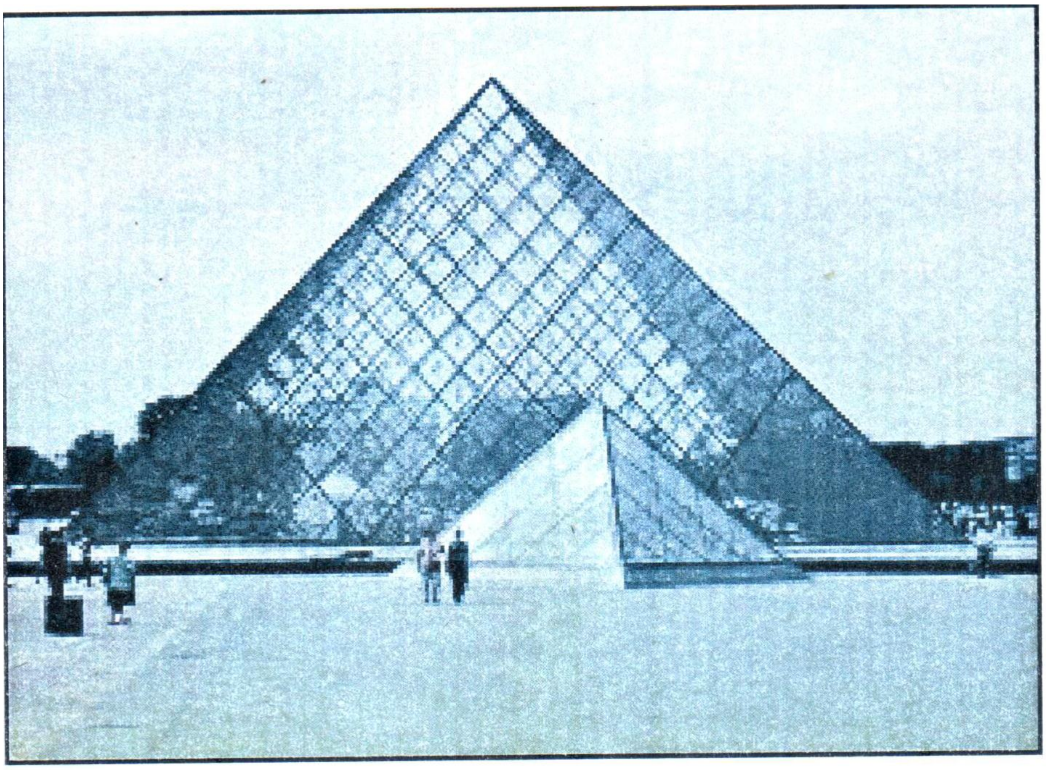
DEPARTMENT OF EDUCATION

GRADE 8

MATHEMATICS

STRAND 3



MEASUREMENTS (1)

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PAPUA NEW GUINEA

GRADE 8

MATHEMATICS

STRAND 3

MEASUREMENTS (1)

SUB-STRAND 1:	**LENGTH**
SUB-STRAND 2:	**AREA**
SUB-STRAND 3:	**SURFACE AREA**
SUB-STRAND 4:	**VOLUME AND CAPACITY**

Acknowledgements

We acknowledge the contributions of all Secondary and Upper Primary Teachers who in one way or another helped to develop this Course.

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MR. DEMAS TONGOGO
Principal- FODE

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SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education's Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans' harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course..

....

DR. UKE KOMBRA, PhD
Acting Secretary for Education

STRAND 3: MEASUREMENTS (1)

Dear Student,

This is the Third Strand of the Grade 8 Mathematics Course. It is based on the NDOE Upper Primary Mathematics Syllabus and Curriculum Framework for Grade 8.

This Strand consists of four Sub-strands:

Sub-strand 1: Length
Sub-strand 2: Area
Sub-strand 3: Surface Area
Sub-strand 4: Volume and Capacity

Sub-strand 1- **Length** - You will estimate and measure lengths and distances. You will also learn the rules and formula in finding the perimeters of simple and composite shapes. And lastly, you will solve problems involving perimeters by applying the rules.

Sub-strand 2- **Area** – You will investigate the area of circles.

Sub-strand 3- **Surface Area** – You will Investigate surface areas of rectangular prisms, triangular prisms, cylinders and pyramids and apply surface area rules.

Sub-strand 4- **Volume and Capacity** - You will investigate volumes of simple solids to determine rules and apply capacity and volume measurements in problem solving.

You will find that each lesson has reading materials to study, worked examples to help you and a practice exercise for you to complete. The answers to practice exercises are given at the end of each sub-strand.

All lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy studying this Strand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.
Step 2: When you complete Lesson 1, do Practice Exercise 1.
Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of the SUB-STRAND 1.
Step 4: Then, revise Lesson 1 and correct your mistakes, if any.
Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) like this:

$\boxed{\surd}$ Lesson 1: Units of Length

Go on to the next Lesson. Repeat the same process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Content Page 3, like this $\boxed{\surd}$ .This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand Test 1 in Assignment 3.

Then go on to the next Sub-strand. Repeat the same process until you complete all of the four Sub-strands in Strand 3.

<u>Assignment:</u> (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test for that Sub-strand in your Assignment. The Course book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand Test. The Assignment tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Workbook will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all of the Assignments.

SUB-STRAND 1

LENGTH

Lesson 1:	**Units of Length**
Lesson 2:	**Estimating Lengths and Distances**
Lesson 3:	**Measuring Lengths and Distances**
Lesson 4:	**Perimeter of Simple Shapes**
Lesson 5:	**Perimeter of Composite Shapes**
Lesson 6:	**Solving Problems Involving Perimeter**

SUB-STRAND 1: LENGTH

Introduction

The most basic and convenient basis for early measurement of length were parts of the human body. For instance, the Egyptian **cubit** was recognized as the most widespread and first recorded unit of linear measure around 3000BC. It was derived from the length of an arm from the tip of an elbow to the top of an extended middle finger (about 524 mm).

The earliest units for measuring lengths also included: palm, hand span, digit, human span, foot, yard, pace, and mile.

Many centuries ago, there were no standard units of length. This means the measures varied from person to person because people differed in body build.

Around 1500 AD, some of the most common English units for lengths were:

1 foot	=	12 inches
1 yard	=	3 feet
1 pace	=	5 feet
1 furlong	=	125 paces
1 mile	=	8 furlongs
1 leage	=	12 furlongs

Because it was sometimes difficult to remember the tables of equivalent measures, and because of the nature of our number system, the Metric System of measures was devised and adopted in 1791.

In this system the following prefixes are used to convey the meanings and relationships among measures.

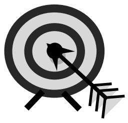
kilo means 1000 times	deci means $\frac{1}{10}$ or 0.1
hekto means 100 times	centi means $\frac{1}{100}$ or 0.01
Deka means 10 times	milli means $\frac{1}{1000}$ or 0.001

More of this will be explained and discussed as you go along the lessons in this sub-strand.

In this sub-strand, you will estimate and measure lengths and distances, find perimeters of simple and composite polygons and solve problems involving perimeters.

