Mental Math

Fact Learning
Mental Computation
Estimation

Grade 5
Teacher's Guide

Prince Edward Island
CANADA
Department of Education
English Programs

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Mental Computation
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2007

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Mental Math in the Elementary Mathematics Curriculum

Mental math in this guide refers to fact learning, mental computation, and computational estimation. The Atlantic Canada Mathematics Curriculum supports the acquisition of these skills through the development of thinking strategies across grade levels.

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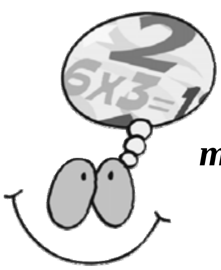
Pre-Operational Skills

Many children begin school with a limited understanding of number and number relationships. Counting skills, which are essential for ordering and comparing numbers, are an important component in the development of number sense. Counting on, counting back, concepts of more and less, and the ability to recognize patterned sets, all mark advances in children's development of number ideas.



Basic facts are mathematical operations for which some students may not be conceptually prepared.

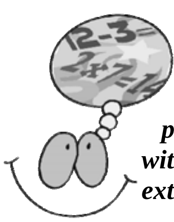
Basic facts are mathematical operations for which some students may not be conceptually prepared. As a minimum, the following skills should be in place before children are expected to acquire basic facts.

- Students can immediately name the number that comes after a given number from 0-9, or before a given number from 2-10.
- When shown a familiar arrangement of dots $\leq$ 10 on ten frames, dice, or dot cards, students can quickly identify the number without counting.

- For numbers $\leq$ 10 students can quickly name the number that is one-more, one-less; two-more, two-less. (the concept of less tends to be more problematic for children and is related to strategies for the subtraction facts)

Mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades.

Curriculum Outcomes	Thinking Strategies
Grade 1 **B7-** use mental strategies to find sums to 18 and differences from 18 or less **B8-** memorize simple addition and/or subtraction facts from among those for which the total is 10 or less **C5-** use number patterns to help solve addition and subtraction sentences	**P. 28** • Doubles Facts for addition and subtraction facts **P. 36** • Using patterns to learn the facts • Commutative property (3+2 = 2+3)

Grade 2 **B5-** develop and apply strategies to learn addition and subtraction facts **B11-** estimate the sum or difference of two 2-digit numbers  ***Fact learning is a mental exercise with an oral and/or visual prompt; the focus is oral, rather than paper-and pencil; drills should be short with immediate feedback over an extended period of time.***	**P. 22** • Doubles plus 1 • Make 10 (“bridging to 10”) • Two-apart facts; double in-between • Subtraction as “think addition” • Compensation • Balancing for a constant difference **P. 30** (Estimation) • Rounding both numbers to the nearest 10 • Round one number up and one number down • Front-end estimation

Grade 3 **B11/12-** mentally add and subtract two-digit and one-digit numbers, and rounded numbers. **B9-** continue to estimate in addition and subtraction situations **B10-** begin to estimate in multiplication and division situations **C3 -** use and recognize the patterns in a multiplication table	**P. 34** • Make 10 • Compatible numbers (“partner” numbers) • Front-end addition • Back up through ten (“counting on”) • Compensation • Balancing for a constant difference **P. 28** • Commutative property for multiplication (3x2 = 2x3) • Division as “think multiplication” • Helping facts

Curriculum Outcomes	Thinking Strategies
Grade 4 **B9 -** demonstrate a knowledge of the multiplication facts to 9 x 9 **B14 -** estimate the product or quotient of 2- or 3-digit numbers and single digit numbers **B15 -** mentally solve appropriate addition and subtraction computations **B16 -** mentally multiply 2-digit numbers by 10 or 100 **C2 -** apply the pattern identified when multiplying by increasing powers of 10	**P. 32** • Doubles • Clock-facts for 5s • Patterns for 9s • Helping facts **P. 36** (Estimation) • Rounding • Front-end • Clustering of Compatibles **P. 38** • Compatibles for division **P. 40** • Front-end addition • Compensation • Up through 100 (counting on) • Back down through 100 (counting back) • Compatible numbers • Place-value-change strategy for mentally multiplying by 10, 100

Curriculum Outcomes	Thinking Strategies
Grade 5 **B10-** estimate sums and differences involving decimals to thousandths **B11-** estimate products and quotients of two whole numbers **B12-** estimate products and quotients of decimal numbers by single-digit whole numbers **B15-** multiply whole numbers by 0.1, 0.01, and 0.001 mentally **C2-** recognize and explain the pattern in dividing by 10, 100, 1000 and in multiplying by 0.1, 0.01 and 0.001 **B13-** perform appropriate mental multiplications with facility  ***By grade 5, students should possess a variety of strategies to compute mentally. It is important to recognize that these strategies develop and improve over the years with regular practice.***	**P. 40 to 41** (Estimation) • Rounding one up, one down • Looking for compatibles that make approximately 10, 100, 1000 • Front-end **P. 44** • Place-value-change strategy for mentally multiplying by 10, 100, 1000 • “Halve-double” strategy for multiplication • Front-end multiplication • Compensation **P. 46 to 50** • Place-value-change strategy for mentally dividing by 10, 100, 1000 • Place-value-change strategy for mentally multiplying by 0.1, 0.01, 0.001

Grade 6 **B9-** estimate products and quotients involving whole numbers only, whole numbers and decimals, and decimals only **B10-** divide numbers by 0.1, 0.01, and 0.001 mentally **C2-** use patterns to explore division by 0.1, 0.01, and 0.001 **B11-** calculate sums and differences in relevant contexts using the most appropriate method	**P. 40** (Estimation) • Rounding one up, one down for multiplication • Front-end method for multiplication and division **P. 42 and 50** • Place-value-change strategy for mentally dividing by 0.1, 0.01, 0.001 **P. 44** • Compensation in multiplication • Front-End

Students should perform mental computations with facility using strategies outlined in the Mental Math Guides.

Definitions and Connections

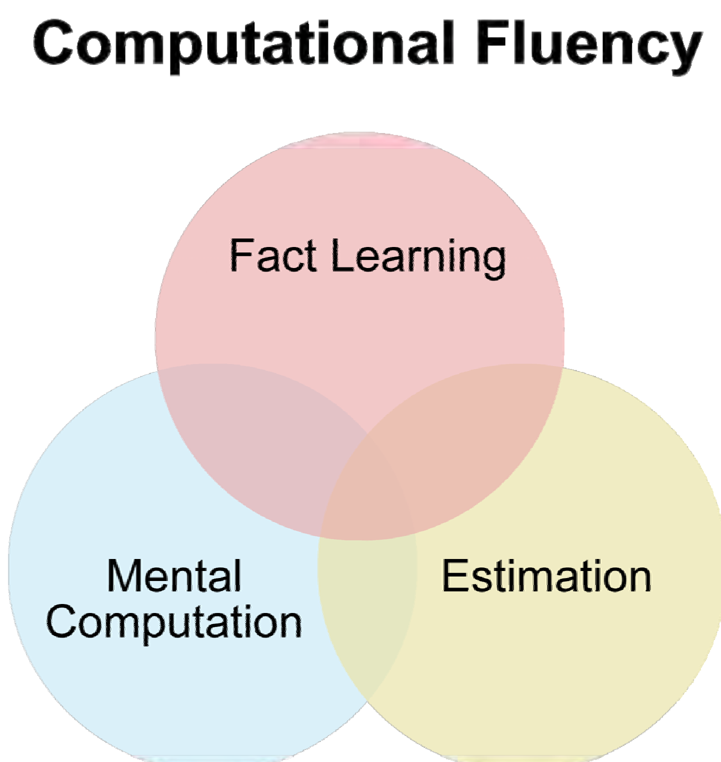
Fact learning refers to the acquisition of the 100 number facts relating to the single digits 0-9 in each of the four operations. Mastery is defined by a correct response in 3 seconds or less.

Mental computation refers to using strategies to get exact answers by doing most of the calculations in one's head. Depending on the number of steps involved, the process may be assisted by quick jottings of sub-steps to support short term memory.

Computational estimation refers to using strategies to get approximate answers by doing calculations mentally.

Students develop and use thinking strategies to recall answers to basic facts. These are the foundation for the development of other mental calculation strategies. When facts are automatic, students are no longer using strategies to retrieve them from memory.

Basic facts and mental calculation strategies are the foundations for estimation. Attempts at estimation are often thwarted by the lack of knowledge of the related facts and mental math strategies.



Rationale

In modern society, the development of mental computation skills needs to be a goal of any mathematical program for two important reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people still need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

In modern society, the development of mental computation skills needs to be a goal of any mathematics program.

Besides being the foundation of the development of number and operation sense, fact learning is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these are numerical. Without a command of the basic facts, it is very difficult to detect these patterns and relationships. As well, nothing empowers students more with confidence, and a level of independence in mathematics, than a command of the number facts.

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Teaching Mental Computation Strategies

The development of mental math skills in the classroom should go beyond drill and practice by providing exercises that are meaningful in a mathematical sense. All of the strategies presented in this guide emphasize learning based on an understanding of the underlying logic of mathematics.

While learning addition, subtraction, multiplication and division facts, for instance, students learn about the properties of these operations to facilitate mastery. They apply the commutative property of addition and multiplication, for example, when they discover that 3 + 7 is the same as 7 + 3 or that 3 x 7 = 7 x 3. Knowing this greatly reduces the number of facts that need to be memorized. They use the distributive property when they learn that 12 x 7 is the same as (10 + 2) x 7 = (7 x 10) + (2 x 7) which is equal to 70 + 14 = 84.

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Understanding our base ten system of numeration is key to developing computational fluency. At all grades, beginning with single digit addition, the special place of the number 10 and its multiples is stressed. In addition, students are encouraged to add to make 10 first, and then add beyond the ten. Addition of ten and multiples of ten is emphasized, as well as multiplication by 10 and its multiples.

Connections between numbers and the relationship between number facts should be used to facilitate learning. The more connections that are established, and the greater the understanding, the easier it is to master facts. In multiplication, for instance, students learn that they can get to 6 x 7 if they know 5 x 7, because 6 x 7 is one more group of 7.

Introducing Thinking Strategies to Students

In general, a strategy should be introduced in isolation from other strategies. A variety of practice should then be provided until it is mastered, and then it should be combined with other previously learned strategies. Knowing the name of a strategy is not as important as knowing how it works. That being said, however, knowing the names of the strategies certainly aids in classroom communication. In the mental math guides for each grade, strategies are consistently named; however, in some other resources, you may find the same strategy called by a different name.

When introducing a new strategy, use the chalkboard, overhead or LCD projector, to provide students with an example of a computation for which the strategy works. Are there any students in the class who already have a strategy for doing the computation in their heads? If so, encourage them to explain the strategy to the class with your help. If not, you could share the strategy yourself.

