

Your driver's average speed for a trip can be determined by the following formula:

$$S = \frac{\text{Miles}}{\text{Time}}$$

(time in hours & parts of hours)

Example:

Total miles = 660; Driving time = 9 ¼ hours

660 ÷ 9.25 = 71 mph average speed

Slide to Stop Speed

Determines the initial speed of a vehicle at the point where observable skid marks first appear on the roadway surface

$$S = \sqrt{30 \cdot (f) \cdot (d)}$$

Speed Based on Skid Mark Length

<u>Known Data</u>

Skid Dx = 105 feet

u = .75

n = .80

m = -.03

f = .57

$$S = \sqrt{30 \cdot (f) \cdot (d)}$$

S = √30 (.57) (105)

S = √1795.5

S = 42.4 mph

(speed for 105 ft of skid length on this particular surface; assumes no speed remaining at end of skid)

However, if vehicle still traveling 20 mph at end of skid (impact), need to <u>combine</u> (not add) these speeds.

Combined Speed Formula

$$S = \sqrt{S_1^2 + S_2^2 + S_3^2 + etc.....}$$

Example:

Slide to Stop Speed = 42.4 mph

Impact Speed = 20 mph

$$S = \sqrt{42.4^2 + 20^2}$$

$$S = \sqrt{1797.76 + 400}$$

$$S = \sqrt{2197.76}$$

S = 46.9 mph

Critical Speed

- The speed at which a vehicle exceeds the tires' ability to maintain traction (starts to slide sideways while moving forward).
- The radius measurement must be adjusted to determine the path of the vehicle's center of mass before you proceed.

(calculated radius minus ½ of vehicle's width)

Critical Speed

Use this equation when the super-elevation (crown) of the road is 10% (.10) or less.

$$S = (3.86)\sqrt{R(\mu \pm e)}$$

S = speed

μ = coefficient of friction (level surface)

e = super-elevation

R = radius (feet)

3.68 = Mathematical Constant

Critical Speed Example

Known Data

Speed Limit = 30 mph

Yaw Radius = 163 ft

u = .70

e = +.05

Was car speeding around curve?

$$S = (3.86)\sqrt{R(\mu \pm e)}$$

S = (3.86) √163 (.70 + .05)

S = (3.86) √163 (.75)

S = (3.86) √122.25

S = (3.86) x (11.06)

S = 42.7 mph

Continuation of Presentation

To convert a velocity to a speed

$$S = \frac{V}{1.466}$$

V = Velocity in feet per seconds (ft/sec)
S = Speed in miles per hour (mph)
1.466 = Mathematical Constant

To Convert Inches to Feet

$$\text{Feet} = \frac{\text{Inches}}{(12)}$$

Example: 81 inches ÷ 12 = 6.75 ft

To Convert Feet to Inches

$$Inches = Feet\,(12)$$

Example: 6.75 feet x 12 = 81 inches

To Convert Minutes to Parts of an Hour

$$\textit{Parts of an Hour} = \textit{Minutes} \div 60$$

Examples:

15 mins ÷ 60 = .25 hour (¼)