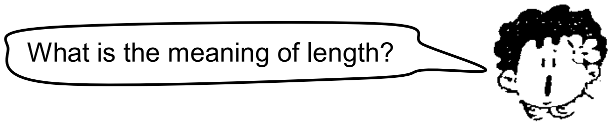
Lesson 1: Units of Length

You learnt about the different units of length in your Grade 7 Mathematics Strand 3.



In this lesson, you will:

- define length and distance.
- identify linear units of measurements and instruments used to measure length
- convert units of length from larger to smaller unit and vice versa.



Length is the measurement of how long an object or something is from one end to the other end.

Distance is the length between two points or positions.

For example:

1. The distance between two points is the length of a line joining them.
2. The length of a ship or a car is the measurement along its greatest dimension.
3. A piece, often of a standard size, that is normally measured along its greatest dimension: **a length of cloth.**
4. A measure used as a unit to estimate distances**: won the race by a length.**

Man has devised instruments to help him measure lengths. To measure lengths and distances he uses **a meterstick, a yardstick, a tape measure, a ruler** made of steel or other materials.

You learnt about some of these measuring instruments and their uses in Grade 7.

Here again are some of them.

To measure shorter lengths, you use a ruler.

To measure body measurements like the arm and the waist, you use the dress maker's tape measure.

To measure longer lengths like the height and width of a classroom you used the carpenter's tape measure or the meterstick.

To measure very short distances or lengths, man has devised **calipers** which can measure up to one hundredths of a millimetre. By inventing very sensitive instruments, he can now measure lengths which are so small that they are not seen by the naked eye.

Below are pictures of some of the above mentioned measuring instruments.

Ruler

Dress maker's Tape measure

Carpenter's tape measure

Caliper

Standard units were necessary for accurate and precise measurements. The development of standard units was crucial for progress.

The **metre** is the basic unit of length or distance. The table below shows the relationship of the different metric units of length to the metre.

Unit	Symbol	Value
kilometre	km	1000 m
hectometre	hm	100 m
decametre	dam	10 m
metre	m	1m
decimetre	dm	0.1 m
centimetre	cm	0.01 m
millimetre	mm	0.001 m

Note that each unit is ten times as long as the next smaller unit, or the one below it.

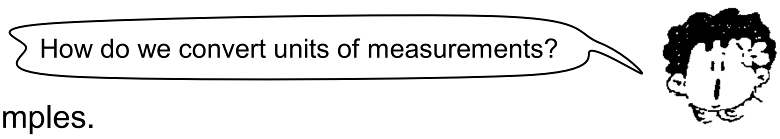
If the table of equivalents is arranged from the smallest to the largest, the relations of consecutive units are all in tens.

Look at the table below:

Table of Unit Equivalence		
10 millimetres (mm)	=	1 centimetre (cm)
10 centimetres (cm)	=	1 decimetre (dm)
10 decimetres (dm)	=	1 metre (m)
10 metres (m)	=	1 decametre (dam)
10 decametres (dam)	=	1 hectometre (hm)
10 hectometres (hm)	=	1 kilometre (km)

Conversion of Units of Measurements

Measures cannot be added or subtracted unless they are expressed in the same units. For example you cannot add millimetres, centimetres and metres directly. They should all first be changed to the same unit before they can be added or subtracted.



Here are some examples.

Example 1

Find the sum of 13 kilometres and 60 metres

Solution: The sum can be expressed in two ways, in metres or kilometres.

a) To find the sum in metres, express 13 km in metres

Using the table of equivalence, 1 km = 1000 m

So, 13 km = 13 x 1000

= 13 000 m

Therefore, 13 000 m + 60 m = **13 060 m**

b) To find the sum in kilometres, express 60 m into km

Using the table of equivalence, 1 m = $\frac{1}{1000}$ km or 0.001 km

So, 60 m = 60 x 0.001

= 0.06 km

Therefore, 13 km + 0.06 km = **13.06 km**

The two answers in both ways are equivalent. That is,

13 060 m = 13.06 km

As the example shows, we change measures to either a larger or smaller unit.

- To change from a larger unit to a smaller unit, multiply by the number of smaller units in each large unit.

- To change from a smaller unit to a larger unit, divide by the conversion factor.

A simple rule to follow is:

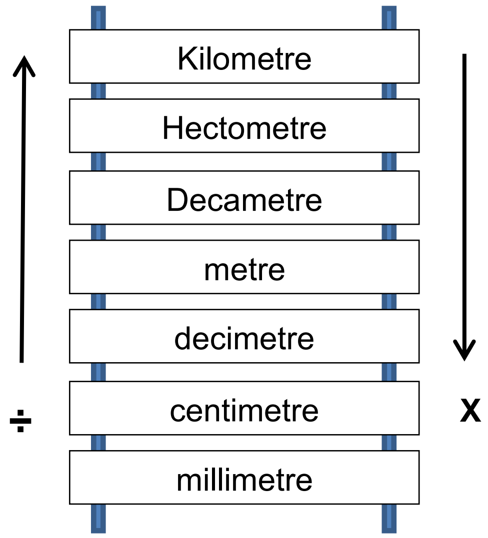
LARGE to small	➡ **MULTIPLY**
Small to LARGE	➡ **DIVIDE**

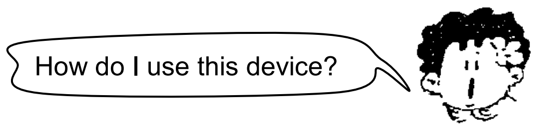
Hence, if you can mentally arrange the units from the largest to the smallest, in a ladder-like arrangement, **going down** the ladder will require **multiplication**; **going up** the ladder will require **division**.

Conversion of units in the Metric system is easy because the conversion factor is always a 10 or a power of 10.

Below is a device to help you convert from one unit of length to another.

CONVERSION LADDER





To use the device, determine what direction the conversion or change will take, whether downward or upward. Count how many steps it will take to get from the initial unit, to the unit of measure called for. Each step means a conversion factor of 10.

For example:

1. 25 km = __________m

   Solution:

   From km to m is 3 steps down. Hence multiply by 10 x 10 x 10 or 1000.

   Therefore, 25 km = 25 x 1000 m = **25 000 m**

2. 375 cm = __________m

   Solution:

   From cm to m is 2 steps up. Hence, divide by 10 x 10 or 100.

   Therefore, 375 cm = (375 ÷ 100) m = **3.75 m**

3. 290 mm = __________cm

   Solution:

   From mm to cm is 1 step up. Hence, divide by 10.

   Therefore, 290 mm = (290 ÷ 10) cm = **29 cm**

4. How many metres are there in 1.8 km?

   Solution:

   From km to m is 3 steps down. Hence multiply by 10 x 10 x 10 or 1000.

   Therefore, 1.8 km = 1.8 x 1000 m = **18 000 m**

5. The distance between Hongkong and San Francisco is 11 097 km. How far is this in metres?

   Solution:

   From km to m is 3 steps down. Hence multiply by 1000.