Explaining the strategy should include anything that will help students see its pattern, logic, and simplicity. That might be concrete materials, diagrams, charts, or other visuals. The teacher should also “think aloud” to model the mental processes used to apply the strategy and discuss situations where it is most appropriate and efficient as well as those in which it would not be appropriate at all.

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In the initial activities involving a strategy, you should expect to have students do the computation the way you modeled it. Later, however, you may find that some students employ their own variation of the strategy. If it is logical and efficient for them, so much the better. Your goal is to help students broaden their repertoire of thinking strategies and become more flexible thinkers; it is not to prescribe what they must use.

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You may find that there are some students who have already mastered the simple addition, subtraction, multiplication and division facts with single-digit numbers. Once a student has mastered these facts, there is no need to learn new strategies for them. In other words, it is not necessary to re-teach a skill that has been learned in a different way.

On the other hand, most students can benefit from the more difficult problems even if they know how to use the written algorithm to solve them. The emphasis here is on mental computation and on understanding the place-value logic involved in the algorithms. In other cases, as in multiplication by 5 (multiply by 10 and divide by 2), the skills involved are useful for numbers of all sizes.

Practice and Reinforcement

In general, it is the frequency rather than the length of practice that fosters retention. Thus daily, brief practices of 5-10 minutes are most likely to lead to success.

In general, it is the frequency rather than the length of practice that fosters retention. Thus daily, brief practices of 5-10 minutes are most likely to lead to success. Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises should be varied in type, and focus as much on the discussion of how students obtained their answers as on the answers themselves.

The selection of appropriate exercises for the reinforcement of each strategy is critical. The numbers should be ones for which the strategy being practiced most aptly applies and, in addition to lists of number expressions, the practice items should often include applications in contexts such as money, measurements and data displays. Exercises should be presented with both visual and oral prompts and the oral prompts that you give should expose students to a variety of linguistic descriptions for the operations. For example, 5 + 4 could be described as:

- the sum of 5 and 4
- 4 added to 5
- 5 add 4
- 5 plus 4
- 4 more than 5
- 5 and 4 etc.

Response Time

- *Basic Facts*

In the curriculum guide, fact mastery is described as a correct response in 3 seconds or less and is an indication that the student has committed the facts to memory. This 3-second-response goal is a guideline for teachers and does not need to be shared with students if it will cause undue anxiety. Initially, you would allow students more time than this as they learn to apply new strategies, and reduce the time as they become more proficient.

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- *Mental Computation Strategies*

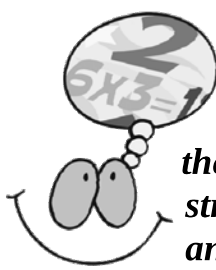
With other mental computation strategies, you should allow 5 to 10 seconds, depending on the complexity of the mental activity required. Again, in the initial stages, you would allow more time, and gradually decrease the wait time until students attain a reasonable time frame. While doing calculations in one's head is the principal focus of mental computation strategies, sometimes in order to keep track, students may need to record some sub-steps in the process. This is particularly true in computational estimation when the numbers may be rounded. Students may need to record the rounded numbers and then do the calculations mentally for these rounded numbers.

In many mental math activities it is reasonable for the teacher to present a mental math problem to students, ask for a show of hands, and then call on individual students for a response. In other situations, it may be more effective when all students participate simultaneously and the teacher has a way of checking everyone's answers at the same time. Individual response boards or student dry-erase boards are tools which can be used to achieve this goal.

Struggling Students and Differentiated Instruction

It is imperative that teachers identify the best way to maximize the participation of all students in mental math activities.

It is imperative that teachers identify the best way to maximize the participation of all students in mental math activities. Undoubtedly there will be some students who experience considerable difficulty with the strategies assigned to their grade and who require special consideration. You may decide to provide these students with alternative questions to the ones you are expecting the others to do, perhaps involving smaller or more manageable numbers. Alternatively, you may just have the student complete fewer questions or provide more time.



There may be students in the upper grades who do not have command of the basic facts. For the teacher, that may mean going back to strategies at a lower grade level to build success, and accelerating them vertically to help students catch up.

There may be students in the upper grades who do not have command of the basic facts. For the teacher, that may mean going back to strategies at a lower grade level to build success, and accelerating them vertically to help students catch up. For example, if the students are in grade 6 and they don't yet know the addition facts, you can find the strategies for teaching them in the grade 2 Mental Math Guide and the grade 2 Curriculum Guide. The students, however, are more intellectually mature, so you can immediately apply those same strategies to tens, hundreds, and thousands, and to estimation of whole numbers and decimal sums.

The more senses you can involve when introducing the facts, the greater the likelihood of success for all students, but especially for students experiencing difficulty.

Many of the thinking strategies supported by research and outlined in the curriculum advocate for a variety of learning modalities.
For example:

- Visual (images for the addition doubles; hands on a clock for the "times-five" facts)

- Auditory (silly sayings and rhymes: "6 times 6 means dirty tricks; 6 x 6 is 36")

- Patterns in Number (the product of an even number multiplied by 5 ends in 0 and the tens digit is one less than the number being multiplied)

- Tactile (ten frames, base ten blocks)

- Helping Facts (8 x 9 = 72, so 7 x 9 is one less group of 9; 72 - 9 = 63)

Whatever differentiation you make it should be to facilitate the student's development in mental computation, and this differentiation should be documented and examined periodically to be sure it is still necessary.

Combined Grade Classrooms

What you do in these situations may vary from one strategy to another. Sometimes the students may be all doing the same strategy, sometimes with the same size or type of number, sometimes with different numbers. For example, in a combined grade 2-3 class, students might be working on the "make ten" strategy for addition. The teacher would ask the grade 2 students questions such as 9 + 6 or 5 + 8, while the grade 3 students would be given questions such as 25 + 8 or 39 + 6; the same strategy is applied, but at different levels of difficulty.

Other times, you may decide to introduce different strategies at different times on the first day, but conduct the reinforcements at the same time on subsequent days using the appropriate exercises for each grade level.

It is important to remember that there will be students in the lower grade who can master some, or all, the strategies expected for the higher grade, and some students in the higher grade who will benefit from the reinforcement of the strategies from the lower grade.

Assessment

Your assessment of mental computation should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the practice sessions. You should also ask students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where paper-and-pencil responses are weak.

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Timed Tests of Basic Facts

Some of the former approaches to fact learning were based on stimulus-response; that is, the belief that students would automatically give the correct answer if they heard the fact over-and-over again. No doubt, many of us learned our facts this way. These approaches often used a whole series of timed tests of 50 to 100 items to reach the goal.

... the thinking strategy approach prescribed by our curriculum is to teach students strategies that can be applied to a group of facts with mastery being defined as a correct response in 3 seconds or less.

In contrast, the thinking strategy approach prescribed by our curriculum is to teach students strategies that can be applied to a group of facts with mastery being defined as a correct response in 3 seconds or less. The traditional timed test would have limited use in assessing this goal. To be sure, if you gave your class 50 number facts to be answered in 3 minutes and some students completed all, or most, of them correctly, you would expect that these students know their facts. However, if other students only completed some of these facts and got many of those correct, you wouldn't know how long they spent on each question and you wouldn't have the information you need to assess the outcome. You could use these sheets in alternative ways, however.

For example:

- Ask students to quickly circle the facts which they think are "hard" for them and just complete the others. This type of self assessment can provide teachers with valuable information about each student's level of confidence and perceived mastery.
- Ask students to circle and complete only the facts for which a specific strategy would be useful. For example, circle and complete all the "double-plus-1" facts.
- Ask them to circle all the "make ten" facts and draw a box around all "two-apart" facts. This type of activity provides students with the important practice in strategy selection and allows the teacher to assess whether or not students recognize situations for which a particular strategy works.

Parents and Guardians: Partners in Developing Mental Math Skills

Parents and guardians are valuable partners in reinforcing the strategies you are developing in school. You should help parents understand the importance of these strategies in the overall development of their children's mathematical thinking, and encourage them to have their children do mental computation in natural situations at home and out in the community. Through various forms of communication, you should keep parents abreast of the strategies you are teaching and the types of mental computations they should expect their children to be able to do.

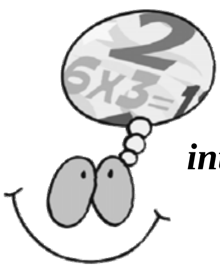
Fact Learning

A. Fact Learning - Addition and Subtraction

- **Reviewing Addition and Subtraction Facts and Fact Learning Strategies**

  At the beginning of grade 5, it is important to ensure that students review the addition and subtraction facts with sums/minuends to 18 and the fact learning strategies addressed in previous grades. All subtraction facts can be completed using a “think addition” strategy, especially by students who know their addition facts very well.

  Students can use these facts and strategies when doing mental math addition with numbers in the 10s, 100s, 1000s and 10 000s in grade 5. Further information about these strategies can be found in the grade 3 mental math teacher’s guide.



At the beginning of grade 5, it is important to review the addition and subtraction facts to 18 and the fact learning strategies introduced in previous grades.

Examples

The following are the addition fact strategies with examples, and examples of the same facts applied to 10s, 100, and 1000s:

a) **Doubles Facts**: 4 + 4, 40 + 40, 400 + 400, and 4000 + 4000

b) **Plus One Facts**: (next number) 5 + 1, 50 + 10, 500 + 100, 5000 + 1000

c) **Plus Two Facts**: (2-more-than facts) 7 + 2, 70 + 20, 700 + 200, 7000 + 2000

d) **Plus Three Facts**: 6 + 3, 60 + 30, 600 + 300, 6000 + 3000

e) **Near Doubles**: (1-apart facts) 3 + 4, 30 + 40, 300 + 400, 3000 + 4000

f) **Plus Zero Facts**: (no-change) 8 + 0, 80 + 0, 800 + 0, 8000 + 0

g) **Doubles Plus 2 Facts**: (double in-between or 2-apart facts) 5 + 3, 50 + 30, 500 + 300, 5000 + 3000

h) **Make 10 Facts**: 9 + 6, 90 + 60, 900 + 600; 8 + 4, 80 + 40, 800 + 400

I) **Make 10 Extended**: (with a 7) 7+ 4, 70 + 40, 700 + 400, 7000 + 4000

A thinking strategy is a way of thinking that helps complete a fact quickly. For a strategy to be a thinking strategy, it must be done mentally and it must be efficient. Students who have mastered the number facts no longer rely on thinking strategies to recall them.

B. Fact Learning – Multiplication and Division

Most students will have mastered the multiplication facts with products to 81 by the end of grade 4. These facts, and the thinking strategies used to acquire them, should be reviewed and reinforced throughout the year. The related division facts will be mastered in grade 5.

- **Multiplication Fact Learning Strategies**
  Review of the multiplication facts and the related fact learning strategies should be done at the beginning of grade 5. Most students will have mastered the multiplication facts with products to 81 by the end of grade 4.This year, students will then apply these strategies to the related division facts and work toward mastery.

