GREAT MATHEMATICIANS

OF THE 20th CENTURY

Contents

Pavel Sergeevich Aleksandrov

Born: 7 May 1896 in Bogorodsk (also called Noginsk), Russia
Died: 16 Nov 1982 in Moscow, Russia (USSR)

Like most Russian mathematicians there are different ways to transliterate **Aleksandrov**'s name into the Roman alphabet. The most common way, other than Aleksandrov, is to write it as **Alexandroff.**

Pavel Sergeevich Aleksandrov's father Sergej Aleksandrovich Aleksandrov was a medical graduate from Moscow University who had decided not to follow an academic career but instead had chosen to use his skills in helping people and so he worked as a general practitioner in Yaroslavskii. Later he worked in more senior positions in a hospital in Bogorodskii, which is where he was when Pavel Sergeevich was born.

When Pavel Sergeevich was one year old his father moved to Smolensk State hospital, where he was to earn the reputation of being a very fine surgeon, and the family lived from this time in Smolensk. The city of Smolensk is on the Dnieper River 420 km west of Moscow. Pavel Sergeevich's early education was from his mother, Tsezariya Akimovna Aleksandrova, who applied all her considerable talents to bringing up and educating her children. It was from her that Aleksandrov learnt French and also German. His home was one that was always filled with music as his brothers and sisters all had great talent in that area.

The fine start which his mother gave him meant that he always excelled at the grammar school in Smolensk which he attended. His mathematics teacher Alexsander Romanovich Eiges soon realised that his pupil had a remarkable talent for the subject and ([3] and [4]):-

> *... at grammar school he studied celestial mechanics and mathematical analysis. But his interest was mainly directed towards fundamental problems of mathematics: the foundations of geometry and non-euclidean geometry. Eiges had a proper appreciation of his pupil and exerted a decisive influence on his choice of a career in mathematics.*

In 1913 Aleksandrov graduated from the grammar school being dux of the school and winning the gold medal. Certainly at this time he had already decided on a career in mathematics, but he had not set his sights as high as a university teacher, rather he was aiming to become a secondary school teacher of mathematics. Eiges was the role model who he was aspiring to match at this stage, for Eiges had done more than teach Aleksandrov mathematics, he had also influenced his tastes in literature and the arts.

Aleksandrov entered Moscow University in 1913 and immediately he was helped by Stepanov. Stepanov, who was working at Moscow University, was seven years older than Aleksandrov but his home was also in Smolensk and he often visited the Aleksandrov home there. Stepanov was an important influence on Aleksandrov at this time and suggested that Aleksandrov join Egorov's seminar even in the first year of his studies in Moscow. In Aleksandrov's second year of study he came in contact with Luzin who had just returned to Moscow. Aleksandrov wrote (see for example [3] or [4]):-

> *After Luzin's lecture I turned to him for advice on how best to continue my mathematical studies and was struck most of all by Luzin's kindness to the man addressing him - an 18-year old student ... I then became a student of Luzin, during his most creative period ... To see Luzin in those years was to see a display of what is called an inspired relationship to science. I learnt not only mathematics from him, I received also a lesson in what makes a true scholar and what a university professor can and should be. Then, too, I saw that the pursuit of science and the raining of young people in it are two facets of one and the same activity - that of a scholar.*

Aleksandrov proved his first important result in 1915, namely that every non-denumerable Borel set contains a perfect subset. It was not only the result which was important for set theory, but also the methods which Aleksandrov used which turned out to be one of the most useful methods in descriptive set theory. After Aleksandrov's great successes Luzin did what many a supervisor might do, he realised that he had one of the greatest mathematical talents in Aleksandrov so he thought that it was worth asking him to try to solve the biggest open problem in set theory, namely the continuum hypothesis.

After Aleksandrov failed to solve the continuum hypothesis (which is not surprising since it can neither be proved or disproved as was shown by Cohen in the 1960s) he thought he was not capable of a mathematical career. Aleksandrov went to Novgorod-Severskii and became a theatre producer. He then went to Chernikov where, in addition to theatrical work, he lectured on Russian and foreign languages, becoming friends with poets, artists and musicians. After a short term in jail in 1919 at the time of the Russian revolution, Aleksandrov returned to Moscow in 1920. Luzin and Egorov had built up an impressive research group at the University of Moscow which the students called 'Luzitania' and they, together with Privalov and Stepanov, were very welcoming to Aleksandrov on his return.

It was not an immediate return to Moscow for Aleksandrov, however, for he spent 1920-21 back home in Smolensk where he taught at the University. During this time he worked on his research, going to Moscow about once every month to keep in touch with the mathematicians there and to prepare himself for his examinations. At around this time Aleksandrov became friendly with Urysohn, who was a member of 'Luzitania', and the friendship would soon develop into a major mathematical collaboration.

After taking his examinations in 1921, Aleksandrov was appointed as a lecturer at Moscow university and lectured on a variety of topics including functions of a real variable, topology and Galois theory. In July 1922 Aleksandrov and Urysohn went to spend the summer at Bolshev, near to Moscow, where they began to study concepts in topology. Hausdorff, building on work by Fréchet and others, had created a theory of topological and metric spaces in his famous book *Grundzüge der Mengenlehre* published in 1914. Aleksandrov and Urysohn now began to push the theory forward with work on countably compact spaces producing results of fundamental importance. The notion of a compact space and a locally compact space is due to them.

In the summers of 1923 and 1924 Aleksandrov and Urysohn visited Göttingen and impressed Emmy Noether, Courant and Hilbert with their results. The mathematicians in Göttingen were particularly impressed with their results on when a topological space is metrisable. In the summer of 1924 they also visited Hausdorff in Bonn and he was fascinated to hear the major new directions that the two were taking in topology. However while visiting Hausdorff in Bonn ([3] and [4]):-

> *Every day Aleksandrov and Urysohn swam across the Rhine - a feat that was far from being safe and provoked Hausdorff's displeasure.*

Aleksandrov and Urysohn then visited Brouwer in Holland and Paris in August 1924 before having a holiday in the fishing village of Bourg de Batz in Brittany. Of course mathematicians continue to do mathematics while on holiday and they were both working hard. On the morning of 17 August Urysohn began to write a new paper but tragically he drowned while swimming in the Atlantic later that day. Aleksandrov determined that no ideas of his great friend and collaborator should be lost and he spent part of 1925 and 1926 in Holland working with Brouwer on preparing Urysohn's paper for publication.

The atmosphere in Göttingen had proved very helpful to Aleksandrov, particularly after the death of Urysohn, and he went there every summer from 1925 until 1932. He became close friends with Hopf and the two held a topological seminar in Göttingen. Of course Aleksandrov also taught in Moscow University and from 1924 he organised a topology seminar there. At Göttingen, Aleksandrov also lectured and participated in Emmy Noether's seminar. In fact Aleksandrov always included Emmy Noether and Hilbert among his teachers, as well as Brouwer in Amsterdam and Luzin and Egorov in Moscow.

From 1926 Aleksandrov and Hopf were close friends working together. They spent some time in 1926 in the south of France with Neugebauer. Then Aleksandrov and Hopf spent the academic year 1927-28 at Princeton in the United States. This was an important year in the development of topology with Aleksandrov and Hopf in Princeton and able to collaborate with Lefschetz, Veblen and Alexander. During their year in Princeton, Aleksandrov and Hopf planned a joint multi-volume work on *Topology* the first volume of which did not appear until 1935. This was the only one of the three intended volumes to appear since World War II prevented further collaboration on the remaining two volumes. In fact before the joint work with Hopf appeared in print, Aleksandrov had begun yet another important friendship and collaboration.

In 1929 Aleksandrov's friendship with Kolmogorov began and they ([3] and [4]):-

> *... journeyed a lot along the Volga, the Dnieper, and other rivers, and in the Caucuses, the*

Crimea, and the south of France.

The year 1929 marks not only the beginning of the friendship with Kolmogorov but also the appointment of Aleksandrov as Professor of Mathematics at Moscow University. In 1935 Aleksandrov went to Yalta with Kolmogorov, then finished the work on his *Topology* book in the nearby Crimea and the book was published in that year. The 'Komarovski' period also began in that year ([3] and [4]):-

> *Over the last forty years, many of the events in the history of mathematics in the University of Moscow have been linked with Komarovka, a small village outside Moscow. Here is the house owned since* 1935 *by Aleksandrov and Kolmogorov. Many famous foreign mathematicians also visited Komarovka - Hadamard, Fréchet, Banach, Hopf, Kuratowski, and others.*

In 1938-1939 a number of leading mathematicians from the Moscow University, among them Aleksandrov, joined the Steklov Mathematical Institute of the USSR Academy of Sciences but at the same time they kept their positions at the University.

Aleksandrov wrote about 300 scientific works in his long career. As early as 1924 he introduced the concept of a locally finite covering which he used as a basis for his criteria for the metrisability of topological spaces. He laid the foundations of homology theory in a series of fundamental papers between 1925 and 1929. His methods allowed arguments of combinatorial and algebraic topology to be applied to point set topology and brought together these areas. Aleksandrov's work on homology moved forward with his homological theory of dimension around 1928-30

Aleksandrov was the first to use the phrase 'kernel of a homomorphism' and around 1940-41 he discovered the ingredients of an exact sequence. He worked on the theory of continuous mappings of topological spaces. In 1954 he organised a seminar on this last topic aimed at first year students at Moscow University and in this he showed one of the aspects of his career which was of major importance to him, namely the education of students. This is described in ([3] and [4]):-

> *To the training of these students and those who came after them, Aleksandrov literally devoted all his strength. His influence on the class of young men studying topology under him was never purely mathematical, however real and significant that was. There were physical days exercise on topological walks, in long outings lasting several days by boat, ... in swimming across the Volga or other broad stretches of water, in skiing excursions lasting for hours on the slopes outside Moscow, slopes to which Aleksandrov gave striking, fantastic names...*

Many honours were given to Aleksandrov for his outstanding contribution to mathematics. He was president of the Moscow Mathematical Society from 1932 to 64, vice president of the International Congress of Mathematicians from 1958 to 62, a corresponding member of the USSR Academy of Sciences from 1929 and a full member from 1953. Many other societies elected Aleksandrov to membership including the Göttingen Academy of Sciences, the Austrian Academy of Sciences, the Leopoldina Academy in Halle, the Polish Academy of Sciences, the National Academy of Sciences of the United States, the London Mathematical Society, the American Philosophical Society, and the Dutch Mathematical Society.

He edited several mathematical journals, in particular the famous Soviet Journal *Uspekhi Matematicheskikh Nauk*, and he received many Soviet awards, including the Stalin Prize in 1943 and five Orders of Lenin.

Today the Department of General Topology and Geometry of Moscow State University is Russia's leading centre of research in set-theoretic topology. After Aleksandrov's death in November 1982, his colleagues from the Department of Higher Geometry and Topology, in which he had held the chair, sent a letter to Moscow University's rector A A Logunov proposing that one of Aleksandrov's former students should become Head of the Department, to preserve Aleksandrov's scientific school. On 28 December 1982 the rector issued a circular creating the Department of general topology and Geometry. Vitaly Vitalievich Fedorchuk was elected Head of the Department.

Also in memory of Aleksandrov's contributions to topology at Moscow University and his work with the Moscow Mathematical Society, there is an annual topological symposium *Aleksandrov Proceedings* held every May.

Luitzen Egbertus Jan Brouwer

Born: 27 Feb 1881 in Overschie (now a suburb of Rotterdam), Netherlands
Died: 2 Dec 1966 in Blaricum, Netherlands

L E J Brouwer is usually known by this form of his name with full initials, but he was known to his friends as Bertus, an abbreviation of the second of his three forenames. He attended high school in Hoorn, a town on the Zuiderzee north of Amsterdam. His performance there was outstanding and he completed his studies by the age of fourteen. He had not studied Greek or Latin at high school but both were required for entry into university, so Brouwer spent the next two years studying these topics. During this time his family moved to Haarlem, just west of Amsterdam, and it was in the Gymnasium there in 1897 that he sat the entrance examinations for the University of Amsterdam.

Korteweg was the professor of mathematics at the University of Amsterdam when Brouwer began his studies, and he quickly realised that in Brouwer he had an outstanding student. While still an undergraduate Brouwer proved original results on continuous motions in four dimensional space and Korteweg encouraged him to present them for publication. This he did, and it became his first paper published by the Royal Academy of Science in Amsterdam in 1904. Other topics which interested Brouwer were topology and the foundations of mathematics. He learnt something of these topics from lectures at the university but he also read many works on the topics on his own.

He obtained his master's degree in 1904 and in the same year married Lize de Holl who was eleven years older that Brouwer and had a daughter from a previous marriage. After the marriage, which would produce no children, the couple moved to Blaricum, near Amsterdam. Three years later Lize qualified as a pharmacist and Brouwer helped her in many ways from doing bookkeeping to serving in the chemists shop. However, Brouwer did not gain the affection of his step-daughter and relations between them was strained.

From an early stage Brouwer was interested in the philosophy of mathematics, but he was also fascinated by mysticism and other philosophical questions relating to human society. He published his own ideas on this topic in 1905 in his treatise *Leven, Kunst, en Mystiek* (Life, art, and mysticism). In this work he [1]:-

> *... considers as one of the important moving principles in human activity the transition from goal to means, which after some repetitions may result in activities opposed to the original goal.*

Brouwer's doctoral dissertation, published in 1907, made a major contribution to the ongoing debate between Russell and Poincaré on the logical foundations of mathematics. His doctoral thesis [13]:-

> *... revealed the twin interests in mathematics that dominated his entire career; his fundamental concern with critically assessing the foundations of mathematics, which led to his creation of intuitionism, and his deep interest in geometry, which led to his seminal work in topology ...*

He quickly discovered that his ideas on the foundations of mathematics would not be readily accepted [13]:-

> *Brouwer quickly found that his philosophical ideas sparked controversy. Korteweg, his thesis advisor, had not been pleased with the more philosophical aspects of the thesis, and had even demanded that several parts of the original draft be cut from the final presentation. Korteweg urged Brouwer to concentrate on more "respectable" mathematics, so that the young man might enhance his mathematical reputation and thus secure an academic career. Brouwer was fiercely independent and did not follow in anybody's footsteps, but he apparently took his teacher's advice ...*

Brouwer continued to develop the ideas of his thesis in *The Unreliability of the Logical Principles* published in 1908.

The research which Brouwer now undertook was in two areas. He continued his study of the logical foundations of mathematics and he also put a very large effort into studying various problems which he attacked because they appeared on Hilbert's list of problems proposed at the Paris International Congress of Mathematicians in 1900. In particular Brouwer attacked Hilbert's fifth problem concerning the theory of continuous groups. He addressed the International Congress of Mathematicians in Rome in 1908 on the topological foundations of Lie groups. However, after studying Schönflies' report on set theory, he wrote to Hilbert:-

> *I discovered all of a sudden that the Schoenfliesian investigations concerning topology of the plane, on which I had relied in the fullest way, could not be taken as correct in all parts, so that my group-theoretic results also became doubt.*

In 1909 he was appointed as an privatdocent at the University of Amsterdam. He gave his inaugural lecture on 12 October 1909 on 'The nature of geometry' in which he outlined his research programme. A couple of months later he made an important visit to Paris, around Christmas 1909, and there met Poincaré, Hadamard and Borel. Prompted by discussions in Paris, he began working on the problem of the invariance of dimension.

Brouwer was elected to the Royal Academy of Sciences in 1912 and, in the same year, was appointed extraordinary professor of set theory, function theory and axiomatics at the University of Amsterdam; he would hold the post until he retired in 1951. Hilbert wrote a warm letter of recommendation which helped Brouwer to gain his chair in 1912. Despite the substantial contributions he had made to topology by this time, Brouwer chose to give his inaugural professorial lecture on intuitionism and formalism. In the following year Korteweg resigned his chair so that Brouwer could be appointed as ordinary professor.

Although he had helped Brouwer to obtain his chair in Amsterdam, in 1919 Hilbert tried to tempt him away with an offer of a chair in Göttingen. He was also offered the chair at Berlin in the same year. These must have been tempting offers, but despite their attractions Brouwer turned them down. Perhaps the exceptional way he was treated by Amsterdam, mentioned in the following quote by Van der Waerden, helped him make these decisions.

Van der Waerden, who studied at Amsterdam from 1919 to 1923, wrote about Brouwer as a lecturer (see for example [14]):-

> *Brouwer came* [*to the university*] *to give his courses but lived in Laren. He came only once a week. In general that would have not been permitted - he should have lived in Amsterdam - but for him an exception was made. ... I once interrupted him during a lecture to ask a question. Before the next week's lesson, his assistant came to me to say that Brouwer did not want questions put to him in class. He just did not want them, he was always looking at the blackboard, never towards the students. ... Even though his most important research contributions were in topology, Brouwer never gave courses on topology, but always on -- and only on -- the foundations of intuitionism. It seemed that he was no longer convinced of his results in topology because they were not correct from the point of view of intuitionism, and he judged everything he had done before, his greatest output, false according to his philosophy. He was a very strange person, crazy in love with his philosophy.*

As is mentioned in this quotation, Brouwer was a major contributor to the theory of topology and he is considered by many to be its founder. The status of the subject when he began his research is well described in [13]:-

> *When Brouwer was beginning his career as a mathematician, set-theoretic topology was in a primitive state. Controversy surrounded Cantor's general set theory because of the set-theoretic paradoxes or contradictions. Point set theory was widely applied in analysis and somewhat less widely applied in geometry, but it did not have the character of a unified theory. There were some perceived benchmarks. For example; the generally held view that dimension was invariant under one-to-one continuous mappings ...*

He did almost all his work in topology early in his career between 1909 and 1913. He discovered characterisations of topological mappings of the Cartesian plane and a number of fixed point theorems. His first fixed point theorem, which showed that an orientation preserving continuous one-one mapping of the sphere to itself always fixes at least one point, came out of his researches on Hilbert's fifth problem. Originally proved for a 2-dimensional sphere, Brouwer later generalised the result to spheres in *n* dimensions. Another result of exceptional importance was proving the invariance of dimension.

As well as proving theorems of major importance in topology, Brouwer also developed methods which have become standard tools in the subject. In particular he used simplicial approximation, which approximated continuous mappings by piecewise linear ones. He also introduced the idea of the degree of a mapping, generalised the Jordan curve theorem to n-dimensional space, and defined topological spaces in 1913.

Van der Waerden, in the above quote, said that Brouwer would not lecture on his own topological results since they did not fit with mathematical intuitionism. In fact Brouwer is best known to many mathematicians as the founder of the doctrine of mathematical intuitionism, which views mathematics as the formulation of mental constructions that are governed by self-evident laws. His doctrine differed substantially from the formalism of Hilbert and the logicism of Russell. His doctoral thesis in 1907 attacked the logical foundations of mathematics and marks the beginning of the Intuitionist School. His views had more in common with those of Poincaré and if one asks which side of the debate between Russell and Poincaré he came down on then it would have with the latter.

In his 1908 paper *The Unreliability of the Logical Principles* Brouwer rejected in mathematical proofs the Principle of the Excluded Middle, which states that any mathematical statement is either true or false. In 1918 he published a set theory developed without using the Principle of the Excluded Middle *Founding Set Theory Independently of the Principle of the Excluded Middle. Part One, General Set Theory.* His 1920 lecture *Does Every Real Number Have a Decimal Expansion?* was published in the following year. The answer to the question of the title which Brouwer gives is "no". Also in 1920 he published *Intuitionistic Set Theory,* then in 1927 he developed a theory of functions *On the Domains of Definition of Functions* without the use of the Principle of the Excluded Middle.

His constructive theories were not easy to set up since the notion of a set could not be taken as a basic concept but had to be built up using more basic notions which, in Brouwer's case, were choice sequences. Loosely speaking, that the elements of a set had property p, meant to Brouwer that he had a construction which allowed him to decide after a finite number of steps whether each element of the set had property p. Such ideas are fundamental to theoretical computer science today.

The later part of Brouwer's career contains some controversial episodes. He had been appointed to the editorial board of *Mathematische Annalen* in 1914 but in 1928 Hilbert decided that Brouwer was becoming too powerful, particularly since Hilbert felt that he himself did not have long to live (in fact he lived until 1943). He tried to remove Brouwer from the board in a way which was not compatible with the way the board was set up. Brouwer vigorously opposed the move and he was strongly supported by other board members such as Einstein and Carathéodory. In the end Hilbert managed to get his own way but it was a devastating episode for Brouwer who was left mentally broken; see [26] for details.

In 1935 Brouwer entered local politics when he was elected as Neutral Party candidate for the municipal council of Blaricum. He continued to serve on the council until 1941. He was also active setting up a new journal and he became a founding editor of *Compositio Mathematica* which began publication in 1934.

Further controversy arose due to his actions in World War II. Brouwer was active in helping the Dutch resistance, and in particular he supported Jewish students during this difficult period. However, in 1943 the Germans insisted that the students sign a declaration of loyalty to Germany and Brouwer encouraged his students to do so. He afterwards said that he did so in order that his students might have a chance to complete their studies and to work for the Dutch resistance against the Germans. However, after Amsterdam was liberated, Brouwer was suspended from his post for a few months because of his actions. Again he was deeply hurt and considered emigration.

After retiring in 1951, Brouwer lectured in South Africa in 1952, and the United States and Canada in 1953. His wife died in 1959 at the age of 89 and Brouwer, who himself was 78, was offered a one year post in the University of British Columbia in Vancouver; he declined. In 1962, despite being well into his 80s, he was offered a post in Montana. He died in 1966 in Blaricum as the result of a traffic accident.

Kneebone writes in [3] about Brouwer's contributions to the philosophy of mathematics:-

> *Brouwer is most famous ... for his contribution to the philosophy of mathematics and his attempt to build up mathematics anew on an Intuitionist foundation, in order to meet his own searching*

> *criticism of hitherto unquestioned assumptions. Brouwer was somewhat like Nietzsche in his ability to step outside the established cultural tradition in order to subject its most hallowed presuppositions to cool and objective scrutiny; and his questioning of principles of thought led him to a Nietzschean revolution in the domain of logic. He in fact rejected the universally accepted logic of deductive reasoning which had been codified initially by Aristotle, handed down with very little change into modern times, and very recently extended and generalised out of all recognition with the aid of mathematical symbolism.*

Kneebone also writes in [3] about the influence that Brouwer's views on the foundations of mathematics had on his fellow mathematicians:-

> *Brouwer's projected reconstruction of the whole edifice of mathematics remained a dream, but his ideal of constructivism is now woven into our whole fabric of mathematical thought, and it has inspired, as it still continues to inspire, a wide variety of inquiries in the constructivist spirit which have led to major advances in mathematical knowledge.*

Despite failing to convert mathematicians to his way of thinking, Brouwer received many honours for his outstanding contributions. We mentioned his election to the Royal Dutch Academy of Sciences above. Other honours included election to the Royal Society of London, the Berlin Academy of Sciences, and the Göttingen Academy of Sciences. He was awarded honorary doctorates the University of Oslo in 1929, and the University of Cambridge in 1954. He was made Knight in the Order of the Dutch Lion in 1932.