   Therefore, 11 097 km = 11 097 x 1000 m = **11 097 000 m**

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

1. Perform the indicated operation:

   a) 32 x 100 = ___________

   b) 3.7 x 1000 = ___________

   c) 15 700 ÷ 1000 = ___________

   d) 36.8 x 100 = ___________

   e) 3435 ÷ 1000 = ___________

2. Convert each of the following:

   a) 19 km = ___________mm

   b) 1.5 m = ____________cm

   c) 5 m = _____________km

   d) 520 mm = _____________ km

3. Solve each of the following.

   a) Which is longer: 290 mm or 2.9 m?

   b) Which of these measures is the smallest? 48 cm, 149 mm, 2.5 dm

   c) How many metres are there in 1.8 kilometres?

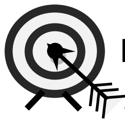
   d) A mango tree has a height of 5 metres. What is the height in kilometres?

   e) The length of the blackboard is 1.6 metres. What is the length in centimetres?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 2: Estimating Lengths and Distances

In the previous lesson, you defined length and identified the different units for measuring it. You also learnt to change one unit of length to another from smaller unit to larger unit.



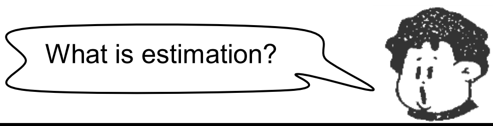
In this lesson, you will:

- define estimation
- estimates lengths and distances .

In real life, in many situations, you do not need to calculate the exact answer to questions but you can do an approximate calculation in your head by observation and visualization. Estimation is a very useful way to check calculations and ensure that you have not made a careless mistake.

Estimation is a skill for life. As you walk around and live wouldn't it be good if you could estimate, how much a bill was which product was the best value for money and make other estimates such as lengths and angles?

Also, wouldn't it be good if you could **quickly guess** how many people were in a room, how many cars are in the parking lot, how many books are on the shelf, or even how many sunflowers are in the field? In estimation, you are not talking about exact answers, but answers that are good or close enough to the right answer.



- **Estimation means finding a value or number that is close enough to the right answer, usually with some thought or calculation involved.**
- **An estimate is an answer that should be close to an exact answer.**

You make estimates every day.

Examples

1. You estimate how long it will take you to walk to school.
2. You estimate how much money you will need to buy some things at the store.

Sometimes you must estimate because it is impossible to know the exact answer. When you predict the future, for example, you have to estimate since it is impossible to know exactly what will happen. A weather forecaster's prediction is an estimate of what will happen in the future.

Sometimes you estimate because finding the exact answer is not practical. For example, you could estimate the number of books in your library instead of actually counting them.

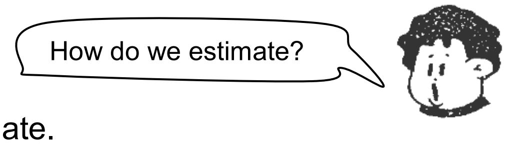
Sometimes you estimate because finding an exact answer is not worth the trouble. For example, you might estimate the cost of several items at the store to be sure you have enough money. There is no need to find the exact answer until you pay for the items.

It is a very useful skill to be able to estimate how many things you can see, or how long something is or how big something is and so on.

Examples:

1. Ken's estimation of 400 bricks to build the wall was very good, as there were only 12 bricks left over.
2. The driver of a car is estimating distances all the time.
3. A house painter when giving a quote estimates the amount of paint he will need and the time it will take him to do the job.

Whenever you measure an object it is a good idea to estimate what your measurement will be beforehand.



Here are some ways to estimate.

1. Leading – digit estimation

A digit is a single number. e.g. 4

One way to estimate is to adjust each number in a problem before you estimate. You may adjust each number as follows:

1. Keep the first non-zero digit of the number.
2. Replace the other digits of the number with zeros.

Then make your estimate using these adjusted numbers.

This way of estimating is called **leading – digit estimation.**

Example 1

What is the cost of 5 kilograms of tiger prawns at K74 per kilogram?

Solution:

Using leading-digit estimation, the tiger prawns cost about K70 per kilogram.

So, 5 kilograms will cost about 5 x K70 or K350.

2. Rounding Off

Rounding off is another way to adjust numbers and make them simpler and easier to work with. Estimation with rounded numbers is usually more accurate than leading-digit estimation.

You learnt in Grade 7 Strand 3 that numbers are often rounded to the nearest multiple of 1000, 100, 10 and so on.

The next few examples show the steps to follow in rounding a number.

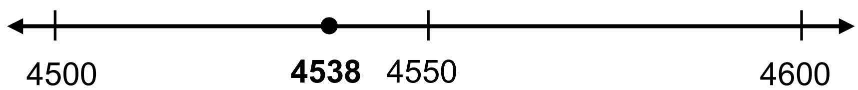
Example 2

Round off 4538 to the nearest hundred.

Solution:

Step 1:	Find the digit in the place you are rounding to.	4$\underline{5}$38
Step 2:	Rewrite the number, replacing all digits to the right of this digit with zeros. This is the lower number.	4$\underline{5}$00
Step 3:	Add 1 to the digit in the place you are rounding to. If the sum is 10, write 0 to the left of the digit. This is the higher number.	4$\underline{6}$00
Step 4:	Is the number you are rounding closer to the lower number or to the higher number?	
Step 5:	Round to the closer of the two numbers. If it is halfway between the lower and the higher number, round up to the higher number.	4500

This can be shown also using the number line.



4538 is closer to 4500 than it is to 4600.

Therefore, 4538 rounded to the nearest hundred is 4500.

Now you will learn to estimate lengths and distances.

As you have learnt, **length** is the measurement of how long an object is from end to end.

The measure of a line is its **length**. A line has no width. The **distance** between two points is the **length of the interval** joining them. When we measure **length**, we want to know how long an interval is. An **interval** is part of a line. If the object is small, we can use a ruler to measure its length.

Sometimes you may not have a ruler, meterstick, or a tape measure handy. When this happens, you can estimate lengths by using the lengths of common objects and distances that you know well.

Some examples of personal references for length are given below. A good personal reference is something that you see or use often, and so, you don't forget.

For example:

A wooden pencil is not a good personal reference for length because it gets shorter as it is sharpened.

Here are the examples:

About 1 millimetre

- Thickness of a 5 toea coin
- Thickness of the point of a thumbtack
- Thickness of a thin edge of a paper match

About 1 centimetre

- Thickness of a crayon
- Width of the head of a thumbtack
- Thickness of a pattern block

About 1 metre

- One big step (for an adult)
- Width of a front door
- Tip of the nose to tip of the thumb with arm extended (for an adult)

About 1 kilometre

- 1 000 big steps (for an adult)
- Length of ten football fields

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1. The thickness of a 20 toea coin is about 2 millimetres. Estimate the length of each interval to the nearest millimetre. Check by measurement using your ruler to see how close your estimates are.

   a)

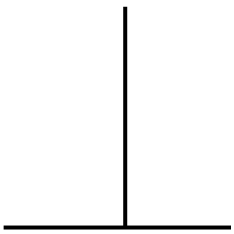
   b)

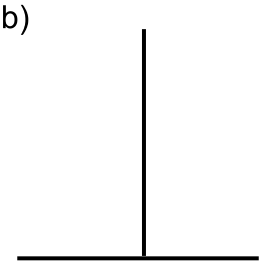
   c)