  Following are the strategies to be introduced by the teacher, in sequence, starting at grade 3 and continuing through grade 6 for those students who need them. An understanding of the commutative or "turnaround" property in multiplication greatly reduces the number of facts to be mastered.

- **x2 Facts** (with turnarounds): 2x2, 2x3, 2x4, 2x5, 2x6, 2x7, 2x8, 2x9
  These are directly related to the addition doubles and teachers need to make this connection clear. For example, 3 + 3 is double 3 (6); 3 x 2 and 2 x 3 are also double 3.

- **Nifty Nines** (with turnarounds): 6x9, 7x9, 8x9, 9x9
  There are two patterns in the nine-times table that students should discover:

  1. When you multiply a number by 9, the digit in the tens place in the product is one less than the number being multiplied. For example in 6 x 9, the digit in the tens place of the product will be 5.

  2. The two digits in the product must add up to 9. So in this example, the number that goes with 5 to make nine is 4. The answer, then, is 54.

Some students might also figure out their 9-times facts by multiplying first by 10, and then subtracting. For example, for 7 x 9 or 9 x 7, you could think “7 tens is 70, so 7 nines is 70 -7, or 63.

- **Fives Facts** (with turnarounds): 5x3, 5x4, 5x5, 5x6, 5x7

  It is easy to make the connection to the multiplication facts involving 5s using an analog clock. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 can be made. This is why you may see the Five Facts referred to as the “clock facts.” This would be the best strategy for students who know how to tell time on an analog clock, a specific outcome from the grade 3 curriculum.
  You should also introduce the two patterns that result when numbers are multiplied by 5:

  1. For even numbers multiplied by 5, the answer always ends in zero, and the digit in the tens place is half the other number. So, for 8 x 5 = 40.

  2. For odd numbers multiplied by 5, the product always ends in 5, and the digit in the tens place is half of the number that comes before the other number. So, for 5 x 9 = 45.

The more senses you can involve when introducing the facts, the greater the likelihood of success for all students, but especially for students experiencing difficulty.

- **Ones Facts** (with turnarounds): 1x1, 1x2, 1x3, 1x4, 1x5, 1x6, 1x7, 1x8, 1x9
  While the ones facts are the “no change” facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. For example 6 × 1 means six groups of 1 or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means one group of 6. It is important to avoid teaching arbitrary rules such as “any number multiplied by one is that number”. Students will come to this rule on their own given opportunities to develop understanding.

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- **The Tricky Zeros Facts**
  As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero. Teachers must help students understand the meaning of the number sentence. For example: 6 × 0 means "six 0s or "six sets of nothing." This could be shown by drawing six boxes with nothing in each box. 0 × 6 means "zero sets of 6." Ask students to use counters or blocks to build two sets of 6, then 1 set of 6 and finally zero sets of 6 where they don't use any counters or blocks. They will quickly realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero". Students will come to this rule on their own, given opportunities to develop understanding.

- **Threes Facts** (with turnarounds): 3x3, 3x4, 3x6, 3x 7, 3x8, 3x9
  The strategy here, is for students to think "times 2, plus another group". So for 7 x 3 or 3 x 7, the student should think "7 times 2 is 14 , plus 7 more is 21".

- **Fours Facts** (with turnarounds): 4x4, 4x6, 4x7, 4x8, 4x9

  One strategy that works for any number multiplied by 4 is "double-double". For example, for 6 x 4, you would double the 6 (12) and then double again (24). Another strategy that works any time one (or both) of the factors is even, is to divide the even number in half, then multiply, and then double your answer. So, for 7 x 4, you could multiply 7 x 2 (14) and then double that to get 28. For 16 x 9, think 8 x 9 (72) and 72 + 72 = 70 + 70 (140) plus 4 = 144.

- **The Last Six Facts**

  After students have worked on the above seven strategies for learning the multiplication facts, there are only six facts left to be learned and their turnarounds: 6 × 6; 6 × 7; 6 × 8; 7 × 7; 7 × 8 and 8 × 8. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. You should put each fact before them and ask for their suggestions.

Multiplication Facts With Products to 81 –
Clustered by Thinking Strategy and in Sequence

Facts With 2
(addition doubles)

2x1	1x2
2x2	
2x3	3x2
2x4	4x2
2x5	5x2
2x6	6x2
2x7	7x2
2x8	8x2
2x9	9x2

Facts With 10
(Not officially a “basic fact”, but included here since our number system is base-ten)

10x1	1x10
10x2	2x10
10x3	3x10
10x4	4x10
10x5	5x10
10x6	6x10
10x7	7x10
10x8	8x10
10x9	9x10
10x10	

Facts With 5
(Clock Facts)

5x1	1x5
5x2	2x5
5x3	3x5
5x4	4x5
5x5	
5x6	6x5
5x7	7x5
5x8	8x5
5x9	9x5

Facts With 9
(Patterns)

9x1	1x9
9x2	2x9
9x3	3x9
9x4	4x9
9x5	5x9
9x6	6x9
9x7	7x9
9x8	8x9
9x9	

Facts With 1
(no change facts)

1x1	
1x2	2x1
1x3	3x1
1x4	4x1
1x5	5x1
1x6	6x1
1x7	7x1
1x8	8x1
1x9	9x1

Facts With 0
(Facts with zero have products of zero)

0x0	
0x1	1x0
0x2	2x0
0x3	3x0
0x4	4x0
0x5	5x0
0x6	6x0
0x7	7x0
0x8	8x0
0x9	9x0

Square Facts
(These facts (and others like them) form square arrays)

3x3
4x4
6x6
7x7
8x8

Facts With 4
(Double-Double)

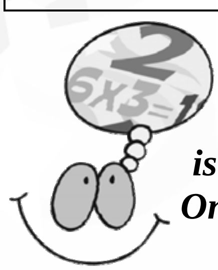
4x1	1x4
4x2	2x4
4x3	3x4
4x4	
4x5	5x4
4x6	6x4
4x7	7x4
4x8	8x4
4x9	9x4

Times-3 Facts
(Double-plus 1 more set)

3x6	6x3
3x7	7x3
3x8	8x3

Last 6 Facts

6x7	7x6
6x8	8x6
7x8	8x7



After students have mastered each cluster of multiplication facts, it is appropriate to have them learn the corresponding division facts. One strategy for learning the division facts is “think multiplication”.

Division Facts With Dividends to 81 – Clustered by Thinking Strategy and in Sequence

Facts With 2
(addition doubles)

2÷1	2÷2
4÷2	
6÷3	6÷2
8÷4	8÷2
10÷5	10÷2
12÷6	12÷2
14÷7	14÷2
16÷8	16÷2
18÷9	18÷2

Facts With 10
(Not officially a “basic fact”, but included here since our number system is base-ten)

10÷10	10÷1
20÷10	20÷2
30÷10	30÷3
40÷10	40÷4
50÷10	50÷5
60÷10	60÷6
70÷10	70÷7
80÷10	80÷8
90÷10	90÷9
100÷10	

Facts With 5
(Clock Facts)

5÷1	5÷5
10÷2	10÷5
15÷3	15÷5
20÷4	20÷5
25÷5	
30÷6	30÷5
35÷7	35÷5
40÷8	40÷5
45÷9	45÷5

Facts With 9
(Patterns)

9÷1	9÷9
18÷2	18÷9
27÷3	27÷9
36÷4	36÷9
45÷5	45÷9
54÷6	54÷9
63÷7	63÷9
72÷8	72÷9
81÷9	

Facts With 1
(no change facts)

1÷1	
2÷2	2÷1
3÷3	3÷1
4÷4	4÷1
5÷5	5÷1
6÷6	6÷1
7÷7	7÷1
8÷8	8÷1
9÷9	9÷1

Facts With 0
(Facts with zero have products of zero)

0÷0	
0÷1	1÷0
0÷2	2÷0
0÷3	3÷0
0÷4	4÷0
0÷5	5÷0
0÷6	6÷0
0÷7	7÷0
0÷8	8÷0
0÷9	9÷0

Square Facts
(These facts (and others like them) form square arrays)

9÷3
16÷4
36÷6
49÷7
64÷8

Facts With 4
(Double-Double)

8÷2	8÷4
12÷3	12÷4
16÷4	
20÷5	20÷4
24÷6	24÷4
28÷7	28÷4
32÷8	32÷4
36÷9	36÷4

Times-3 Facts
(Double-plus 1 more set)

18÷6	18÷3
21÷7	21÷3
24÷8	24÷3

Last 6 Facts

42÷6	42÷7
48÷8	48÷6
56÷7	56÷8

Mental Computation

C. Mental Computation – Addition

Your goal for teaching mental computation should be to show students a wide variety of mental methods, provide opportunities where each method can be employed, and encourage students to use mental methods regularly to improve their skills.

- **Front-End Addition (Extension)**
  This strategy involves adding the highest place values and then adding the sums of the next place value(s). In Grade 4, the Front-End Addition strategy was extended to numbers in the thousands. In grade 5, it includes decimal 10ths and 100ths.

Examples

For 37 + 26, think: “30 and 20 is 50 and 7 and 6 is 13; 50 plus 13 is 63.”

For 450 + 380, think, “400 and 300 is 700, 50 and 80 is 130; 700 plus 130 is 830.”

For 3300 + 2800, think, “3000 and 2000 is 5000, 300 and 800 is 1100; 500 plus 1100 is 6100.”

For 2 070 + 1 080, think, “2000 and 1000 is 3000, 70 and 80 is 150, and 3000 and 150 is 3150.”

Practice Items

a) Numbers in the 10s, 100s and 1000s

45 + 38 =	34 + 18 =	53 + 29 =
15 + 66 =	74 + 19 =	190 + 430 =
340 + 220 =	470 + 360 =	607 + 304 =
3500 + 2300 =	5400 + 3400 =	6800 + 2100 =
2900 + 6000 =	3700 + 3200 =	7500 + 2400 =
8800 + 1100 =	2700 + 7200 =	6300 + 4400 =

b) Numbers in the 10ths and 100ths

4.6 + 3.2 =	5.4 + 3.7 =	1.85 + 2.25 =
3.3 + 2.4 =	6.6 + 2.5 =	0.36 + 0.43 =
1.5 + 1.5 =	0.75 + 0.05 =	0.45 + 0.44 =

Add your own practice items

- **Break Up and Bridge (Extension)**
  This strategy is similar to front-end addition except that you begin with all of the first number and then add on parts of the second number beginning with the largest place value. In Grade 4, the Break Up and Bridge strategy was extended to include numbers in the hundreds. In grade 5, it includes numbers in the 1000s as well as decimal 10ths and 100ths.

Examples

For 45 + 36, think, “45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81.”

For 537 + 208, think, “537 and 200 is 737, and 737 plus 8 is 745.”

