- Divorced women are less likely to remarry than divorced men.
- There are proportionately less never married women than men.
- The average age of first marriage is lower for women than men.

How can we explain gender differences in behavior?

This class will explore the effect of differential fecundity on gender roles.

Differential fecundity: Women are fecund for a shorter period of their lives than men.

- By itself, differential fecundity has no implication for gender roles.
- Other than age, we need to generate some additional heterogeneity among men.
- First, let individuals have different levels of income which potential spouses value. Wealthier older men may use their wealth to try to compete with younger men in the marriage market.
- Second, we can introduce search frictions into the marriage market. If search frictions are large, a young woman who is matched with an older man may prefer to marry the older man rather than rejecting the match.


## 1 Gender neutrality

Consider a society with a constant population and a sex ratio of one.

Each individual lives for two periods as adults and one period as children.

Women are fecund when they are young and infertile when they are old.

Men are fecund over their entire lives.

Individuals marry only to have children.

So older single women will not want to marry.

Per period return to being single to zero.

Older married couples who cannot have any children will also receive a return of zero.

Each fertile couple will have a boy and a girl per period.

These children will become adults in the next period and make their own decisions.

Each member of a fertile couple will receive a per period return of $\gamma$.

Divorced individuals, widows or widowers suffer a disutility of $\beta<\gamma$.

Marriage is a public good.

The discount rate is also set to zero.

Consider an equilibrium in which young men and young women marry each other.

Each young married adult will receive a payoff of $\gamma$ for the period.

Will the older husband like to divorce his wife and remarry a younger woman?

His payoff from divorce and remarriage is $\gamma-\beta>0$.

If a young woman marries him, her life time utility is:

$$
\gamma-\beta
$$

If she marries a young man and he divorces her later, her life time utility is

If her marriage to a young man lasts two periods, her life time utility will be

$$
\gamma
$$

She will be strictly better off marrying this young man rather than an older man.

If an older man choose to divorce his current wife and reenter the marriage market, he will suffer a disutility loss of $\beta$ and not find any young woman being willing to marry him.

The equilibrium outcome in this society is that all young adults will marry each other and remained married for two periods.

## 2 Random Matching in the Marriage

 MarketAdd random matching in the marriage market.

There will be at most one match between an eligible man and an eligible woman per period.

Single older women will not be interested in participating in the marriage market because they will have not gain from participating.

Single young women who are randomly matched with older men will be willing to marry them to obtain a life time payoff of $\gamma-\beta>0$.

Single young women may prefer to marry young men but may be unable to match with them due to the random matching assumption.

There is a stationary equilibrium in which young women will marry both young and old men.

Young men who are unsuccessful in meeting a young woman will reenter the marriage market.

Young couples who marry will divorce in the second period.

Divorced men will reenter the marriage market. Some divorced men will remarry.

Let $n$ be the number of young men and women who enter the marriage market in period $t$.

Let $o$ be the number of older men who are also in the marriage market in period $t$.

Total number of men in the marriage market is $n+o$.

Due to random matching, an eligible man will meet an eligible woman with probability

$$
\frac{n}{n+o}
$$

The number of young men who will successfully meet a woman and marry her is

$$
\left(\frac{n}{n+o}\right) n=\frac{n^{2}}{n+o}
$$

The number of young men who will remain single and reenter the marriage market in period $t+1$ is

$$
\left(1-\frac{n}{n+o}\right) n=\frac{n o}{n+o}
$$

Consider the behavior of older men who married when young. Their payoff from divorce in reentering the marriage market is

$$
\begin{equation*}
d=-\beta+\frac{n \gamma}{n+o} \tag{1}
\end{equation*}
$$

If this payoff is greater than zero, they will divorce their wife and reenter the marriage market.

Let us assume for the time being that $d>0$.

The number of divorce men who will reenter the marriage market at period $t+1$ is

$$
\frac{n^{2}}{n+o}
$$

The total number of older men in the marriage market will consists of never married older men and divorced men:

$$
\begin{aligned}
o & =\frac{n o}{n+o}+\frac{n^{2}}{n+o} \\
& =n
\end{aligned}
$$

Using $o=n$ and (1), $d>0$ as long as:

$$
-\beta+\frac{n \gamma}{n+n}>0
$$

which reduces to:

$$
\begin{equation*}
\gamma>2 \beta \tag{2}
\end{equation*}
$$

When (2) holds, there is an equilibrium in which young women will marry both young men and older men.
$\frac{1}{2}$ of the young women will marry young men and be divorced when old.
$\frac{1}{4}$ of the young women will marry never married older men.
$\frac{1}{4}$ of the young women will marry divorced older men.
These divorced men who remarry will have two wives over their lifetime.
$\frac{1}{4}$ of the men will never marry.