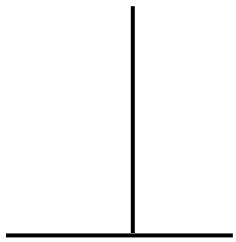
   d)

   e)

2. For which diagram do you think the horizontal and vertical lengths are the same?

   a) 

   b) 

   c) 

3. Estimate the lengths of each line to the nearest centimetres. Check your answer by direct measurement.

   a)

   b)

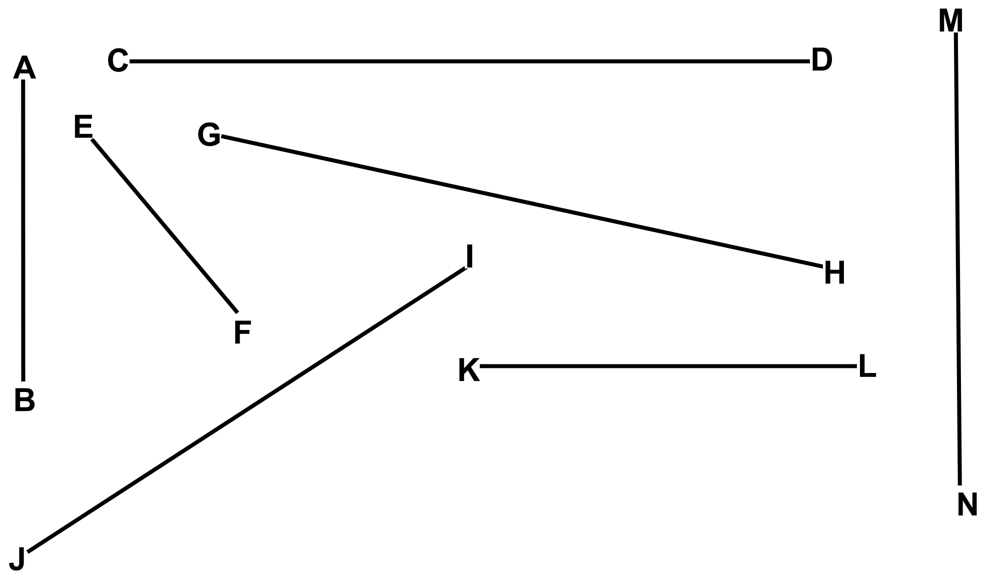
   c)

   d)

   e)

   f)

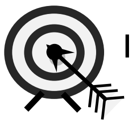
4. Estimate the lengths of lines to the nearest millimeter. Check your answers by direct measurements using your ruler.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 3: Measuring Lengths and Distances

You learnt the meaning of estimation and to estimate length and distances in Lesson 2.



In this lesson, you will:

- measure a given length or distance by actual measurement
- compare estimates with actual measurements.

First, let us differentiate **estimating** lengths and distances from **measuring** lengths and distances.

- **Estimating lengths and distances is using your judgment to guess what the amount or measurement is without using the measuring instruments.**
- **Measuring length and distances is the actual measurement using the instruments to read the actual lengths.**

If you are measuring length then you need to:

- choose the right instruments or equipment
- know how to read measurements correctly.

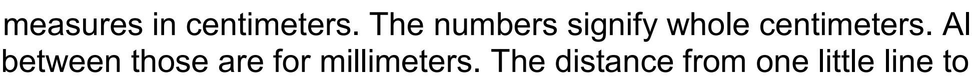
There are many different types of instruments that you can use to measure lengths and distances. The instrument or equipment you choose will vary depending on what you want to measure.

Once you have selected the right instrument then you should always make sure that you start measuring from the right place.

In most cases, measuring instrument will show the starting point as a line marked with a 0. You should line this up with one end of the object you are measuring.

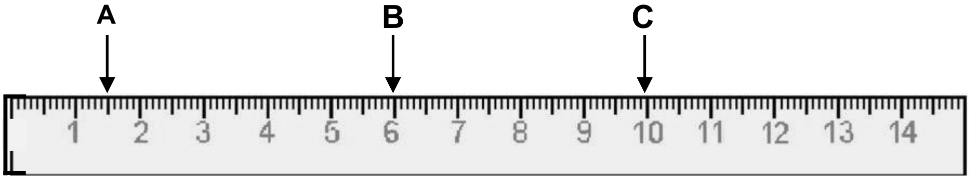
Measure the lines using the ruler.

Look at the ruler, there are 10 millimeters in each centimeter.



This ruler measures in centimeters. The numbers signify whole centimeters. All the little lines between those are for millimeters. The distance from one little line to the next line is 1 millimeter. We write 1 mm. Millimeters are pretty tiny.

When reading a ruler, you want to find out how far the item is from zero. When reading a ruler, you must locate the zero marking. This may vary depending on the ruler. On some rulers, the zeros start at the end, but on others it starts about $\frac{1}{5}$ of a centimetre from the end. If you do not check your ruler beforehand you will not be getting an accurate measurement.

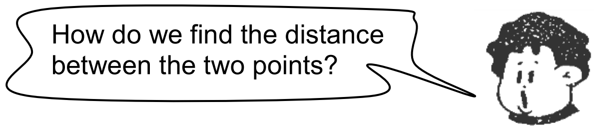


Point A is $1\frac{1}{2}$ cm or $1\frac{1}{2}$ cm away from zero.

Point B is 6 cm or 6 cm away from zero.

Point C is 10 cm or 10 cm away from zero.

Now suppose we have to find the distance from Point A to Point B. What answer do we get?



As you have learnt in the previous lesson, the distance between two points is the length of the interval joining them. Thus, we say, the interval from Point A to Point B is the length of line AB.

To find AB,(the distance from Point A to Point B), subtract $1\frac{1}{2}$ cm from 6 cm.

Hence we have, $\quad AB = 6 \text{ cm} - 1\frac{1}{2} \text{ cm}$

$$= 4\frac{1}{2} \text{ cm}$$

This means that the length of the interval AB is $4\frac{1}{2}$ cm.

The same can be done if you find the distance from Point B to Point C and from Point A to Point C.

Here are the solutions:

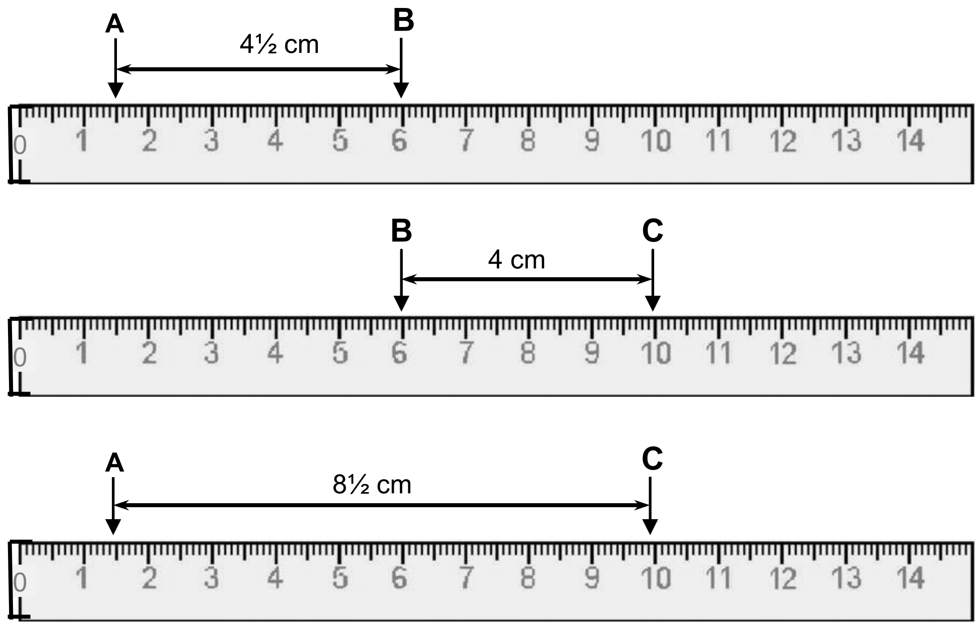
For the interval from Point B to Point C, we have BC = 10 cm – 6 cm