Élie Joseph Cartan

Born: 9 April 1869 in Dolomieu (near Chambéry), Savoie, Rhône-Alpes, France
Died: 6 May 1951 in Paris, France

Élie Cartan's mother was Anne Cottaz and his father was Joseph Cartan who was a blacksmith. The family were very poor and in late 19th century France it was not possible for children from poor families to obtain a university education. It was Élie's exceptional abilities, together with a lot of luck, which made a high quality education possible for him. When he was in primary school he showed his remarkable talents which impressed the young school inspector, later important politician, Antonin Dubost. Dubost was at this time employed as an inspector of primary schools and it was on a visit to the primary school in Dolomieu, in the French Alps, that he discovered the remarkable young Élie. Dubost was able to obtain state funds which paid for Élie to attend the Lycée in Lyons, where he completed his school education with distinction in mathematics. The state stipend was extended to allow him to study at the École Normale Supérieure in Paris.

Cartan became a student at the École Normale Supérieure in 1888 and obtained his doctorate in 1894. He was then appointed to the University at Montpellier where he lectured from 1894 to 1896. Following this he was appointed as a lecturer at the University of Lyon, where he taught from 1896 to 1903. In 1903 Cartan was appointed as a professor at the University of Nancy and he remained there until 1909 when he moved to Paris. His appointment in 1909 was as a lecturer at the Sorbonne but three years later he was appointed to the Chair of Differential and Integral Calculus in Paris. He was appointed as Professor of Rational Mechanics in 1920, and then Professor of Higher Geometry from 1924 to 1940. He retired in 1940.

He married Marie-Louise Bianconi in 1903 and they had four children, one of them Henri Cartan was to produce brilliant work in mathematics. Two other sons died tragically. Jean, a composer, died of tuberculosis at the age of 25 while their son Louis was a member of the Resistance fighting in France against the occupying German forces. After his arrest in February 1943 the family received no further news but they feared the worst. Only in May 1945 did they learn that he had been beheaded by the Nazis in December 1943. By the time they received the news of Louis' murder by the Germans, Cartan was 75 years old and it was a devastating blow for him. Their fourth child was a daughter.

Cartan worked on continuous groups, Lie algebras, differential equations and geometry. His work achieved a synthesis between these areas. He added greatly to the theory of continuous groups which had been initiated by Lie. His doctoral thesis of 1894 contains a major contribution to Lie algebras where he completed the classification of the semisimple algebras over the complex field which Killing had essentially found. However, although Killing had shown that only certain exceptional simple algebras were possible, he had not proved that in fact these algebras exist. This was shown by Cartan in his thesis when he constructed each of the exceptional simple Lie algebras over the complex field. He later classified the semisimple Lie algebras over the real field and found all the irreducible linear representations of the simple Lie algebras. He turned to the theory of associative algebras and investigated the structure for these algebras over the real and complex field. Wedderburn would complete Cartan's work in this area.

He then turned to representations of semisimple Lie groups. His work is a striking synthesis of Lie theory, classical geometry, differential geometry and topology which was to be found in all Cartan's work. He applied Grassmann algebra to the theory of exterior differential forms. He developed this theory between 1894 and 1904 and applied his theory of exterior differential forms to a wide variety of problems in differential geometry, dynamics and relativity. Dieudonné writes in [1]:-

> *He discussed a large number of examples, treating them in an extremely elliptic style that was made possible only by his uncanny algebraic and geometric insight and that has baffled two generations of mathematicians.*

In 1945 he published the book *Les systèmes différentiels extérieurs et leurs applications géométriques.*

By 1904 Cartan was writing papers on differential equations and in many ways this work is his most impressive. Again his approach was totally innovative and he formulated problems so that they were invariant and did not depend on the particular variables or unknown functions. This enabled Cartan to define what the general solution of an arbitrary differential system really is but he was not only interested in the general solution for he also studied singular solutions. He did this by moving from a given system to a new associated system whose general solution gave the singular solutions to the original system. He failed to show that all singular solutions were given by his technique, however, and this was not achieved until four years after his death.

From 1916 on he published mainly on differential geometry. Klein's *Erlanger Programm* was seen to be inadequate as a general description of geometry by Weyl and Veblen and Cartan was to play a major role. He examined a space acted on by an arbitrary Lie group of transformations, developing a theory of moving frames which generalises the kinematical theory of Darboux. In fact this work led Cartan to the notion of a fibre bundle although he does not give an explicit definition of the concept in his work.

Cartan further contributed to geometry with his theory of symmetric spaces which have their origins in papers he wrote in 1926. It developed ideas first studied by Clifford and Cayley and used topological methods developed by Weyl in 1925. This work was completed by 1932 and so provides [1]:-

> *... one of the few instances in which the initiator of a mathematical theory was also the one who brought it to completion.*

Cartan then went on to examine problems on a topic first studied by Poincaré. By this stage his son, Henri Cartan, was making major contributions to mathematics and Élie Cartan was able to build on theorems proved by his son. Henri Cartan said [9]:-

> [*My father*] *knew more than I did about Lie groups, and it was necessary to use this knowledge for the determination of all bounded circled domains which admit a transitive group. So we wrote an article on the subject together* [*Les transformations des domaines cerclés bornés, C. R. Acad. Sci. Paris* 192 (1931), 709-712]. *But in general my father worked in his corner, and I worked in mine.*

Cartan discovered the theory of spinors in 1913. These are complex vectors that are used to transform three-dimensional rotations into two-dimensional representations and they later played a fundamental role in quantum mechanics. Cartan published the two volume work *Leçons sur la théorie des spineurs* in 1938.

He is certainly one of the most important mathematicians of the first half of the 20^{th} century. Dieudonné writes in [1]:-

> *Cartan's recognition as a first rate mathematician came to him only in his old age; before* 1930 *Poincaré and Weyl were probably the only prominent mathematicians who correctly assessed his uncommon powers and depth. This was due partly to his extreme modesty and partly to the fact that in France the main trend of mathematical research after* 1900 *was in the field of function theory, but chiefly to his extraordinary originality. It was only after* 1930 *that a younger generation started to explore the rich treasure of ideas and results that lay buried in his papers. Since then his influence has been steadily increasing, and with the exception of Poincaré and Hilbert, probably no one else has done so much to give the mathematics of our day its present shape and viewpoints.*

For his outstanding contributions Cartan received many honours, but as Dieudonné explained in the above quote, these did not come until late in career. He received honorary degrees from the University of Liege in 1934, and from Harvard University in 1936. In 1947 he was awarded three honorary degrees from the Free University of Berlin, the University of Bucharest and the Catholic University of Louvain. In the following year he was awarded an honorary doctorate by the University of Pisa. He was elected a Fellow of the Royal Society of London on 1 May 1947, the Accademia dei Lincei and the Norwegian Academy. Elected to the French Academy of Sciences on 9 March 1931 he was vice-president of the Academy in 1945 and President in 1946.

Israil Moiseevic Gelfand

Born: 2 Sept 1913 in Krasnye Okny, Odessa, Ukraine, Russia
Died: 5 Oct 2009 in New Brunswick, N.J., USA

Israil Gelfand went to Moscow at the age of 16, in 1930, before completing his secondary education. There he took on a variety of different jobs such as door keeper at the Lenin library, but he also began to teach mathematics.

There were many different institutes in Moscow where mathematics was taught in evening classes and Gelfand taught elementary mathematics in various of these institutes, then a little later progressing to teach more advanced mathematics. While he did this evening teaching he also attended lectures at Moscow University, the first course he attended being the theory of functions of a complex variable by Lavrentev.

In 1932 Gelfand was admitted as a research student under Kolmogorov's supervision. His work was in functional analysis and he was fortunate to be in a strong school of functional analysis so he received much support from other mathematicians such as A E Plessner and L A Lyusternik. Gelfand presented his thesis *Abstract functions and linear operators* in 1935 which contains important results, but is perhaps even more important for the methods that he used, studying functions on normed spaces by applying linear functionals to them and using classical analysis to study the resulting functions.

Gelfand's next major achievement was the theory of commutative normed rings which he created and studied in his D.Sc. thesis submitted in 1938. The importance of this work is brought out in [20] and [21]:-

> *... it was* [*Gelfand*] *who brought to light the fundamental concept of a maximal ideal which made it possible to unite previously uncoordinated facts and to create an interesting new theory. Gelfand's theory of normed rings revealed close connections between Banach's general functional analysis and classical analysis.*

During the time that he was carrying out this work for his D.Sc., Gelfand taught at the USSR Academy of Sciences. He held a post at the Academy from 1935 until 1941 when he was appointed as professor at Moscow State University.

In joint work with Naimark in the early 1940s, Gelfand worked on noncommutative normed rings with an involution. They showed that these rings could always be represented as a ring of linear operators on a Hilbert space.

It is impossible to do any justice to the range of work covered by Gelfand in this short article. However, we should mention some of the main strands of his work. One important area which he started work on in the early 1940s was the theory of representations of non-compact groups. This work followed on from the representation theory of finite groups by Frobenius and Schur and the representation theory of compact groups by Weyl.

Another important area of his work is that on differential equations where he worked on the inverse Sturm-Liouville problem. He saw the importance of the work of Sobolev and Schwartz on the theory of generalised functions and distributions, and he developed this theory in a series of monographs. He worked on computational mathematics, developing general methods for solving the equations of mathematical physics by numerical means. In this area he also worked on difference operators.

Gelfand's work on group representations led him to study integral geometry (a term due to Blaschke) which in turn he was led to by a study of the Radon transform. Between 1968 and 1972 Gelfand produced a series of important papers on the cohomology of infinite dimensional Lie algebras.

From 1958 onwards Gelfand became interested in problems in biology and medicine. In 1960, together with Fomin and other scientists, he set up the Institute of Biological Physics of the USSR Academy of Sciences. In particular he became interested in cell biology and also became interested in experimental work as well as the theoretical work which was his first interest. His work in biology is described in [9] and [10] as follows:-

> *On the basis of actual biological results, he developed important general principles of the organisation of control in complex multi-cell systems. These ideas, apart from their biological significance, served as a starting point for the creation of new methods of finding an extremum, which were succesfully applied to problems of X-ray structural analysis, problems of recognition, ...*

Gelfand's interests were certainly not confined to research despite his incredible record of having published over 500 papers in mathematics, applied mathematics and biology. He established a correspondence school in mathematics which [4]:-

> *... helped to bring rich mathematical experiences to students all over the Soviet Union.*

His style of teaching is described in [20] and [21]:-

> *One of the characteristic features of Israil Moiseevic's activities has been the extremely close bond between his research work and his teaching. The formulation of new problems and unexpected questions, a tendency to look at even well known things from a new point of view characterises Gelfand as a teacher, regardless of whether at a given moment he is holding a conversation with schoolchildren or with his own colleagues.*

In 1973 Gelfand was awarded the Order of Lenin for the third time ([9] and [10]):-

> *For services in the development of mathematics, the training of scientific specialists and in celebration of his sixtieth birthday ...*

This is only one of a very large number of honours which have been given to Gelfand over many years of outstanding contributions. He was president of the Moscow Mathematical Society during 1968-70. He was elected an honorary member of the American National Academy of Science, the American Academy of Arts and Sciences, the Royal Irish Academy, the American Mathematical Society, the London Mathematical Society. He has been awarded many honorary doctorates including one from the University of Oxford.

In 1989-90 Gelfand taught at Harvard University and in 1990 he also taught at Massachusetts Institute of Technology. In that same year, 1990, he emigrated to the United States where he became Distinguished Visiting Professor at Rutgers. He also holds a chair in the departments of mathematics and biology at the Center for Mathematics, Science, and Computer Education in the Institute for Discrete Mathematics and Computer Science at Rutgers University.

In 1992 Gelfand set up a programme in the United States similar to the correspondence school in mathematics which he had run in Russia. The Gelfand Outreach Program [4]:-

> *... fosters mathematical excellence in high school students.*

In 1994 Gelfand was awarded a MacArthur Fellowship from the John T and Catherine D MacArthur Foundation. The MacArthur Fellowships are [4]:-

> *... no-strings-attached awards that are intended to foster creativity in a wide range of human endeavours. The award to Gelfand is* $375,000, *to be paid over five years.*

Let us mention two other honours given to Gelfand. In 1989 he received the Kyoto Prize from the Inamori Foundation, an international award to honor those who have contributed significantly to the scientific, cultural, and spiritual betterment of mankind. The announcement states:-

> *Izrail Moiseevich Gelfand is one of the highest authorities in modern mathematical sciences. Through his pioneering and monumental work in mathematical sciences, especially in functional analysis - which has experienced tremendous development this century and not only affected other areas of mathematics but has also provided indispensable mathematical tools for the physics of elementary particles and quantum mechanics - he has brought up and inspired many prominent mathematicians in the course of his creative career. He has provided key ideas and deep insights to the whole of mathematical sciences and made outstanding contributions to the*

advancement of the field.

In 2005 Gelfand received the Leroy P Steele Prize of the American Mathematical Society for Lifetime Achievement:-

> *... for profoundly influencing many fields of research through his own work and through his interactions with other mathematicians and students.*

Kurt Gödel

Born: 28 April 1906 in Brünn, Austria-Hungary (now Brno, Czech Republic)
Died: 14 Jan 1978 in Princeton, New Jersey, USA

Kurt Gödel's father was Rudolf Gödel whose family were from Vienna. Rudolf did not take his academic studies far as a young man, but had done well for himself becoming managing director and part owner of a major textile firm in Brünn. Kurt's mother, Marianne Handschuh, was from the Rhineland and the daughter of Gustav Handschuh who was also involved with textiles in Brünn. Rudolf was 14 years older than Marianne who, unlike Rudolf, had a literary education and had undertaken part of her school studies in France. Rudolf and Marianne Gödel had two children, both boys. The elder they named Rudolf after his father, and the younger was Kurt.

Kurt had quite a happy childhood. He was very devoted to his mother but seemed rather timid and troubled when his mother was not in the home. He had rheumatic fever when he was six years old, but after he recovered life went on much as before. However, when he was eight years old be began to read medical books about the illness he had suffered from, and learnt that a weak heart was a possible complication. Although there is no evidence that he did have a weak heart, Kurt became convinced that he did, and concern for his health became an everyday worry for him.

Kurt attended school in Brünn, completing his school studies in 1923. His brother Rudolf said:-

> *Even in High School my brother was somewhat more one-sided than me and to the astonishment of his teachers and fellow pupils had mastered university mathematics by his final Gymnasium years. ... Mathematics and languages ranked well above literature and history. At the time it was rumoured that in the whole of his time at High School not only was his work in Latin always given the top marks but that he had made not a single grammatical error.*

Gödel entered the University of Vienna in 1923 still without having made a definite decision whether he wanted to specialise in mathematics or theoretical physics. He was taught by Furtwängler, Hahn, Wirtinger, Menger, Helly and others. The lectures by Furtwängler made the most impact on Gödel and because of them he decided to take mathematics as his main subject. There were two reasons: Furtwängler was an outstanding mathematician and teacher, but in addition he was paralysed from the neck down so lectured from a wheel chair with an assistant who wrote on the board. This would make a big impact on any student, but on Gödel who was very conscious of his own health, it had a major influence. As an undergraduate Gödel took part in a seminar run by Schlick which studied Russell's book *Introduction to mathematical philosophy*. Olga Taussky-Todd, a fellow student of Gödel's, wrote:-

> *It became slowly obvious that he would stick with logic, that he was to be Hahn's student and not Schlick's, that he was incredibly talented. His help was much in demand.*

He completed his doctoral dissertation under Hahn's supervision in 1929 submitting a thesis proving the completeness of the first order functional calculus. He became a member of the faculty of the University of Vienna in 1930, where he belonged to the school of logical positivism until 1938. Gödel's father died in 1929 and, having had a successful business, the family were left financially secure. After the death of her husband, Gödel's mother purchased a large flat in Vienna and both her sons lived in it with her. By this time Gödel's older brother was a successful radiologist. We mentioned above that Gödel's mother had a literary education and she was now able to enjoy the culture of Vienna, particularly the theatre accompanied by Rudolf and Kurt.

Gödel is best known for his proof of "Gödel's Incompleteness Theorems". In 1931 he published these results in *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme*. He proved fundamental results about axiomatic systems, showing in any axiomatic mathematical system there are propositions that cannot be proved or disproved within the axioms of the system. In particular the consistency of the axioms cannot be proved. This ended a hundred years of attempts to establish axioms which would put the whole of mathematics on an axiomatic basis. One major attempt had been by Bertrand Russell with *Principia Mathematica* (1910-13). Another was Hilbert's formalism which was dealt a severe blow by Gödel's results. The theorem did not destroy the fundamental idea of formalism, but it did demonstrate that any system would have to be more comprehensive than that envisaged by Hilbert. Gödel's results were a landmark in 20^{th}-century mathematics, showing that mathematics is not a finished object, as had been believed. It also implies that a computer can never be programmed to answer all mathematical questions.

Gödel met Zermelo in Bad Elster in 1931. Olga Taussky-Todd, who was at the same meeting, wrote:-

> *The trouble with Zermelo was that he felt he had already achieved Gödel's most admired result himself. Scholz seemed to think that this was in fact the case, but he had not announced it and perhaps would never have done so. ... The peaceful meeting between Zermelo and Gödel at Bad Elster was not the start of a scientific friendship between two logicians.*

Submitting his paper on incompleteness to the University of Vienna for his habilitation, this was accepted by Hahn on 1 December 1932. Gödel became a Privatdozent at the University of Vienna in March 1933.

Now 1933 was the year that Hitler came to power. At first this had no effect on Gödel's life in Vienna; he had little interest in politics. In 1934 Gödel gave a series of lectures at Princeton entitled *On undecidable propositions of formal mathematical systems.* At Veblen's suggestion Kleene, who had just completed his Ph.D. thesis at Princeton, took notes of these lectures which have been subsequently published. However, Gödel suffered a nervous breakdown as he arrived back in Europe and telephoned his brother Rudolf from Paris to say he was ill. He was treated by a psychiatrist and spent several months in a sanatorium recovering from depression.

Despite the health problems, Gödel's research was progressing well and he proved important results on the consistency of the *axiom of choice* with the other axioms of set theory in 1935. However after Schlick, whose seminar had aroused Gödel's interest in logic, was murdered by a National Socialist student in 1936, Gödel was much affected and had another breakdown. His brother Rudolf wrote:-

> *This event was surely the reason why my brother went through a severe nervous crisis for some time, which was of course of great concern, above all for my mother. Soon after his recovery he received the first call to a Guest Professorship in the USA.*

He visited Göttingen in the summer of 1938, lecturing there on his set theory research. He returned to Vienna and married Adele Porkert in the autumn of 1938. In fact he had met her in 1927 in Der Nachtfalter night club in Vienna. She was six years older than Gödel and had been married before and both his parents, but particularly his father, objected to the idea that they marry. She was not the first girl that Gödel's parents had objected to, the first he had met around the time he went to university was ten years older than him.

In March 1938 Austria had became part of Germany but Gödel was not much interested and carried on his life much as normal. He visited Princeton for the second time, spending the first term of session 1938-39 at the Institute for Advanced Study. The second term of that academic year he gave a beautiful lecture course at Notre Dame. Most who held the title of privatdozent in Austria became paid lecturers after the country became part of Germany but Gödel did not and his application made on 25 September 1939 was given an unenthusiastic response. It seems that he was thought to be Jewish, but in fact this was entirely wrong, although he did have many Jewish friends. Others also mistook him for a Jew, and he was once attacked by a gang of youths, believing him to be a Jew, while out walking with his wife in Vienna.

When the war started Gödel feared that he might be conscripted into the German army. Of course he was also convinced that he was in far too poor health to serve in the army, but if he could be mistaken for a Jew he might be mistaken for a healthy man. He was not prepared to risk this, and after lengthy negotiation to obtain a U.S. visa he was fortunate to be able to return to the United States, although he had to travel via Russia and Japan to do so. His wife accompanied him.

In 1940 Gödel arrived in the United States, becoming a U.S. citizen in 1948 (in fact he believed he had found an inconsistency in the United States Constitution, but the judge had more sense than to listen during his interview!). He was an ordinary member of the Institute for Advanced Study from 1940 to 1946 (holding year long appointments which were renewed every year), then he was a permanent member until 1953. He held a chair at Princeton from 1953 until his death, holding a contract which explicitly stated that he had no lecturing duties. One of Gödel's closest friends at Princeton was Einstein. They each had a high regard for the other and they spoke frequently. It is unclear how much Einstein influenced Gödel to work on relativity, but he did indeed contribute to that subject.

He received the Einstein Award in 1951, and National Medal of Science in 1974. He was a member of the National Academy of Sciences of the United States, a fellow of the Royal Society, a member of the Institute of France, a fellow of the Royal Academy and an Honorary Member of the London Mathematical Society. However, it says much about his feelings towards Austria that he refused membership of the Academy of Sciences in Vienna, then later when he was elected to honorary membership he again refused the honour. He also refused to accept the highest National Medal for scientific and artistic achievement that Austria offered him. He certainly felt bitter at his own treatment but equally so about that of his family.

Gödel's mother had left Vienna before he did, for in 1937 she returned to her villa in Brno where she was openly critical of the National Socialist regime. Gödel's brother Rudolf had remained in Vienna but by 1944 both expected German defeat, and Rudolf's mother joined him in Vienna. In terms of the treaty negotiated after the war between the Austrians and the Czechs, she received one tenth of the value for her villa in Brno. It was an injustice which infuriated Gödel; in fact he always took such injustices as personal even although large numbers suffered in the same way.

After settling in the United States, Gödel again produced work of the greatest importance. His masterpiece *Consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory* (1940) is a classic of modern mathematics. In this he proved that if an axiomatic system of set theory of the type proposed by Russell and Whitehead in *Principia Mathematica* is consistent, then it will remain so when the axiom of choice and the generalized continuum-hypothesis are added to the system. This did not prove that these axioms were independent of the other axioms of set theory, but when this was finally established by Cohen in 1963 he built on these ideas of Gödel.

Concerns with his health became increasingly worrying to Gödel as the years went by. Rudolf, Gödel's brother, was a medical doctor so the medical details given by him in the following will be accurate. He wrote:-

> *My brother had a very individual and fixed opinion about everything and could hardly be convinced otherwise. Unfortunately he believed all his life that he was always right not only in mathematics but also in medicine, so he was a very difficult patient for doctors. After severe bleeding from a duodenal ulcer ... for the rest of his life he kept to an extremely strict* (*over strict?*) *diet which caused him slowly to lose weight.*

Adele, Gödel's wife, was a great support to him and she did much to ease the tensions which troubled him. However she herself began to suffer health problems, having two strokes and a major operation. Towards the end of his life Gödel became convinced that he was being poisoned and, refusing to eat to avoid being poisoned, essentially starved himself to death [3]:-

> *A slight person and very fastidious, Gödel was generally worried about his health and did not travel or lecture widely in later years. He had no doctoral students, but through correspondence and personal contact with the constant succession of visitors to Princeton, many people benefited from his extremely quick and incisive mind. Friend to Einstein, von Neumann and Morgenstern, he particularly enjoyed philosophical discussion.*

He died [18]:-

> *... sitting in a chair in his hospital room at Princeton, in the afternoon of* 14 *January* 1978.

It would be fair to say that Gödel's ideas have changed the course of mathematics [3]:-

> *... it seems clear that the fruitfulness of his ideas will continue to stimulate new work. Few mathematicians are granted this kind of immortality.*

Felix Hausdorff

Born: 8 Nov 1868 in Breslau, Germany (now Wrocław, Poland)
Died: 26 Jan 1942 in Bonn, Germany

Felix Hausdorff's father was Louis Hausdorff, who was a merchant dealing in textiles, and his mother was Hedwig Tietz; both were Jewish. Felix was born into a wealthy family and this had quite an influence on his life and career since he never had the problem of having to work to support himself financially. Felix was stll a young boy when the family moved from Breslau to Leipzig, and it was in Leipzig that he grew up. At school he had wide interests and, in addition to mathematics, he was attracted to literature and music. In fact he wanted to pursue a career in music as a composer but his parents put pressure on him to give up the idea of becoming a composer. They achieved this, but only after quite an effort for Felix had his heart set on the idea, and after this he turned towards mathematics as the subject to study at university.