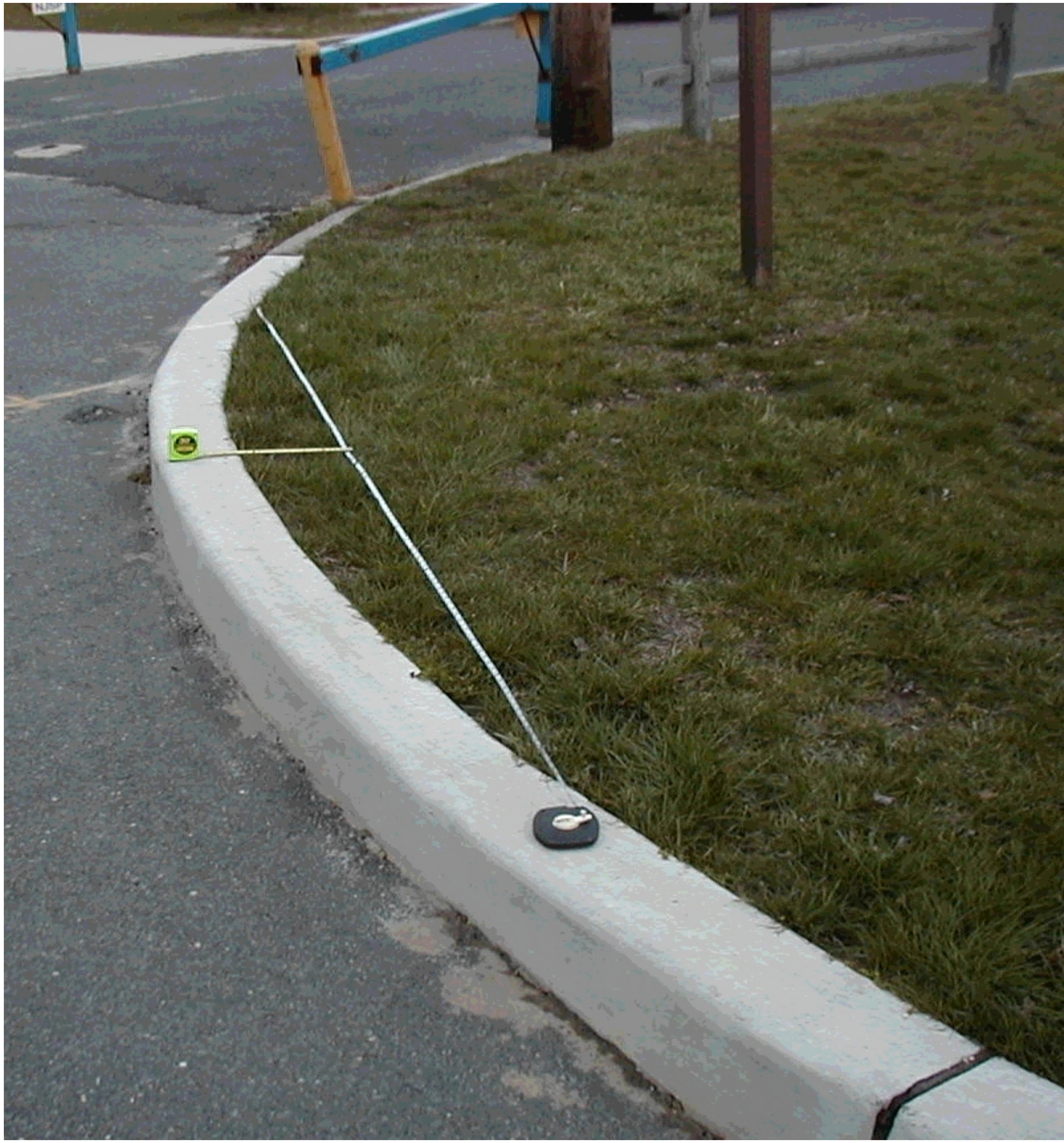
30 mins ÷ 60 = .50 hour (½)

38 mins ÷ 60 = .63 hour , etc

Determining the Radius of Curves

Method to measure radius of curve:

- Chord and middle ordinate
- Will work for any size curve
- Also used to determine speed of <u>yaw marks</u>