$$= 4 \text{ cm}$$

For the interval from Point A to Point C, we have AC = 10 cm - $1\frac{1}{2}$ cm

$$= 8\frac{1}{2} \text{ cm.}$$

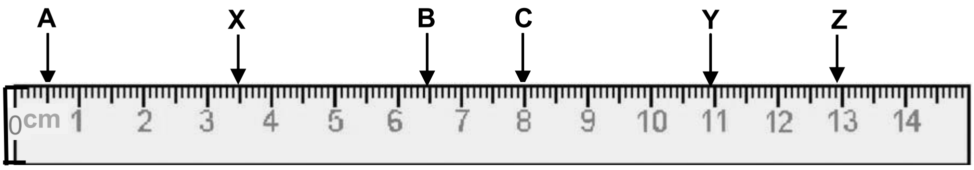
For a better understanding, see the diagrams below showing the three intervals.



NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. Refer to the diagram and write down the distance of each letter in centimeters as indicated by the arrow from zero.



Answer: A = _____ X = _____

B = _____ Y = _____

C = _____ Z = _____

2. Using the diagram in Question 1, find the length of each of the following intervals.

   a) AB = _______
   b) AC = _______
   c) AY = _______
   d) XY = _______
   e) BY = _______

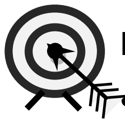
3. Measure each of the following lengths to the nearest centimetres.

   (a)
   (b)
   (c)
   (d)
   (e)

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 4: Perimeter of Simple Shapes

You learnt to estimate and measure lengths and distances in the previous lessons.



In this lesson, you will:

- define perimeter
- find the perimeter of simple polygons
- discover rules in finding the perimeter of simple polygons.

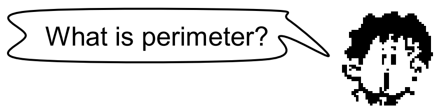
Sometimes we want to know the length or distance around a shape.

For example, in the picture the students are measuring the length and width of the table. If the length of the table is 125 units and the width is 90 units, how many units around is the table?

By adding the units, we get:

$$125 + 90 + 125 + 90 = 430 \text{ units}$$

This distance around the edges of the table is what we call the **perimeter** of the table.



Perimeter means the measure or distance around a polygon.

The perimeter can be determined by actual measurement or by computation when necessary measures are given. The general formula for finding the perimeter of any polygon is

Perimeter = measure of side x number of sides

When the polygon is regular (all the sides of the polygon are equal). If not, the formula to be applied is

$$\textbf{Perimeter} = s_1 + s_2 + s_3 + s_4 + \ldots + s_x$$

In general, the perimeter of a polygon is the sum of the lengths of all its sides.

Let us state the formula of the perimeters of simple polygons.

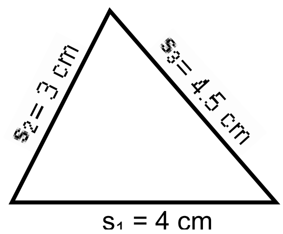
First, we have the triangles.

You have learnt that there are three kinds of triangles, namely the equilateral, isosceles and scalene triangles. For all these three triangles the formula for the perimeter is:

$$\boxed{\textbf{Perimeter} = s_1 + s_2 + s_3}$$

Example 1

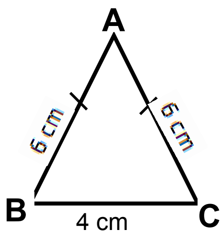
The perimeter of the triangle given in the figure is:



P = 4 cm + 3 cm + 4.5 cm
= 11.5 cm

Example 2

What kind of triangle is $\triangle$**ABC**?



Solution:

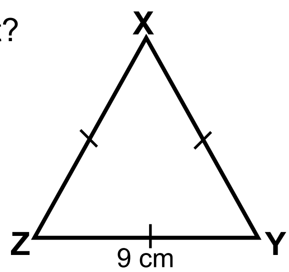
As the marks on the two legs suggest, it is an isosceles triangle.

You may find the perimeter as follows:

$P = s_1 + s_2 + s_3$
= 6 cm + 6 cm + 4 cm
= 16 cm

Example 3

What do the marks on the three sides of the triangle suggest?



Solution:

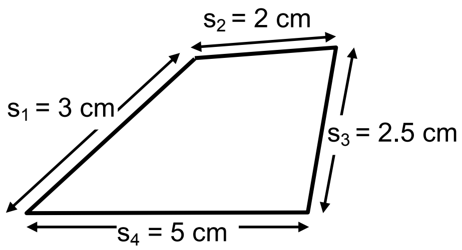
The triangle is an equilateral triangle. Hence, all the sides are equal in length.

You may find the perimeter as follows:

$P = s_1 + s_2 + s_3$ or $P = 3s$
= 9 cm + 9 cm + 9 cm = 3 x 9 cm
= 27 cm = 27 cm

What about Quadrilaterals? How do we find their perimeters?

For irregular quadrilaterals, the perimeter is found by the formula;

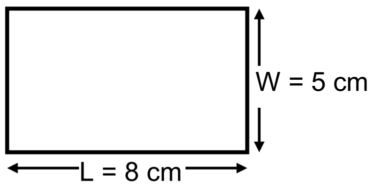


Perimeter = $s_1 + s_2 + s_3 + s_4$

P = 3 cm + 2 cm + 2.5 cm + 5 cm
= 12.5 cm

The perimeter of the irregular quadrilateral is 12.5 cm.

For a rectangle, the formula of the perimeter in symbols, if **L** means length and **W** means width, then



Perimeter = 2L + 2W

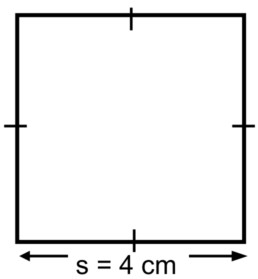
P = 2(8 cm) + 2(5 cm)
= 16 cm + 10 cm
= 26 cm

The perimeter of the rectangle is 26 cm.

What about a square and a rhombus? How do you describe their sides?