The life time utility of a young woman is independent of her marriage partner and is:

$$
\begin{equation*}
\gamma-\beta \tag{3}
\end{equation*}
$$

$\frac{1}{4}$ of men never marry and receive a life time utility of 0 .
$\frac{1}{4}$ of men will marry once when old and receive a utility of $\gamma$.
$\frac{1}{4}$ of men will marry once when young and receive a life time utility of $\gamma-\beta$.
$\frac{1}{4}$ of men will marry twice and receive a life time utility of $2 \gamma-\beta$. The expected utility of a young man who marries is:

$$
\begin{aligned}
\frac{\gamma-\beta}{2}+\frac{2 \gamma-\beta}{2} & =\frac{3 \gamma}{2}-\beta \\
& >\gamma
\end{aligned}
$$

Since the expected utility from marriage is larger than $\gamma$, a young man will never voluntarily choose not to marry.

A young man who enters the marriage market does not know what his marital outcomes will be. So his life time expected utility is:

$$
\begin{equation*}
\frac{1}{4} 0+\frac{1}{4}(\gamma-\beta)+\frac{1}{4}(2 \gamma-\beta)+\frac{1}{4} \gamma=\gamma-\frac{\beta}{2} \tag{4}
\end{equation*}
$$

Comparing (4) with (3), men are better off than women in this society.

However they are both worse off than in a society without divorce and remarriage where all young adults marry each other.

The drop in social welfare comes from the fact that when a married young man thinks about whether he should divorce his wife when old, he see a private benefit from divorcing his wife. His divorce creates
a negative externality for his wife and other men in the marriage market which he does not take into account.

The predicted gender difference in welfare is specific to this model. In particular, this model does not allow redistribution of resources between men and women even though fecund women are scarce in the marriage market.

From a positive or predictive point of view, this model is substantially better than the benchmark gender neutral model.

# 3 Heterogenous Individuals and Transferable Utility 

We will now consider a completely opposite model, one without random matching and with transferable utilities.

Consider a society with the same demographic structure as the benchmark society.

Let all young adults have the same initial skill $L$.

With probability $p<\frac{1}{2}$, an old adult will have skill $H>L$.

An individual with skill $s$ in a particular period will derive a utility from labor income of $s$.

Labor income is not shared.

For the rest of this section, we ignore the individuals' payoffs from labor market earnings.

Individuals may derive more utility from the marriage market.

The utility from participating in the marriage market is in addition to the utility from labor income.

Older single women will receive no additional utility from marriage and therefore be willing to remain single.

Older eligible men will prefer to marry if they can find a young spouse.

All young women will marry.

Since all young women are identical, let the first period marital output with a husband of skill $S$ be $m(S)$.
$m(S)$ does not include labor earnings of the husband.
$m(S)$ is divided between the couple.
The output from marriage is zero in the second period even if the marriage persists.

We assume that $m(S)$ is increasing in $S$.

The marriage may also end in divorce and both parties will suffer a divorce cost of $\beta<m(L)$.

The marriage may also end if the young woman married an older man and becomes a widow in the second period. She will also suffer a cost of $\beta$ in the second period.

The marriage market clears by transfers between market participants such that any individual can marry if the individual is willing to pay the transfer required to do so.

Individuals can agree to pre-nuptial (state contingent) contracts in marriage. They will agree in the first period what transfers will occur if the couple divorces in the second period.

Let $R$ be the expected lifetime utility that a young woman gets from the marriage market.

Consider an eligible older man of skill $S$. His gain in marriage is:

$$
\begin{equation*}
V(S)=m(S)-\beta-R \tag{5}
\end{equation*}
$$

The marriage produces $m(S)$.

He has to pay her $\beta$ in anticipation of her being a widow.
$R$ is her net gain from marriage.

He will only marry if $V(S)>0 . V_{S}>0$.

So if an older eligible man of skill $L$ marries, an older eligible man of skill $H$ will also marry.