Hausdorff studied at Leipzig University under Heinrich Bruns and Adolph Mayer, graduating in 1891 with a doctorate in applications of mathematics to astronomy. His thesis was titled *Zur Theorie der astronomischen Strahlenbrechung* and studied refraction and extinction of light in the atmosphere. He published four papers on astronomy and optics over the next few years and he submitted his habilitation thesis to Leipzig in 1895, also based on his research into astronomy and optics. His methods were based on those of Bruns who had developed his own method of determining refraction and extinction, based on an idea of Bessel.

However Hausdorff's main interests were in literature and philosophy and his circle of friends consisted almost entirely of writers and artists, such as the composer Max Reger, rather than scientists. He also seemed keen to make a name for himself in the world of literature, more so than in the world of mathematics, and he published his literary work under the pseudonym of Paul Mongré. In 1897 he published his first literary work *Sant' Ilario: Thoughts from Zarathustra's Country* which was a work of 378 pages. He published a philosophy book *Das Chaos in kosmischer Auslese* (1898) which is a critique of metaphysics contrasting the empirical with the transcendental world that he rejected. His next major literary work was a book of poem *Ekstases* (1900) which deals with nature, life, death and erotic passion, and in addition he wrote many articles on philosophy and literature. As Segal writes in [6]:-

> *As the child of a wealthy family, he did not have to worry about making a career as a mathematician; for him, mathematics, both as research and as a subject to teach, was more an avocation than anything else.*

Hausdorff married Charlotte Sara Goldschmidt in Leipzig in 1899. Charlotte and her sister Edith were from Jewish parents but had converted to Lutheranism. Although still a Privatdozent, Hausdorff was well off, so marriage at this stage in his career presented no financial difficulties. In 1902 he was promoted to an extraordinary professorship of mathematics at Leipzig and turned down the offer of a similar appointment at Göttingen. This clearly indicates that at this time Hausdorff was keener to remain in his literary and artistic circle in Leipzig than he was to progress his career in mathematics. He continued his literary interests and in 1904 published a farce *Der Arzt seiner Ehre.* In many ways this marked the end of his literary interests but this farce was performed in 1912 and was very successful.

After 1904 Hausdorff began working in the area for which he is famous, namely topology and set theory. He introduced the concept of a partially ordered set and from 1901 to 1909 he proved a series of results on ordered sets. In 1907 he introduced special types of ordinals in an attempt to prove Cantor's continuum hypothesis. He also posed a generalisation of the continuum hypothesis by asking if 2 to the power $\aleph_a$ was equal to $\aleph_{a+1}$. Hausdorff proved further results on the cardinality of Borel sets in 1916.

Hausdorff taught at Leipzig until 1910 when he went to Bonn. It was Study who in many ways motivated Hausdorff to become more involved in both mathematical research and also in developing his career in mathematics. Partly the lack of mathematical drive in his early career had been due to his extreme modesty, so his friendship with Study was an important factor in turning him towards important problems and his subsequent rise to fame. Having encouraged Hausdorff to move to Bonn, Study encouraged him to move again in 1913, this time to become an ordinary professorship at Greifswalf. A year later, in 1914, Hausdorff published his famous text *Grundzüge der Mengenlehre* which builds on work by Fréchet and others to created a theory of topological and metric spaces. Earlier results on topology fitted naturally into the framework set up by Hausdorff as Katetov explains in [1]:-

> [*Hausdorff's*] *broad approach, his aesthetic feeling, and his sense of balance may have played a substantial part. He succeeded in creating a theory of topological and metric spaces into which the previous results fitted well, and he enriched it with many new notions and theorems. From the modern point of view, the Grundzüge contained, in addition to other special topics, the beginnings of the theories of topological and metric spaces, which are now included in all textbooks on the subject.*

The *Grundzüge* was republished in revised form in 1927 and 1937. The 1914 edition was reprinted in 1949 and 1965 by Chelsea, the 1927 edition was published in 1937 in Russian, and the 1937 edition was translated into English and also published by Chelsea in 1957.

Hausdorff returned to Bonn in 1921, by this time an emminent mathematician, and he worked there until 1935 when he was forced to retire by the Nazi regime. Although as early as 1932 he sensed the oncoming calamity of Nazism he made no attempt to emigrate while it was still possible. He swore the necessary oath to Hitler in November 1934 but by the following January a new law forced him to give up his position. He continued to undertake research in topology and set theory but the results could not be published in Germany. Certainly he wanted to continue research and wished to emigrate for in 1939 he wrote to Courant asking if he could find a research fellowship for him. Sadly Courant could not do so.

As a Jew his position became more and more difficult. In 1941 he was scheduled to go to an internment camp but managed to avoid being sent. Erich Bessel-Hagen, the only colleague from Bonn who kept in touch with Hausdorff after his forced retirement, wrote in a letter to a friend in the summer of 1941 (see [24] and [6]):-

> *I often had great anxiety about the Hausdorffs. Mrs Hausdorff was for a long time seriously ill from an old ailment - I don't know what it is. Scarcely was she over the worst than there came the agitation about the intended internment of the Jews. Here the procedure was mad. In the early part of the year, old nuns were forcibly driven out of a cloister on the Kreuzberg; these poor old women who never harmed anyone and only carried on a retiring life devoted to their pious usages ... Now all Jews still living in Bonn will be compulsorily interned in this stolen building; they must either auction their things, or place them for preservation in "faithful" hands.*

Bonn University requested that the Hausdorffs be allowed to remain in their home and this was granted. By October 1941 they were forced to wear the "yellow star" and around the end of the year they were informed that they would be sent to Cologne. Bessel-Hagen wrote that he knew this was (see [24] and [6]):-

> *... a preliminary to deportation to Poland. And what one hears concerning the accommodation and treatment of Jews there is completely unimaginable.*

They were not sent to Cologne but in January 1942 they were informed that they were to be interned in Endenich. Together with his wife and his wife's sister, he committed suicide on 26 January. He wrote to a friend on Sunday 25 January (see [24] and [6]):-

> *Dear Friend Wollstein*
> *By the time you receive these lines, we three will have solved the problem in another way - in the way which you have continually attempted to dissuade us. ...*
> *What has been done against the Jews in recent months arouses well-founded anxiety that we will no longer be allowed to experience a bearable situation. ...*
> *Forgive us, that we still cause you trouble beyond death; I am convinced that you will do what you are able to do (and which perhaps is not very much). Forgive us also our desertion! We wish you and all our friends will experience better times*
> *Yours faithfully*

Felix Hausdorff

On the night of Sunday 25 January all three took barbiturates. Both Hausdorff and his wife Charlotte were dead by the morning of the 26 January. Edith, Charlotte's sister, survived for a few days in a coma before dying.

We have mentioned above Hausdorff's early work on astronomy, his work on philosophy, and his literature. We also mentioned his work on ordered sets and his masterpiece on set theory and topology *Grundzüge der Mengenlehre* (1914). Let us add that one famous paradoxical result, namely that half a sphere and one third of a sphere can be congruent to each other, is contained in this work (see [28] for details). Let us now examine other important contributions made by Hausdorff. In 1919 he introduced the notion of Hausdorff dimension in the seminal paper *Dimension und äusseres Mass.* The idea was a generalisation of one which had been introduced five years earlier by Carathéodory but Hausdorff realised that Carathéodory's construction made sense, and was useful, for defining fractional dimensions. Hausdorff's paper includes a proof that the dimension of the middle-third Cantor set is log 2/log 3. Chatterji writes [10]:-

> *Within the mathematical work of Hausdorff the two publications devoted explicitly to measure theory occupy a significant place: they are not only important for measure theory but have also contributed fundamentally to its development. It is not well known that throughout his life Hausdorff had been interested in various fundamental problems of measure and integration theory and had made important contributions at different times. This becomes quite evident if one studies his lecture notes and other Nachlass papers.*

One such lecture course was given on probability theory by Hausdorff in Bonn in the summer of 1923. He studied the Gaussian law of errors, limit theorems and problems of moments, and set theory and the strong law of large numbers, which he based on measure theory.

David Hilbert

Born: 23 Jan 1862 in Königsberg, Prussia (now Kaliningrad, Russia)
Died: 14 Feb 1943 in Göttingen, Germany

David Hilbert attended the gymnasium in his home town of Königsberg. After graduating from the gymnasium, he entered the University of Königsberg. There he went on to study under Lindemann for his doctorate which he received in 1885 for a thesis entitled *Über invariante Eigenschaften specieller binärer Formen, insbesondere der Kugelfunctionen.* One of Hilbert's friends there was Minkowski, who was also a doctoral student at Königsberg, and they were to strongly influence each others mathematical progress.

In 1884 Hurwitz was appointed to the University of Königsberg and quickly became friends with Hilbert, a friendship which was another important factor in Hilbert's mathematical development. Hilbert was a member of staff at Königsberg from 1886 to 1895, being a Privatdozent until 1892, then as Extraordinary Professor for one year before being appointed a full professor in 1893.

In 1892 Schwarz moved from Göttingen to Berlin to occupy Weierstrass's chair and Klein wanted to offer Hilbert the vacant Göttingen chair. However Klein failed to persuade his colleagues and Heinrich Weber was appointed to the chair. Klein was probably not too unhappy when Weber moved to a chair at Strasbourg three years later since on this occasion he was successful in his aim of appointing Hilbert. So, in 1895, Hilbert was appointed to the chair of mathematics at the University of Göttingen, where he continued to teach for the rest of his career.

Hilbert's eminent position in the world of mathematics after 1900 meant that other institutions would have liked to tempt him to leave Göttingen and, in 1902, the University of Berlin offered Hilbert Fuchs' chair. Hilbert turned down the Berlin chair, but only after he had used the offer to bargain with Göttingen and persuade them to set up a new chair to bring his friend Minkowski to Göttingen.

Hilbert's first work was on invariant theory and, in 1888, he proved his famous Basis Theorem. Twenty years earlier Gordan had proved the finite basis theorem for binary forms using a highly computational approach. Attempts to generalise Gordan's work to systems with more than two variables failed since the computational difficulties were too great. Hilbert himself tried at first to follow Gordan's approach but soon realised that a new line of attack was necessary. He discovered a completely new approach which proved the finite basis theorem for any number of variables but in an entirely abstract way. Although he proved that a finite basis existed his methods did not construct such a basis.

Hilbert submitted a paper proving the finite basis theorem to *Mathematische Annalen.* However Gordan was the expert on invariant theory for *Mathematische Annalen* and he found Hilbert's revolutionary approach difficult to appreciate. He refereed the paper and sent his comments to Klein:-

> *The problem lies not with the form ... but rather much deeper. Hilbert has scorned to present his thoughts following formal rules, he thinks it suffices that no one contradict his proof ... he is content to think that the importance and correctness of his propositions suffice. ... for a comprehensive work for the Annalen this is insufficient.*

However, Hilbert had learnt through his friend Hurwitz about Gordan's letter to Klein and Hilbert wrote himself to Klein in forceful terms:-

> *... I am not prepared to alter or delete anything, and regarding this paper, I say with all modesty, that this is my last word so long as no definite and irrefutable objection against my reasoning is raised.*

At the time Klein received these two letters from Hilbert and Gordan, Hilbert was an assistant lecturer while Gordan was the recognised leading world expert on invariant theory and also a close friend of Klein's. However Klein recognised the importance of Hilbert's work and assured him that it would appear in the

Annalen without any changes whatsoever, as indeed it did.

Hilbert expanded on his methods in a later paper, again submitted to the *Mathematische Annalen* and Klein, after reading the manuscript, wrote to Hilbert saying:-

> *I do not doubt that this is the most important work on general algebra that the Annalen has ever published.*

In 1893 while still at Königsberg Hilbert began a work *Zahlbericht* on algebraic number theory. The German Mathematical Society requested this major report three years after the Society was created in 1890. The *Zahlbericht* (1897) is a brilliant synthesis of the work of Kummer, Kronecker and Dedekind but contains a wealth of Hilbert's own ideas. The ideas of the present day subject of 'Class field theory' are all contained in this work. Rowe, in [18], describes this work as:-

> *... not really a Bericht in the conventional sense of the word, but rather a piece of original research revealing that Hilbert was no mere specialist, however gifted. ... he not only synthesized the results of prior investigations ... but also fashioned new concepts that shaped the course of research on algebraic number theory for many years to come.*

Hilbert's work in geometry had the greatest influence in that area after Euclid. A systematic study of the axioms of Euclidean geometry led Hilbert to propose 21 such axioms and he analysed their significance. He published *Grundlagen der Geometrie* in 1899 putting geometry in a formal axiomatic setting. The book continued to appear in new editions and was a major influence in promoting the axiomatic approach to mathematics which has been one of the major characteristics of the subject throughout the 20th century.

Hilbert's famous 23 Paris problems challenged (and still today challenge) mathematicians to solve fundamental questions. Hilbert's famous speech *The Problems of Mathematics* was delivered to the Second International Congress of Mathematicians in Paris. It was a speech full of optimism for mathematics in the coming century and he felt that open problems were the sign of vitality in the subject:-

> *The great importance of definite problems for the progress of mathematical science in general ... is undeniable. ...* [*for*] *as long as a branch of knowledge supplies a surplus of such problems, it maintains its vitality. ... every mathematician certainly shares ..the conviction that every mathematical problem is necessarily capable of strict resolution ... we hear within ourselves the constant cry: There is the problem, seek the solution. You can find it through pure thought...*

Hilbert's problems included the continuum hypothesis, the well ordering of the reals, Goldbach's conjecture, the transcendence of powers of algebraic numbers, the Riemann hypothesis, the extension of Dirichlet's principle and many more. Many of the problems were solved during this century, and each time one of the problems was solved it was a major event for mathematics.

Today Hilbert's name is often best remembered through the concept of Hilbert space. Irving Kaplansky, writing in [2], explains Hilbert's work which led to this concept:-

> *Hilbert's work in integral equations in about* 1909 *led directly to 20th-century research in functional analysis* (*the branch of mathematics in which functions are studied collectively*)*. This work also established the basis for his work on infinite-dimensional space, later called Hilbert space, a concept that is useful in mathematical analysis and quantum mechanics. Making use of his results on integral equations, Hilbert contributed to the development of mathematical physics by his important memoirs on kinetic gas theory and the theory of radiations.*

Many have claimed that in 1915 Hilbert discovered the correct field equations for general relativity before Einstein but never claimed priority. The article [11] however, shows that this view is in error. In this paper the authors show convincingly that Hilbert submitted his article on 20 November 1915, five days before Einstein submitted his article containing the correct field equations. Einstein's article appeared on 2 December 1915 but the proofs of Hilbert's paper (dated 6 December 1915) do not contain the field equations.

As the authors of [11] write:-

> *In the printed version of his paper, Hilbert added a reference to Einstein's conclusive paper and a concession to the latter's priority: "The differential equations of gravitation that result are, as*

it seems to me, in agreement with the magnificent theory of general relativity established by Einstein in his later papers". If Hilbert had only altered the dateline to read "submitted on 20 November 1915, revised on [*any date after 2 December 1915, the date of Einstein's conclusive paper*]*," no later priority question would have arisen.*

In 1934 and 1939 two volumes of *Grundlagen der Mathematik* were published which were intended to lead to a 'proof theory', a direct check for the consistency of mathematics. Gödel's paper of 1931 showed that this aim is impossible.

Hilbert contributed to many branches of mathematics, including invariants, algebraic number fields, functional analysis, integral equations, mathematical physics, and the calculus of variations. Hilbert's mathematical abilities were nicely summed up by Otto Blumenthal, his first student:-

In the analysis of mathematical talent one has to differentiate between the ability to create new concepts that generate new types of thought structures and the gift for sensing deeper connections and underlying unity. In Hilbert's case, his greatness lies in an immensely powerful insight that penetrates into the depths of a question. All of his works contain examples from far-flung fields in which only he was able to discern an interrelatedness and connection with the problem at hand. From these, the synthesis, his work of art, was ultimately created. Insofar as the creation of new ideas is concerned, I would place Minkowski higher, and of the classical great ones, Gauss, Galois, and Riemann. But when it comes to penetrating insight, only a few of the very greatest were the equal of Hilbert.

Among Hilbert's students were Hermann Weyl, the famous world chess champion Lasker, and Zermelo.

Hilbert received many honours. In 1905 the Hungarian Academy of Sciences gave a special citation for Hilbert. In 1930 Hilbert retired and the city of Königsberg made him an honorary citizen of the city. He gave an address which ended with six famous words showing his enthusiasm for mathematics and his life devoted to solving mathematical problems:-

Wir müssen wissen, wir werden wissen - *We must know, we shall know.*

Heinz Hopf

Born: 19 Nov 1894 in Gräbschen (near Breslau), Germany (now Wrocław, Poland)
Died: 3 June 1971 in Zollikon, Switzerland

Heinz Hopf's father was Wilhelm Hopf and his mother was Elizabeth Kirchner. Wilhelm Hopf was from a Jewish family. He joined Heinrich Kirchner at his brewery in Breslau in 1887. Wilhelm married Elizabeth, Heinrich Kirchner's eldest daughter, in 1892 and by that time he owned the brewery firm. They had two children, the eldest Hedwig was born in 1893 while Heinz was born in the following year. Elizabeth Hopf was a Protestant and, in 1895, Wilhelm converted to his wife's religion.

Heinz attended Dr Karl Mittelhaus' school from 1901 until 1904 and following this he began his studies at the König-Wilhelm Gymnasium in Breslau. He attended the Gymnasium until 1913 and it was at this school that his talent for mathematics first became clear to his teachers. In his other subjects, however, his results were less good and it is probable that he devoted too much time to sport, he was particularly fond of swimming and tennis, and not enough to his academic subjects. He left the Gymnasium with the mathematics report stating:-

> *He has shown an extraordinary gift in this topic, especially in the algebraic direction.*

In April 1913 Hopf entered the Silesian Friedrich Wilhelms University in Breslau to read for a degree in mathematics. There he was taught by Kneser, Schmidt, and Rudolf Sturm. He also attended lectures by Dehn and Steinitz who taught at the polytechnic in Breslau. However, his studies were interrupted by the outbreak of World War I in 1914. He immediately enlisted and for the duration of the war he fought on the Western front as a lieutenant. During a fortnight's leave from military service in 1917 Hopf went to a class by Schmidt on set theory at the University of Breslau. From that time on he knew that he wanted to undertake research in mathematics. He wrote in [13] about the influence Schmidt's lectures had on him :-

> *I was fascinated; this fascination - of the power of the method of the mapping degree - has never left me since, but has influenced major parts of my work. And when I look for the cause of this effect, I see particularly two things: firstly, Schmidt's vividness and enthusiasm in his lecture, and secondly my own increased receptiveness during a fortnight off after many years of military service.*

After the war Hopf returned to his studies in Breslau but after about a year he left and went to the University of Heidelberg. By this time Schmidt had left Breslau and it appears that Hopf wanted to go to Heidelberg to be with his sister who had begun her studies there in the previous year. At Heidelberg Hopf took courses in philosophy and psychology as well as attending courses by Perron and Stäckel. In 1920 Hopf went to study for his doctorate at the University of Berlin where Schmidt was now teaching. He attended several courses by Schur in Berlin and he received his doctorate in 1925 with a thesis, supervised by Schmidt, studying the topology of manifolds. Among other results, he classified simply connected Riemannian 3-manifolds of constant curvature in this thesis. It was an impressive piece of work which received the following praise from Schmidt in his report (see for example [11]):-

> *The boldness of the questions deserves as much admiration as the surprising results of the solutions. But the most beautiful thing in the thesis is the method of proving, which is, particularly rarely found in works in that area, abstract and comprehensible in every step, and which, due to the abstractness, shows equally clearly the richness of the concrete geometric imagination.*

Bieberbach and Schmidt examined him in mathematics, while Planck examined him in physics.

Hopf went to Göttingen in 1925 where he met Emmy Noether. Her contributions would play an important part in Hopf's developing ideas. Perhaps even more significant was the fact that Aleksandrov was also spending time in Göttingen and Hopf wrote in [13]:-

> *My most important experience in Göttingen was to meet Pavel Aleksandrov. The meeting soon became friendship; not only topology, not only mathematics was discussed; it was a fortunate and also a very happy time, not restricted to Göttingen but continued on many joint journeys.*

During this year in Göttingen Hopf worked on his habilitation thesis which was completed by the autumn of 1926. The thesis contains a different proof of the fact just shown by Lefschetz that for any closed manifold the sum of the indices of a generic vector field is a topological invariant, namely the Euler characteristic. Aleksandrov and Hopf spent some time in 1926 in the south of France with Neugebauer. Then the two spent the academic year 1927-28 at Princeton in the United States. This was an important year in the development of topology with Aleksandrov and Hopf in Princeton and able to collaborate with Lefschetz, Veblen and Alexander. During their year in Princeton, Aleksandrov and Hopf planned a joint multi-volume work on *Topology* the first volume of which did not appear until 1935. This was the only one of the three intended volumes to appear since World War II prevented further collaboration on the remaining two volumes.

Hopf married Anja von Mickwitz in October 1928. He was offered an assistant professorship by Princeton in December 1929 but he rejected the offer. In 1930 Weyl left his chair in the ETH in Zurich to take up a chair at Göttingen and in 1931 Hopf was approached to see if he was interested in accepting this chair. In part the offer had been prompted by a very positive recommendation which Schur had sent to Zurich:-

> *Hopf is an excellent lecturer, a mathematician of strong temperament and strong influence, a leading example in his discipline ... I cannot wish you a better colleague in respect to his manners, his education and his sympathetic nature.*

Hopf replied to the approach of the ETH in Zurich indicating that he would accept a formal offer:-

> *A call to Switzerland, to the beautiful city of Zurich, could indeed tempt and honour me, particularly to such a famous chair. I therefore declare that I am in principle willing to accept such an offer.*

However, before receiving the formal offer from Zurich, Hopf received the offer of a chair at Freiburg but he waited for the Zurich offer and accepted it. He took up his duties in Zurich in April 1931. The next few years were not easy ones for Hopf. After the Nazis came to power in Germany in 1933, Hopf's father, being Jewish, came under increasing pressure. Hopf continued to visit his parents in Breslau up until 1939. Seeing the difficulties that his father faced Hopf arranged for his parents to receive immigration papers for Switzerland. However, his father fell ill and could not travel.

Hopf was able to provide refuge in Switzerland for friends who had to flee Germany under the Nazis. In particular Schur came for a while before finally going to Palestine in 1939. Hopf's own position became more difficult, however, for he was still a German citizen. Lefschetz, realising Hopf's difficulties, invited him to Princeton but Hopf refused. Then in 1943 he was told to move back to Germany or he would lose his German citizenship. Faced with this he had little choice but to quickly apply for Swiss citizenship, which was soon granted.

With the end of World War II Hopf was able to help his German friends again. He did much more than this, however, for he put much energy into trying to re-establish a mathematical community in Germany. His visit to the research centre in Oberwolfach in August 1946 was part of his efforts. Soon after the Oberwolfach visit, Hopf went to the United States where he spent six months and there he renewed many old friendships. He was offered professorships by many of the most prestigious of the American universities but, after careful consideration, he decided to remain loyal to Zurich.