For 5300 plus 2400, think, “5300 and 2000 (from the 2400) is 7300 and 7300 plus 400 (from the rest of 2400) is 7700.”

For 3.6 plus 5.3, think, “3.6 and 5 (from the 5.3) is 8.6 and 8.6 plus 0.3 (the rest of 5.3) is 8.9.”

Practice Items

a) Numbers in the 10s and 100s

37 + 42 =	72 + 21 =	88 + 16 =
74 + 42 =	325 + 220 =	301 + 435 =
747 + 150 =	142 + 202 =	370 + 327 =

b) Numbers in the 1000s

7700 + 1200 =	4100 + 3600 =	5700 + 2200 =
7300 + 1400 =	2800 + 6100 =	3300 + 3400 =
5090 + 2600 =	17 400 + 1300 =	

c) Numbers in the 10ths and 100ths

4.2 + 3.5 =	6.3 + 1.6 =	4.2 + 3.7 =
6.1 + 2.8 =	0.32 + 0.56 =	2.08 + 3.2 =
4.15 + 3.22 =	5.43 + 2.26 =	6.03 + 2.45 =
15.45 + 1.25 =	43.30 + 7.49 =	70.32 + 9.12 =

Add your own practice items

- **Finding Compatibles (Extension)**

  Compatible numbers are sometimes referred to as friendly numbers or nice numbers in other professional resources. This strategy for addition involves looking for pairs of numbers that combine to make a sum that will be easy to work with. Some examples of common compatible numbers include 1 and 9; 40 and 60; 75 and 25 and 300 and 700.

Compatible numbers are sometimes referred to as friendly numbers or nice numbers ... Some examples of common compatible numbers include 1 and 9; 40 and 60; 75 and 25 and 300 and 700.

In grade 4, this strategy was extended to 100s. In grade 5, it includes numbers in the 10ths and 100ths.

Examples

For 3 + 8 + 7 + 6 + 2, think, “3 and 7 is 10, 8 and 2 is 10, so 10 and 10 and 6 is 26.”

For 25 + 47 + 75, think, “25 and 75 is 100, so 100 and 47 is 147.”

For 400 + 720 + 600, think, “400 and 600 is 1000, so the sum is 1720.”

For 3000 + 7000 + 2400, think, “3000 and 7000 is 10 000, so 10 000 and 2400 is 12 400.”

Practice Items

a) Numbers in the 10s, 100s & 1000s

11 + 59 =	33 + 27 =	60 + 30 + 40 =
75 + 95 + 25 =	80 + 20 + 79 =	40 + 72 + 60 =
90 + 86 + 10 =	125 + 25=	475 + 25 =
625 + 75 =	290 + 510 =	300 + 437 + 700 =
800 + 740 + 200 =		900 + 100 + 485 =
4400 + 1600 + 3000 =		9000 + 3300 + 1000=
3250 + 3000 + 1750 =		2200 + 2800 + 600 =
3000 + 300 + 700 + 2000 =		3400 + 5600=

b) Numbers in the 10ths and 100ths are.

0.6 + 0.9 + 0.4 + 0.1 =	0.2 + 0.4 + 0.8 + 0.6 =
0.7 + 0.1 + 0.9 + 0.3 =	0.2 + 0.4 + 0.3 + 0.8 + 0.6 =
0.4 + 0.5 + 0.6 + 0.2 + 0.5 =	0.25 + 0.50 + 0.75 =
0.80 + 0.26 =	0.45 + 0.63 =

Add your own practice items

In the initial activities involving a strategy, you should expect to have students do the computation the way you modeled it. Later, however, you may find that some students employ their own variation of the strategy. If it is logical and efficient for them, so much the better.

- **Compensation (Extension)**

  This strategy involves changing one number in a sum to a nearby ten, hundred, thousand, or decimal tenth or hundredth, carrying out the addition using that changed number, and then adjusting the answer to compensate for the original change. Students should understand that the reason a number is changed is to make it more compatible and easier to work with. They must also remember to adjust their answer to account for the change that was made.

Examples

For 52 + 39, think,“ 52 plus 40 is 92, but I added 1 too many to take me to the next 10, so I subtract one from my answer to get 91.”

For 345 + 198, think, “345 + 200 is 545, but I added 2 too many; so I subtract 2 from 545 to get 543.”

For 4500 plus 1900, think, “4500 + 2000 is 6500 but I added 100 too many; so, I subtract 100 from 6500 to get 6400.”

For 0.54 plus 0.29, think, “0.54 + 0.3 is 0.84 but I added 0.01 too many; so, I subtract 0.01 from 0.84 to compensate, to get 0.83.”

Practice Items

a) Numbers in the 10s and 100s

56 + 8 =	72 + 9 =	44 + 27 =
14 + 58 =	21 + 48 =	255 + 49 =
371 + 18 =	125 + 49 =	504 + 199 =
354 + 597 =	826 + 99 =	676 + 197 =
304 + 399 =	526 + 799 =	

b) Numbers in the 1000s

1300 + 800 =	5400 + 2900 =	6421 + 1900 =
3450 + 4800 =	2330 + 5900 =	15 200 + 2900 =
4621 + 3800 =	2111 + 4900 =	2050 + 6800 =

c) Numbers in the 10ths and 100ths

0.71 + 0.09 =	0.56 + 0.08 =	0.32 + 0.19 =
0.44 + 0.29 =	0.17 + 0.59 =	2.31 + 0.99 =
4.52 + 0.98 =	1.17 + 0.39 =	25.34 + 0.58 =

Add your own practice items

- **Make 10s, 100s, or 1000s (Extension)**

  Make Ten is a thinking strategy introduced in grade 2 for addition facts which have an 8 or a 9 as one of the addends. It involves taking part of the other number and adding it to the 8 or 9 to make a 10 and then adding on the rest.

  For example:

  For 8 + 6, you take 2 from the 6 and give it to the 8 to make 10 + 4. In Grade 4, this strategy was extended to “make 100s” and “make 1000s.” At the grade 5 level, additional practice with this strategy will be beneficial.

Examples

For 58 + 6, think, “58 plus 2 (from the 6) is 60, and 60 plus 4 (the other part of 6)
is 64.”

For 350 + 59, think, “350 plus 50 is 400, and 400 plus 9 is 409.”

For 7400 + 790, think, “7400 plus 600 is 8000, and 8000 plus 190 is 8190.”

Practice Items

58 + 6 =	5 + 49 =	29 + 3 =
38 + 5 =	680 + 78 =	490 + 18 =
170 + 40 =	570 + 41 =	450 + 62 =
630 + 73 =	560 + 89 =	870 + 57 =
780 + 67 =	2800 + 460 =	5900 + 660 =
1700 + 870 =	8900 + 230 =	3500 + 590 =
2200 + 910 =	3600 + 522 =	4700 + 470 =

Add your own practice items

D. Mental Computation – Subtraction

- **Back Down Through 10/100/1000**

This strategy extends one of the strategies students learned in Grade 3 for fact learning. It involves subtracting a part of the subtrahend to get to the nearest ten or hundred, or thousand and then subtracting the rest of the subtrahend. It was introduced in grade 3 for subtraction of 2-digit numbers and extended to numbers in the 100s in grade 4. In grade 5, this strategy includes numbers in the 1000s.

Examples

For 15 - 8, think, “15 subtract 5 (one part of the 8) is 10, and 10 subtract 3 (the other part of the 8) is 7.”

For 74 - 6, think, “74 subtract 4 (one part of the 6) is 70 and 70 subtract 2 (the other part of the 6) is 68.”

For 530 - 70, think, “530 subtract 30 (one part of the 70) is 500 and 500 subtract 40 (the other part of the 70) is 460.”

For 8600- 700, think, “8600 subtract 600(one part of the 700) is 8000 and 8000 subtract 100(the rest of the 700) is 7900.”

Practice Items

a) Numbers in the 10s and 100s

74 - 7 =	97 - 8 =	53 - 5 =
420 - 60 =	340 -70 =	630 - 60 =
540 - 70 =	760 - 70 =	320 - 50 =

b) Numbers in the 1000s

9200 - 500 =	4700 - 800 =	6100 - 300 =
7500 - 700 =	800 - 600 =	4200 - 800 =
9500 - 600 =	3400 - 700 =	2300 - 600 =

Add your own practice items

Situations must be regularly provided to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. It is recommended that regular, maybe daily, practice be provided.

- **Up Through 10/100/1000 (Extension)**
  This strategy is an extension of the “Up through 10” strategy that students learned in Grade 3 to help master the subtraction facts. It can also be thought of as, “counting on to subtract” (See Fact Learning - Subtraction in this booklet).

  To apply this strategy, you start with the smaller number (the subtrahend) and keep track of the distance to the next 10, 100, 1000 and then add this amount to the rest of the distance to the greater number (the minuend). In grade 5, the strategy is extended to numbers involving tenths and hundredths.

Examples

For 613 - 594, think, “It’s 6 from 594 to 600 and then 13 more to get to 613; that’s 19 altogether.”

For 84 - 77, think, “It’s 3 from 77 to 80 and 4 more to 84; so that’s 7 altogether.”

For 2310 - 1800, think, “It’s 200 from 1800 to 2000 then 310 more, so that’s 510 in all.”

For 12.4 - 11.8, think: “It’s 2 tenths to get to 12 from 11.8 and then 4 more tenths, so that’s 6 tenths, or 0.6 altogether.”

For 6.12 - 5.99, think, “It’s one hundredth from 5.99 to 6.00 and then twelve more hundredths to get to 6.12 ; So the difference is 1 hundredth plus 12 hundredths, or 0.13.”

Practice Items

a) Numbers in the 10s and 100s

11 -7 =	17 - 8 =	13 - 6 =
12 - 8 =	15 - 6 =	16 - 7 =
95 -86 =	67 -59 =	46 - 38 =
88 - 79 =	62 - 55 =	42 - 36 =
715 - 698 =	612 - 596 =	817 - 798 =
411 - 398 =	916 - 897 =	513 - 498 =
727 - 698 =	846 - 799 =	631 - 597 =

b) Numbers in the 1000s

5170 - 4800 =	3210 - 2900 =	8220 - 7800 =
9130 - 8950 =	2400 - 1800 =	4195 - 3900 =
7050 - 6750 =	1280 - 900 =	8330 - 7700 =

c) Numbers in the 10ths and 100ths

15.3 - 14.9 =	27.2 - 26.8 =	19.1 - 18.8 =
45.6 - 44.9 =	23.5 - 22.8 =	50.1 - 49.8 =
34.4 - 33.9 =	52.8 - 51.8 =	70.3 - 69.7 =
3.25 - 2.99 =	5.12 - 4.99 =	4.05 - 3.98 =
3.24 - 2.99 =	8.04 - 7.98 =	6.53 - 5.97 =
24.12 - 23.99 =	36.11 - 35.98 =	100.72 - 99.98 =

Add your own practice items

- **Compensation (Extension)**
  This strategy for subtraction was first introduced in grade 4. It involves changing the subtrahend (the amount being subtracted) to the nearest 10 or 100, carrying out the subtraction, and then adjusting the answer to compensate for the original change. In grade 5, the strategy is extended to numbers in the 1000s.