Since the four sides of a square and a rhombus are congruent, their perimeter is found by the formula:

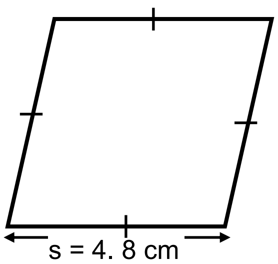
Perimeter = 4s



P = 4 (4 cm)
= 16 cm

The perimeter of the square is 16 cm.

Just like a square, a rhombus has four congruent sides.

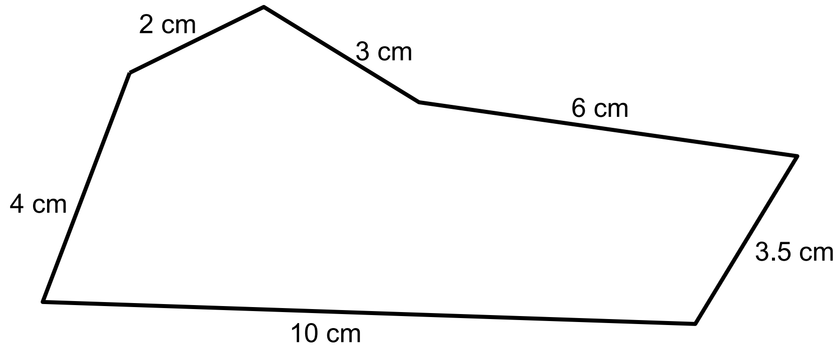


Perimeter = 4s

P = 4 (4.8 cm)
= 19.2 cm

The perimeter of the rhombus is 19.2 cm.

Now, how do you describe this polygon? How do you find its perimeter?



Since the sides of the polygon are of different measures and there are six sides, the perimeter is found by the formula

$$\boxed{\textbf{Perimeter} = s_1 + s_2 + s_3 + s_4 + s_5 + s_6}$$

Hence, you have P = 2 cm + 3 cm + 6 cm + 3.5 cm + 10 cm + 4 cm

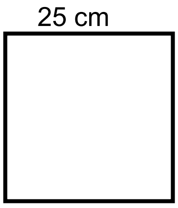
= 28. 5 cm

The perimeter of the polygon is 28.5 cm.

Now look at the following examples.

Example 1

A handkerchief is 25 cm on each side. How much lace is needed if an allowance of 2 cm is needed at the corners?



Solution: Since, each side is 25 cm long,

P = 4s

= 4 x 25 cm

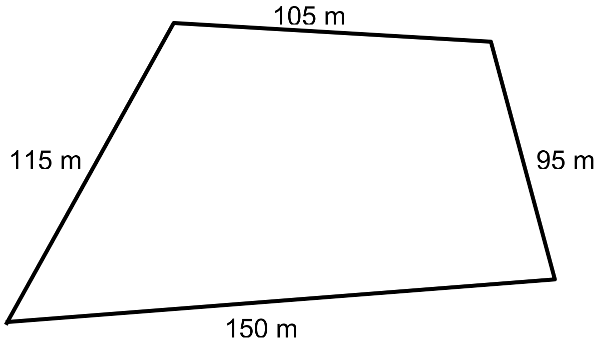
= 100 cm

Adding the allowance of 2 cm, we have, (100 + 2)cm = 102 cm.

Therefore, a length of lace 102 cm long is needed.

Example 2

The shape and lengths of the sides of Uncle Bob's field are given in the diagram below. How much barbed wire is needed to fence it?



Solution:

The perimeter is found by using the formula:

$$\text{Perimeter} = s_1 + s_2 + s_3 + s_4$$
$$= 115 \text{ m} + 105 \text{ m} + 95 \text{ m} + 150 \text{ m}$$
$$= 465 \text{ m}$$

Uncle Bob needs 465 m of barbed wire to fence his field.

Example 3

Mrs. Johnson decided to sew red ribbon along the edges of the table cloth she is making. If the table cloth measures 1.75 m by 2.85 m, at least how many metres of ribbon should she prepare?

Solution:

Find the perimeter of the table cloth. Use the formula P = 2L + 2W

- Length = 2.85 m $\Rightarrow$ estimated length is 3 m
- Width = 1.75 m $\Rightarrow$ estimated width is 2 m
- P = 2(2.85 m) + 2(1.75 m) $\Rightarrow$ estimated P = 2(3 m) + 2(2 cm)

  = 5.7 m + 3.5 m = 6 m + 4 m

  = 9.2 m = 10 m

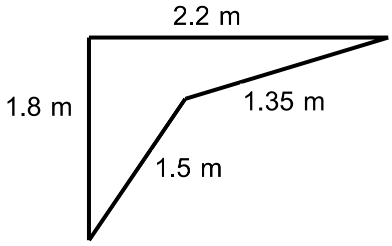
Mrs Johnson should prepare at least 10 m of ribbon.

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. Find the perimeters of these shapes.

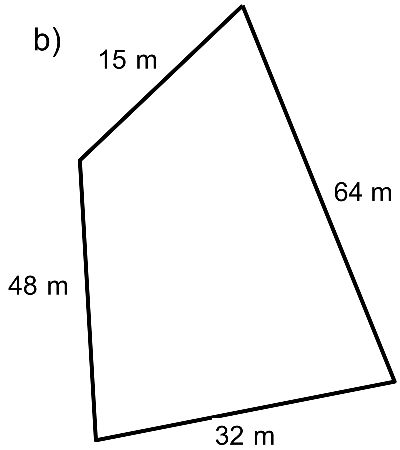
a)



Working out:

Answer: ___________

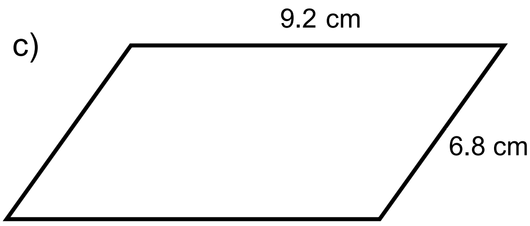
b)



Working out:

Answer: ___________

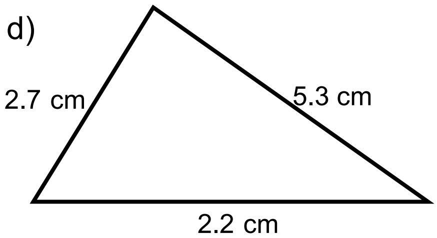
c)



Working out:

Answer: ___________

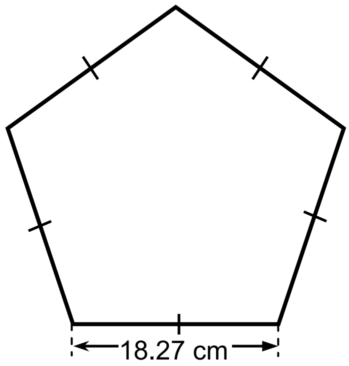
d)



Working out:

Answer: ___________

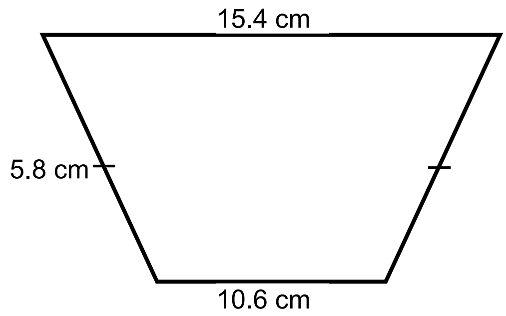
e)



Working out:

Answer: ___________

f)



Working out:

Answer: ___________

---

2. Sam estimated that he needs at least 1 metre of wooden sidings for the picture frame he plans to make.

   What would be the dimensions of the largest frame he could make if the frame is a

   a) square?