Over the next few years he enjoyed invitations to lecture at leading international conferences, and he visited many places including Paris, Brussels, Rome and Oxford. He spent the academic year 1955-56 with his wife in the United States.

Most of Hopf's work was in algebraic topology where he can be thought of as continuing Brouwer's work. He studied homotopy classes and vector fields producing a formula about the integral curvature.

Hopf extended Lefschetz's fixed point formula in work which he undertook in 1928. It is in this 1928 paper that he first explicitly used homology groups. His work on the homology of manifolds, undertaken in

Princeton in 1927-28, led to his definition of the intersection ring by defining a product on cycles by their intersection. This idea was later seen to be connected to cohomology.

He defined what is now known as the 'Hopf invariant' in 1931. This was done in his work on maps between spheres of different dimensions which cannot be distinguished homologically so required the introduction of a new invariant. In 1939 he examined the homology of a compact Lie group. This was to attack questions posed to him by Élie Cartan. The ideas which he introduced in this investigation led to him defining what is today called a Hopf algebra.

In the early 1940s Hopf published [11]:-

> *The paper Fundamentalgruppe und zweite Bettische Gruppe* [*which*] *is legitimately regarded to be the beginning of homological algebra. It opened the way for the definition for the homology and cohomology of a group. This step was made independently at different places shortly after the paper became known ...*

The honours which Hopf received are almost too numerous to list. He was President of the International Mathematical Union from 1955 until 1958. He received honorary doctorates from many universities including Princeton, Freiburg, Manchester, Sorbonne, Brussels, and Lausanne. He was awarded many prizes including the Gauss-Weber medal and the Lobachevsky award. He was elected to honorary membership of many learned societies throughout the world.

Freudenthal gives this description of Hopf in [1]:-

> *Hopf was a short, vigorous man with cheerful, pleasant features. His voice was well modulated, and his speech slow and strongly articulated. His lecture style was clear and fascinating; in personal conversation he conveyed stimulating ideas.*

Frei and Stammbach in [11] pay this tribute to Hopf:-

> *Without doubt Heinz Hopf was one of the most distinguished mathematicians of the twentieth century. His work is closely linked with the emergence of algebraic topology; it is most decisively thanks to his early works that this area established itself as a new and important branch of mathematics. his work has influenced profoundly the evolution not only of topology but of a large part of mathematics. But Heinz Hopf was not only a gifted researcher: he was also an excellent teacher and a personality of the highest integrity. at the same time, he effervesced with charm and subtle humour.*

Andrey Nikolaevich Kolmogorov

Born: 25 April 1903 in Tambov, Tambov province, Russia
Died: 20 Oct 1987 in Moscow, Russia (USSR)

Andrei Nikolaevich Kolmogorov's parents were not married and his father took no part in his upbringing. His father Nikolai Kataev, the son of a priest, was an agriculturist who was exiled. He returned after the Revolution to head a Department in the Agricultural Ministry but died in fighting in 1919. Kolmogorov's mother also, tragically, took no part in his upbringing since she died in childbirth at Kolmogorov's birth. His mother's sister, Vera Yakovlena, brought Kolmogorov up and he always had the deepest affection for her.

In fact it was chance that had Kolmogorov born in Tambov since the family had no connections with that place. Kolmogorov's mother had been on a journey from the Crimea back to her home in Tunoshna near Yaroslavl and it was in the home of his maternal grandfather in Tunoshna that Kolmogorov spent his youth. Kolmogorov's name came from his grandfather, Yakov Stepanovich Kolmogorov, and not from his own father. Yakov Stepanovich was from the nobility, a difficult status to have in Russia at this time, and there is certainly stories told that an illegal printing press was operated from his house.

After Kolmogorov left school he worked for a while as a conductor on the railway. In his spare time he wrote a treatise on Newton's laws of mechanics. Then, in 1920, Kolmogorov entered Moscow State University but at this stage he was far from committed to mathematics. He studied a number of subjects, for example in addition to mathematics he studied metallurgy and Russian history. Nor should it be thought that Russian history was merely a topic to fill out his course, indeed he wrote a serious scientific thesis on the owning of property in Novgorod in the 15th and 16th centuries. There is an anecdote told by D G Kendall in [10] regarding this thesis, his teacher saying:-

> *You have supplied one proof of your thesis, and in the mathematics that you study this would perhaps suffice, but we historians prefer to have at least ten proofs.*

Kolmogorov may have told this story as a joke but nevertheless jokes are only funny if there is some truth in them and undoubtedly this is the case here.

In mathematics Kolmogorov was influenced at an early stage by a number of outstanding mathematicians. P S Aleksandrov was beginning his research (for the second time) at Moscow around the time Kolmogorov began his undergraduate career. Luzin and Egorov were running their impressive research group at this time which the students called 'Luzitania'. It included M Ya Suslin and P S Urysohn, in addition to Aleksandrov. However the person who made the deepest impression on Kolmogorov at this time was Stepanov who lectured to him on trigonometric series.

It is remarkable that Kolmogorov, although only an undergraduate, began research and produced results of international importance at this stage. He had finished writing a paper on operations on sets by the spring of 1922 which was a major generalisation of results obtained by Suslin. By June of 1922 he had constructed a summable function which diverged almost everywhere. This was wholly unexpected by the experts and Kolmogorov's name began to be known around the world. The authors of [7] and [8] note that:-

> *Almost simultaneously* [*Kolmogorov*] *exhibited his interest in a number of other areas of classical analysis: in problems of differentiation and integration, in measures of sets etc. In every one of his papers, dealing with such a variety of topics, he introduced an element of originality, a breadth of approach, and a depth of thought.*

Kolmogorov graduated from Moscow State University in 1925 and began research under Luzin's supervision in that year. It is remarkable that Kolmogorov published eight papers in 1925, all written while he was still an undergraduate. Another milestone occurred in 1925, namely Kolmogorov's first paper on probability appeared. This was published jointly with Khinchin and contains the 'three series' theorem as well as results on inequalities of partial sums of random variables which would become the basis for martingale inequalities and the stochastic calculus.

In 1929 Kolmogorov completed his doctorate. By this time he had 18 publications and Kendall writes in [10]:-

> *These included his versions of the strong law of large numbers and the law of the iterated logarithm, some generalisations of the operations of differentiation and integration, and a contribution to intuitional logic. His papers ... on this last topic are regarded with awe by specialists in the field. The Russian language edition of Kolmogorov's collected works contains a retrospective commentary on these papers which* [*Kolmogorov*] *evidently regarded as marking an important development in his philosophical outlook.*

An important event for Kolmogorov was his friendship with Aleksandrov which began in the summer of 1929 when they spent three weeks together. On a trip starting from Yaroslavl, they went by boat down the Volga then across the Caucasus mountains to Lake Sevan in Armenia. There Aleksandrov worked on the topology book which he co-authored with Hopf, while Kolmogorov worked on Markov processes with continuous states and continuous time. Kolmogorov's results from his work by the Lake were published in 1931 and mark the beginning of diffusion theory. In the summer of 1931 Kolmogorov and Aleksandrov made another long trip. They visited Berlin, Göttingen, Munich, and Paris where Kolmogorov spent many hours in deep discussions with Paul Lévy. After this they spent a month at the seaside with Fréchet

Kolmogorov was appointed a professor at Moscow University in 1931. His monograph on probability theory *Grundbegriffe der Wahrscheinlichkeitsrechnung* published in 1933 built up probability theory in a rigorous way from fundamental axioms in a way comparable with Euclid's treatment of geometry. One success of this approach is that it provides a rigorous definition of conditional expectation. As noted in [10]:-

> *The year* 1931 *can be regarded as the beginning of the second creative stage in Kolmogorov's life. Broad general concepts advanced by him in various branched of mathematics are characteristic of this stage.*

After mentioning the highly significant paper *Analytic methods in probability theory* which Kolmogorov published in 1938 laying the foundations of the theory of Markov random processes, they continue to describe:-

> *... his ideas in set-theoretic topology, approximation theory, the theory of turbulent flow, functional analysis, the foundations of geometry, and the history and methodology of mathematics.* [*His contributions to*] *each of these branches ...* [*is*] *a single whole, where a serious advance in one field leads to a substantial enrichment of the others.*

Aleksandrov and Kolmogorov bought a house in Komarovka, a small village outside Moscow, in 1935. Many famous mathematicians visited Komarovka: Hadamard, Fréchet, Banach, Hopf, Kuratowski, and others. Gnedenko and other graduate students went on ([7] and [8]):-

> *... mathematical outings* [*which*] *ended in Komarovka, where Kolmogorov and Aleksandrov treated the whole company to dinner. Tired and full of mathematical ideas, happy from the consciousness that we had found out something which one cannot find in books, we would return in the evening to Moscow.*

Around this time Malcev and Gelfand and others were graduate students of Kolmogorov along with Gnedenko who describes what it was like being supervised by Kolmogorov ([7] and [8]):-

> *The time of their graduate studies remains for all of Kolmogorov's students an unforgettable period in their lives, full of high scientific and cultural strivings, outbursts of scientific progress and a dedication of all one's powers to the solutions of the problems of science. It is impossible to forget the wonderful walks on Sundays to which* [*Kolmogorov*] *invited all his own students* (*graduates and undergraduates*)*, as well as the students of other supervisors. These outings in the environs of Bolshevo, Klyazma, and other places about 30-35 kilometres away, were full of discussions about the current problems of mathematics* (*and its applications*)*, as well as discussions about the questions of the progress of culture, especially painting, architecture and*

> *literature.*

In 1938-1939 a number of leading mathematicians from the Moscow University joined the Steklov Mathematical Institute of the USSR Academy of Sciences while retaining their positions at the University. Among them were Aleksandrov, Gelfand, Kolmogorov, Petrovsky, and Khinchin. The Department of Probability and Statistics was set up at the Institute and Kolmogorov was appointed as Head of Department.

Kolmogorov later extended his work to study the motion of the planets and the turbulent flow of air from a jet engine. In 1941 he published two papers on turbulence which are of fundamental importance. In 1954 he developed his work on dynamical systems in relation to planetary motion. He thus demonstrated the vital role of probability theory in physics.

We must mention just a few of the numerous other major contributions which Kolmogorov made in a whole range of different areas of mathematics. In topology Kolmogorov introduced the notion of cohomology groups at much the same time, and independently of, Alexander. In 1934 Kolmogorov investigated chains, cochains, homology and cohomology of a finite cell complex. In further papers, published in 1936, Kolmogorov defined cohomology groups for an arbitrary locally compact topological space. Another contribution of the highest significance in this area was his definition of the cohomology ring which he announced at the International Topology Conference in Moscow in 1935. At this conference both Kolmogorov and Alexander lectured on their independent work on cohomology.

In 1953 and 1954 two papers by Kolmogorov, each of four pages in length, appeared. These are on the theory of dynamical systems with applications to Hamiltonian dynamics. These papers mark the beginning of KAM-theory, which is named after Kolmogorov, Arnold and Moser. Kolmogorov addressed the International Congress of Mathematicians in Amsterdam in 1954 on this topic with his important talk *General theory of dynamical systems and classical mechanics.*

N H Bingham [10] notes Kolmogorov's major part in setting up the theory to answer the probability part of Hilbert's Sixth Problem "to treat ... by means of axioms those physical sciences in which mathematics plays an important part; in the first rank are the theory of probability and mechanics" in his 1933 monograph *Grundbegriffe der Wahrscheinlichkeitsrechnung.* Bingham also notes:-

> *... Paul Lévy writes poignantly of his realisation, immediately on seeing the "Grundbegriffe", of the opportunity which he himself had neglected to take. A rather different perspective is supplied by the eloquent writings of Mark Kac on the struggles that Polish mathematicians of the calibre Steinhaus and himself had in the 1930s, even armed with the "Grundbegriffe", to understand the* (*apparently perspicuous*) *notion of stochastic independence.*

If Kolmogorov made a major contribution to Hilbert's sixth problem, he completely solved Hilbert's Thirteenth Problem in 1957 when he showed that Hilbert was wrong in asking for a proof that there exist continuous functions of three variables which could not be represented by continuous functions of two variables.

Kolmogorov took a special interest in a project to provide special education for gifted children [10]:-

> *To this school he devoted a major proportion of his time over many years, planning syllabuses, writing textbooks, spending a large number of teaching hours with the children themselves, introducing them to literature and music, joining in their recreations and taking them on hikes, excursions, and expeditions. ...* [*Kolmogorov*] *sought to ensure for these children a broad and natural development of the personality, and it did not worry him if the children in his school did not become mathematicians. Whatever profession they ultimately followed, he would be content if their outlook remained broad and their curiosity unstifled. Indeed it must have been wonderful to belong to this extended family of* [*Kolmogorov*].

Such an outstanding scientist as Kolmogorov naturally received a whole host of honours from many different countries. In 1939 he was elected to the USSR Academy of Sciences. He received one of the first State Prizes to be awarded in 1941, the Lenin Prize in 1965, the Order of Lenin on six separate occasions, and the Lobachevsky Prize in 1987. He was also elected to the many other academies and societies including the Romanian Academy of Sciences (1956), the Royal Statistical Society of London (1956), the Leopoldina Academy of Germany (1959), the American Academy of Arts and Sciences (1959), the London Mathematical

Society (1959), the American Philosophical Society (1961), The Indian Statistical Institute (1962), the Netherlands Academy of Sciences (1963), the Royal Society of London (1964), the National Academy of the United States (1967), the French Academy of Sciences (1968).

In addition to the prizes mentioned above, Kolmogorov was awarded the Balzan International Prize in 1962. Many universities awarded him an honorary degree including Paris, Stockholm, and Warsaw.

Kolmogorov had many interests outside mathematics, in particular he was interested in the form and structure of the poetry of the Russian author Pushkin.

Nikolai Nikolaevich Luzin

Born: 9 Dec 1883 in Irkutsk, Russia
Died: 25 Feb 1950 in Moscow, Russia (USSR)

Nikolai Nikolaevich Luzin was born in Irkutsk, and his birthplace was not, as is incorrectly stated in a number of sources, Tomsk. Nikolai's father was a businessman, half Russian and half Buryat. Nikolai was the only son of his parents and the family moved to Tomsk when he was about eleven years old so that he could attend the Gymnasium there.

One might expect that Nikolai would have shown a special talent for mathematics at the Gymnasium, but this was far from the case ([15] and [16]):-

> *This was because the system of instruction ... was based on mechanical memory: it was required to learn the theorems by heart and to reproduce their proofs exactly. For Luzin this was torture. His progress in mathematics at the Gymnasium became worse and worse, so that his father was obliged to engage a tutor ...*

Fortunately the tutor was a talented young man who quickly discovered that, despite Luzin's poor performance in mathematics, he could solve hard problems but often using a novel method that the tutor had never seen before. Soon the tutor had shown Luzin that mathematics was not a subject where one had to learn long lists of facts, but a topic where creativity and imagination played a major role.

In 1901 Luzin left the Gymnasium and at this time his father sold his business and the family moved to Moscow. There Luzin entered the Faculty of Physics and Mathematics at Moscow University intending to train to become an engineer. At first Luzin lived in the new family home in Moscow, but Luzin's father began to gamble on the stock exchange with the money he had made from the sale of his business. The family soon hit hard times as Luzin's father lost all their savings and the family had to leave their home. Luzin, together with a friend, moved into a room owned by the widow of a doctor. His friend soon became involved with the Revolution and was forced into hiding. Luzin stayed on by himself in the room but he clearly got on well with the owners since he later, in 1908, married the widow's daughter.

At Moscow University Luzin studied under Bugaev, learning from him the theory of functions which was to influence greatly the direction his research would eventually take. However he was only an average student who seemed to show little flair for mathematics. However, although Luzin appeared to lack talent in mathematics, one of his teachers Egorov spotted his great talent, invited him to his home, and began to set him hard problems.

There was a mathematics student at the university, Pavel Florensky, who experienced a crisis after graduating and turned to religion and the study of theology. This had a major effect on Luzin, who was a close friend of Florensky, as we shall describe below.

After graduating in the autumn of 1905 Luzin seemed unsure whether to devote himself to mathematics. In fact Luzin's crisis had hit him in the spring of 1905 and, on 1 May 1906, Luzin wrote to Florensky from Paris where Egorov had sent him five months earlier in an attempt to get him through the crisis (see [9]):-

> *You found me a mere child at the University, knowing nothing. I don't know how it happened, but I cannot be satisfied any more with analytic functions and Taylor series ... it happened about a year ago. ... To see the misery of people, to see the torment of life, to wend my way home from a mathematical meeting ... where, shivering in the cold, some women stand waiting in vain for dinner purchased with horror - this is an unbearable sight. It is unbearable, having seen this, to calmly study (*in fact to enjoy*) science. After that I could not study only mathematics, and I wanted to transfer to the medical school. ... I have been here about five months, but have only recently begun to study.*

Luzin was not only upset by seeing the prostitutes, he also says in the letter how he had been affected by the 'terrible days' of the 1905 Revolution. There are letters from Egorov at this time pleading with Luzin not to give up mathematics. After returning to Russia, Luzin studied medicine and theology as well as mathematics. However in April 1908 he wrote of the joy he was finding in number theory (see [9]):-

> *It is a mysterious area that envelops me deeper and deeper.*

In the same letter he says that he has just married and:-

> *... my wife is also very interested and shares my commitment to the search for the profound truths of life.*

Largely Luzin's crisis seems to have been solved by Florensky to whom Luzin wrote in July 1908:-

> *Two times I was very close to suicide - then I came ... looking to talk with you, and both times I felt as if I had leaned on a pillar and with this feeling of support I returned home ... I owe my interest in life to you...*

His interest in mathematics slowly returned but it was not until 1909 that Luzin seems to have finally committed himself completely to mathematics. Under Egorov's supervision he worked on his master's thesis. In 1910 he was appointed as assistant lecturer in Pure Mathematics at Moscow University. He worked for a year with Egorov and they went on to publish joint papers on function theory which mark the beginnings of the Moscow school of function theory.

In 1910 Luzin travelled abroad visiting Göttingen where he was influenced by Edmund Landau. He returned to Moscow in 1914 and he completed his thesis *The integral and trigonometric series* which he submitted in 1915. After his oral examination he was awarded a doctorate, despite having submitted his thesis for the Master's Degree. Egorov was extraordinarily impressed by the work and had pressed for the award of the doctorate, but it was written in a style quite different from the accepted Russian style of the time. Some of the results were not rigorously proved but were justified using phrases such as 'it seems to me' and 'I am convinced'. Other mathematicians were not so impressed at the time, for example Steklov wrote comments in the margin such as 'it seems to him, but it doesn't seem to me' and 'Göttingen chatter'.

However, the work was of fundamental importance as is stated in [15] and [16]:-

> *The influence of Luzin's dissertation on the future development of the theory of functions cannot be overestimated. Its fundamental results, deep methods of investigation and fundamental statements of problems put it into the ranks of works with which it is difficult to compare any dissertation or monograph of the time.*

In 1914 Luzin and his wife separated for a short time and again Florensky seems to have helped them through the difficult time. He wrote to Luzin's wife (see [9]):-

> *Nikolai Nikolaevich is a very sweet and fine person; but in personal relationships he is not at all mature, especially in intuitively perceiving the hidden currents of life. ... You will have to take the relationship in hand and create a family tone, simplicity. Instead, as I perceive it ... you have established the tone of an acquaintanceship rather than a family.*

Florensky seems to have given good advice since Luzin and his wife returned to a successful marriage.

In 1917 Luzin was appointed as Professor of Pure Mathematics at Moscow University just before the Revolution. The Revolution caused Luzin to rethink some of the same thoughts as he had done at the time of his crisis and again he exchanged letters with Florensky. By this stage, however, his mathematical career was extremely successful and the second crisis did not materialise.

Over the next ten years Luzin and Egorov built up an impressive research group at the University of Moscow which the students called 'Luzitania'. The first students included P S Aleksandrov, M Ya Suslin, D E Menshov and A Ya Khinchin. The next students included P S Urysohn, A N Kolmogorov, N K Bari, rnik and N G Shnirelman. In 1923 P S Novikov and L V Keldysh joined the group.

Another of the members of the Luzitania research group at this time was Lavrentev. In fact Lavrentev draws the following picture of the group:-

> *Whereas Egorov was reserved and formal, Luzin was extroverted and theatrical, inspiring real devotion among these students and young colleagues. ... There was intense camaraderie ... inspired by Luzin.*

Luzin's main contributions are in the area of foundations of mathematics and measure theory. He also made significant contributions to descriptive set topology. In the theory of boundary properties of analytic functions he proved an important result in 1919 on the invariance of sets of boundary points under conformal mappings. He also studied, together with Privalov, boundary uniqueness properties of analytic functions.

From 1917 onwards, Luzin studied descriptive set theory. He stated the fundamental problem ([15] and [16]):-

> *The aim of set theory is a question of great importance: can we regard a line atomistically as a set of points: incidentally this question is not new, but goes back to the Greeks.*

Much of Luzin's work on set theory involved the study of effective sets, that is sets which can be constructed without the axiom of choice. Keldysh describes this work in [12] and [13]:-

> *... Luzin proceeded from the point of view of the French school* (*Borel, Lebesgue*), *which greatly influenced him. But whereas the French had analysed set-theoretical constructions carried out with the help of the Axiom of Choice, Luzin went considerably further and considered difficulties arising within the theory of effective sets. The study of effective sets that he embarked upon was pursued intensively for more than two decades and led to the solution of many important problems of set theory ...*

Luzin's school was at its peak during the years 1922 to 1926, but then Luzin concentrated on writing his second monograph on the theory of functions and spent less time with the young mathematicians in the school. Many of these mathematicians turned to other topics such as topology, differential equations, and functions of a complex variable.

In 1927 Luzin was elected as a member of the USSR Academy of Sciences. Two years later he became a full member of first the Department of Philosophy, then to the Department of Pure Mathematics. He worked from this time until his death in the USSR Academy of Sciences. From 1935 he headed the Department of the Theory of Functions of Real Variables at the Steklov Institute.

In 1931 Luzin himself turned to a new area when he began to study differential equations and their application to geometry and to control theory. His work in this area led him to study the bending of surfaces which is described in [15] and [16]:-

> *The bending of a surface on a principal base is a continuous bending of a surface under which the conjugacy of the net of certain curves on the surface is preserved. ... Finikov had derived differential equations that determine all principal on a given surface, and Byushgens had obtained differential equations that determine surfaces which have a given linear element and admit a bending on a principal base. However, the question of solubility of these equations, in general, remained unclear. ... no example was found in which the equations ... were insoluble ... up to* 1938, *when Luzin, by means of a subtle analysis of these equations, established that the existence of a principal base is rather rare.*

It has been drawn to our attention by [19], that in 1936, Luzin was the victim of a violent political campaign organized by the Soviet authorities through the newspaper *Pravda*. He was accused of anti-Soviet propaganda and sabotage by publishing all his important results abroad and only minor papers in Soviet journals. The aim was obviously to get rid of Luzin as a representative of the old pre-Soviet mathematical school of Moscow: his master, Egorov, had been himself the victim of such a campaign in 1930 (based on his religious sympathies) and died shortly after in 1931 in despair and misery. A contemporary record of the "Luzin affair" has been miraculously preserved and recently edited in Moscow by Demidov and Levchin [3], [23]. It shows that Luzin had had a narrow escape from a tragic fate as the Soviet authorities may have feared the international consequences of a too strong attack on a scientist so famous abroad. The main visible consequence of the Luzin affair was that, from this precise moment, Soviet mathematicians began to publish almost exclusively in Soviet journals and in Russian.