Examples

For 17 – 9, think, “I can change 9 to 10 and then subtract 17 – 10; that gives me 7, but I only need to subtract 9, so I’ll add 1 back on. My answer is 8.”

For 56 - 18, think, “I can change 18 to 20 and then subtract 56 - 20; that gives me 36, but I only need to subtract 18, so I’ll add 2 back on. My answer is 38.”

For 756 - 198, think: “756 - 200 = 556, and 556 + 2 = 558”

For 5760 - 997, think: 5760 - 1000 is 4760; but I subtracted 3 too many; so, I add 3 to 4760 to compensate to get 4763.

For 3660 - 996, think: 3660 - 1000 + 4 = 2664.

Practice Items

a) Numbers in the 10s

15 - 8 =	17 - 9 =	23 - 8 =
74 - 9 =	84 - 7 =	92 - 8 =
65 - 9 =	87 - 9 =	73 - 7 =

b) Numbers in the 100s

673 - 99 =	854 - 399 =	953 - 499 =
775 - 198 =	534 - 398 =	647 - 198 =
641 - 197 =	802 - 397 =	444 - 97 =
765 - 99 =	721 - 497 =	513 - 298 =

C) Numbers in the 1000s

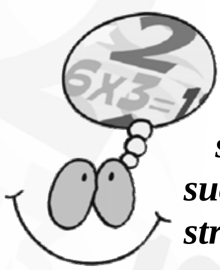
8620 - 998 =	4100 - 994 =	5700 - 397 =
9850 - 498 =	3720 - 996 =	2900 - 595 =
4222 - 998 =	7310 - 194 =	75 316 - 9900

Add your own practice items

- **Balancing For a Constant Difference (New)**
  This strategy for subtraction involves adding or subtracting the same amount from both the subtrahend and the minuend to get a ten, hundred or thousand in order to make the subtraction easier. This strategy needs to be carefully introduced to convince students that it works because the two numbers are the same distance apart as the original numbers.

  Examining pairs of numbers on a number line such as a metre stick can help students understand the logic of the strategy. For example, the difference or distance between the numbers 66 and 34 (66 - 34) on a number line is the same as the difference between 70 and 38, and it’s easier to mentally subtract the second pair of numbers.

  Because both numbers change, many students may need to record at least the first changed number to keep track.



Adding or subtracting the same amount from both numbers maintains the distance between them and makes the mental subtraction easier. Examining pairs of numbers on a number line such as a metre stick can help students understand the logic of this strategy.

Examples

1) For 87 -19, think, “Add 1 to both numbers to get 88 - 20, so 68 is the answer.”

   For 76 - 32, think, “Subtract 2 from both numbers to get 74 -30, so the answer is 44.”

2) For 345 - 198, think, “Add 2 to both numbers to get 347 - 200; the answer is 147.”

   For 567 - 203, think, “Subtract 3 from both numbers to get 564 -200; so the answer is 364.”

3) For 8.5 - 1.8, think, “Add 2 tenths to both numbers to get 8.5 - 2.0; That’s 6.7.”

   For 5.4 - 2.1, think, “Subtract 1 tenth from both numbers to get 5.3 - 2.0 or 3.3.”

4) For 6.45 - 1.98, think, “Add 2 hundredths to both numbers to get 6.47 - 2.00, so 4.47 is the answer.”

For 5.67 - 2.03, think, “Subtract 3 hundredths from both numbers to get 5.64 - 2.00. The answer is 3.64.”

Practice Items

a) Numbers in the 10s and 100s

85 - 18 =	42 - 17 =	36 - 19 =
78 - 19 =	67 -18 =	75 - 38 =
88 - 48 =	94 - 17 =	45 - 28 =
67 - 32 =	88 - 43 =	177 - 52 =
649 - 299 =	563 - 397 =	823 - 298 =
912 - 797 =	737 - 398 =	456 - 198=
631 - 499 =	811 - 597 =	628 - 298 =
736 - 402 =	564 - 303 =	577 - 102 =
948 - 301 =	437 - 103 =	819 - 504 =

b) Numbers in the 10ths and 100ths

6.4 - 3.9 =	7.6 - 4.2 =	8.7 - 5.8 =
4.3 - 1.2 =	9.1 - 6.7 =	5.0 - 3.8 =
6.3 - 2.2 =	4.7 - 1.9 =	12.5 - 4.3 =
15.3 - 5.7 =	8.36 - 2.99 =	7.45 - 1.98 =
5.40 - 3.97 =	6.25 - 2.01 =	8.53 - 6.02 =

Add your own practice items

- **Break Up and Bridge**
  With this subtraction strategy, first introduced in grade 4, you start with the larger number (the minuend) and subtract the highest place value of the second number first (the subtrahend), and then the rest of the subtrahend. The strategy is extended to numbers in the 1000s in grade 5.

Examples

For 92 - 26, think, “92 subtract 20 (from the 26) is 72 and 72 subtract 6 is 66.”

For 745 - 203, think, “745 subtract 200 (from the 203) is 545 and 545 minus 3 is 542.”

For 8369 - 204, think, “8369 subtract 200 is 8169 and minus 4 (the rest of the 204) is 8165 .

Practice Items

a) Numbers in the 10s and 100s

79 - 37 =	93 - 72 =	98 - 22 =
79 - 41=	74 - 15 =	77 - 15 =
95 - 27 =	85 - 46 =	67 - 42 =
56 - 31 =	86 - 54 =	156 - 47 =
736 - 301 =	848 - 207 =	927 - 605 =
632 - 208 =	741 - 306 =	758 - 205 =
928 - 210 =	847 - 412 =	746 - 304 =

b) Numbers in the 1000s

9275 - 8100 =	6350 - 4200 =	8461 - 4050 =
10 270 - 8100 =	15 100 - 3003 =	4129 - 2005 =
3477 - 1060 =	38 500 -10 400 =	137 400 - 6100 =

Add your own practice items

E. Mental Computation – Multiplication and Division

- **Using Multiplication Facts for Tens, Hundreds and Thousands**
  This strategy applies to tens, hundreds and thousands (with one non-zero digit in the number) multiplied by a 1-digit number. It could also be used for 2-digit by 2-digit multiplication when both numbers end in zero. Students multiply the non-zero digits as if they were multiplication facts, and then attach the appropriate place value name to the result.

Examples

For 3 × 70, think, "3 times 7 tens is 21 tens, or 210."

For 6 × 900, think: "6 times 9 hundreds is 54 hundreds, or 5400."

For 30 × 80, think, "Tens by tens is hundreds, so 3 tens by 8 tens is 24 hundreds, or 2 400."

For 60 × 20, think, "Tens by tens is hundreds.6 tens by 2 tens is 12 hundreds, or 1200."

Practice Items

4 x 30 =	8 x 40 =	9 x 30 =
6 x 50 =	70 x7 =	90 x 40 =
60 × 20 =	30 × 50 =	90 × 60 =
40 × 40 =	70 × 90 =	10 × 400 =
30 × 600 =	80 × 200 =	100 × 50 =
700 × 30 =	900 × 50 =	6 × 200 =
8 × 600 =	9 × 800 =	300 × 4 =
40 × 80 =	60 × 20 =	30 × 50 =
90 × 60 =	40 × 40 =	70 × 90 =
10 × 400 =	30 × 600 =	80 × 200 =
100 × 50 =	700 × 30 =	900 × 50 =

Add your own practice items

- **Multipying by 10,100, and 1000 Using a Place-Value-Change Strategy (Extension)**
  This strategy is first introduced in grade 4 where students multiply by 10 and 100 mentally by applying their understanding of how the place values change. For example, when 8 is multiplied by 10 for a product of 80, the 8 ones becomes 8 tens, an increase of 1 place value. When 28 is multiplied by 100 for a product of 2800, the 2 tens increase two places to 2 thousands and the 8 ones increases two places to 8 hundred. All the place values of the number being multiplied increase one place when multiplying by 10, two places when multiplying by 100, and 3 places when multiplying by 1000.

  Some students will see the pattern that one zero gets attached to a number that is multiplied by 10, two zeros get attached to a number multiplied by 100, and 3 zeros get attached to a number multiplied by 1000. However, understanding the "place-value-change strategy" will be more meaningful than the "attach zeros strategy" and will produce more correct answers, especially when decimal 10ths, 100ths and 1000ths are involved later in the year.

Later, when students are working with decimals, such as 100 × 0.12, using the "place-value-change strategy" will be more meaningful than the "attach-zeros strategy" and will more likely produce correct answers."

Examples

For 24 x 10, the 2 tens increases one place to 2 hundreds and the 4 ones increases one place to 4 tens; 240

For 36 x 100, the 3 tens increases two places to 3 thousands and the 6 ones increases two places to 6 hundreds; 3600.

For 37 x 1000, the 3 tens will increase to 3 ten-thousands or 30 000, and the 7 tens will increase to 7 thousands. 30 000 plus 7000 is 37 000

Practice Items

10 × 53 =	10 × 34 =	87 × 10 =
10 × 20 =	47 × 10 =	78 × 10 =
92 × 10 =	10 × 66 =	40 × 10 =
100 × 7 =	100 × 2 =	100 × 15 =
100 × 74 =	100 × 39 =	37 × 100 =
10 × 10 =	55 × 100 =	100 × 83 =
100 × 70 =	40 × 100 =	100 x 22 =
1000 × 6 =	1000 × 14 =	83 × 1000 =
$73 × 1000 =	$20 × 1000 =	16 × $1000 =
$3 × 10 =	$7 × 10 =	$50 × 10 =
10 × 3.3 =	4.5 × 10 =	0.7 × 10 =
8.3 × 10 =	7.2 × 10 =	10 × 4.9 =
100 × 2.2 =	100 × 8.3 =	100 × 9.9 =
7.54 × 10 =	8.36 x10 =	10 × 0.3 =
100 × 0.12 =	100 × 0.41 =	100 × 0.07 =
3.78 × 100 =	1000 × 2.2 =	1000 × 43.8 =
1000 × 5.66 =	8.02 × 1000 =	0.04 × 1000 =
3 m = _____mm	7m = ____mm	4.2m = ___mm
6.2m = ____mm	6cm = ___mm	9km = ____m
7.7km = ____m	3dm = ____mm	3dm = ____ cm
5m = ___ cm	8m = ___cm	3m =___cm

Add your own practice items

- **Dividing by 10, 100, and 1000 Using a Place-Value-Change Strategy (New)**
  Again, it is important for students to explore the patterns that result when numbers are divided by 10, 100 or 1000. Calculators are useful tools for this exploratory work and should be used to help students understand that all the place values of the number being divided decrease one place when dividing by 10, two places when dividing by 100, and three places when dividing by 1000.