   Answer: ___________

   b. rectangle?

   Answer: ___________

   c. hexagon?

   Answer: ___________

3. A school yard is a quadrilateral in shape. Its sides are 147 m, 155 m, 138 m and 145 m.

   a) What is its perimeter?

   Answer: ___________

   b) If you can walk 65 m per minute about how many minutes will it take you to walk around the school once?

   Answer: ___________

---

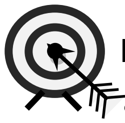
4. Mr. Jones would like to fence his orchard of young fruit trees with barbed wire to keep away stray cows. The orchard measures 405 m by 585 m. If a roll contains 55 m of barbed wire, how many rolls will he need?

   Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 5: Perimeters of Composite Shapes

You defined and derived the formula of the perimeter of simple shapes and polygons in the last lesson. You also learnt to calculate the perimeters of simple shapes and polygons using the formula.

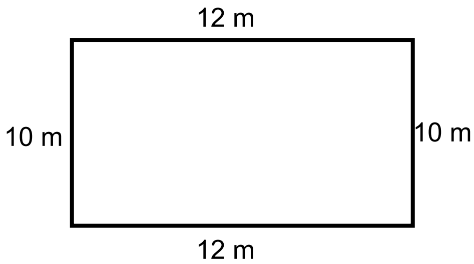


In this lesson, you will:

- define composite shapes
- determine the perimeter of composite shapes.

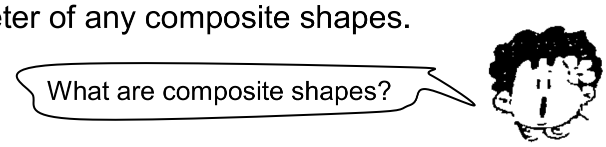
You know that the perimeter of a shape is the distance around its edge. And that in general, finding the perimeter just means adding the lengths of all its sides.

For example, think of a farmer walking starting from one corner of his orchard measuring 12 m by 10 m. Imagine that the farmer walks all the way around the sides of the orchard. What distance will he have walked?



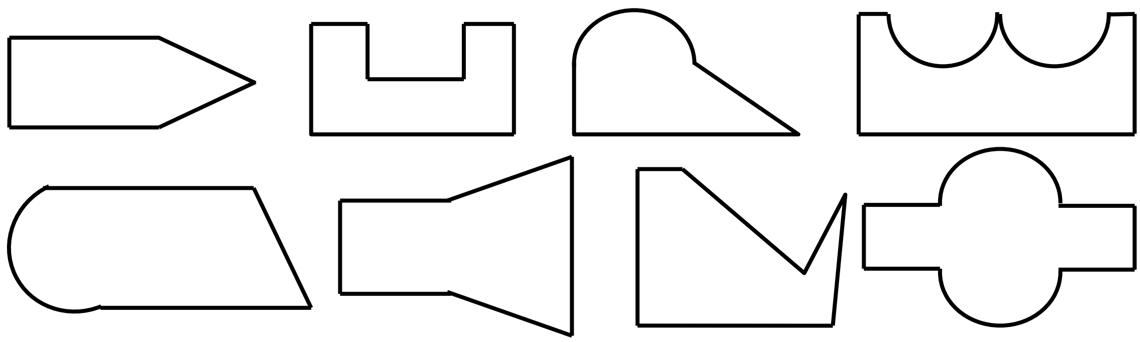
Perimeter = 12 m + 10 m + 12 m + 10 m
= 44 m

Now you know how to calculate the perimeter of a simple shape, you can now calculate the perimeter of any composite shapes.



Composite shapes are the results of combining shapes, either by adding parts or taking parts away. They are figures made up of a number of shapes. They are figures that can be divided into more than one of the basic shapes like rectangles, squares, triangles, trapezoids or circles.

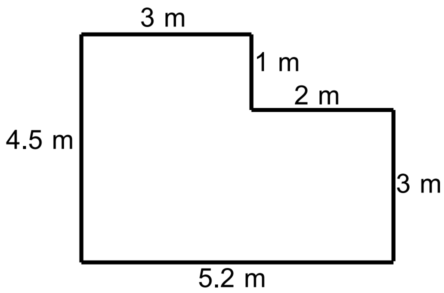
Here are some examples of composite shapes.



The perimeter of a composite shape is the total distance or length around the shape.

Example 1

The perimeter of this L-shaped figure below is:



$P = s_1 + s_2 + s_3 + s_4 + s_5 + s_6$

$= 4.5\text{ m} + 3\text{m} + 1\text{ m} + 2\text{ m} + 3\text{ m} + 5.2\text{ m}$

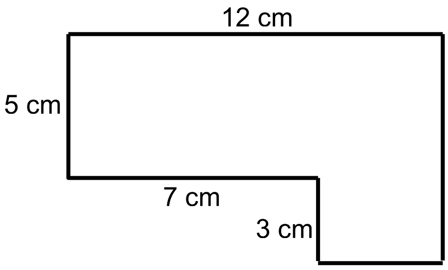
= 18.7 m

Sometimes the shape is so complicated.

In this case, you need to break the shape down into individual shapes like rectangles, squares, triangles and the like. Then calculate the missing lengths and be particularly careful to include all the sides.

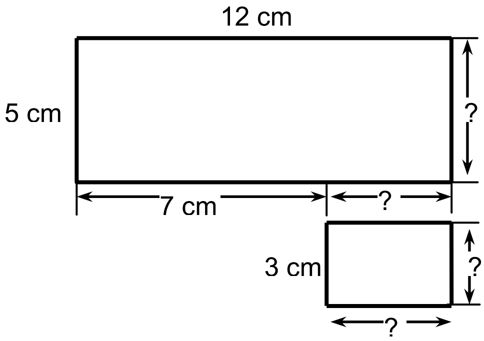
Example 2

Find the perimeter of the shape below.



Solution:

Step 1: Divide the shape into rectangles.



Step 2: You know that the opposite sides of a rectangle are equal.

So you can work out the lengths of the missing sides.

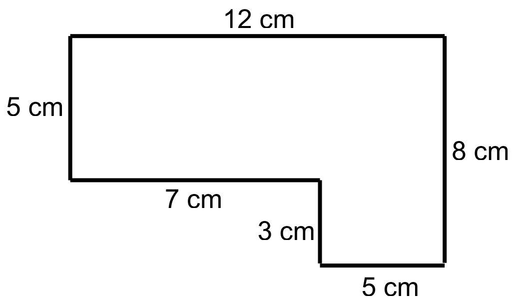
You know that the top side is 12 cm long, therefore the opposite side of this rectangle must be equal to this.

You know the length of the part of this side is 7 cm, therefore the missing length is

$$12\text{ cm} - 7\text{ cm} = 5\text{ cm}$$

This means that the side opposite this must also be 5 cm long.