Luzin always had an interest in the history of mathematics and late in his career he wrote important articles on Newton and on Euler.

As a teacher his remarkable talents are described by Kuznetsov ([15] or [16]):-

> *His presentation was always very elegant and at first sight apparently unnecessarily simple - the result of his great pedagogic talent. The solution of the large problems that he undertook is distinguished by their subtlety, elegance, and simplicity of presentation.*

Keldysh and Novikov wrote in [14]:-

> *Thanks to his exceptional intuition and his ability to see deeply into the heart of a question, Luzin frequently predicted mathematical facts whose proof turned out to be possible only after many years and required the creation of completely new mathematical methods. He was one of the outstanding mathematicians and thinkers of our time ...*

John von Neumann

Born: 28 Dec 1903 in Budapest, Hungary
Died: 8 Feb 1957 in Washington D.C., USA

John von Neumann was born János von Neumann. He was called Jancsi as a child, a diminutive form of János, then later he was called Johnny in the United States. His father, Max Neumann, was a top banker and he was brought up in a extended family, living in Budapest where as a child he learnt languages from the German and French governesses that were employed. Although the family were Jewish, Max Neumann did not observe the strict practices of that religion and the household seemed to mix Jewish and Christian traditions.

It is also worth explaining how Max Neumann's son acquired the "von" to become János von Neumann. In 1913 Max Neumann purchased a title but did not change his name. His son, however, used the German form von Neumann where the "von" indicated the title.

As a child von Neumann showed he had an incredible memory. Poundstone, in [8], writes:-

> *At the age of six, he was able to exchange jokes with his father in classical Greek. The Neumann family sometimes entertained guests with demonstrations of Johnny's ability to memorise phone books. A guest would select a page and column of the phone book at random. Young Johnny read the column over a few times, then handed the book back to the guest. He could answer any question put to him* (*who has number such and such?*) *or recite names, addresses, and numbers in order.*

In 1911 von Neumann entered the Lutheran Gymnasium. The school had a strong academic tradition which seemed to count for more than the religious affiliation both in the Neumann's eyes and in those of the school. His mathematics teacher quickly recognised von Neumann's genius and special tuition was put on for him. The school had another outstanding mathematician one year ahead of von Neumann, namely Eugene Wigner.

World War I had relatively little effect on von Neumann's education but, after the war ended, Béla Kun controlled Hungary for five months in 1919 with a Communist government. The Neumann family fled to Austria as the affluent came under attack. However, after a month, they returned to face the problems of Budapest. When Kun's government failed, the fact that it had been largely composed of Jews meant that Jewish people were blamed. Such situations are devoid of logic and the fact that the Neumann's were opposed to Kun's government did not save them from persecution.

In 1921 von Neumann completed his education at the Lutheran Gymnasium. His first mathematics paper, written jointly with Fekete the assistant at the University of Budapest who had been tutoring him, was published in 1922. However Max Neumann did not want his son to take up a subject that would not bring him wealth. Max Neumann asked Theodore von Kármán to speak to his son and persuade him to follow a career in business. Perhaps von Kármán was the wrong person to ask to undertake such a task but in the end all agreed on the compromise subject of chemistry for von Neumann's university studies.

Hungary was not an easy country for those of Jewish descent for many reasons and there was a strict limit on the number of Jewish students who could enter the University of Budapest. Of course, even with a strict quota, von Neumann's record easily won him a place to study mathematics in 1921 but he did not attend lectures. Instead he also entered the University of Berlin in 1921 to study chemistry.

Von Neumann studied chemistry at the University of Berlin until 1923 when he went to Zurich. He achieved outstanding results in the mathematics examinations at the University of Budapest despite not attending any courses. Von Neumann received his diploma in chemical engineering from the Technische Hochschule in Zürich in 1926. While in Zurich he continued his interest in mathematics, despite studying chemistry, and interacted with Weyl and Pólya who were both at Zurich. He even took over one of Weyl's courses when he was absent from Zurich for a time. Pólya said [18]:-

Johnny was the only student I was ever afraid of. If in the course of a lecture I stated an unsolved problem, the chances were he'd come to me as soon as the lecture was over, with the complete solution in a few scribbles on a slip of paper.

Von Neumann received his doctorate in mathematics from the University of Budapest, also in 1926, with a thesis on set theory. He published a definition of ordinal numbers when he was 20, the definition is the one used today.

Von Neumann lectured at Berlin from 1926 to 1929 and at Hamburg from 1929 to 1930. However he also held a Rockefeller Fellowship to enable him to undertake postdoctoral studies at the University of Göttingen. He studied under Hilbert at Göttingen during 1926-27. By this time von Neumann had achieved celebrity status [8]:-

By his mid-twenties, von Neumann's fame had spread worldwide in the mathematical community. At academic conferences, he would find himself pointed out as a young genius.

Veblen invited von Neumann to Princeton to lecture on quantum theory in 1929. Replying to Veblen that he would come after attending to some personal matters, von Neumann went to Budapest where he married his fiancée Marietta Kovesi before setting out for the United States. In 1930 von Neumann became a visiting lecturer at Princeton University, being appointed professor there in 1931.

Between 1930 and 1933 von Neumann taught at Princeton but this was not one of his strong points [8]:-

His fluid line of thought was difficult for those less gifted to follow. He was notorious for dashing out equations on a small portion of the available blackboard and erasing expressions before students could copy them.

In contrast, however, he had an ability to explain complicated ideas in physics [3]:-

For a man to whom complicated mathematics presented no difficulty, he could explain his conclusions to the uninitiated with amazing lucidity. After a talk with him one always came away with a feeling that the problem was really simple and transparent.

He became one of the original six mathematics professors (J W Alexander, A Einstein, M Morse, O Veblen, J von Neumann and H Weyl) in 1933 at the newly founded Institute for Advanced Study in Princeton, a position he kept for the remainder of his life.

During the first years that he was in the United States, von Neumann continued to return to Europe during the summers. Until 1933 he still held academic posts in Germany but resigned these when the Nazis came to power. Unlike many others, von Neumann was not a political refugee but rather he went to the United States mainly because he thought that the prospect of academic positions there was better than in Germany.

In 1933 von Neumann became co-editor of the *Annals of Mathematics* and, two years later, he became co-editor of *Compositio Mathematica.* He held both these editorships until his death.

Von Neumann and Marietta had a daughter Marina in 1936 but their marriage ended in divorce in 1937. The following year he married Klára Dán, also from Budapest, whom he met on one of his European visits. After marrying, they sailed to the United States and made their home in Princeton. There von Neumann lived a rather unusual lifestyle for a top mathematician. He had always enjoyed parties [8]:-

Parties and nightlife held a special appeal for von Neumann. While teaching in Germany, von Neumann had been a denizen of the Cabaret-era Berlin nightlife circuit.

Now married to Klára the parties continued [18]:-

The parties at the von Neumann's house were frequent, and famous, and long.

Ulam summarises von Neumann's work in [35]. He writes:-

In his youthful work, he was concerned not only with mathematical logic and the axiomatics of set theory, but, simultaneously, with the substance of set theory itself, obtaining interesting results in measure theory and the theory of real variables. It was in this period also that he

> *began his classical work on quantum theory, the mathematical foundation of the theory of measurement in quantum theory and the new statistical mechanics.*

His text *Mathematische Grundlagen der Quantenmechanik* (1932) built a solid framework for the new quantum mechanics. Van Hove writes in [36]:-

> *Quantum mechanics was very fortunate indeed to attract, in the very first years after its discovery in* 1925*, the interest of a mathematical genius of von Neumann's stature. As a result, the mathematical framework of the theory was developed and the formal aspects of its entirely novel rules of interpretation were analysed by one single man in two years* (1927-1929).

Self-adjoint algebras of bounded linear operators on a Hilbert space, closed in the weak operator topology, were introduced in 1929 by von Neumann in a paper in *Mathematische Annalen* . Kadison explains in [22]:-

> *His interest in ergodic theory, group representations and quantum mechanics contributed significantly to von Neumann's realisation that a theory of operator algebras was the next important stage in the development of this area of mathematics.*

Such operator algebras were called "rings of operators" by von Neumann and later they were called W*-algebras by some other mathematicians. J Dixmier, in 1957, called them "von Neumann algebras" in his monograph *Algebras of operators in Hilbert space (von Neumann algebras).* In the second half of the 1930's and the early 1940s von Neumann, working with his collaborator F J Murray, laid the foundations for the study of von Neumann algebras in a fundamental series of papers.

However von Neumann is know for the wide variety of different scientific studies. Ulam explains [35] how he was led towards game theory:-

> *Von Neumann's awareness of results obtained by other mathematicians and the inherent possibilities which they offer is astonishing. Early in his work, a paper by Borel on the minimax property led him to develop ... ideas which culminated later in one of his most original creations, the theory of games.*

In game theory von Neumann proved the minimax theorem. He gradually expanded his work in game theory, and with co-author Oskar Morgenstern, he wrote the classic text *Theory of Games and Economic Behaviour* (1944).

Ulam continues in [35]:-

> *An idea of Koopman on the possibilities of treating problems of classical mechanics by means of operators on a function space stimulated him to give the first mathematically rigorous proof of an ergodic theorem. Haar's construction of measure in groups provided the inspiration for his wonderful partial solution of Hilbert's fifth problem, in which he proved the possibility of introducing analytical parameters in compact groups.*

In 1938 the American Mathematical Society awarded the Bôcher Prize to John von Neumann for his memoir *Almost periodic functions and groups.* This was published in two parts in the *Transactions of the American Mathematical Society,* the first part in 1934 and the second part in the following year. Around this time von Neumann turned to applied mathematics [35]:-

> *In the middle* 30*'s, Johnny was fascinated by the problem of hydrodynamical turbulence. It was then that he became aware of the mysteries underlying the subject of non-linear partial differential equations. His work, from the beginnings of the Second World War, concerns a study of the equations of hydrodynamics and the theory of shocks. The phenomena described by these non-linear equations are baffling analytically and defy even qualitative insight by present methods. Numerical work seemed to him the most promising way to obtain a feeling for the behaviour of such systems. This impelled him to study new possibilities of computation on electronic machines ...*

Von Neumann was one of the pioneers of computer science making significant contributions to the development of logical design. Shannon writes in [29]:-

Von Neumann spent a considerable part of the last few years of his life working in [*automata theory*]. *It represented for him a synthesis of his early interest in logic and proof theory and his later work, during World War II and after, on large scale electronic computers. Involving a mixture of pure and applied mathematics as well as other sciences, automata theory was an ideal field for von Neumann's wide-ranging intellect. He brought to it many new insights and opened up at least two new directions of research.*

He advanced the theory of cellular automata, advocated the adoption of the bit as a measurement of computer memory, and solved problems in obtaining reliable answers from unreliable computer components.

During and after World War II, von Neumann served as a consultant to the armed forces. His valuable contributions included a proposal of the implosion method for bringing nuclear fuel to explosion and his participation in the development of the hydrogen bomb. From 1940 he was a member of the Scientific Advisory Committee at the Ballistic Research Laboratories at the Aberdeen Proving Ground in Maryland. He was a member of the Navy Bureau of Ordnance from 1941 to 1955, and a consultant to the Los Alamos Scientific Laboratory from 1943 to 1955. From 1950 to 1955 he was a member of the Armed Forces Special Weapons Project in Washington, D.C. In 1955 President Eisenhower appointed him to the Atomic Energy Commission, and in 1956 he received its Enrico Fermi Award, knowing that he was incurably ill with cancer.

Eugene Wigner wrote of von Neumann's death [18]:-

When von Neumann realised he was incurably ill, his logic forced him to realise that he would cease to exist, and hence cease to have thoughts ... It was heartbreaking to watch the frustration of his mind, when all hope was gone, in its struggle with the fate which appeared to him unavoidable but unacceptable.

In [5] von Neumann's death is described in these terms:-

... his mind, the amulet on which he had always been able to rely, was becoming less dependable. Then came complete psychological breakdown; panic, screams of uncontrollable terror every night. His friend Edward Teller said, "I think that von Neumann suffered more when his mind would no longer function, than I have ever seen any human being suffer."

Von Neumann's sense of invulnerability, or simply the desire to live, was struggling with unalterable facts. He seemed to have a great fear of death until the last... No achievements and no amount of influence could save him now, as they always had in the past. Johnny von Neumann, who knew how to live so fully, did not know how to die.

It would be almost impossible to give even an idea of the range of honours which were given to von Neumann. He was Colloquium Lecturer of the American Mathematical Society in 1937 and received the its Bôcher Prize as mentioned above. He held the Gibbs Lectureship of the American Mathematical Society in 1947 and was President of the Society in 1951-53.

He was elected to many academies including the Academia Nacional de Ciencias Exactas (Lima, Peru), Academia Nazionale dei Lincei (Rome, Italy), American Academy of Arts and Sciences (USA), American Philosophical Society (USA), Instituto Lombardo di Scienze e Lettere (Milan, Italy), National Academy of Sciences (USA) and Royal Netherlands Academy of Sciences and Letters (Amsterdam, The Netherlands).

Von Neumann received two Presidential Awards, the Medal for Merit in 1947 and the Medal for Freedom in 1956. Also in 1956 he received the Albert Einstein Commemorative Award and the Enrico Fermi Award mentioned above.

Peierls writes [3]:-

He was the antithesis of the "long-haired" mathematics don. Always well groomed, he had as lively views on international politics and practical affairs as on mathematical problems.

Emmy Amalie Noether

Born: 23 March 1882 in Erlangen, Bavaria, Germany
Died: 14 April 1935 in Bryn Mawr, Pennsylvania, USA

Emmy Noether's father Max Noether was a distinguished mathematician and a professor at Erlangen. Her mother was Ida Kaufmann, from a wealthy Cologne family. Both Emmy's parents were of Jewish origin and Emmy was the eldest of their four children, the three younger children being boys.

Emmy Noether attended the Höhere Töchter Schule in Erlangen from 1889 until 1897. She studied German, English, French, arithmetic and was given piano lessons. She loved dancing and looked forward to parties with children of her father's university colleagues. At this stage her aim was to become a language teacher and after further study of English and French she took the examinations of the State of Bavaria and, in 1900, became a certificated teacher of English and French in Bavarian girls schools.

However Noether never became a language teacher. Instead she decided to take the difficult route for a woman of that time and study mathematics at university. Women were allowed to study at German universities unofficially and each professor had to give permission for his course. Noether obtained permission to sit in on courses at the University of Erlangen during 1900 to 1902. Then, having taken and passed the matriculation examination in Nürnberg in 1903, she went to the University of Göttingen. During 1903-04 she attended lectures by Blumenthal, Hilbert, Klein and Minkowski.

In 1904 Noether was permitted to matriculate at Erlangen and in 1907 was granted a doctorate after working under Paul Gordan. Hilbert's basis theorem of 1888 had given an existence result for finiteness of invariants in n variables. Gordan, however, took a constructive approach and looked at constructive methods to arrive at the same results. Noether's doctoral thesis followed this constructive approach of Gordan and listed systems of 331 covariant forms.

Having completed her doctorate the normal progression to an academic post would have been the habilitation. However this route was not open to women so Noether remained at Erlangen, helping her father who, particularly because of his own disabilities, was grateful for his daughter's help. Noether also worked on her own research, in particular she was influenced by Fischer who had succeeded Gordan in 1911. This influence took Noether towards Hilbert's abstract approach to the subject and away from the constructive approach of Gordan.

Noether's reputation grew quickly as her publications appeared. In 1908 she was elected to the Circolo Matematico di Palermo, then in 1909 she was invited to become a member of the Deutsche Mathematiker-Vereinigung and in the same year she was invited to address the annual meeting of the Society in Salzburg. In 1913 she lectured in Vienna.

In 1915 Hilbert and Klein invited Noether to return to Göttingen. They persuaded her to remain at Göttingen while they fought a battle to have her officially on the Faculty. In a long battle with the university authorities to allow Noether to obtain her habilitation there were many setbacks and it was not until 1919 that permission was granted. During this time Hilbert had allowed Noether to lecture by advertising her courses under his own name. For example a course given in the winter semester of 1916-17 appears in the catalogue as:-

> *Mathematical Physics Seminar: Professor Hilbert, with the assistance of Dr E Noether, Mondays from* 4-6*, no tuition.*

Emmy Noether's first piece of work when she arrived in Göttingen in 1915 is a result in theoretical physics sometimes referred to as Noether's Theorem, which proves a relationship between symmetries in physics and conservation principles. This basic result in the general theory of relativity was praised by Einstein in a letter to Hilbert when he referred to Noether's

> *penetrating mathematical thinking.*

It was her work in the theory of invariants which led to formulations for several concepts of Einstein's general theory of relativity.

At Göttingen, after 1919, Noether moved away from invariant theory to work on ideal theory, producing an abstract theory which helped develop ring theory into a major mathematical topic. *Idealtheorie in Ringbereichen* (1921) was of fundamental importance in the development of modern algebra. In this paper she gave the decomposition of ideals into intersections of primary ideals in any commutative ring with ascending chain condition. Lasker (the world chess champion) had already proved this result for polynomial rings.

In 1924 B L van der Waerden came to Göttingen and spent a year studying with Noether. After returning to Amsterdam van der Waerden wrote his book *Moderne Algebra* in two volumes. The major part of the second volume consists of Noether's work.

From 1927 on Noether collaborated with Helmut Hasse and Richard Brauer in work on non- commutative algebras.

In addition to teaching and research, Noether helped edit *Mathematische Annalen*. Much of her work appears in papers written by colleagues and students, rather than under her own name.

Further recognition of her outstanding mathematical contributions came with invitations to address the International Mathematical Congress at Bologna in 1928 and again at Zurich in 1932. In 1932 she also received, jointly with Artin, the Alfred Ackermann-Teubner Memorial Prize for the Advancement of Mathematical Knowledge.

In 1933 her mathematical achievements counted for nothing when the Nazis caused her dismissal from the University of Göttingen because she was Jewish. She accepted a visiting professorship at Bryn Mawr College in the USA and also lectured at the Institute for Advanced Study, Princeton in the USA.

Weyl in his Memorial Address [28] said:-

> *Her significance for algebra cannot be read entirely from her own papers, she had great stimulating power and many of her suggestions took shape only in the works of her pupils and co-workers.*

In [26] van der Waerden writes:-

> *For Emmy Noether, relationships among numbers, functions, and operations became transparent, amenable to generalisation, and productive only after they have been dissociated from any particular objects and have been reduced to general conceptual relationships.*

Roger Penrose

Born: 8 Aug 1931 in Colchester, Essex, England

Roger Penrose's parents, Lionel Sharples Penrose and Margaret Leathes, were both medically trained. Margaret was a doctor while Lionel was a medical geneticist who was elected a Fellow of the Royal Society. He was involved with a project called the Colchester survey which aimed to discover whether inherited factors or environmental factors were the most significant in determining if someone would be likely to suffer from mental heath problems. He was in Colchester carrying out this work at the time Roger was born. Roger's brother, Oliver Penrose, had been born two years earlier. Oliver went on to become professor of mathematics first at the Open University, then at Heriot-Watt University in Edinburgh, Scotland. Roger also had a younger brother Jonathan who went on to become a lecturer in psychology. Jonathan was British Chess Champion ten times between 1958 and 1969 and, many argue, was the most naturally talented British chess player of all time.

In 1939 Roger's father went to the United States with his family but as all the indications pointed towards the outbreak of war, he decided not to return to England with his family but accepted an appointment in a hospital in London, Ontario, Canada. Roger attended school in London, Ontario but although it was during this period that he first became interested in mathematics it was not his schooling which stimulated this interest, rather it was his family. He writes ([2] or [3]):-

> *I remember making various polyhedra when I was about ten ...*

Roger's father became Director of Psychiatric Research at the Ontario Hospital in London Ontario, but he was very interested in mathematics, particularly geometry, while Roger's mother was also interested in geometry. Roger's brother Oliver ([2] or [3]):-

> *... was two years older than I was, but four years ahead in school. He knew a lot about mathematics at a young age and took a great interest in both mathematics and physics.*

In 1945, after the World War II ended, the Penrose family returned to England. Roger's father was appointed as Professor of Human Genetics at University College London and Roger attended University College School in London. Then his interest in mathematics began to increase but his family saw him following in his father's footsteps and taking up a medical career. However, as was typical in schools at this time, biology and mathematics were alternatives at the University College School with pupils having to choose one or the other ([2] or [3]):-

> *... I remember an occasion when we had to decide which subjects to do in the final two years. Each of us would go up to see the headmaster, one after the other, and he said "Well, what subjects do you want to do when you specialise next year". I said "I'd like to do biology, chemistry and mathematics" and he said "No, that's impossible - you can't do biology and mathematics at the same time, we just don't have that option". Since I had no desire to lose my mathematics I said "Mathematics, physics and chemistry". My parents were rather annoyed when I got home; my medical career had disappeared in one stroke.*

Penrose entered University College London which he was entitled to do without paying fees since his father was professor there. He was awarded a B.Sc. degree with First Class Honours in Mathematics and then decided to go to Cambridge to undertake research in pure mathematics. He was following in the footsteps of his older brother Oliver who had also taken his undergraduate degree at University College London and had gone to Cambridge to undertake research but Oliver had chosen physics. Roger, however, was set on research in mathematics and on entering St John's College he began research in algebraic geometry supervised by Hodge. However, after one year of study at Cambridge, finding that his interests were not particularly central to those of Hodge, he changed his supervisor to John Todd. Penrose was awarded his Ph.D. for his work in algebra and geometry from the University of Cambridge in 1957 but by this time he had already become interested in physics. He described how three courses which he attended during his first year at Cambridge influenced him ([2] or [3]):-

> *I remember going to three courses, none of which had anything to do with the research I was supposed to be doing. One was a course by Hermann Bondi on general relativity which was fascinating ... Another was a course by Paul Dirac on quantum mechanics which was beautiful in a completely different way ... And the third course ... was a course on mathematical logic by Steen. I learnt about Turing machines and Gödel's theorem ...*

The first major influence prompting his interest in physics had been Dennis Sciama, a physicist friend of his brother. Penrose said ([2] or [3]):-

> [*Sciama*] *was very influential on me. He taught me a great deal of physics, and the excitement of doing physics came through; he was that kind of person, who conveyed the excitement of what was currently going on in physics ...*

While at Cambridge working towards his doctorate he began to publish articles on semigroups, and on rings of matrices. In 1955 he published *A generalized inverse for matrices* in the *Proceedings of the Cambridge Philosophical Society.* In this paper Penrose defined a generalized inverse X of a complex rectangular (or possibly square and singular) matrix A to be the unique solution to the equations $AXA = A$, $XAX = X$, $(AX)^T = AX$, $(XA)^T = XA$. He used this generalized inverse for problems such as solving systems of matrix equations, and finding a new type of spectral decomposition. His second publication of 1955 was *A note on inverse semigroups* published in the same journal and co-authored with Douglas Munn. An inverse semigroup is a generalisation of a group and continues to be the subject of many research papers. This early paper gave several alternative definitions. In the following year Penrose published *On best approximation solutions of linear matrix equations* which used the generalized inverse of a matrix to find the best approximate solution X to $AX = B$ where A is rectangular and non-square or square and singular.