Examples

For, 60 ÷ 10, think, “The 6 tens will decrease to 6 ones; therefore, the answer is 6.”

For, 500 ÷ 10, think: “The 5 hundreds will decrease to 5 tens; therefore, the answer is 50.”

For, 7 500 ÷ 100, think, The 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75.”

For, 75 000 ÷ 1000; think, “The 7 ten thousands will decrease to 7 tens and the 5 thousands will decrease to 5 ones; therefore, the answer is 75. “

Practice Items

70 ÷ 10 =	90 ÷ 10 =	40 ÷ 10 =
200 ÷ 10 =	800 ÷ 10 =	100 ÷ 10 =
400 ÷ 100 =	900 ÷ 100 =	6000 ÷ 100 =
4200 ÷ 100 =	7600 ÷ 100 =	8500 ÷ 100 =
9700 ÷ 100 =	4400 ÷ 100 =	10 000 ÷ 100 =
600 pennies = $____	1800 pennies = $____	5600 pennies = $____
82 000 ÷ 1000 =	98 000 ÷ 1000 =	12 000 ÷ 1000 =
66 000 ÷ 1000 =	70 000 ÷ 1000 =	100 000 ÷ 1000 =
430 000 ÷ 1000 =	104 000 ÷ 1000 =	4 500 ÷ 1000 =
77 000m = _____km	84 000m = ____km	7 700m = _____km

Add your own practice items

- **Multiplying by 0.1, 0.01, and 0.001 Using a Place-Value-Change Strategy (New)**
  By now, students should have a good understanding of how place values change when multiplying and dividing by 10, 100 and 1000. Now they will apply this same strategy to decimal tenths, hundredths and thousandths. By exploring the patterns that result when numbers are multiplied by these fractional amounts they will see that all the place values of the number being multiplied decrease one place when multiplying by 0.1, two places when multiplying by 0.01 and three places when multiplying by 0.001

All the place values of the number being multiplied decrease one place when multiplying by 0.1, two places when multiplying by 0.01 and three places when multiplying by 0.001.

Examples

For 5 × 0.1, think, “The 5 ones will decrease one place to 5 tenths, therefore the answer is 0.5.”

For, 0.4 × 0.1, think, “The 4 tenths will decrease one place to 4 hundredths, therefore the answer is 0.04.”

For 5 × 0.01, think, “The 5 ones will decrease two places to 5 hundredths, so the answer is 0.05.”

For, 0.4 × 0.01, think, “The 4 tenths will decrease two places to 4 thousandths, therefore the answer is 0.004.”

For 5 × 0.001, think, “The 5 ones will decrease three places to 5 thousandths; so, the answer is 0.005.”

Practice Items

$6 \times 0.1 =$	$8 \times 0.1 =$	$3 \times 0.1 =$
$9 \times 0.1 =$	$1 \times 0.1 =$	$12 \times 0.1 =$
$72 \times 0.1 =$	$136 \times 0.1 =$	$406 \times 0.1 =$
$0.7 \times 0.1 =$	$0.5 \times 0.1 =$	$0.1 \times 10 =$
$1.6 \times 0.1 =$	$0.1 \times 84 =$	$0.1 \times 3.2 =$
$6 \times 0.01 =$	$8 \times 0.01 =$	$1.2 \times 0.01 =$
$0.5 \times 0.01 =$	$0.4 \times 0.01 =$	$0.7 \times 0.01 =$
$2.3 \times 0.01 =$	$3.9 \times 0.01 =$	$10 \times 0.01 =$
$100 \times 0.01 =$	$330 \times 0.01 =$	$46 \times 0.01 =$
$3 \times 0.001 =$	$7 \times 0.001 =$	$80 \times 0.001 =$
$21 \times 0.001 =$	$45 \times 0.001 =$	$12 \times 0.001 =$
$62 \times 0.001 =$	$9 \times 0.001 =$	$75 \times 0.001 =$
4mm =_____m	9mm = _____m	6m = ______km

Add your own practice items

- **Front End Multiplication - The Distributive Principle (New)**
  This strategy is useful when multiplying 2-, 3-, and 4-digit numbers by 1-digit numbers. It involves calculating the product of the highest place value and the 1-digit number, and then adding this to the sub-product(s) of the other place values and the 1-digit number.

Examples

For 3 × 62, think, “6 tens times 3 is 18 tens (180) and 3 times 2 is 6 for a total of 186.”

For 706 x 4, think, “7 hundreds times 4 is 28 hundreds (2800) and 6 times 4 is 24 for a total of 2824.”

For 5 × 6100, think, “6 thousand times 5 is 30 thousands, and 5 times 100 is 500; so 30 000 plus 500 is 30 500.”

For 3.2 × 6, think, “3 times 6 is 18 and 6 times 2 tenths is 12 tenths or 1 and 2 tenths; so 18 plus 1.2 is 19.2.”

For 62 × 0.2, think: “60 times 2 tenths is 120 tenths or 12; and 2 tenths times 2 is 4 tenths or 0.4; so 12 plus 0.4 is 12.4.”

For 47 × 0.3, think, “40 times 3 tenths is 120 tenths or 12; and 7 times 3tenths is 21 tenths or 2.1; so 12 plus 2.1 is 14.1”

Practice Items

53 × 3 =	32 × 4 =	41 × 6 =
29 × 2 =	83 × 3 =	75 × 3 =
3 × 503 =	209 × 9 =	703 × 8 =
606 × 6 =	503 × 2 =	8 04 × 6 =
309 × 7 =	122 × 4 =	320 × 3 =
6 × 3100 =	5 × 5100 =	2 × 4300 =
3 × 3200 =	2 × 4300 =	7 × 2100 =
4.6 × 2 =	36 × 0.2 =	8.3 × 5 =
43 × 0.5 =	96 × 0.3 =	83 × 0.9 =
7.9 × 6 =	3.7 × 4 =	52 × 0.4 =
8.9 × 5 =	75 × 0.8 =	3.3 × 7 =

Add your own practice items

- **Compensation (New for Multiplication)**
  This strategy for multiplication involves changing one of the factors to a ten, hundred or thousand, carrying out the multiplication, and then adjusting the answer to compensate for the change that was made. This strategy could be used when one of the factors is near a ten, hundred or thousand.

Examples

For 6 × 39, think, “6 groups of 40 is 240. 6 groups of 39 would be 6 less; 240 - 6 = 234.”

For 7 × 198, think, “7 times 200 is 1400, but this is 14 more than it should be because there were 2 extra in each of the 7 groups; 1400 subtract 14 is 1368.”

For 19 × 60, think, “20 groups of 60 is 1200, so 19 groups of 60 would be 60 less; 1200 - 60 = 1140.”

Practice Items

6 × 39 =	8 × 29 =	5 × 49 =
2 × 79 =	6 × 89 =	7 × 59 =
4 × 49 =	9 × 69 =	8 x 32 =
5 × 399 =	3 × 199 =	4 × 198 =
9 × 198 =	8 × 698 =	7 × 598 =
29 × 50 =	39 × 40 =	89 × 20 =
49 × 90 =	79 × 30 =	59 × 60 =

Add your own practice items

Estimation

Estimation – Addition, Subtraction, Multiplication and Division

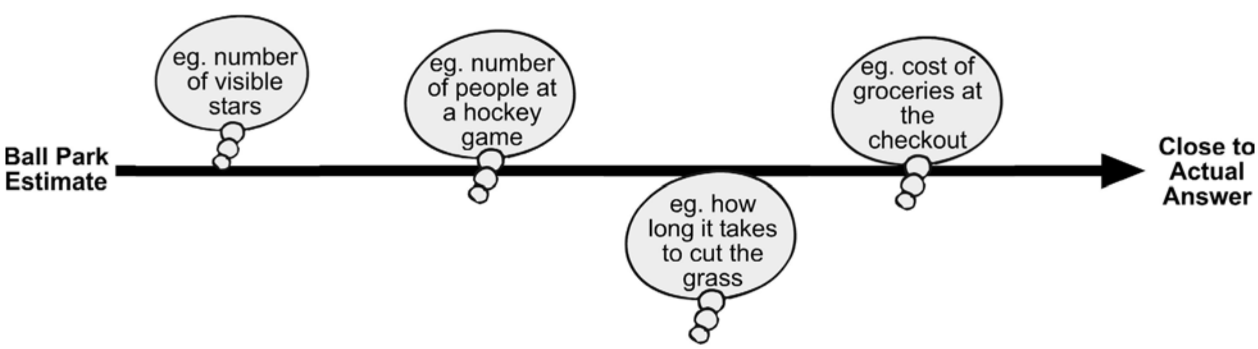
When asked to estimate, students often try to do the exact computation and then “round” their answer to produce an estimate that they think their teacher is looking for. Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Estimates can be very broad and general, or they can be quite close to the actual answer. It all depends on the reason for estimating in the first place, and these reasons can vary in context and according to the needs of the individual at the time.

Help students identify situations outside of school where they would estimate distances, number, temperature, length of time and discuss how accurate their estimates needed to be. Place these situations on an estimation continuum with broad, ball-park estimates at one end and estimates that are very close to the actual answer at the other.

For example:



In mathematics, it is essential that estimation strategies are used by students before attempting pencil/paper or calculator computations to help them determine whether or not their answers are reasonable.

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When teaching estimation strategies, it is important to use words and phrases such as, *about, almost, between, approximately, a little more than, a little less than, close to* and *near*.

- **Rounding in Addition and Subtraction (Continued from Grade 4)**
  With this strategy for addition and subtraction, you start with the highest place values in each number, round them to the closest 10, 100 or 1000, and then add or subtract the rounded numbers.

Example

To estimate 378 + 230, think, “378 rounds to 400 and 230 rounds to 200; so, 400 plus 200 is 600.”

To estimate 4276 + 3937, think, “4276 rounds to 4000 and 3937 rounds to 4000, so 4000 plus 4000 is 8000.”

To estimate 594 - 203, think, “594 rounds to 600 and 203 rounds to 200, so 600 subtract 200 is 400.”

To estimate 6237 - 2945, think, “6 237 rounds to 6000 and 2945 rounds to 3000, so 6000 subtract 3000 is 3000.”

Practice Items

28 + 57 =	41 + 34 =	123 + 62 =
303 + 49 =	137 + 641 =	223 + 583 =
6110 + 3950 =	4460 + 7745 =	1370 + 6410 =
36 - 22 =	43 - 8 =	54 - 18 =
834 - 587 =	947 - 642 =	780 - 270 =
4807 - 1203 =	7856 - 1250 =	5029 - 4020 =

Students should estimate automatically whenever faced with a calculation. Facility with basic facts and mental math strategies is key to estimation

- **Rounding With "Fives"**

a) *Addition*

When the digit 5 is involved in the rounding procedure for numbers in the 10s, 100s, and 1000s, the number can be rounded up or down depending upon the effect the rounding will have in the overall calculation. For example, if both numbers to be added are about 5, 50, or 500, it is better to round one number up and one number down to minimize the effect the rounding will have in the estimation.