Will an older man of skill $S$ in a current marriage divorce his wife to remarry? He will do so only if he can afford to pay for the divorce costs of him and his wife:

$$
\begin{align*}
D(S) & =V(S)-2 \beta  \tag{6}\\
& =m(S)-3 \beta-R>0
\end{align*}
$$

If $D(S)<0$, he will choose not to divorce his wife. $D_{S}>0$. If husband of skill $S$ is willing to divorce his wife and remarry, then a single older man with the same skill will also marry.

Put another way, if a single older man of skill $S$ chooses not to marry, a husband of the same skill will not divorce his current wife to remarry.

We will study an equilibrium in which high skill husbands want to divorce their wives but low skill men do not.

The return to marriage for a young man is then:

$$
\begin{align*}
U & =m(L)-R+p D(H)+(1-p) 0  \tag{7}\\
& =m(L)-R+p D(H)
\end{align*}
$$

$m(L)-R$ is the first period return from marriage.
$p D(H)+(1-p) 0$ is the expected second period payoff from being married in the first period.

Consider an equilibrium in which some young men remain single. The return to being single is:

$$
\begin{equation*}
Y=0+p V(H)+(1-p) \max (V(L), 0) \tag{8}
\end{equation*}
$$

0 is his first period return from being single in the marriage market.
$p V(H)+(1-p) \max (V(L), 0)$ is his expected second period return.

We will now argue that $V(L) \leq 0$ in equilibrium.

Assuming that $V(L) \leq 0$ (to be verified later), reduces to:

$$
\begin{align*}
Y & =p V(H)  \tag{9}\\
& =p(m(H)-\beta-R)
\end{align*}
$$

For young men to be indifferent between marriage or not, the return to marriage for a young man must be zero. That is, $U=Y$, which implies

$$
\begin{align*}
R & =m(L)+p[D(H)-V(H)]  \tag{10}\\
& =m(L)-2 p \beta
\end{align*}
$$

For high skill husbands to be willing to divorce their wives and remarry, (6) and (10) implies:

$$
\begin{align*}
D(H) & =m(H)-3 \beta-R>0  \tag{11}\\
-(m(L)-2 p \beta) & >0  \tag{12}\\
m(H)-m(L) & >\beta(3-2 p)
\end{align*}
$$

$$
m(H)-3 \beta-(m(L)-2 p \beta)>0
$$

(12) is a necessary condition for high skill husbands to be willing to divorce their wives and remarry. This condition also means that children is a normal good.

For single low skill older men to choose not to marry, $V(L) \leq 0$ and (10) imply:

$$
\begin{align*}
V(L) & =m(L)-\beta-R \leq 0  \tag{13}\\
m(L)-\beta-(m(L)-2 p \beta)) & \leq 0 \\
-\beta(1-2 p) & \leq 0
\end{align*}
$$

Since $p<\frac{1}{2}$, (13) is automatically satisfied.
(10) and (9) imply:

$$
\begin{align*}
Y & =p(m(H)-\beta-m(L)+2 p \beta)  \tag{14}\\
& =p(m(H)-m(L)-\beta(1-2 p))
\end{align*}
$$

Consider the case where $\beta=0$. In this case, we have:

$$
\begin{aligned}
& R=m(L) \\
& Y=p(m(H)-m(L))
\end{aligned}
$$

Women obtain all the rents from marriage from the man who is indifferent between marriage and being single.

The marginal man is the low skilled man and he gives away all the gains from marriage, $m(L)$.

The high skilled man only has to bid $R=m(L)$ to be sure that he will attract a spouse.

If $p(m(H)-m(L))>m(L)$, men are better off than women in the marriage market even though women are relatively scarce.

The expected lifetime gains to entering the marriage market for a man and a woman together is:

$$
R+Y=m(L)+p(m(H)-m(L))
$$

Consider an alternative society in which divorce is forbidden. There are two potential marriage patterns. First, men may decide to marry only when they are old. The total expected gains to marriage per couple is:

$$
p m(H)+(1-p) m(L)<R+Y
$$

Second, all individuals marry when they are young and each couple will gain $m(L)<R+Y$.

In either case, social welfare is higher in the society with divorce and remarriage.

Divorce is welfare improving.

The welfare gain obtained here is due to (1) an increase in the number of high output marriages (high skilled old men and young women), and (2) the transfers between men and women correctly price the relative scarcity of fecund women in the marriage market.