Penrose spent the academic year 1956-57 as an Assistant Lecturer in Pure Mathematics at Bedford College, London and was then appointed as a Research Fellow at St John's College, Cambridge. This was a three year post and during its tenure he married Joan Isabel Wedge in 1959. Before the fellowship ended Penrose had been awarded a NATO Research Fellowship which enabled him to spend the years 1959-61 in the United States, first at Princeton and then at Syracuse University. Back in England, Penrose spent the following two years 1961-63 as a Research associate at King's College, London before returning to the United States to spend the year 1963-64 as a Visiting Associate Professor at the University of Texas at Austin.

In 1964 Penrose was appointed as a Reader at Birkbeck College, London and two years later he was promoted to Professor of Applied Mathematics there. In 1973 he was appointed Rouse Ball Professor of Mathematics at the University of Oxford and he continued to hold this until he became Emeritus Rouse Ball Professor of Mathematics in 1998. In that year he was appointed Gresham Professor of Geometry at Gresham College, London.

Beginning in 1959, Penrose published a series of important papers on cosmology. The first was *The apparent shape of a relativistically moving sphere* while in 1960 he published *A spinor approach to general relativity.* This latter paper was described as follows:-

> *An elegant and detailed exposition ... of the mathematical apparatus of gravitation theory, with emphasis on the geometrical theory of the Riemann tensor.*

As well as important papers on cosmology, Penrose continues to publish papers on pure mathematics. Together with Henry Whitehead and Christopher Zeeman he published *Imbedding of manifolds in euclidean space* in 1961. Among other results, the authors prove in this paper that, if $0 < 2m \leq n$, then every closed $(m-1)$-connected n-manifold can be imbedded in $\mathbf{R}^{2n-m+1}$. This time with Ezra Newman, Penrose published *An approach to gravitational radiation by a method of spin coefficients* in the following year in which they show that:-

> *... the two-component spinor formalism leads to the consideration of a tetrad in space-time consisting of two real null-vectors and two complex conjugate ones.*

In 1965, using topological methods, Penrose proved an important theorem which, under conditions which he called the existence of a trapped surface, proved that a singularity must occur in a gravitational collapse. Basically under these conditions space-time cannot be continued and classical general relativity breaks down. Penrose looked for a unified theory combining relativity and quantum theory since quantum effects become dominant at the singularity.

One of Penrose's major breakthroughs was his introduction of twistor theory in an attempt to unite relativity and quantum theory. This is a remarkable mathematical theory combining powerful algebraic and geometric methods. Together with Wolfgang Rindler, Penrose published this first volume of *Spinors and space-time* in 1984. This volume covered two-spinor calculus and relativistic fields while the second volume covering spinor and twistor methods in space-time geometry appeared two years later.

It is for a number of outstanding popular books that Penrose is perhaps best known. He published *The Emperor's New Mind : Concerning computers, minds, and the laws of physics* in 1989. In the following year the book was awarded the Rhone-Poulenc Science Book Prize. Sklar, reviewing the book, writes that its aim is:-

> *... to expound and critically attack one recent view of the nature of mind ... taken as reducing mental activity to the carrying out of an algorithmic process, and to propose that a more adequate theory of mind will have to be founded on an as yet not existing physical theory adequate to the known nature of the material world. In the process of the argument elegant expositions, at a level suitable for the unlearned but reasonably sophisticated reader, are given of a wide variety of topics ranging from the nature of algorithms and abstract computability, through results on undecidability and incompleteness, the basic structures of classical physics, the basic structures and philosophical puzzles in quantum mechanics, the basic features of entropic asymmetry and its relation to cosmological structure, the search for an adequate quantum theory of gravity, to some of the results of neuro-anatomy and research into the functioning of the brain.*

In 1994 Penrose published *Shadows of the mind : A search for the missing science of consciousness* which continues to develop the topic of *The emperor's new mind.* In 1996 Penrose and Hawking published *The nature of space and time.* This book is a record of a debate between the two at the Isaac Newton Institute of Mathematical Sciences at the University of Cambridge in 1994. Each of the two gave three lectures given alternately so that each could respond to the other's arguments, and then, in a final session, there is a debate between the two. We quote from Penrose's contribution since he states clearly his own position, and that of Hawking:-

> *At the beginning of this debate Stephen said that he thinks that he is a positivist, whereas I am a Platonist. I am happy with him being a positivist, but I think that the crucial point here is, rather, that I am a realist. Also, if one compares this debate with the famous debate of Bohr and Einstein, some seventy years ago, I should think that Stephen plays the role of Bohr, whereas I play Einstein's role! For Einstein argued that there should exist something like a real world, not necessarily represented by a wave function, whereas Bohr stressed that the wave function doesn't describe a "real" microworld but only "knowledge" that is useful for making predictions.*

There is one further aspect of Penrose's work which we must mention. This is his work on non-periodic tilings, an interest which he took up while a graduate student at Cambridge. His first attempts led to success but with a large number of tiles. Further work over many years led to Penrose discovering that he could find non-periodic tilings with only six tiles, then finally he achieved the seemingly impossible with finding non-periodic tilings with only two tiles. By non-periodic we mean that the tilings are not invariant under any translation. Here are some properties of the tiling: in any finite tiled region, only one tiling is possible; in an infinite tiling of the plane, any tiling of a region that occurs is repeated infinitely often elsewhere in the plane and must reoccur within twice the diameter of the region from where you first found it. In fact the tiling of any finite region will eventually appear in every Penrose tiling.

In addition to Penrose's main appointments which we have mentioned above, he also held a number of visiting and part-time posts. He held visiting positions at Yeshiva, Princeton and Cornell during 1966-67 and 1969. From 1983 until 1987 he was Lovett Professor at Rice University in Houston. He then became Distinguished Professor of Physics and Mathematics at Syracuse University in New York until 1993 when he became Francis and Helen Pentz Distinguished Professor of Physics and Mathematics at Pennsylvania State University.

Penrose has received many honours for his contributions. He was elected a Fellow of the Royal Society of London (1972) and a Foreign Associate of the United States National Academy of Sciences (1998). We mentioned the Science Book Prize (1990) which he received for The Emperor's New Mind but this is only one of many prizes. Others include the Adams Prize from Cambridge University; the Wolf Foundation Prize for Physics (jointly with Stephen Hawking for their understanding of the universe): the Dannie Heinemann Prize from the American Physical Society and the American Institute of Physics; the Royal Society Royal Medal; the Dirac Medal and Medal of the British Institute of Physics; the Eddington Medal of the Royal Astronomical Society; the Naylor Prize of the London Mathematical Society; and the Albert Einstein Prize and Medal of the Albert Einstein Society. In 1994 he was knighted for services to science.

In 2000 he received the Order of Merit. He was awarded the De Morgan Medal by the London Mathematical Society in 2004. Part of the citation reads:-

> *His deep work on General Relativity has been a major factor in our understanding of black holes. His development of Twistor Theory has produced a beautiful and productive approach to the classical equations of mathematical physics. His tilings of the plane underlie the newly discovered quasi-crystals.*

The Royal Society awarded Penrose their Copley Medal in 2005. The announcement reads:-

> *Sir Roger Penrose, OM, FRS has been awarded the Royal Society's Copley medal the world's oldest prize for scientific achievement for his exceptional contributions to geometry and mathematical physics. Sir Roger, Emeritus Rouse Ball Professor of Mathematics at the University of Oxford, has made outstanding contributions to general relativity theory and cosmology, most notably for his work on black holes and the Big Bang.*

Martin Rees, President of the Royal Society, explained Penrose's exceptional contributions which led to the award:-

> *Roger has been producing original and important scientific ideas for half a century. His work is characterised by exceptional geometrical and physical insight. He applied new mathematical techniques to Einstein's theory, and led the renaissance in gravitation theory in the* 1960*s. His novel ideas on space and time and his concept of 'twistors' are increasingly influential. Even his recreations have had intellectual impact: for instance the 'impossible figures' popularised in Escher's artwork, and the never-repeating patterns of 'Penrose tiling'. He has influenced and stimulated a wide public through his lectures, and his best-selling and wide-ranging books.*

On receiving the award, Penrose said:-

> *The award of the Royal Society's Copley Medal came as a complete surprise to me. It is an extraordinary honour, this being the Royal Society's oldest and most distinguished award, first given just* 200 *years before I was born. I feel most humbled for my name to be added to that enormously distinguished list of previous recipients.*

Several universities have awarded Penrose an honorary degree including New Brunswick University (1992), the University of Surrey (1993), the University of Bath (1994), the University of London (1995), the University of Glasgow (1996), Essex University (1996), the University of St Andrews (1997), Santiniketon University (1998), Warsaw University (2005), Katholieke Universiteit Leuven (2005) and the University of York (2006).

Lev Semenovich Pontryagin

Born: 3 Sept 1908 in Moscow, Russia
Died: 3 May 1988 in Moscow, Russia (USSR)

Lev Semenovich Pontryagin's father, Semen Akimovich Pontryagin was a civil servant. Pontryagin's mother, Tat'yana Andreevna Pontryagina, was 29 years old when he was born and she was a remarkable woman who played a crucial role in his path to becoming a mathematician. Perhaps the description of 'civil servant', although accurate, gives the wrong impression that the family were reasonably well off. In fact Semen Akimovich's job left the family without enough money to allow them to give their son a good education and Tat'yana Andreevna worked using her sewing skills to help out the family finances.

Pontryagin attended the town school where the standard of education was well below that of the better schools but the family's poor circumstances put these well out of reach financially. At the age of 14 years Pontryagin suffered an accident and an explosion left him blind. This might have meant an end to his education and career but his mother had other ideas and devoted herself to help him succeed despite the almost impossible difficulties of being blind. The help that she gave Pontryagin is described in [1] and [2]:-

> *From this moment Tat'yana Andreevna assumed complete responsibility for ministering to the needs of her son in all aspects of his life. In spite of the great difficulties with which she had to contend, she was so successful in her self-appointed task that she truly deserves the gratitude ... of science throughout the world. For many years she worked, in effect, as Pontryagin's secretary, reading scientific works aloud to him, writing in the formulas in his manuscripts, correcting his work and so on. In order to do this she had, in particular, to learn to read foreign languages. Tat'yana Andreevna helped Pontryagin in all other respects, seeing to his needs and taking very great care of him.*

It is not unreasonable to pause for a moment and think about how Tat'yana Andreevna, with no mathematical training or knowledge, made by her determination and extreme efforts a major contribution to mathematics by allowing Pontryagin to become a mathematician against all the odds. There must be many other non-mathematicians, perhaps many of whom are unrecorded by history, who have also by their unselfish acts allowed mathematics to flourish. As we try to show in this archive, the development of mathematics depends on a wide number of influences other than the talents of the mathematicians themselves: political influences, economic influences, social influences, and the acts of non-mathematicians like Tat'yana Andreevna.

But how does one read a mathematics paper without knowing any mathematics? Of course it is full of mysterious symbols and Tat'yana Andreevna, not knowing their mathematical meaning or name, could only describe them by their appearance. For example an intersection sign became a 'tails down' while a union symbol became a 'tails up'. If she read '*A* tails right *B*' then Pontryagin knew that *A* was a subset of *B*!

Pontryagin entered the University of Moscow in 1925 and it quickly became apparent to his lecturers that he was an exceptional student. Of course that a blind student who could not make notes yet was able to remember the most complicated manipulations with symbols was in itself truly remarkable. Even more remarkable was the fact that Pontryagin could 'see' (if you will excuse the bad pun) far more clearly than any of his fellow students the depth of meaning in the topics presented to him. Of the advanced courses he took, Pontryagin felt less happy with Khinchin's analysis course but he took a special liking to Aleksandrov's courses. Pontryagin was strongly influenced by Aleksandrov and the direction of Aleksandrov's research was to determine the area of Pontryagin's work for many years. However this was as much to do with Aleksandrov himself as with his mathematics ([1] and [2]):-

> *Aleksandrov's personal charm, his attention and helpfulness influenced the formation of Pontryagin's scientific interests to a remarkable extent, as much in fact as the personal abilities and inclinations of the young scholar himself.*

The year 1927 was the year of the death of Pontryagin's father. By 1927, although he was still only 19 years old, Pontryagin had begun to produce important results on the Alexander duality theorem. His main tool was to use link numbers which had been introduced by Brouwer and, by 1932, he had produced the most significant of these duality results when he proved the duality between the homology groups of bounded closed sets in Euclidean space and the homology groups in the complement of the space.

Pontryagin graduated from the University of Moscow in 1929 and was appointed to the Mechanics and Mathematics Faculty. In 1934 he became a member of the Steklov Institute and in 1935 he became head of the Department of Topology and Functional Analysis at the Institute.

Pontryagin worked on problems in topology and algebra. In fact his own description of this area that he worked on was:-

> *... problems where these two domains of mathematics come together.*

The significance of this work of Pontryagin on duality ([1] and [2]):-

> *... lies not merely in its effect on the further development of topology; of equal significance is the fact that his theorem enabled him to construct a general theory of characters for commutative topological groups. This theory, historically the first really exceptional achievement in a new branch of mathematics, that of topological algebra, was one of the most fundamental advances in the whole of mathematics during the present century...*

One of the 23 problems posed by Hilbert in 1900 was to prove his conjecture that any locally Euclidean topological group can be given the structure of an analytic manifold so as to become a Lie group. This became known as Hilbert's Fifth Problem. In 1929 von Neumann, using integration on general compact groups which he had introduced, was able to solve Hilbert's Fifth Problem for compact groups. In 1934 Pontryagin was able to prove Hilbert's Fifth Problem for abelian groups using the theory of characters on locally compact abelian groups which he had introduced.

Among Pontryagin's most important books on the above topics is *topological groups* (1938). The authors of [1] and [2] rightly assert:-

> *This book belongs to that rare category of mathematical works that can truly be called classical - book which retain their significance for decades and exert a formative influence on the scientific outlook of whole generations of mathematicians.*

In 1934 Cartan visited Moscow and lectured in the Mechanics and Mathematics Faculty. Pontryagin attended Cartan's lecture which was in French but Pontryagin did not understand French so he listened to a whispered translation by Nina Bari who sat beside him. Cartan's lecture was based around the problem of calculating the homology groups of the classical compact Lie groups. Cartan had some ideas how this might be achieved and he explained these in the lecture but, the following year, Pontryagin was able to solve the problem completely using a totally different approach to the one suggested by Cartan. In fact Pontryagin used ideas introduced by Morse on equipotential surfaces.

Pontryagin's name is attached to many mathematical concepts. The essential tool of cobordism theory is the Pontryagin-Thom construction. A fundamental theorem concerning characteristic classes of a manifold deals with special classes called the Pontryagin characteristic class of the manifold. One of the main problems of characteristic classes was not solved until Sergei Novikov proved their topological invariance.

In 1952 Pontryagin changed the direction of his research completely. He began to study applied mathematics problems, in particular studying differential equations and control theory. In fact this change of direction was not quite as sudden as it appeared. From the 1930s Pontryagin had been friendly with the physicist A A Andronov and had regularly discussed with him problems in the theory of oscillations and the theory of automatic control on which Andronov was working. He published a paper with Andronov on dynamical systems in 1932 but the big shift in Pontryagin's work in 1952 occurred around the time of Andronov's death.

In 1961 he published *The Mathematical Theory of Optimal Processes* with his students V G Boltyanskii, R V Gamrelidze and E F Mishchenko. The following year an English translation appeared and, also in 1962, Pontryagin received the Lenin prize for his book. He then produced a series of papers on differential games which extends his work on control theory. Pontryagin's work in control theory is discussed in the historical survey [3].

Another book by Pontryagin *Ordinary differential equations* appeared in English translation, also in 1962.

Pontryagin received many honours for his work. He was elected to the Academy of Sciences in 1939, becoming a full member in 1959. In 1941 he was of one the first recipients of the Stalin prizes (later called the State Prizes). He was honoured in 1970 by being elected Vice-President of the International Mathematical Union.

Sergei Lvovich Sobolev

Born: 6 Oct 1908 in St Petersburg, Russia
Died: 3 Jan 1989 in Leningrad (now St Petersburg), Russia (USSR)

Sergei L'vovich Sobolev's father, Lev Aleksandrovich Sobolev, was an important layer and barrister. His mother, Nataliya Georgievna, played an important role in Sobolev's upbringing, particularly after the death of Sobolev's father when Sobolev was 14 years old. He studied at the Khar'kov Workers' Technical School preparing to enter the high school which he did in 1922 around the time of his father's death.

The high school which he entered was called the 190th School of Leningrad at the time although previously it had been called the Lentovskii High School. In [5] (see also [4]) it is explained that this school :-

> *... was founded during the First Russian Revolution by the foremost St Petersburg teachers for pupils who had been excluded from the State Schools and Technical Colleges because of their participation in the revolutionary movement.*

After graduating from high school in 1925, Sobolev entered the Physics and Mathematics Faculty of Leningrad State University where his talents were quickly spotted by Smirnov who had returned to Leningrad three years earlier. Sobolev became interested in differential equations, a topic which would dominate his research throughout his life, and even at this stage in his career he produced new results which he published.

By 1929 Sobolev had completed his university education and he began to teach in a number of different educational establishments. For example his first appointment was in 1929 at the Theoretical Department of the Seismological Institute of the USSR Academy of Sciences. However, in addition, he taught at the Leningrad Electrotechnic Institute in 1930-31.

In 1932 the Steklov Institute of Physics and Mathematics was divided into separate Departments of Mathematics and of Physics. Vinogradov headed the Mathematics Department and invited Sobolev to join the Department. By this time, however, Sobolev had already ([4] and [5]):-

> *... published a number of profound papers in which he put forward a new method for the solution of an important class of partial differential equations.*

Working with Smirnov, Sobolev studied functionally invariant solutions of the wave equation. These methods allowed them to find closed form solutions to the wave equation describing the oscillations of an elastic medium. The methods also led them to a complete theory of Rayleigh surface waves and Sobolev went on to solve problems on diffraction. Sobolev was honoured for this outstanding work by election as Corresponding Member of the USSR Academy of Sciences in 1933.

On 28 April 1934, at the general meeting of the Division of Mathematical and Natural Sciences of the USSR Academy of Sciences, a decision was taken to split the Departments of the Steklov Institute of Physics and Mathematics, which had been created two years earlier, into two independent Institutes, the Steklov Mathematical Institute and the Lebedev Physical Institute. In the same year the Steklov Mathematical Institute was moved from Leningrad to Moscow and Sobolev went with the new Institute to Moscow. By 1935 Sobolev was head of the Department of the Theory of Differential Equations at the Institute.

During the 1930s Sobolev introduced notions which were fundamental in the development of several different areas of mathematics [22]:-

> *The study of Sobolev function spaces, which he introduced in the 1930s, immediately became a whole area of functional analysis. Sobolev's notion of generalised function* (*distribution*) *turned out to be especially important; with further developments by Schwartz and Gelfand, it became one of the central notions of mathematics.*

While working in Moscow, Sobolev built on the standard variational method for solving elliptic boundary value problems by introducing these Sobolev function spaces. He gave inequalities on the norms on these spaces which were important in the theory of embedding function spaces. He applied his methods to solve difficult problems in mathematical physics.

In 1939 Sobolev was elected a full member of the USSR Academy of Sciences. He was only 31 years of age at the time of his election which was a remarkable achievement. It made him the youngest full member of the Academy of Sciences and in fact he remained the youngest member for quite a few years.

At the beginning of World War II, the Steklov Mathematical Institute was moved from Moscow to Kazan. In the October of that year Sobolev was appointed as Director of the Institute and in the spring of 1943 he supervised the move of the Institute back to Moscow. Sobolev became one of the first recipients of a Stalin prize (later called a State prize) in the first presentation of these prizes in 1941. His period as Director of the Institute ended in February 1944.

A new area of his research involved the study of the motion of a fluid in a rotating vessel. He was led to study a number of new problems which ([4] and [5]):-

> *... led him to lay the foundations of the theory of operators in a space with an indefinite metric, and to introduce new ideas in the spectral theory of operators. These ideas in the main concern generalised solutions of non-classical boundary value problems.*

In 1950 he published his famous text *Applications of functional analysis in mathematical physics* (in Russian). An English translation was published by the American Mathematical Society in 1963. Schwartz's book on the the theory of distributions appeared in the same year.

In the early 1950s Sobolev's work turned towards computational mathematics and in 1952 he became head of the first department of computational mathematics in the Soviet Union when he organised the first such department at Moscow State University. However in 1956 he joined with a number of colleagues in proposing ways in which the large areas of Russia in the east could be opened up with educational initiatives. The scheme was to set up a number of Institutes for Scientific research to balance the large number of high quality educational establishments in the east of the Soviet Union.

After the plan was approved, Sobolev spent some time in Moscow recruiting staff and organising the establishment of an Institute in Novosibirsk. In [4] and [5] his contribution to the Institute in Novosibirsk is stressed:-

> *During the difficult formative years of the Institute Sobolev, by his excellent example, infused his young colleagues with the best habits for scientific work. In ten years, under the leadership of Sobolev, the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences has become one of the greatest centres for the mathematical sciences of international status.*

In 1958 Sobolev was part of the Soviet delegation to the International Mathematical Union, the delegation being led by Vinogradov, and Sobolev attended the International Congress at Edinburgh that year and gave an invited address on partial differential equations. From 1960 until 1978 Sobolev, in addition to his work at the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences, was a professor at Novosibirsk University.

During the 1960s much of Sobolev's research was directed towards numerical methods, in particular to interpolation. Although interpolation for functions of a single variable was well worked out, the problem of interpolation in many dimensions was largely unsolved. Sobolev applied his theories of generalised functions and of embeddings of function spaces to cubature formulae, the multi-dimensional analogues of quadrature formulae for functions of one variable. Again a major text by Sobolev *Introduction to the theory of cubature formulae* has been extremely influential in this area.

Sobolev received many honours for his fundamental contributions to mathematics. He was elected to many scientific societies, including the USSR Academy of Sciences, the Académie des Sciences de France, and the Accademia Nazionale dei Lincei. He was awarded many prizes, including three State Prizes and the 1988 M V Lomonosov Gold Medal from the USSR Academy of Sciences.

Andrei Nikolaevich Tikhonov

Born: 30 Oct 1906 in Gzhatska, Smolensk, Russia
Died: 8 Nov 1993 in Moscow, Russia

Like most Russian mathematicians there are different ways to transliterate **Andrei Nikolaevich Tikhonov**'s name into the Roman alphabet. The most common way, other than Andrei Nikolaevich Tikhonov, is to write it as **Andrey Nikolayevich Tychonoff.**

Andrei Nikolaevich Tikhonov attended secondary school as a day pupil and entered the Moscow University in 1922, the year in which he completed his school education. His studied in the Mathematics Department of the Faculty of Mathematics and Physics at Moscow University and made remarkable progress, having his first paper published in 1925 while he was still in the middle of his undergraduate course.

This first work was related to results of Aleksandrov and Urysohn on conditions for a topological space to be metrisable. However he did not stop there and continued his investigations in topology. By 1926 he had discovered the topological construction which is today named after him, the Tikhonov topology defined on the product of topological spaces. Aleksandrov, recalling in [4] how he failed to appreciate the significance of Tikhonov's ideas at the time he proposed them, remembered:-

> *... very well with what mistrust he met Tikhonov's proposed definition. How was it possible that a topology introduced by means of such enormous neighbourhoods, which are only distinguished from the whole space by a finite number of the coordinates, could catch any of the essntial characteristics of a topological product?*

Tikhonov certainly had given the right definition and this idea, which was counterintuitive to even as great a topologist as Aleksandrov, allowed Tikhonov to go on and prove such important topological results as the product of any set of compact topological spaces is compact.