Examples

For 45 + 65, think, "Since both numbers involve 5s, it would be best to round to 40 + 70 to get 110."

For 4520 + 4610, think, "Since both numbers are both close to 500, it would be best to round to 4000 + 5000 to get 9000."

Practice Items

a)	35 + 55 =	45 + 31 =	26 + 35 =
	250 + 650 =	653 + 128 =	179 + 254 =
	384 + 910 =	137 + 641 =	798 + 387 =
	530 + 660 =	350 + 550 =	450 + 319 =
	2500 + 4500 =	4550 + 4220 =	6810 + 1550 =
	5184 + 2958 =	4867 + 6219 =	7760 + 3140 =

Add your own practice items

b) *Subtraction*

For subtraction, the process of estimation is similar to addition, except for situations where both numbers are close to 5, 50, or 500. In these situations, both numbers should be rounded up. If you round one number up and one down, it will increase the difference between the two numbers and your estimate will be farther from the actual answer.

Examples

To estimate 594 - 203, think, “594 rounds to 600 and 203 rounds to 200; so, 600 - 200 is 400.”

To estimate 6237 - 2945, think, “6237 rounds to 6000 and 2945 rounds to 3000; so, 6 000 - 3000 is 3 000.”

To estimate 5549 - 3487, think, “Both numbers are close to 500, so round both up; 6000 - 4000 is 2000.”

Practice Items

427 - 192 =	984 - 430 =	872 - 389 =
594 - 313 =	266 - 94 =	843 - 715 =
834 - 587 =	947 - 642 =	782 - 277 =
4768 - 3068 =	6892 - 1812 =	7368 - 4817 =
4807 - 1203 =	7856 - 1250 =	5029 - 4020 =
8876 - 3640 =	9989 - 4140 =	1754 - 999 =

Add your own practice items

Ongoing practice in computational estimation is a key to developing understanding of numbers and number operations and increasing mental process skills.

- **Rounding in Multiplication (New)**
  To estimate the product of a 2-digit or 3-digit factor by a single-digit factor, the multi- digit factor is rounded to the nearest 10 or 100 and then multiplied by the single-digit factor.

Examples

To estimate 7 × 64, think, “64 rounds to 60, and 7 times 60 is 420.”

To estimate 8 × 693, think, “693 rounds to 700, and 8 times 700 is 560.”

Practice Items

4 × 59 =	7 × 22 =	8 × 61 =
9 × 43 =	295 × 6 =	7 × 402 =
889 × 3 =	5 × 503 =	2 × 888 =
7 × 821 =	1 × 795 =	712 × 4 =

To estimate the product of two 2-digit numbers where the number in the ones place in both numbers is 5 or more, the smaller factor should be rounded up and the larger factor down.

Example

To estimate 76 × 36, round 36 (the smaller factor) up to 40, and round 76 (the larger factor down to 70) which equals 70 × 40 = 2800. This produces a closer estimate than rounding to 80 × 40 or 80 × 30.

Practice Items

57 × 29 =	49 × 28 =	38 × 27 =
66 × 57 =	87 × 19 =	36 × 58 =
27 × 68 =	87 × 37 =	96 × 78 =

Add your own practice items

Computational estimation is a mental activity; therefore, regular oral practice, accompanied by the sharing of strategies must be provided.

- **Adjusted Front End Estimation for Addition and Subtraction (Extended to Decimal 10ths and 100ths)**
  This strategy begins with a front-end estimate and then making an adjustment by considering some or all the values in the other place values. This will result in a more accurate estimate.

Examples

To estimate 437 + 545, think, “400 plus 500 is 900, but this can be adjusted by thinking 30 and 40 is 70, so the adjusted estimate would be 90 + 70 = 970.”

To estimate 3237 + 2125, think, “3000 plus 2000 is 5000, and 200 plus 100 is 300, so the adjusted estimate is 5300.”

To estimate 382 - 116, think, “300 subtract 100 is 200, and 80 - 10 is 70, so the adjusted estimate is 270.”

To estimate 6237 - 2954, think, “6000 subtract 2000 is 4000, and 954 would account for about another 1000; therefore, the adjusted estimate is 6000 - 2000 - 1000 or 3000.”

To estimate 8.64 + 5.28, think, “8 plus 5 is 13, and 0.64 plus 0.28 would account for another 1 whole; therefore, the adjusted estimate is 8 + 5 + 1= 14.”

To estimate 7.12 - 3.89, think, “7 - 3, but 0.89 would account for about another 1 whole; therefore the adjusted estimate is 7 - 3 - 1 = 3.”

Practice Items

a) Estimating Sums

251 + 445 =	589 + 210 =	320 + 275 =
642 + 264 =	519 + 180 =	148 + 450 =
5695 + 2450 =	4190 + 1850 =	4550 + 3445 =
5240 + 3790 =	1910 + 5125 =	7.45 + 1.56 =
5.89 + 2.10 =	3.20 + 2.75 =	6.43 + 2.67 =
3.19 + 2.81 =	3.48 + 4.50 =	$2.45 + $4.60 =
$3.95 + $4.07 =	$2.73 - $2.22 =	

b) Estimating Differences

645 - 290 =	720 - 593 =	834 - 299 =
935 - 494 =	6210 - 2987 =	8040 - 5899 =
9145 - 4968 =	7120 - 4975 =	6148 - 3920 =
7.43 - 4.95 =	5.29 - 2.99 =	6.18 - 1.97 =
8.05 - 4.92 =	8.11 - 4.98 =	9.21 - 5.99 =
$6.45 - $5.98 =	$7.20 - $5.97 =	$8.34 - $2.99 =

Add your own practice items

Estimation must be used with all computations, but when an exact answer is required, students need to decide whether it is more appropriate to use a mental strategy, pencil and paper, or some form of technology.

Appendixes

Appendix 1

Thinking Strategies in Mental Math

Mental math proficiency represents one important dimension of mathematical knowledge. Not all individuals will develop rapid mental number skills to the same degree. Some will find their strength in mathematics through other avenues, such as visual or graphic representations or creativity in solving problems. But mental math has a clear place in school mathematics. It is an area where many parents and families feel comfortable offering support and assistance to their children.

The following table identifies all of the thinking strategies in *Mental Math: Fact Learning, Mental Computation and Estimation* and the grade level in which they are first introduced. These strategies are then extended and developed in subsequent years.

For example, Front End Addition involving 2-digit numbers is first introduced in grade 2, continued in grade 3, extended to 3-digit numbers in grade 4, and to decimal tenths, hundredths, and thousandths in grades 5 and 6. The teachers guide for each grade level contains a complete description of each strategy with examples and practice items.

Strategy	Description
Grade 1	
Pre-Operation • Patterned Set Recognition • Part-Part-Whole Relationships • Counting On and Back • Next Number • Ten-Frame Visualization for Numbers 0-10 • One More/One Less, Two More/Two Less Relationships	• Students are able to identify common configuration sets of numbers such as the dots on a standard die, dominoes and dot cards without counting. • Recognition of two parts in a whole. Leads to the understanding that numbers can be decomposed into component parts. • Students can count on and back from a given number 0-9 • Students are able to immediately state the number that comes after any given number from 0-9. • Students can visualize the standard ten-frame representation of numbers and answer questions from their visual memories. • Students are presented with a number and asked for the number that is *one more*, *one less*, *two more*, or *two less* than the number.
Addition Facts to 10 • Doubles • Plus 1 Facts • Plus 2 Facts • Plus 3 Facts	• Doubles posters created as visual images • *Next number* facts • Ten-frame, skip counting, 2-more-than relationship, counting on • Ten-frame, 2-more-than plus 1, counting on
Subtraction Facts With Minuends to 10 • Think-Addition • Ten Frame Visualization • Counting Back	• For 9 - 3, think, *"3 plus what equals 9?"* • Visualize the minuend on a ten-frame, and remove the subtrahend, to determine the difference. • For -1, -2, -3 facts
Adding 10 to a Number	For numbers 11-20
Grade 2	
Addition Facts to 18 • Near Doubles • 2-Aparts • Plus zero • Make 10	• Double the smaller number and add 1 • Double the number in between • *No change* facts • For facts with 8 or 9 as addends. Eg. 7 + 9 is the same as 10 + 6
Subtraction Facts With Minuends to 18 • Up Through 10 • Back Down Through 10	• For 13 - 8, think, *"From 8 up to 10 is **2**, and then 3 more is **5**."* • For 14 - 6, think, *"14 - 4 gets me to 10, and then 2 more brings me to 8."*
Addition facts extended to numbers in the 10s	2-Apart Facts: 3 + 5 is double 4, so 30 + 50 is double 40.
Front-End Addition	Highest place values are totaled first and then added to the sum of the remaining place values.
Finding Compatibles	Looking for pairs of numbers that add easily, particularly, numbers that add to 10.
Compensation	One or both numbers are changed to make the addition easier and the answer adjusted to compensate for the change.
Rounding in Addition and Subtraction (5 or 50 not involved in rounding process until grade 4)	Round to nearest 10.