Radius Equation

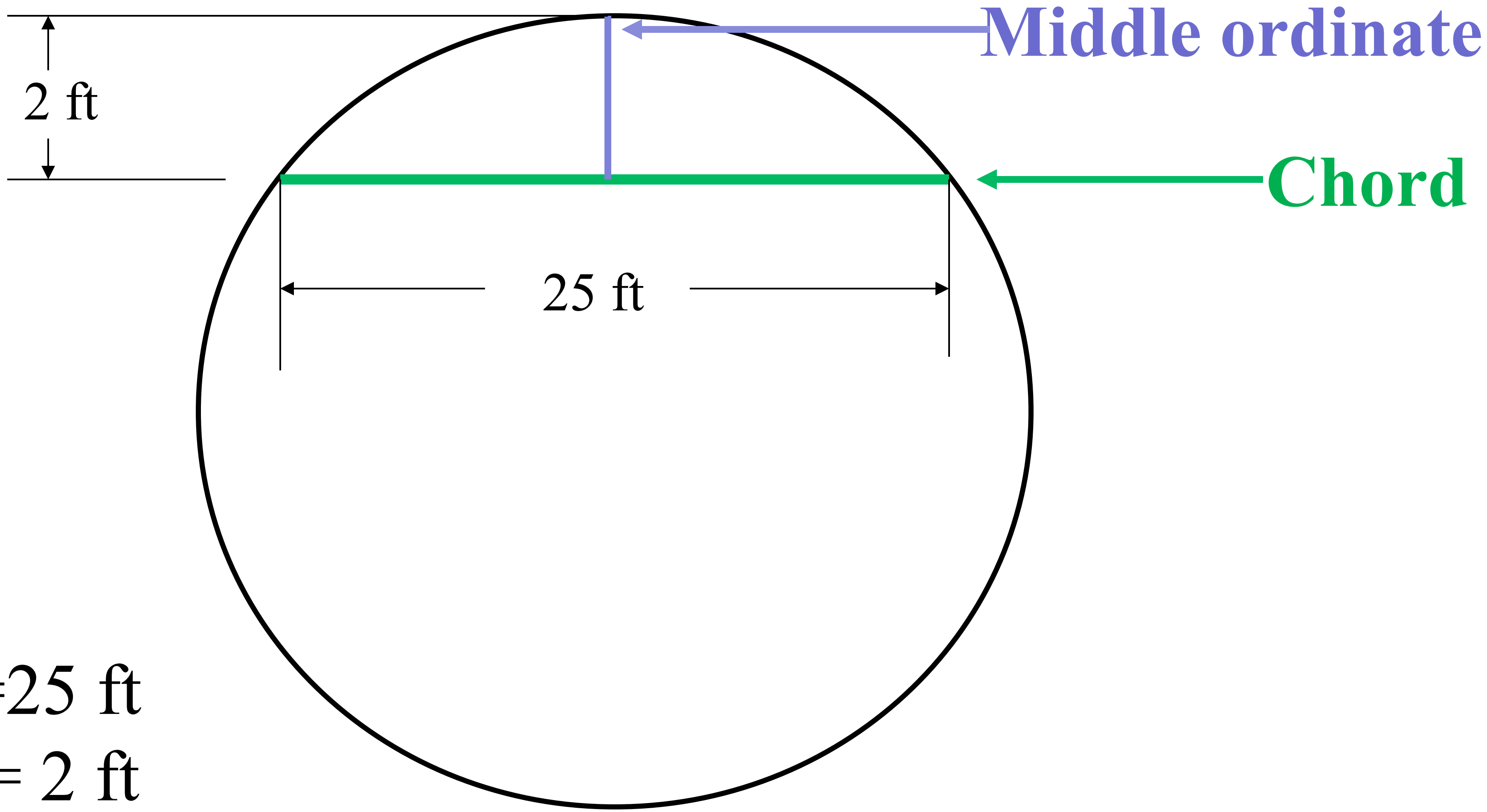
$$R = \frac{C^2}{8M} + \frac{M}{2}$$

R = Radius
C = Length of Chord
M = Middle Ordinate

The Radius Equation is derived from the Pythagorean Theorem

$$c^2 = a^2 + b^2$$

Radius Exercise



C =25 ft

M = 2 ft

Radius Exercise

Known Data

C = 25 ft
M = 2 ft

$$R = \frac{C^2}{8M} + \frac{M}{2}$$

$$R = \frac{25^2}{8(2)} + \frac{2}{2}$$

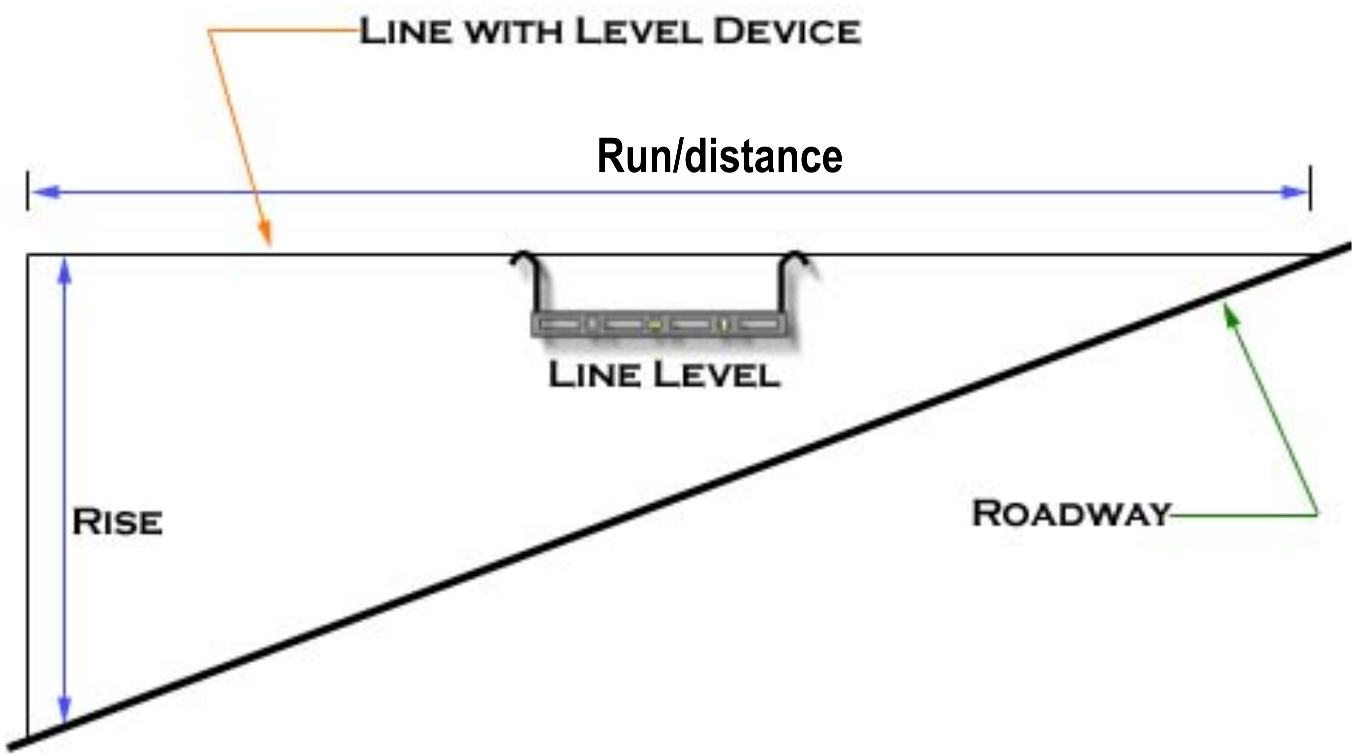
$$R = \frac{625}{16} + \frac{2}{2}$$

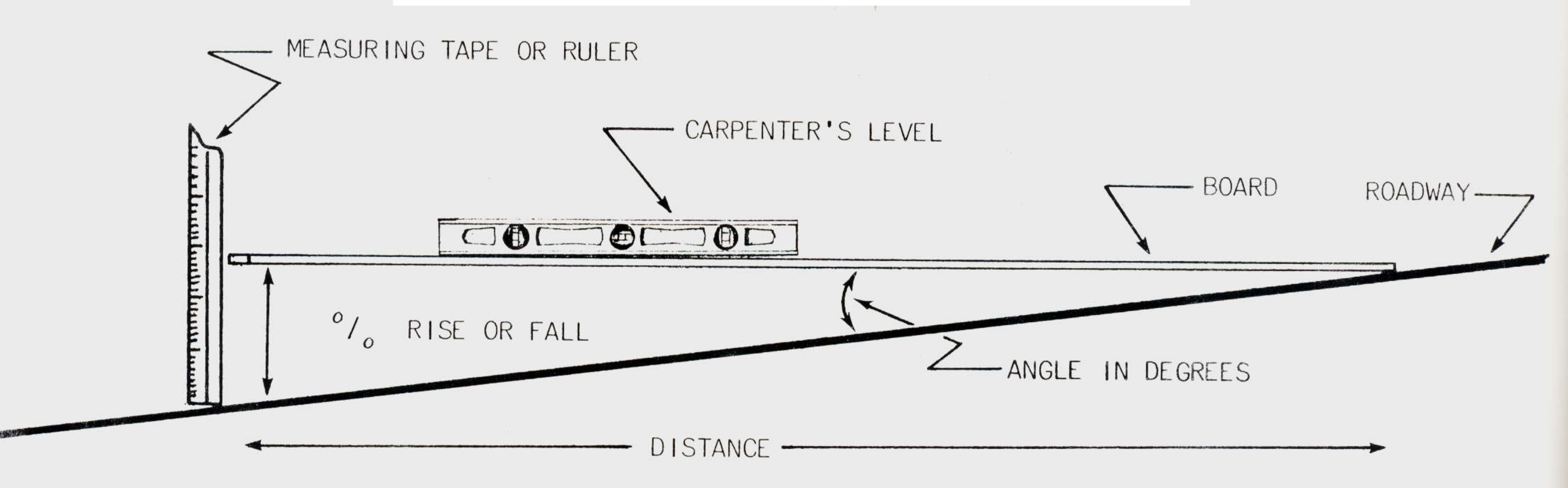
$$R = 39.06 + 1$$

$$R = 40.06\,ft$$

Measuring the Grade of a Road

Roadway grades (slopes) are measured by the rise or fall of the surface per foot of horizontal distance (run) – parallel to the center line of the road





m = Percentage grade (+ or -)

$$m = \frac{\text{Rise or Fall}}{\text{Distance (or Run)}}$$

To Convert Percent to Degrees

Degrees = Percent (ARC-TAN)

Inverse Tangent Key on Scientific Calculator

To Convert Degrees to Percent

Percent = Degrees (Tangent)

Key on Scientific Calculator

Grade Exercise

- You measure a distance of 100 feet along the center line of a roadway and determine that the rise is 2 feet.
- What is the grade?

$$m = \frac{\text{rise}}{\text{distance}}$$

$$m = \frac{2}{100}$$

$$m = .02$$

To change to a % grade

$$.02 \times 100 = 2\% grade$$

How many degrees is 2 % ?