Few mathematicians have gained a worldwide reputation before they even start their research careers but this was essentially how it was for Tikhonov. His results on the Tikhonov topology of products were achieved before he graduated in 1927. With this impressive record he became a research student at Moscow University in 1927. It might be thought that someone who had clearly such an intuitive grasp of topological ideas would be only too pleased to use his talents in that area. Tikhonov, however, had equal talents for other areas of mathematics. The range of his work is summarised in [3]:-

> *We owe to Tikhonov deep and fundamental results in a wide range of topics in modern mathematics. His first-class achievements in topology and functional analysis, in the theory of ordinary and partial differential equations, in the mathematical problems of geophysics and electrodynamics, in computational mathematics and in mathematical physics are all widely known. Tikhonov's scientific work is characterised by magnificent achievements in very abstract fields of so-called pure mathematics, combined with deep investigations into the mathematical disciplines directly connected with practical requirements.*

In fact Tikhonov's work led from topology to functional analysis with his famous fixed point theorem for continuous maps from convex compact subsets of locally convex topological spaces in 1935. These results are of importance in both topology and functional analysis and were applied by Tikhonov to solve problems in mathematical physics.

He defended his habilitation thesis in 1936 on *Functional equations of Volterra type and their applications to mathematical physics.* The thesis applied an extension of Émile Picard's method of approximating the solution of a differential equation and gave applications to heat conduction, in particular cooling which obeys the law given by Josef Stefan and Boltzmann. After successfully defending his thesis, Tikhonov was appointed as a professor at Moscow University in 1936 and then, three years later, he was elected as a

Corresponding Member of the USSR Academy of Sciences.

Tikhonov's approach to problems in mathematical physics is described in [14]:-

> *A characteristic of Tikhonov's research is to combine a concrete theme in natural science with investigations into a fundamental mathematical problem. In discussing some general problem in nature he always knows how to pick out a typical concrete physical problem and to give it a clear mathematical formulation. However, his mathematical investigations are never confined to the solution of a given concrete problem, but serve as the starting point for stating a general mathematical problem that is a broad generalisation of the first problem.*
>
> *The extremely deep investigations of Tikhonov into a number of general problems in mathematical physics grew out of his interest in geophysics and electrodynamics. Thus, his research on the Earth's crust lead to investigations on well-posed Cauchy problems for parabolic equations and to the construction of a method for solving general functional equations of Volterra type. ...*

Tikhonov's work on mathematical physics continued throughout the 1940s and he was awarded the State Prize for this work in 1953. However, in 1948 he began to study a new type of problem when he considered the behaviour of the solutions of systems of equations with a small parameter in the term with the highest derivative. After a series of fundamental papers introducing the topic, the work was carried on by his students.

Another area in which Tikhonov made fundamental contributions was that of computational mathematics ([11] and [12]):-

> *Under his guidance many algorithms for the solution of various problems of electrodynamics, geophysics, plasma physics, gas dynamics, ... and other branches of the natural sciences were evolved and put into practice. ... One of the most outstanding achievemnets in computational mathematics is the theory of homogeneous difference schemes, which Tikhonov developed in collaboration with Samarskii....*

In the 1960s Tikhonov began to produce an important series of papers on ill-posed problems. He defined a class of regularisable ill-posed problems and introduced the concept of a regularising operator which was used in the solution of these problems. Combining his computing skills with solving problems of this type, Tikhonov gave computer implementations of algorithms to compute the operators which he used in the solution of these problems. Tikhonov was awarded the Lenin Prize for his work on ill-posed problems in 1966. In the same year he was elected to full membership of the USSR Academy of Sciences.

Tikhonov's wide interests throughout mathematics led him to hold a number of different chairs at Moscow University, in particular a chair in the Mathematical Physics Faculty and a chair of Computational Mathematics in the Engineering Mathematics Faculty. He also became dean of the Faculty of Computing and Cybernetics at Moscow University. Tikhonov was appointed as Deputy Director of the Institute of Applied Mathematics of the USSR Academy of Sciences, a position he held for many years.

Alan Mathison Turing

Born: 23 June 1912 in London, England
Died: 7 June 1954 in Wilmslow, Cheshire, England

Alan Turing was born at Paddington, London. His father, Julius Mathison Turing, was a British member of the Indian Civil Service and he was often abroad. Alan's mother, Ethel Sara Stoney, was the daughter of the chief engineer of the Madras railways and Alan's parents had met and married in India. When Alan was about one year old his mother rejoined her husband in India, leaving Alan in England with friends of the family. Alan was sent to school but did not seem to be obtaining any benefit so he was removed from the school after a few months.

Next he was sent to Hazlehurst Preparatory School where he seemed to be an 'average to good' pupil in most subjects but was greatly taken up with following his own ideas. He became interested in chess while at this school and he also joined the debating society. He completed his Common Entrance Examination in 1926 and then went to Sherborne School. Now 1926 was the year of the general strike and when the strike was in progress Turing cycled 60 miles to the school from his home, not too demanding a task for Turing who later was to become a fine athlete of almost Olympic standard. He found it very difficult to fit into what was expected at this public school, yet his mother had been so determined that he should have a public school education. Many of the most original thinkers have found conventional schooling an almost incomprehensible process and this seems to have been the case for Turing. His genius drove him in his own directions rather than those required by his teachers.

He was criticised for his handwriting, struggled at English, and even in mathematics he was too interested with his own ideas to produce solutions to problems using the methods taught by his teachers. Despite producing unconventional answers, Turing did win almost every possible mathematics prize while at Sherborne. In chemistry, a subject which had interested him from a very early age, he carried out experiments following his own agenda which did not please his teacher. Turing's headmaster wrote (see for example [6]):-

> *If he is to stay at Public School, he must aim at becoming educated. If he is to be solely a Scientific Specialist, he is wasting his time at a Public School.*

This says far more about the school system that Turing was being subjected to than it does about Turing himself. However, Turing learnt deep mathematics while at school, although his teachers were probably not aware of the studies he was making on his own. He read Einstein's papers on relativity and he also read about quantum mechanics in Eddington's *The nature of the physical world.*

An event which was to greatly affect Turing throughout his life took place in 1928. He formed a close friendship with Christopher Morcom, a pupil in the year above him at school, and the two worked together on scientific ideas. Perhaps for the first time Turing was able to find someone with whom he could share his thoughts and ideas. However Morcom died in February 1930 and the experience was a shattering one to Turing. He had a premonition of Morcom's death at the very instant that he was taken ill and felt that this was something beyond what science could explain. He wrote later (see for example [6]):-

> *It is not difficult to explain these things away - but, I wonder!*

Despite the difficult school years, Turing entered King's College, Cambridge, in 1931 to study mathematics. This was not achieved without difficulty. Turing sat the scholarship examinations in 1929 and won an exhibition, but not a scholarship. Not satisfied with this performance, he took the examinations again in the following year, this time winning a scholarship. In many ways Cambridge was a much easier place for unconventional people like Turing than school had been. He was now much more able to explore his own ideas and he read Russell's *Introduction to mathematical philosophy* in 1933. At about the same time he read von Neumann's 1932 text on quantum mechanics, a subject he returned to a number of times throughout his life.

The year 1933 saw the beginnings of Turing's interest in mathematical logic. He read a paper to the Moral Science Club at Cambridge in December of that year of which the following minute was recorded (see for example [6]):-

> *A M Turing read a paper on "Mathematics and logic". He suggested that a purely logistic view of mathematics was inadequate; and that mathematical propositions possessed a variety of interpretations of which the logistic was merely one.*

Of course 1933 was also the year of Hitler's rise in Germany and of an anti-war movement in Britain. Turing joined the anti-war movement but he did not drift towards Marxism, nor pacifism, as happened to many.

Turing graduated in 1934 then, in the spring of 1935, he attended Max Newman's advanced course on the foundations of mathematics. This course studied Gödel's incompleteness results and Hilbert's question on decidability. In one sense 'decidability' was a simple question, namely given a mathematical proposition could one find an algorithm which would decide if the proposition was true of false. For many propositions it was easy to find such an algorithm. The real difficulty arose in proving that for certain propositions no such algorithm existed. When given an algorithm to solve a problem it was clear that it was indeed an algorithm, yet there was no definition of an algorithm which was rigorous enough to allow one to prove that none existed. Turing began to work on these ideas.

Turing was elected a fellow of King's College, Cambridge, in 1935 for a dissertation *On the Gaussian error function* which proved fundamental results on probability theory, namely the *central limit theorem*. Although the central limit theorem had recently been discovered, Turing was not aware of this and discovered it independently. In 1936 Turing was a Smith's Prizeman.

Turing's achievements at Cambridge had been on account of his work in probability theory. However, he had been working on the decidability questions since attending Newman's course. In 1936 he published *On Computable Numbers, with an application to the Entscheidungsproblem.* It is in this paper that Turing introduced an abstract machine, now called a "Turing machine", which moved from one state to another using a precise finite set of rules (given by a finite table) and depending on a single symbol it read from a tape.

The Turing machine could write a symbol on the tape, or delete a symbol from the tape. Turing wrote [13]:-

> *Some of the symbols written down will form the sequences of figures which is the decimal of the real number which is being computed. The others are just rough notes to "assist the memory". It will only be these rough notes which will be liable to erasure.*

He defined a computable number as real number whose decimal expansion could be produced by a Turing machine starting with a blank tape. He showed that π was computable, but since only countably many real numbers are computable, most real numbers are not computable. He then described a number which is not computable and remarks that this seems to be a paradox since he appears to have described in finite terms, a number which cannot be described in finite terms. However, Turing understood the source of the apparent paradox. It is impossible to decide (using another Turing machine) whether a Turing machine with a given table of instructions will output an infinite sequence of numbers.

Although this paper contains ideas which have proved of fundamental importance to mathematics and to computer science ever since it appeared, publishing it in the *Proceedings of the London Mathematical Society* did not prove easy. The reason was that Alonzo Church published *An unsolvable problem in elementary number theory* in the *American Journal of Mathematics* in 1936 which also proves that there is no decision procedure for arithmetic. Turing's approach is very different from that of Church but Newman had to argue the case for publication of Turing's paper before the London Mathematical Society would publish it. Turing's revised paper contains a reference to Church's results and the paper, first completed in April 1936, was revised in this way in August 1936 and it appeared in print in 1937.

A good feature of the resulting discussions with Church was that Turing became a graduate student at Princeton University in 1936. At Princeton, Turing undertook research under Church's supervision and he returned to England in 1938, having been back in England for the summer vacation in 1937 when he first met Wittgenstein. The major publication which came out of his work at Princeton was *Systems of Logic Based on Ordinals* which was published in 1939. Newman writes in [13]:-

> *This paper is full of interesting suggestions and ideas. ...* [*It*] *throws much light on Turing's*

views on the place of intuition in mathematical proof.

Before this paper appeared, Turing published two other papers on rather more conventional mathematical topics. One of these papers discussed methods of approximating Lie groups by finite groups. The other paper proves results on extensions of groups, which were first proved by Reinhold Baer, giving a simpler and more unified approach.

Perhaps the most remarkable feature of Turing's work on Turing machines was that he was describing a modern computer before technology had reached the point where construction was a realistic proposition. He had proved in his 1936 paper that a universal Turing machine existed [13]:-

> *... which can be made to do the work of any special-purpose machine, that is to say to carry out any piece of computing, if a tape bearing suitable "instructions" is inserted into it.*

Although to Turing a "computer" was a person who carried out a computation, we must see in his description of a universal Turing machine what we today think of as a computer with the tape as the program.

While at Princeton Turing had played with the idea of constructing a computer. Once back at Cambridge in 1938 he starting to build an analogue mechanical device to investigate the Riemann hypothesis, which many consider today the biggest unsolved problem in mathematics. However, his work would soon take on a new aspect for he was contacted, soon after his return, by the Government Code and Cypher School who asked him to help them in their work on breaking the German Enigma codes.

When war was declared in 1939 Turing immediately moved to work full-time at the Government Code and Cypher School at Bletchley Park. Although the work carried out at Bletchley Park was covered by the Official Secrets Act, much has recently become public knowledge. Turing's brilliant ideas in solving codes, and developing computers to assist break them, may have saved more lives of military personnel in the course of the war than any other. It was also a happy time for him [13]:-

> *... perhaps the happiest of his life, with full scope for his inventiveness, a mild routine to shape the day, and a congenial set of fellow-workers.*

Together with another mathematician W G Welchman, Turing developed the *Bombe*, a machine based on earlier work by Polish mathematicians, which from late 1940 was decoding all messages sent by the Enigma machines of the Luftwaffe. The Enigma machines of the German navy were much harder to break but this was the type of challenge which Turing enjoyed. By the middle of 1941 Turing's statistical approach, together with captured information, had led to the German navy signals being decoded at Bletchley.

From November 1942 until March 1943 Turing was in the United States liaising over decoding issues and also on a speech secrecy system. Changes in the way the Germans encoded their messages had meant that Bletchley lost the ability to decode the messages. Turing was not directly involved with the successful breaking of these more complex codes, but his ideas proved of the greatest importance in this work. Turing was awarded the O.B.E. in 1945 for his vital contribution to the war effort.

At the end of the war Turing was invited by the National Physical Laboratory in London to design a computer. His report proposing the Automatic Computing Engine (ACE) was submitted in March 1946. Turing's design was at that point an original detailed design and prospectus for a computer in the modern sense. The size of storage he planned for the ACE was regarded by most who considered the report as hopelessly over-ambitious and there were delays in the project being approved.

Turing returned to Cambridge for the academic year 1947-48 where his interests ranged over many topics far removed from computers or mathematics; in particular he studied neurology and physiology. He did not forget about computers during this period, however, and he wrote code for programming computers. He had interests outside the academic world too, having taken up athletics seriously after the end of the war. He was a member of Walton Athletic Club winning their 3 mile and 10 mile championship in record time. He ran in the A.A.A. Marathon in 1947 and was placed fifth.

By 1948 Newman was the professor of mathematics at the University of Manchester and he offered Turing a readership there. Turing resigned from the National Physical Laboratory to take up the post in Manchester. Newman writes in [13] that in Manchester:-

> *... work was beginning on the construction of a computing machine by F C Williams and T*

Kilburn. The expectation was that Turing would lead the mathematical side of the work, and for a few years he continued to work, first on the design of the subroutines out of which the larger programs for such a machine are built, and then, as this kind of work became standardised, on more general problems of numerical analysis.

In 1950 Turing published *Computing machinery and intelligence* in *Mind.* It is another remarkable work from his brilliantly inventive mind which seemed to foresee the questions which would arise as computers developed. He studied problems which today lie at the heart of artificial intelligence. It was in this 1950 paper that he proposed the Turing Test which is still today the test people apply in attempting to answer whether a computer can be intelligent [1]:-

... he became involved in discussions on the contrasts and similarities between machines and brains. Turing's view, expressed with great force and wit, was that it was for those who saw an unbridgeable gap between the two to say just where the difference lay.

Turing did not forget about questions of decidability which had been the starting point for his brilliant mathematical publications. One of the main problems in the theory of group presentations was the question: given any word in a finitely presented groups is there an algorithm to decide if the word is equal to the identity. Post had proved that for semigroups no such algorithm exist. Turing thought at first that he had proved the same result for groups but, just before giving a seminar on his proof, he discovered an error. He was able to rescue from his faulty proof the fact that there was a cancellative semigroup with insoluble word problem and he published this result in 1950. Boone used the ideas from this paper by Turing to prove the existence of a group with insoluble word problem in 1957.

Turing was elected a Fellow of the Royal Society of London in 1951, mainly for his work on Turing machines in 1936. By 1951 he was working on the application of mathematical theory to biological forms. In 1952 he published the first part of his theoretical study of morphogenesis, the development of pattern and form in living organisms.

Turing was arrested for violation of British homosexuality statutes in 1952 when he reported to the police details of a homosexual affair. He had gone to the police because he had been threatened with blackmail. He was tried as a homosexual on 31 March 1952, offering no defence other than that he saw nothing wrong in his actions. Found guilty he was given the alternatives of prison or oestrogen injections for a year. He accepted the latter and returned to a wide range of academic pursuits.

Not only did he press forward with further study of morphogenesis, but he also worked on new ideas in quantum theory, on the representation of elementary particles by spinors, and on relativity theory. Although he was completely open about his sexuality, he had a further unhappiness which he was forbidden to talk about due to the Official Secrets Act.

The decoding operation at Bletchley Park became the basis for the new decoding and intelligence work at GCHQ. With the cold war this became an important operation and Turing continued to work for GCHQ, although his Manchester colleagues were totally unaware of this. After his conviction, his security clearance was withdrawn. Worse than that, security officers were now extremely worried that someone with complete knowledge of the work going on at GCHQ was now labelled a security risk. He had many foreign colleagues, as any academic would, but the police began to investigate his foreign visitors. A holiday which Turing took in Greece in 1953 caused consternation among the security officers.

Turing died of potassium cyanide poisoning while conducting electrolysis experiments. The cyanide was found on a half eaten apple beside him. An inquest concluded that it was self-administered but his mother always maintained that it was an accident.

Pavel Samuilovich Urysohn

Born: 3 Feb 1898 in Odessa, Ukraine, Russia
Died: 17 Aug 1924 in Batz-sur-Mer, France

Pavel Urysohn is also known as **Pavel Uryson.** His father was a financier in Odessa, the town in which Pavel Samuilovich was born. He came from a family descended from the sixteenth century Rabbi M Jaffe. It was a well-off family and Urysohn received his secondary education in Moscow at a private school there.

In 1915 Urysohn entered the University of Moscow to study physics and in fact he published his first paper in this year. Being interested in physics at this time it is not surprising that this first paper was on a physics topic, and indeed it was, being on Coolidge tube radiation. However his interest in physics soon took second place for after attending lectures by Luzin and Egorov at the University of Moscow he began to concentrate on mathematics.

Urysohn graduated in 1919 and continued his studies there working towards his doctorate. The authors of [8] write:-

> *Luzin was a dynamic mathematician and it was he who persuaded Urysohn to stay on in order to study for a doctorate during* 1919-21.

At this stage Urysohn was interested in analysis, in particular integral equations, and this was the topic of his habilitation. He was awarded his habilitation in June 1921 and, following this, became an assistant professor at the University of Moscow.

Urysohn soon turned to topology. He was asked two questions by Egorov and it was these which occupied him during the summer of 1921. The first question that Egorov posed was to find a general intrinsic topological definition of a curve which when restricted to the plane became Cantor's notion of a continuum which is nowhere dense in the plane. The second of Egorov's questions was a similar one but applied to surfaces, again asking for an intrinsic topological definition.

These were difficult questions which had been around for some time. It was not that Egorov had come up with new questions, rather he was giving the bright young mathematician Urysohn two really difficult problems in the hope that he might come up with new ideas. Egorov was not to be disappointed, for Urysohn attacked the questions with great determination. He did not sit still waiting for inspiration to strike, rather he tried one idea after another to see if it would give him the topological definition of dimension that he was looking for.

A holiday with other young Moscow mathematicians to the village of Burkov, on the banks of the river Kalyazmy near to the town of Bolshev, did not stop him trying to find the "right" definition of dimension. Quite the opposite, it was a good chance for him to think in congenial surroundings, and one morning near the end of August he woke up with an idea in his mind which he felt, even before working through the details, was right. Immediately he told his friend Aleksandrov about his inspiration.

Of course there was a lot of hard work after the moment of inspiration. During the following year Urysohn worked through the consequences building a whole new area of dimension theory in topology. It was an exciting time for the topologists in Moscow for Urysohn lectured on the topology of continua and often his latest results were presented in the course shortly after he had proved them. He published a series of short notes on this topic during 1922. The complete theory was presented in an article which Lebesgue accepted for publication in the *Comptes rendus* of the Academy of Sciences in Paris. This gave Urysohn an international platform for his ideas which immediately attracted the interest of mathematicians such as Hilbert.

Urysohn published a full version of his dimension theory in *Fundamenta mathematicae.* He wrote a major paper in two parts in 1923 but they did not appear in print until 1925 and 1926. Sadly Urysohn had died before even the first part was published. The paper begins with Urysohn stating his aim which was:-

> *To indicate the most general sets that still merit being called "lines" and "surfaces" ...*

In fact Urysohn set out to do far more in this paper than to answer the two questions that Egorov had posed to him. As Crilly and Johnson write [8]:-

> *Not only did he seek definitions of curve and surface, but also definitions of n-dimensional Cantorian manifold and hence of dimension itself. The dimension concept was, in fact, the centre of his attention.*

Although Urysohn did not know of Brouwer's contribution when he worked out the details of his theory of topological dimension, Brouwer had in fact published on that topic in 1913. He had given a global definition, however, and this was in contrast to Urysohn's local definition of dimension. Another important aspect of Urysohn's ideas was the fact that he presented them in the context of compact metric spaces. After Urysohn's death, Aleksandrov argued that although Urysohn's definition of dimension was given for a metric space, it is, nevertheless, completely equivalent to the definition given by Menger for general topological spaces.

Urysohn visited Göttingen in 1923. His reports to the Mathematical Society of Göttingen interested Hilbert and while in Göttingen he learnt of Brouwer's contributions to the area made in the paper of 1913 to which we referred above. Urysohn spotted an error in Brouwer's paper regarding a definition of dimension while he was studying it in Göttingen and easily constructed a counter-example. He met Brouwer at the annual meeting of the German Mathematical Society in Marburg where both gave lectures and Urysohn mentioned Brouwer's error, and his counter-example, in his talk. It was an occasion which made Brouwer begin to think about topology again, for his interests had turned to intuitionism, the subject of his talk at Marburg.

In the summer of 1924 Urysohn set off again with Aleksandrov on a European trip through Germany, Holland and France. Again the two mathematicians visited Hilbert and, by 7 May, they must have left since Hilbert wrote to Urysohn on that day telling him his paper with Aleksandrov was accepted for publication in *Mathematische Annalen* (see below). This letter, given in [11], also thanks Urysohn for caviar he had given Hilbert, and expresses the hope that Urysohn will visit again the following summer.

They then met Hausdorff who was impressed with Urysohn's results. He also wrote a letter to Urysohn which was dated 11 August 1924 (see [11]). The letter discusses Urysohn's metrization theorem and his construction of a universal separable metric space. The construction of a universal metric space, containing an isometric image of any metric space, was one of Urysohn's last results. Like Hilbert, Hausdorff expressed the hope that Urysohn would visit again the following summer. Van Dalen writes in [13] about their final mathematical visit which was to Brouwer:-

> *This time* [*Urysohn and Aleksandrov*] *visited Brouwer, who was most favourably impressed by the two Russians. He was particularly taken with Urysohn, for whom he developed something like the attachment to a lost son.*

After this visit the two mathematicians continued their holiday to Brittany where they rented a cottage. Urysohn drowned in rough seas while on one of their regular swims off the coast.

Urysohn was not only an "inseparable friend" to Aleksandrov but the two collaborated on important publications such as *Zur Theorie der topologischen Räume* published in *Mathematische Annalen* in 1924. Urysohn's main contributions, in addition to the theory of dimension discussed above, are the introduction and investigation of a class of normal surfaces, metrization theorems, and an important existence theorem concerning mapping an arbitrary normed space into a Hilbert space with countable basis. He is remembered particularly for 'Urysohn's lemma' which proves the existence of a certain continuous function taking values 0 and 1 on particular closed subsets.