Grade 3	
Multiplication Facts With Products to 36 • x 2 facts • Fives • Nifty Nines • Ones • Tricky Zeros • Fours • Threes	Introduced early in 3rd reporting period (mid-March) • Related to the addition doubles • Clock facts, patterns • Patterns, helping fact • No change facts • Groups of zero • Double-double • Double plus 1 more set
Break Up and Bridge	With this front-end strategy, you start with all of the first number and add it to the highest place value in the other number, and then add on the rest.
Front-End Estimation for Addition and Subtraction	Add or subtract just the largest place values in each number to produce a “ball park” estimate.
Adjusted Front-End Estimation for Addition and Subtraction	Same as above, except the other place values are considered for a more accurate estimate.
Grade 4	
Make 10s, 100s, 1000s for addition	48 + 36 is the same as 50 + 34 which is 84
Multiplication Facts With Products to 81 • Last Six Facts	Mastery by year-end For facts not already covered by previous thinking strategies
Subtraction facts extended to numbers in the 10s, 100s 100s	Only 1 non-zero digit in each number eg. 600 - 400 =
Compensation (new for subtraction)	For 17-9, think, *“17 - 10 is 7, but I subtracted 1 too many, so the answer is 8."*
Break Up and Bridge (new for subtraction)	For 92 - 26, think, *“92 - 20 is 72 and then 6 more is 66."*
Multiply by 10 and 100 using a place-value-change strategy	The place values for a number multiplied by 100 *increase* 2 places. Eg. 34 x 100; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400

Grade 5	
Division Facts With Dividends to 81 • “Think-Multiplication”	Mastery by year-end For 36 ÷ 6, think *“6 times what equals 36?”*
Balancing for a Constant Difference	Involves changing both numbers in a subtraction sentence by the same amount to make it easier to complete. The difference between the two numbers remains the same. Eg. for 27 - 16, add 3 to each number and think, *“30 - 19 = 11"*
Multiply by 0.1, 0.01, 0.001 using a place-value-change strategy	The place values for a number multiplied by 0.1 *decrease* 1 place. Eg. 34 x 0.1; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Front-End Multiplication (Distributive Principle)	Involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product. 706 x 2 = (700 x 2) + (6 x 2) = 1412
Compensation in Multiplication	Involves changing one factor to a 10 or 100, carrying out the multiplication, and then adjusting the product to compensate for the change. 7 x 198 = 7 x 200 (1400) subtract 14 = 1386
Divide by 10, 100, 1000 using a place-value-change strategy.	The place values for a number divided by 10 *decrease* 1 place. Eg. 34 ÷ 10; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Rounding in Multiplication	Highest place values of factors are rounded and multiplied. When both numbers are close to 5 or 50, one number rounds up and the other down.
Grade 6	
Divide by 0.1, 0.01, 0.001 using a place-value-change strategy	The place values for a number divided by 0.01 *increase* 2 places. Eg. 34 ÷ 0.01; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400
Finding Compatible Factors (Associative Property)	Involves looking for pairs of factors, whose product is easy to work with, usually multiples of 10. For example, for 2 x 75 x 500, think, “2 x 500 = 1000 and 1000 x 75 is 75 000.
Halving and Doubling	One factor is halved and the other is doubled to make the multiplication easier. Students would need to record sub-steps. For example, 500 x 88 = 1000 x 44 = 44 000.
Using division facts for 10s, 100s 1000s	Dividends in the 10s, 100s, and 1000s are divided by single digit divisors. The quotients would have only one digit that wasn’t a zero. For example, for 12 000 ÷ 4, think single digit division facts. 12 ÷ 4 = 3, and thousands divided by ones is thousands, so the answer is 3000.
Partitioning the Dividend (Distributive Property)	The dividend is broken up into two parts that are more easily divided by the divisor. For example, for 372 ÷ 6, think, “(360 + 12) ÷ 6, so 60 + 2 is 62.”

Appendix 2
Mental Math: Fact Learning, Mental Computation, Estimation (Scope and Sequence)

	GRADE 1	GRADE 2	GRADE 3	GRADE 4	GRADE 5	GRADE 6
FACT LEARNING	**Pre-Operation Strategies:** ▸ Patterned Set Recognition for numbers 1-6 (not dependent on counting) ▸ Part-Part-Whole Relationships ▸ Counting on, Counting Back ▸ Next Number ▸ Ten Frame Recognition and Visualization for Numbers 0-10 ▸ One More/One Less and Two More/Two Less Relationships **Addition Facts With Sums to 10 Thinking Strategies:** ▸ Doubles ▸ Plus 1 Facts ▸ Plus 2 Facts ▸ Plus 3 Facts ▸ Ten Frame Facts **Subtraction Facts With Minuends to 10 Thinking Strategies** ▸ Think-Addition ▸ Ten Frame Facts ▸ Counting Back	**Addition and Subtraction Facts** ▸ mastery of facts with sums and minuends to 10 by mid-year ▸ mastery of facts with sums and minuends to 18 by year end **New Thinking Strategies for Addition** ▸ Near Doubles ▸ 2-Apart Facts ▸ Plus 0 Facts ▸ Make 10 Facts **New Thinking Strategies for Subtraction Facts** ▸ Up Through 10 ▸ Back Down Through 10	**Addition** ▸ Review and reinforce facts with sums to 18 and thinking strategies ▸ Addition facts extended to 2-digit numbers. Think *single-digit addition facts* and apply the appropriate place value. **Subtraction** ▸ Review and reinforce facts with minuends to 18 and thinking strategies. ▸ Subtraction facts extended to 2-digit numbers. Think *single-digit subtraction facts* and apply the appropriate place value. **Multiplication Facts (**Products to 36) **Thinking Strategies:** ▸ x2 Facts (related to addition doubles) ▸ x10 Facts (patterns) ▸ x5 Facts (clock facts, patterns) ▸ x9 Facts (patterns, helping facts) ▸ x1 Facts ("no-change" facts) ▸ x0 Facts (products of zero) ▸ x4 Facts (double-double) ▸ x3 Facts (double plus 1 set)	**Addition** Review and reinforce facts to 18 and thinking strategies **Subtraction** ▸ Review and reinforce facts with minuends to 18 and thinking strategies **Multiplication** ▸ Facts With Products to 36-Mastery by Mid-Year ▸ Facts With Products to 81-Mastery by Year End **Thinking Strategies:** ▸ x2 Facts (related to addition doubles) ▸ x10 Facts (patterns) ▸ x5 Facts (clock facts, patterns) ▸ x9 Facts (patterns, helping facts) ▸ x1 Facts ("no-change" facts) ▸ x0 Facts (products of zero) ▸ x4 Facts (double-double) ▸ x3 Facts (double plus 1 set) ▸ Last Six Facts (New; various strategies)	Review Addition and Subtraction Facts With Sums/Minuends to 18 **Multiplication** Review and Reinforce Multiplication Facts With Products to 81 and Thinking Strategies **Division** Division Facts With Dividends to 81-Mastery by Year End Using a "Think-Multiplication" Strategy	▸ Review Addition Subtraction, Multiplication and Division Facts. ▸ Reintroduce thinking strategies to struggling students ▸ See the Mental Math Teacher's Guides for Grades 2-5 for strategies and practice items
MENTAL COMPUTATION	**Addition:** ▸ Adding 10 to a number without counting	**Addition** ▸ Addition facts extended to 2-digit numbers. Think *single-digit addition facts* and apply the appropriate place value. (New) ▸ Front End Addition (2-digit numbers) ▸ Finding Compatibles (single-digit number combinations that make 10) ▸ Compensation (single-digit numbers) **Subtraction** ▸ *Think-Addition* (extended to 2-digit numbers)	**Addition** ▸ Front End Addition (continued from Grade 2) ▸ Break Up and Bridge (New) ▸ Finding Compatibles (single digit numbers that add to 10; 2-digit numbers that add up to 100) ▸ Compensation (extended to 2-digit numbers) **Subtraction** ▸ Back Down Through 10s (extended to subtraction of a single digit from a 2-digit number) ▸ Up Through 10s (extended to 2-digit numbers)	**Addition** ▸ Facts Extended to Addition of Numbers in 10s, 100s, and 1000s ▸ Front End Addition (extended to numbers in 1000s) ▸ Break Up and Bridge (extended to numbers in 100s ▸ Finding Compatibles (extended to numbers in 1000s) ▸ Compensation (extended to numbers in 100s) ▸ Make 10s, 100s, 1000s (Extension) **Subtraction** ▸ Facts Extended to Subtraction of Numbers in 10s, 100s, 1000s ▸ Back Down Through 10s (extended to numbers in 100s) ▸ Up Through 10s (extended to numbers in the 100s) ▸ Compensation (New for Subtraction) ▸ Break Up and Bridge (New for Subtraction) **Multiplication** ▸ Multiplying by 10 and 100 using a "place-value-change" strategy rather than an "attach zeros" strategy	**Addition** ▸ Front End Addition (extended to decimal 10^{ths} and 100^{ths}) ▸ Break Up and Bridge (extended to numbers in 1000s and to decimal 10^{ths} and 100^{ths}) ▸ Finding Compatible (extended to decimal 10^{ths} and $100^{ths)}$ ▸ Compensation (extended to 1000s and to decimal 10^{ths} and $100^{ths)}$ ▸ Make 10s, 100s, 1000s (continued from Grade 4) **Subtraction** ▸ Back Down Through 10s, 100s, 1000s (Extension) ▸ Up Through 10s - (extended to Numbers in 1000s and to decimal 10^{ths} and $100^{ths)}$ ▸ Compensation - (extended to numbers in 1000s) ▸ Balancing for a constant difference (New) ▸ Break Up and Bridge (extended to numbers in 1000s) **Multiplication** ▸ Facts Extended to 10s, 100s and 1000s ▸ Multiplying by 10, 100, 1000 using a "Place-Value-Change" strategy, rather than an "attach zeros" strategy - (continued from Grade 4) ▸ Multiplying by 0.1, 0.01 and 0.001 using a place-value-change strategy (New) ▸ Front End Multiplication (New) ▸ Compensation (New for Multiplication)	**Addition** Practice items provided for review of mental computation strategies for addition. ▸ Front End ▸ Break Up and Bridge ▸ Finding Compatibles ▸ Compensation ▸ Make 10s, 100s, 1000s **Subtraction** ▸ Back Down Through 10s, 100s, 1000s ▸ Up Through 10s, 100s, 1000s ▸ Compensation ▸ Balancing for a Constant Difference (continued From Grade 5) ▸ Break Up and Bridge (extended to numbers in the 10 000s) **Multiplication and Division** ▸ Multiplying and Dividing by 10, 100, 1000 using a "place-value-change" strategy) ▸ Multiplying by 0.1, 0.01, 0.001 (continued from Grade 5) ▸ Dividing by 0.1, 0.01, 0.001 using a "place-value-change" strategy (New) ▸ Front End Multiplication (continued from Grade 5) ▸ Compensation (continued from Grade 5) ▸ Finding Compatible Factors (New) ▸ Halving and Doubling (New) ▸ Using Division Facts for 10s, 100s, 1000s (New) Dividends of 10s, 100s, 1000s divided by single-digit divisors. ▸ Partitioning The Dividend (New)
ESTIMATION		▸ Rounding in Addition and Subtraction (2-digit numbers; 5 is not involved in the rounding procedure until Grade 4)	▸ Front End Addition and Subtraction (New) ▸ Rounding in Addition and Subtraction (extended to 3-digit numbers; 5 or 50 not involved in the rounding procedure until Grade 4) ▸ Adjusted Front End in Addition and Subtraction (New)	▸ Rounding in Addition and Subtraction (extended to 4-Digit Numbers and involving 5, 50 and 500 in the rounding procedure) ▸ Adjusted Front End in Addition and Subtraction (extended to numbers in 1000s)	▸ Rounding in Addition and Subtraction (continued from Grade 4 ▸ Rounding in Multiplication (2-or-3- digit factor by single digit factor; 2-digit by 2-digit) ▸ Adjusted Front End for Addition and Subtraction (extended to decimal 10^{ths} and 100^{ths})	▸ Rounding in Addition and Subtraction (continued From Grade 5) ▸ Rounding in Multiplication (extended from Grade 5 to include 3-digits by 2-digits) ▸ Rounding in Division (New)

Printed by the Document Publishing Centre
Design: Strategic Marketing and Graphic Design
2007

07ED41-17416