.02 arc-tan = 1.15 degrees

Super-elevation

- Super-elevation ("bank" or "crown") is the grade across a roadway.
- Measured in the same way grade is measured:
  - Perpendicularly to the center line of the roadway, or
  - Parallel along the path of travel

$$e = \frac{\text{Rise or Fall}}{\text{Distance (or Run)}}$$

Coefficient of Friction

Definition:

The ratio of the horizontal force (*F*) that's required to slide an object on a level surface - to the object's vertical force (its weight, or *W*)

$$\text{Symbol} = \mu$$

(Greek Term "Mu")

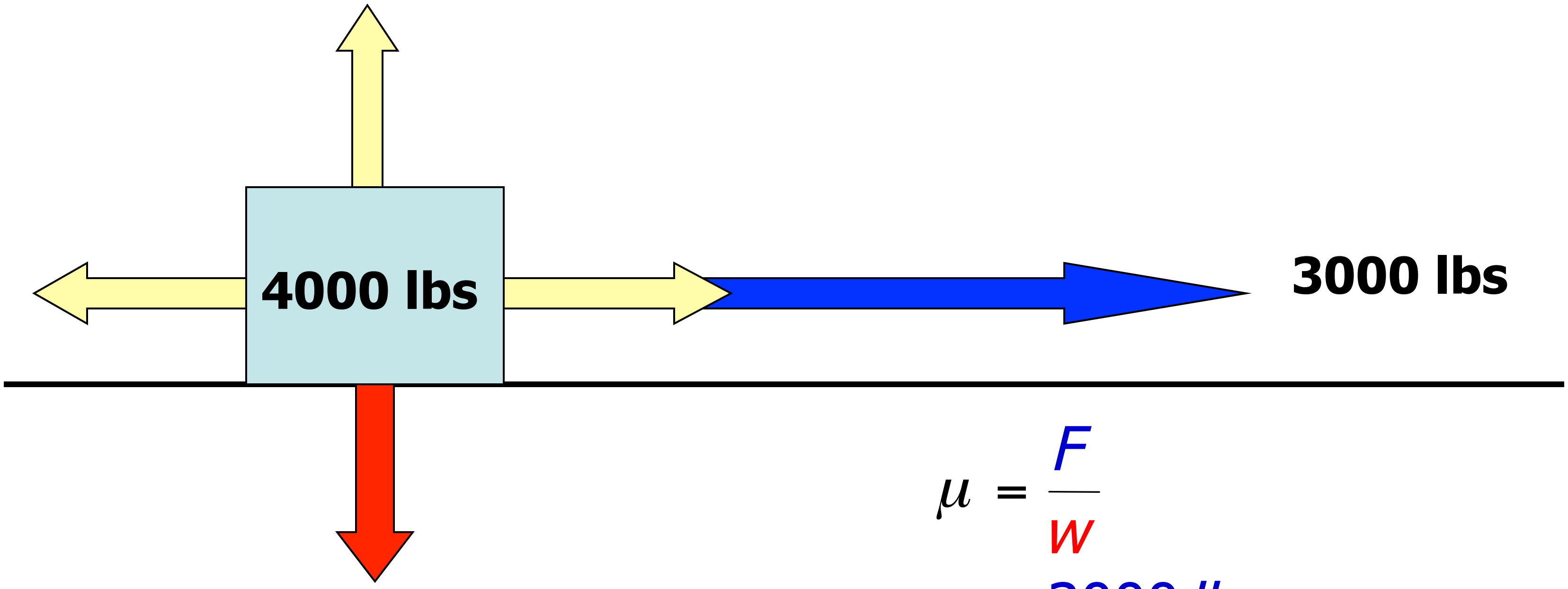
"u" is a surface value – <u>not</u> a vehicle value

Coefficient of Friction

Most coefficient of friction charts assume a level surface with all wheels locked and sliding.

You need to adjust the number taken from a chart to take into account the grade of the road, the number of wheels locked and sliding, and the percent braking of wheels NOT locked and still rolling. The result will be the vehicle's drag factor, otherwise known as its deceleration factor (f) on that surface.

Coefficient of Friction



$$\mu = \frac{F}{W}$$

$$\mu = \frac{3000\,lbs}{4000\,lbs}$$

$$\mu = .75$$

The Relationship Between Coefficient of Friction (*u*) & Drag Factor (*f*)

Coefficient of Friction & Drag Factor

Coefficient of Friction (u) and Drag Factor (f) are only equal when dealing with a level surface and 100% braking

$$f = \mu$$

μ = coefficient of friction (level surface, 100% braking)
f = deceleration or "drag" factor (adjusted for grade and percent braking of each wheel)

Coefficient of Friction & Drag Factor

- To adjust the coefficient of friction for a grade and obtain the correct Deceleration or "Drag" Factor:
  - Add the grade as a decimal to the coefficient of friction if the vehicle skids uphill.
  - Subtract the grade as a decimal from the coefficient of friction if the vehicle skids downhill.
- Adjustment also needed when you have less than all tires locked and sliding.
- All adjustments to the coefficient of friction in order to obtain an adjusted deceleration or "drag" factor must be done prior to using that value in any equations.