After Urysohn's death Brouwer and Aleksandrov made sure that the mathematics he left was properly dealt with. As van Dalen writes [13]:-

> *Brouwer was broken hearted. He decided to look after the scientific estate of Urysohn as a tribute to the genius of the deceased. Together with Aleksandrov he acquitted himself of this task.*

Crilly and Johnson write [8]:-

Considering that he only had three years to devote to topology, he made his mark in his chosen field with brilliance and passion. He transformed the subject into a rich domain of modern mathematics. How much more might he there have been, had he not died so young?

Hermann Klaus Hugo Weyl

Born: 9 Nov 1885 in Elmshorn (near Hamburg), Schleswig-Holstein, Germany
Died: 9 Dec 1955 in Zürich, Switzerland

Hermann Weyl was known as Peter to his close friends. His parents were Anna Dieck and Ludwig Weyl who was the director of a bank. As a boy Hermann had already showed that he had a great talents for mathematics and for science more generally. After taking his Abiturarbeit (high school graduation exam) (see [16]) he was ready for his university studies. In 1904 he entered the University of Munich, where he took courses on both mathematics and physics, and then went on to study the same topics at the University of Göttingen. He was completely captivated by Hilbert. He later wrote:-

> *I resolved to study whatever this man had written. At the end of my first year I went home with the "Zahlbericht" under my arm, and during the summer vacation I worked my way through it - without any previous knowledge of elementary number theory or Galois theory. These were the happiest months of my life, whose shine, across years burdened with our common share of doubt and failure, still comforts my soul.*

His doctorate was from Göttingen where his supervisor was Hilbert. After submitting his doctoral dissertation *Singuläre Integralgleichungen mit besonder Berücksichtigung des Fourierschen Integraltheorems* he was awarded the degree in 1908. This thesis investigated singular integral equations, looking in depth at Fourier integral theorems. It was at Göttingen that he held his first teaching post as a privatdozent, a post he held until 1913. His habilitation thesis *Über gewöhnliche Differentialgleicklungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen* investigated the spectral theory of singular Sturm-Liouville problems. During this period at Göttingen, Weyl made a reputation for himself as an outstanding mathematician who was producing work which was having a major impact on the progress of mathematics. His habilitation thesis was one such piece of work but there was much more. He gave a lecture course on Riemann surfaces in session 1911-12 and out of this course came his first book *Die Idee der Riemannschen Fläche* which was published in 1913. It united analysis, geometry and topology, making rigorous the geometric function theory developed by Riemann. The book introduced for the first time the notion of a [58]:-

> *... two-dimensional differentiable manifold, a covering surface, and the duality between differentials and* 1*-cycles. ...Weyl's idea of a space also included the famous separation property later introduced and popularly credited to Felix Hausdorff* (1914).

L Sario wrote in 1956 that Weyl's 1913 text:-

> *... has undoubtedly had a greater influence on the development of geometric function theory than any other publication since Riemann's dissertation.*

It is rather remarkable that this 1913 text was reprinted in 1997. Weyl himself produced two later editions, the third (and final) of these editions appearing in 1955 covering the same topics as the original text but with a more modern treatment. It was the original 1913 edition, however, which was reprinted in 1997 showing perhaps more fully than the later editions just how significant the original 1913 text was in the development of mathematics.

As a privatdozent at Göttingen, Weyl had been influenced by Edmund Husserl who held the chair of philosophy there from 1901 to 1916. Weyl married Helene Joseph, who had been a student of Husserl, in 1913; they had two sons. Helene, who came from a Jewish background, was a philosopher who was working as a translator of Spanish. Not only did Weyl and his wife share an interest in philosophy, but they shared a real talent for languages. Language for Weyl held a special importance. He not only wrote beautifully in German, but later he wrote stunning English prose despite the fact that, in his own words from a 1939 English text:-

... the gods have imposed upon my writing the yoke of a foreign language that was not sung at my cradle.

From 1913 to 1930 Weyl held the chair of mathematics at Zürich Technische Hochschule. In his first academic year in this new post he was a colleague of Einstein who was at this time working out the details of the theory of general relativity. It was an event which had a large influence on Weyl who quickly became fascinated by the mathematical principles lying behind the theory.

World War I broke out not long after Weyl took up the chair in Zürich. Being a German citizen he was conscripted into the German army in 1915 but the Swiss government made a special request that he be allowed to return to his chair in Zürich which was granted in 1916. In 1917 Weyl gave another course presenting an innovative approach to relativity through differential geometry. The lectures formed the basis of Weyl's second book *Raum-Zeit-Materie* which first appeared in 1918 with further editions, each showing how his ideas were developing, in 1919, 1920, and 1923. These later ideas included a gauge metric (the Weyl metric) which led to a gauge field theory. However Einstein, Pauli, Eddington, and others, did not fully accept Weyl's approach. Also over this period Weyl also made contributions on the uniform distribution of numbers modulo 1 which are fundamental in analytic number theory.

In 1921 Schrödinger was appointed to Zurich where he became a colleague, and soon closest friend, of Weyl. They shared many interests in mathematics, physics, and philosophy. Their personal lives also became entangled as Moore relates in [5]:-

Those familiar with the serious and portly figure of Weyl at Princeton would have hardly recognised the slim, handsome young man of the twenties, with his romantic black moustache. His wife, Helene Joseph, from a Jewish background, was a philosopher and literateuse. Her friends called her Hella, and a certain daring and insouciance made her the unquestioned leader of the social set comprising the scientists and their wives. Anny [*Schrödinger's wife*] *was almost an exact opposite of the stylish and intellectual Hella, but perhaps for that reason* [*Weyl*] *found her interesting and before long she was madly in love with him. ... The special circle in which they lived in Zurich had enjoyed the sexual revolution a generation before* [*the United States*]. *Extramarital affairs were not only condoned, they were expected, and they seemed to occasion little anxiety. Anny would find in Hermann Weyl a lover to whom she was devoted body and soul, while Weyl's wife Hella was infatuated with Paul Scherrer.*

From 1923-38 Weyl evolved the concept of continuous groups using matrix representations. In particular his theory of representations of semisimple groups, developed during 1924-26, was very deep and considered by Weyl himself to be his greatest achievement. The ideas behind this theory had already been introduced by Hurwitz and Schur, but it was Weyl with his general character formula which took them forward. He was not the only mathematician developing this theory, however, for Cartan also produced work on this topic of outstanding importance.

From 1930 to 1933 Weyl held the chair of mathematics at Göttingen where he was appointed to fill the vacancy which arose on Hilbert's retirement. Given different political circumstances it is likely that he would have remained in Göttingen for the rest of his career. However [7]:-

... the rise of the Nazis persuaded him in 1933 *to accept a position at the newly formed Institute for Advanced Study in Princeton, where Einstein also went. Here Weyl found a very congenial working environment where he was able to guide and influence the younger generation of mathematicians, a task for which he was admirably suited.*

One also has to understand that Weyl's wife was Jewish, and this must have played a major role in their decision to leave Germany in 1933. Weyl remained at the Institute for Advanced Study at Princeton until he retired in 1952. His wife Helene died in 1948, and two years later he married the sculptor Ellen Lohnstein Bär from Zürich.

Weyl certainly undertook work of major importance at Princeton, but his most productive period was without doubt the years he spent at Zürich. He attempted to incorporate electromagnetism into the geometric formalism of general relativity. He produced the first unified field theory for which the Maxwell electromagnetic field and the gravitational field appear as geometrical properties of space-time. With his application of group theory to quantum mechanics he set up the modern subject. It was his lecture course on group theory and quantum mechanics in Zürich in session 1927-28 which led to his third major text *Gruppentheorie und Quantenmechanik* published in 1928. John Wheeler writes [55]:-

> *That book has, each time I read it, some great new message.*

More recently attempts to incorporate electromagnetism into general relativity have been made by Wheeler. Wheeler's theory, like Weyl's, lacks the connection with quantum phenomena that is so important for interactions other than gravitation. Wheeler writes about meeting Weyl for the first time in [55]:-

> *Erect, bright-eyed, smiling Hermann Weyl I first saw in the flesh when* 1937 *brought me to Princeton. There I attended his lectures on the Élie Cartan calculus of differential forms and their application to electromagnetism - eloquent, simple, full of insights.*

We have seen above how Weyl's great works were first given as lecture courses. This was a deliberate design by Weyl [55]:-

> *At another time Weyl arranged to give a course at Princeton University on the history of mathematics. He explained to me one day that it was for him an absolute necessity to review, by lecturing, his subject of concern in all its length and breadth. Only so, he remarked, could he see the great lacunae, the places where deeper understanding is needed, where work should focus.*

Many other great books by Weyl appeared during his years at Princeton. These include *Elementary Theory of Invariants* (1935), *The classical groups* (1939), *Algebraic Theory of Numbers* (1940), *Philosophy of Mathematics and Natural Science* (1949), *Symmetry* (1952), and *The Concept of a Riemannian Surface* (1955). There is so much that could be said about all these works, but we restrict ourselves to looking at the contents of *Symmetry* for this perhaps tells us most about the full range of Weyl's interests. Coxeter reviewed the book and his review beautifully captures the spirit of the book:-

> *This is slightly modified version of the Louis Clark Vanuxem Lectures given at Princeton University in* 1951 *... The first lecture begins by showing how the idea of bilateral symmetry has influenced painting and sculpture, especially in ancient times. This leads naturally to a discussion of "the philosophy of left and right", including such questions as the following. Is the occurrence in nature of one of the two enantiomorphous forms of an optically active substance characteristic of living matter? At what stage in the development of an embryo is the plane of symmetry determined? The second lecture contains a neat exposition of the theory of groups of transformations, with special emphasis on the group of similarities and its subgroups: the groups of congruent transformations, of motions, of translations, of rotations, and finally the symmetry group of any given figure. ... the cyclic and dihedral groups are illustrated by snowflakes and flowers, by the animals called Medusae, and by the plans of symmetrical buildings. Similarly, the infinite cyclic group generated by a spiral similarity is illustrated by the Nautilus shell and by the arrangement of florets in a sunflower. The third lecture gives the essential steps in the enumeration of the seventeen space-groups of two-dimensional crystallography ...* [*In the fourth lecture he*] *shows how the special theory of relativity is essentially the study of the inherent symmetry of the four-dimensional space-time continuum, where the symmetry operations are the Lorentz transformations; and how the symmetry operations of an atom, according to quantum mechanics, include the permutations of its peripheral electrons. Turning from physics to mathematics, he gives an extraordinarily concise epitome of Galois theory, leading up to the statement of his guiding principle: "Whenever you have to do with a structure-endowed entity, try to determine its group of automorphisms".*

In 1951 Weyl retired from the Institute for Advanced Study at Princeton. In fact he described the *Symmetry* book as his 'swan song'. After his retirement Weyl and his wife Ellen spent part of their time at Princeton and part at Zurich. He died unexpectedly while in Zurich. He was walking home after posting letters of thanks to those who had wished him well on his seventieth birthday when he collapsed and died.

We must say a little about another aspect of Weyl's work which we have not really mentioned, namely his work on mathematical philosophy and the foundations of mathematics. It is interesting to note what a large number of the references we quote deal with this aspect of his work and its importance is not only in the work itself but also in the extent to which Weyl's ideas on these topics underlies the rest of his mathematical and physical contributions. Weyl was much influenced by Husserl in his outlook and also shared many ideas with Brouwer. Both shared the view that the intuitive continuum is not accurately represented by Cantor's set-theoretic continuum. Wheeler [55] writes:-

> *The continuum ..., Weyl taught us, is an illusion. It is an idealization. It is a dream.*

Weyl summed up his attitude to mathematics, writing:-

> *My own mathematical works are always quite unsystematic, without mode or connection. Expression and shape are almost more to me than knowledge itself. But I believe that, leaving aside my own peculiar nature, there is in mathematics itself, in contrast to the experimental disciplines, a character which is nearer to that of free creative art.*

His often quoted comment:-

> *My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful ...*

although half a joke, sums up his personality.

Norbert Wiener

Born: 26 Nov 1894 in Columbia, Missouri, USA
Died: 18 March 1964 in Stockholm, Sweden

Norbert Wiener's father was Leo Wiener who was a Russian Jew. Because Leo Wiener was such a major influence on his son, we should give some background to his education and career. Leo Wiener attended medical school at the University of Warsaw but was unhappy with the profession, so he went to Berlin where he began training as an engineer. This profession seemed only a little more interesting to him than the medical profession, and he emigrated to the United States having first landed in England. We should note that throughout his education Leo was interested in mathematics and, although he never used his mathematical skills in any jobs he held, it was a deep amateur interest to him all through his life.

Arriving in New Orleans in 1880, Leo tried his hand at various jobs in factories and farms before becoming a school teacher in Kansas City. He progressed from being a language teacher in schools to becoming Professor of Modern Languages at the University of Missouri. While there he met and married Bertha Kahn, who was the daughter of a department store owner. Bertha, from a German Jewish family, was [7]:-

> *... a small woman, healthy, vigorous and vivacious.*

She joined her husband in the boarding house in Columbia, Missouri where their son Norbert was born in the following year.

Not long after Norbert's birth a decision was taken to split the Modern Languages Department at the University of Missouri into separate departments of French and German. Leo was to join the German Department after the split but he lost out in some political manoeuvring so the family left Columbia and they moved to Boston. There Leo brought in money by taking a variety of teaching and other positions and eventually was appointed as an Instructor in Slavic Languages at Harvard. This did not pay well enough to provide for his family, so Leo kept various other positions to augment his salary. He remained at Harvard University for the rest of his career, being eventually promoted to professor.

As a young child Norbert had a nursemaid. When he was about four years old, a second child Constance was born; Wiener's second sister was born on 1901. He writes in [7] about his upbringing:-

> *I was brought up in a house of learning. My father was the author of several books, and ever since I can remember, the sound of the typewriter and the smell of the paste pot have been familiar to me. ... I had full liberty to roam in what was the very catholic and miscellaneous library of my father. At one period or other the scientific interests of my father had covered most of the imaginable subjects of study. ... I was an omnivorous reader ...*

Wiener had problems regarding his schooling, partly because the reading which he had done at home had meant that he was advanced in certain areas but much less so in others. His parents sent him to the Peabody School when he was seven years old and, after worrying about which class he should enter, had him begin in the third grade. After a short time his parents and teachers felt he would be better suited to the fourth grade and he was moved up a year. However, he certainly did not fit into the school in either grade and his teacher had little sympathy with so young a boy in the fourth grade yet lacking certain skills which would be expected the pupils at this stage in their education. He writes [7]:-

> *My chief deficiency was arithmetic. Here my understanding was far beyond my manipulation, which was definitely poor. My father saw quite correctly that one of my chief difficulties was that manipulative drill bored me. He decided to take me out of school and put me on algebra instead of arithmetic, with the purpose of offering a greater challenge and stimulus to my imagination.*

From this time on Wiener's father took over his education and he made rapid progress for so young a child. However, Wiener had problems relating to his movements and was obviously very clumsy. This stemmed partly from poor coordination but also partly for poor eyesight. Advised by a doctor to stop reading for six months to allow his eyes to recover, he still had regular lessons from his father who now taught him to do mathematics in his head. After the six months were up Wiener went back to reading but he had developed some fine mental skills during this period which he retained all his life.

In the autumn of 1903, at age nine, he was sent to school again, this time to Ayer High School. The school agreed to experiment and to find the right level for Wiener who was soon put into senior third year class with pupils who were seven years older than he was. The school only formed part of his education, however, for his father continued to coach him. He graduated in 1906 from Ayer at the age of eleven and celebrated with his eighteen year old fellow students [7]:-

> *I owe a great deal to my Ayer friends. I was given a chance to go through some of the gawkiest stages of growing up in an atmosphere of sympathy and understanding.*

In September 1906, still only eleven years old, Wiener entered Tufts College. Socially a child, he was an adult in educational terms so his student days were not easy ones. Although taking various science courses, he took a degree in mathematics. Wiener's father continued to coach him in mathematics showing complete mastery of undergraduate level topics. In 1909 Wiener graduated from Tufts at age fourteen and entered Harvard to begin graduate studies.

Rather against his father's advice, Wiener began graduate studies in zoology at Harvard. However things did not go too well and by the end of a year a decision was taken, partly by Wiener partly by his father, that he would change topic to philosophy. Having won a scholarship to Cornell he entered in 1910 to begin graduate studies in philosophy. Taking mathematics and philosophy courses, Wiener did not have a successful year and before it was finished his father had made the necessary arrangements to return to Harvard to continue philosophy.

Back at Harvard Wiener was strongly influenced by the fine teaching of Edward Huntington on mathematical philosophy. He received his Ph.D. from Harvard at the age of 18 with a dissertation on mathematical logic supervised by Karl Schmidt. From Harvard Wiener went to Cambridge, England, to study under Russell who told him that in order to study the philosophy of mathematics he needed to know more mathematics so he attended courses by G H Hardy. In 1914 he went to Göttingen to study differential equations under Hilbert, and also attended a group theory course by Edmund Landau. He was influence by Hilbert, Landau and Russell but also, perhaps to an even greater degree, by Hardy. At Göttingen he learned that [7]:-

> *... mathematics was not only a subject to be done in the study but one to be discussed and lived with.*

Wiener returned to the United States a couple of days before the outbreak of World War I, but returned to Cambridge to study further with Russell. Back in the United States he taught philosophy courses at Harvard in 1915, worked for a while for the General Electric Company, then joined Encyclopedia Americana as a staff writer in Albany. While working there he received an invitation from Veblen to undertake war work on ballistics at the Aberdeen Proving Ground in Maryland. Taking about mathematics with his fellow workers while undertaking this war work revived his interest in mathematics. At the end of the war Osgood told him of a vacancy at MIT and he was appointed as an instructor in mathematics.

His first mathematical work at MIT led him to examine Brownian motion. In fact, as Wiener explained in [7], this first work would provide a connecting thread through much of his later studies:-

> *... this study introduced me to the theory of probability. Moreover, it led me very directly to the periodogram, and to the study of forms of harmonic analysis more general than the classical Fourier series and Fourier integral. All these concepts have combined with the engineering preoccupations of a professor of the Mathematical Institute of Technology to lead me to make both theoretical and practical advances in the theory of communication, and ultimately to found the discipline of cybernetics, which is in essence a statistical approach to the theory of communication. Thus, varied as my scientific interests seem to be, there has been a single thread connecting all of them from my first mature work ...*

He attended the International Congress of Mathematicians at Strasbourg in 1920 and while there worked with Fréchet. He returned to Europe frequently in the next few years, visiting mathematicians in England, France and Germany. Especially important was his contacts with Paul Lévy and with Göttingen where his work was seen to have important connections with quantum mechanics. This led to a collaboration with Born.

In 1926 Wiener married Margaret Engemann, and after their marriage Wiener set off for Europe as a Guggenheim scholar. After visiting Hardy in Cambridge he returned to Göttingen where his wife joined him after completing her teaching duties in modern languages at Juniata College in Pennsylvania. Another important year in Wiener's mathematical development was 1931-32 which he spent mainly in England visiting Hardy at Cambridge. There he gave a lecture course on his own contributions to the Fourier integral but Cambridge also provided a base from where he was able to visit many mathematical colleagues on the Continent. Among these were Blaschke, Menger and Frank who invited him to make a visit, while he also met Hahn, Artin and Gödel.

Wiener's papers were hard to read. Sometimes difficult results appeared with hardly a proof as if they were obvious to Wiener, while at other times he would give a lengthy proof of a triviality. Freudenthal writes [1]:-

> *All too often Wiener could not resist the temptation to tell everything that cropped up in his comprehensive mind, and he often had difficulty in separating the relevant mathematics neatly from its scientific and social implications and even from his personal experiences. The reader to whom he appears to be addressing himself seems to alternate in a random order between the layman, the undergraduate student of mathematics, the average mathematician, and Wiener himself.*

Despite the style of his papers, Wiener contributed some ideas of great importance. We have already mentioned above his work in 1921 in Brownian motion. He introduced a measure in the space of one dimensional paths which brings in probability concepts in a natural way. From 1923 he investigated Dirichlet's problem, producing work which had a major influence on potential theory.

Wiener's mathematical ideas were very much driven by questions that were put to him by his engineering colleagues at MIT. These questions pushed him to generalise his work on Browian motion to more general stochastic processes. This in turn led him to study harmonic analysis in 1930. His work on generalised harmonic analysis led him to study Tauberian theorems in 1932 and his contributions on this topic won him the Bôcher Prize in 1933. He received the prize from the American Mathematical Society for his memoir *Tauberian theorems* published in *Annals of Mathematics* in the previous year. The work on Tauberian theorems naturally led him to study the Fourier transform and he published *The Fourier Integral, and Certain of Its Applications* (1933) and *Fourier Transforms* in 1934.

Wiener had an extraordinarily wide range of interests and contributed to many areas in addition to those we have mentioned above including communication theory, cybernetics (a term he coined), quantum theory and during World War II he worked on gunfire control. It is probably this latter work which motivated his invention of the new area of cybernetics which he described in *Cybernetics: or, Control and Communication in the Animal and the Machine* (1948). Freudenthal writes in [1]:-

> *While studying anti-aircraft fire control, Wiener may have conceived the idea of considering the operator as part of the steering mechanism and of applying to him such notions as feedback and stability, which had been devised for mechanical systems and electrical circuits. ... As time passed, such flashes of insight were more consciously put to use in a sort of biological research ...* [*Cybernetics*] *has contributed to popularising a way of thinking in communication theory terms, such as feedback, information, control, input, output, stability, homeostasis, prediction, and filtering. On the other hand, it also has contributed to spreading mistaken ideas of what mathematics really means.*

Wiener himself was aware of these dangers and his wide dealings with other scientists led him to say:-

> *One of the chief duties of the mathematician in acting as an adviser to scientists is to discourage them from expecting too much from mathematics.*

Some of Wiener's publications which we have not mentioned include *Nonlinear Problems in Random Theory* (1958), and *God and Golem, Inc.: A Comment on Certain Points Where Cybernetics Impinges on Religion*

(1964).

We have mentioned above Freudenthal's comments on Wiener's poor writing style. His most famous work *Cybernetics* comes in for special criticism by Freudenthal:-

> *Even measured by Wiener's standards "Cybernetics" is a badly organised work -- a collection of misprints, wrong mathematical statements, mistaken formulas, splendid but unrelated ideas, and logical absurdities. It is sad that this work earned Wiener the greater part of his public renown, but this is an afterthought. At that time mathematical readers were more fascinated by the richness of its ideas than by its shortcomings.*

Freudenthal, in [1], describes both Wiener's appearance and his character:-

> *In appearance and behaviour, Norbert Wiener was a baroque figure, short, rotund, and myopic, combining these and many qualities in extreme degree. His conversation was a curious mixture of pomposity and wantonness. He was a poor listener. His self-praise was playful, convincing and never offensive. He spoke many languages but was not easy to understand in any of them. He was a famously bad lecturer.*

D G Kendall writes [3]:-

> *As a human being Wiener was above all stimulating. I have known some who found the stimulus unwelcome. He could offend publicly by snoring through a lecture and then asking an awkward question in the discussion, and also privately by proffering information and advice on some field remote from his own to an august dinner companion. I like to remember Wiener as I once saw him late at night in Magdalen College, Oxford, surrounded by a spellbound group of undergraduates, talking, endlessly talking.*

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Articles by: *J J O'Connor* and *E F Robertson*
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