Adjusting for Grade

- This equation is used to adjust the level surface coefficient of friction value (μ) according to the grade of the road surface "m". Use (+) for uphill or (-) for down hill

$$f = \mu \pm m$$

f = deceleration factor, adjusted
μ = coefficient of friction (level surface)
m = percent grade (decimal)

Example of Grade Adjustment

- The level surface coefficient of friction value for your road is .70. The collision took place on a 4 % downgrade.
- What is the drag factor?

$$f = \mu \pm m$$

$$f = .70 - .04 = .66$$

Adjusting for Braking Efficiency (percent braking)

If there is one brake (or more) that was not working on a vehicle, the coefficient of friction must be adjusted to represent the lack of (or diminished) braking efficiency on each wheel.

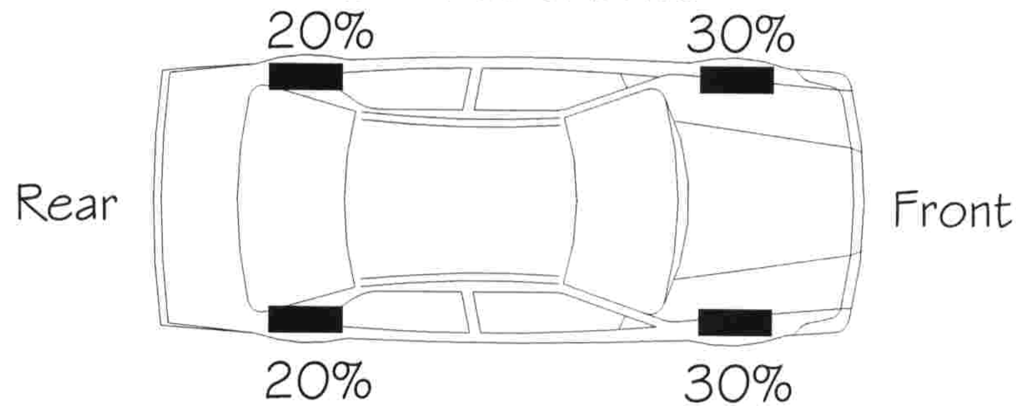
$$f = \mu(\%\,\text{weight})(n)$$

f = deceleration factor (adjusted)
μ = coefficient of friction (level surface)
n = percent braking (decimal)

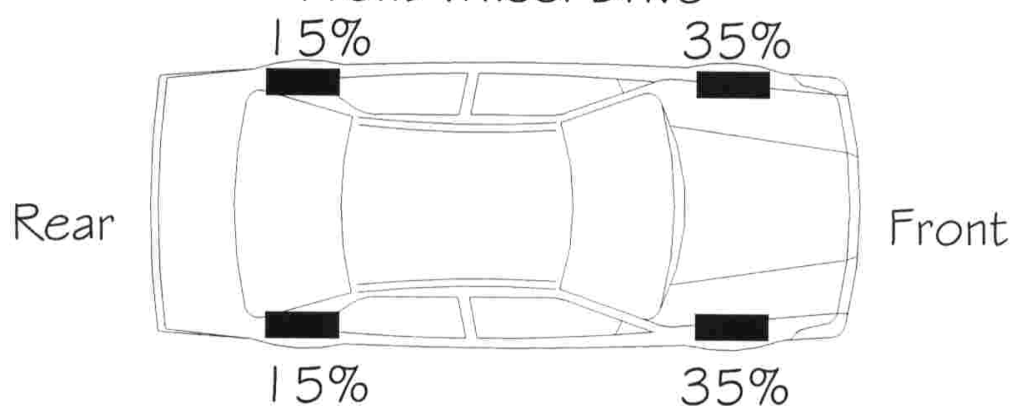
BRAKING EFFICIENCY

BRAKING EFFICIENCY FOR CARS

Rear Wheel Drive



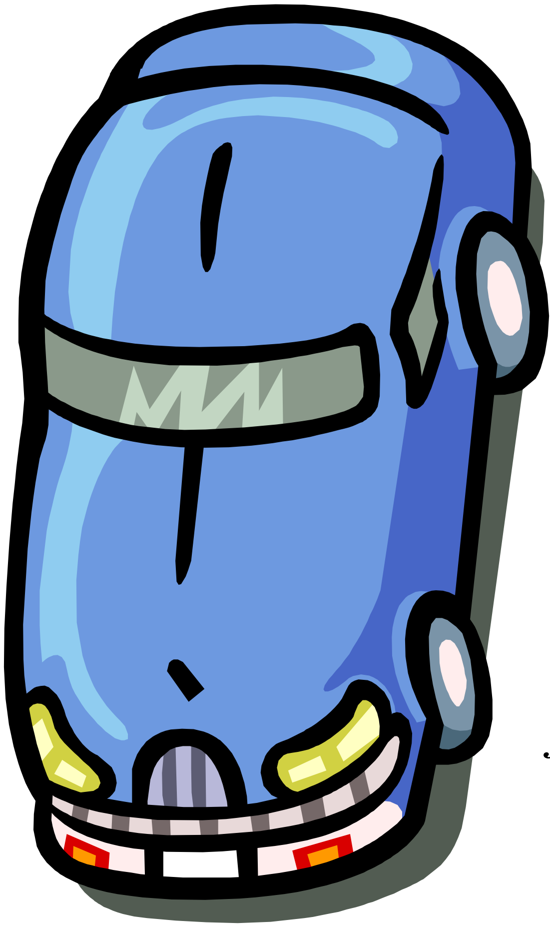
Front Wheel Drive



Braking Efficiency

$$f_{RR} = \mu \cdot \%weight \cdot n$$

$$f_{LR} = \mu \cdot \%weight \cdot n$$



$$f_{RF} = \mu \cdot \%weight \cdot n$$

$$f_{LF} = \mu \cdot \%weight \cdot n$$

$$f_{adj} = f_{RR} + f_{RF} + f_{LR} + f_{LF}$$

Braking Efficiency Example #1

What is the adjusted drag factor for a front wheel drive vehicle with no rear brakes, sliding on a level surface with a coefficient of friction value of .75?

$$f_{LR} = .75 \cdot .15 \cdot 0 = 0$$

$$f_{RR} = .75 \cdot .15 \cdot 0 = 0$$

$$f_{LF} = .75 \cdot .35 \cdot 1 = .2625$$

$$f_{RF} = .75 \cdot .35 \cdot 1 = .2625$$

$$f_{adj} = 0 + .2625 + .2625 + 0 = .525$$

Braking Efficiency Example #2

A front wheel drive vehicle slides with all wheels locked except the left rear, which was braking at ~50% efficiency. The coefficient of friction of the roadway surface was determined to be .75

What is the adjusted drag factor?

$$f_{LR} = .75 \cdot .15 \cdot .50 \qquad f_{RR} = .75 \cdot .15 \cdot 1$$

$$f_{LF} = .75 \cdot .35 \cdot 1 \qquad f_{RF} = .75 \cdot .35 \cdot 1$$

$$f_{adj} = .056 + .262 + .262 + .112 = .692$$

Adjusting for Grade and Percent Braking

This equation is used to adjust the level surface coefficient of friction value (μ) by the percentage of braking (n), and the percentage of grade (m). Use (+) for uphill, use (-) for downhill.

$$f = [(\mu)(n) \pm m]$$

Example of Adjusting for Grade and Percent Braking

Known Data		
Braking (n)	=	70%
Grade (m)	=	-2%
COF (μ)	=	.71

$$f = [(\mu)(n) \pm m]$$

$$f = [(.71)(.70) - .02]$$

$$f = [.49 - .02]$$

$$f = .47$$

Determining Acceleration Rate

This equation will calculate the acceleration (or deceleration) rate of a vehicle when the drag factor is known.

$$a = f \cdot g$$

Gravity Constant (32.2)

Example of Determining Acceleration Rate

$$a = f \cdot g$$

If: $f = .80$

Then: $(.80)\,(32.2) = 25.76\ \text{ft/sec}^2$

(+ if accelerating and – if decelerating)

Test Skids

This equation is used to determine the drag factor for a vehicle by using an exemplar or test vehicle and skidding the vehicle at the same location and in the same direction as the actual crash. This formula is an energy based equation that determines a drag factor when all of the vehicle's energy is used up by skidding.

$$f = \frac{S^2}{30 \cdot D}$$

Constant Velocity

Constant velocity implies that the acceleration rate is zero (not accelerating and not decelerating); therefore, velocity does not change.

$$V = \frac{D\ (\text{feet})}{t\ (\text{secs})}$$

Example: 5280 ft ÷ 55 secs = 96 ft/sec ÷ 1.466 = 65.5 mph

Distance Traveled at Constant Velocity

The **distance** traveled in a given time period at a constant velocity may be determined.

$$D = V \cdot t$$

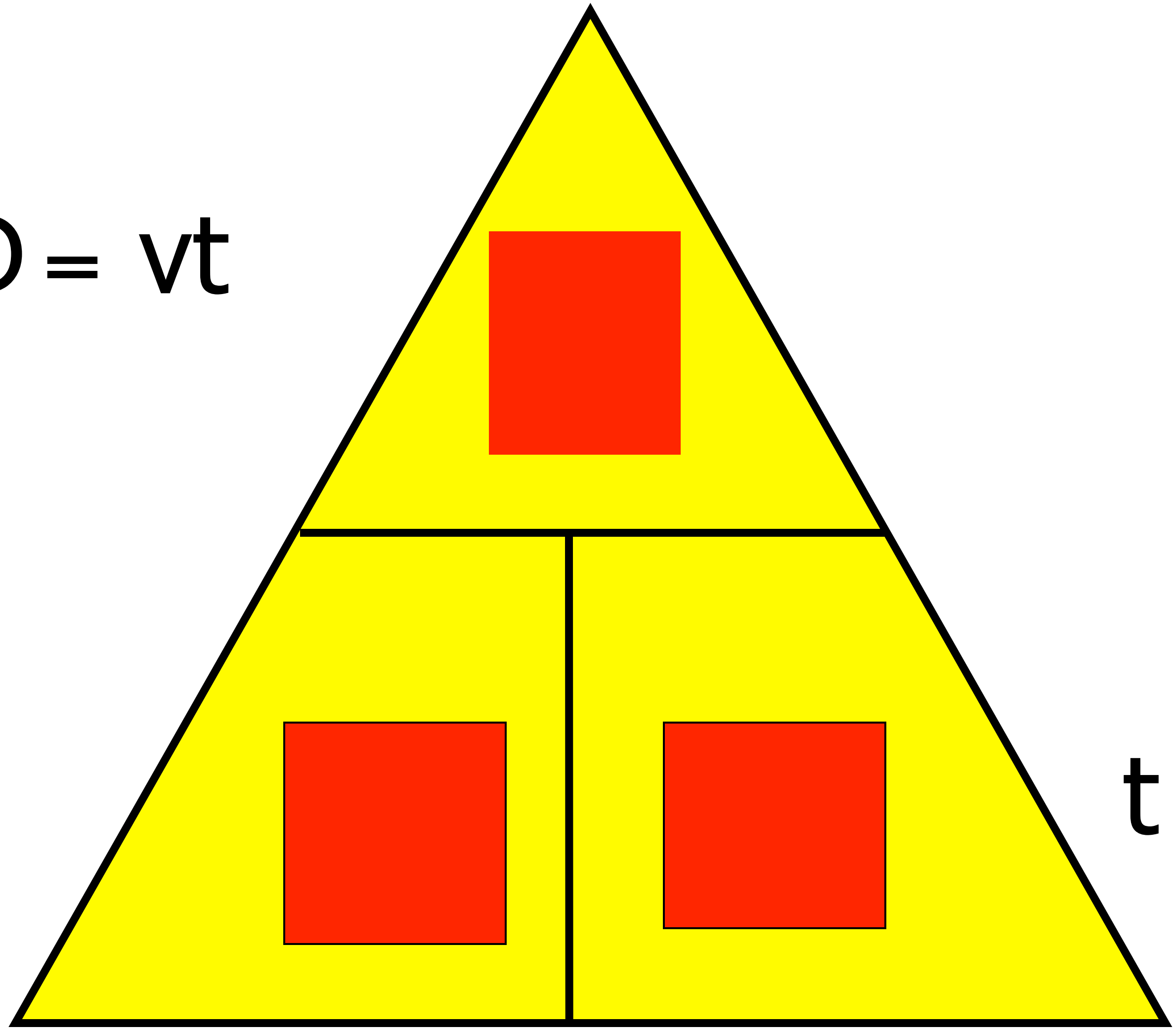
Examples:

60 mph x .25 hours (15 mins) = 15 miles

88 ft/sec x 900 secs (15 mins) = 79,200 feet ÷ 5280 feet = 15 miles

88 ft/sec x 1.5 secs (average reaction time) = 132 feet traveled before anything changes!!!

$$D = vt$$



$$v = \frac{D}{t}$$

$$t = \frac{D}{v}$$

Critical Speed

Use this equation when the super-elevation (crown) of the road is greater than 10%.

$$S = (3.86)\sqrt{\frac{R \cdot f}{1 - \mu \cdot e}}$$

The End

References: “Equations for the Traffic Crash Reconstructionist”, IPTM

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