Step 3: Now you can find the perimeter by adding the lengths of all the sides.



Perimeter = 5 cm + 12 cm + 8 cm + 5 cm + 3 cm + 7 cm

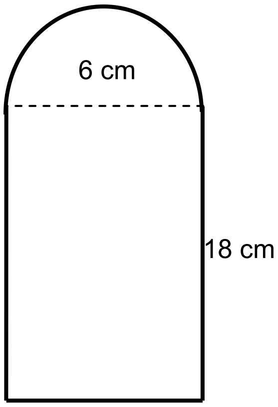
= 40 cm

Therefore, the perimeter of the L-shaped polygon is **40 cm.**

Composite shapes are not always made of polygons or rectangles. There are cases where the composite shape is made up of a combination of polygons and circles. The perimeter of such a combination is the sum of all the sides and curved lengths.

Example 3

Find the perimeter of the figure below.



Solution: The figure is a rectangle with a semi-circle at one end.

To find the perimeter, find the sum of the three sides plus $\frac{1}{2}$ circumference of the circle.

Computation:

Perimeter = 2L + W + $\frac{1}{2}\pi D$

P = (2 x 18 cm) + 6 cm + $\frac{1}{2}$(3.14 x 6 cm)

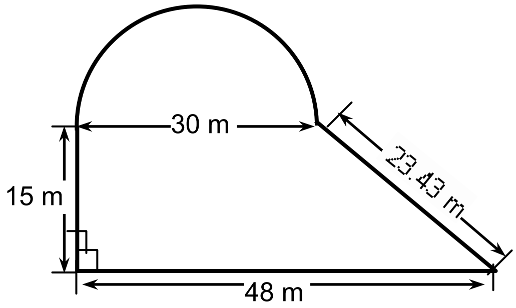
P = 42 cm + $\frac{1}{2}$(18.84 cm)

P = 42 cm + 9.42 cm

P = 51.42 cm

Example 4

Calculate the perimeter of the following shape. Write your answer correct to 2 decimal places.

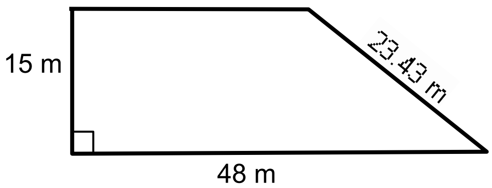


Solution:

The composite shape is made up of a semicircle and a trapezium. To find the perimeter of the shape we have:

Perimeter = Perimeter of Trapezium + Perimeter of semicircle

Step 1: Find the perimeter of the trapezium. Use the formula: $P = s_1 + s_2 + s_3$

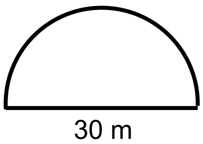


P = 15 m + 23.43 m + 48 m

= 86.43m

Step 2: Find the perimeter of the semicircle.

Perimeter of semicircle = $\frac{1}{2}$(Circumference of circle)



$= \frac{1}{2}(2\pi r)$

$= \frac{1}{2}\ [2(3.14)(15\text{ m})]$

$= (3.14)(15\text{ m})$

= 47.10 m

Step 3: Find the perimeter of the composite shape. Add the two perimeters found in Steps 1 and 2.

Perimeter of composite shape = 86.43 m + 47.10 m

= 133.53 m

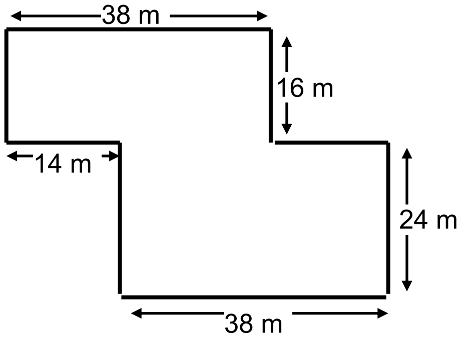
Therefore the perimeter of the composite shape is 133.53 m.

NOW DO PRACTICE EXERCISE 5

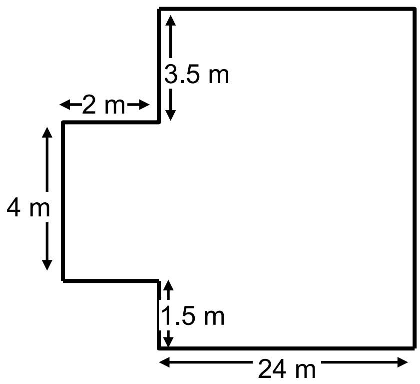
Practice Exercise 5

1. Find the perimeter of each shape below.

a)



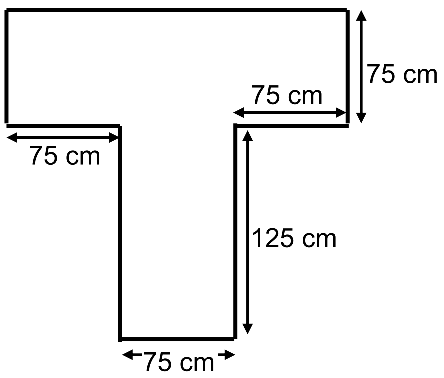
b)



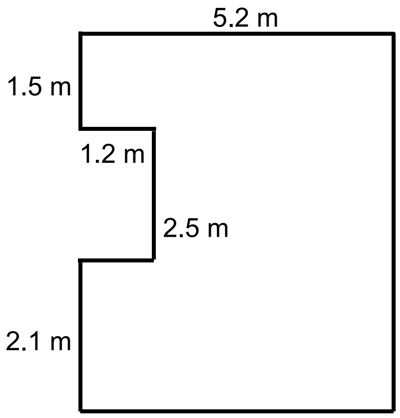
Answer: ___________

Answer: ___________

c)



d)

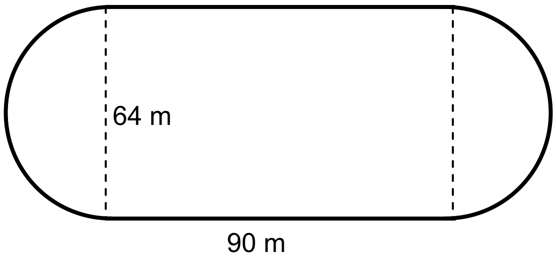


Answer: ___________

Answer: ___________

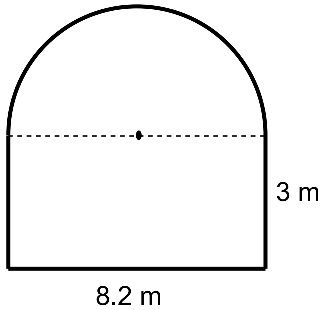
2. Find the perimeter of the following figures correct to 2 decimal places.

(a)



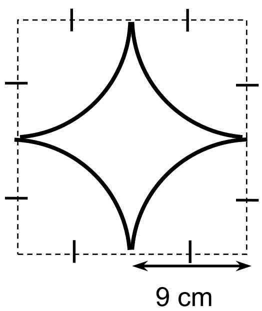
Answer: ___________

(b)



Answer: ___________

(c)

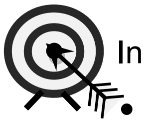


Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 6: Solving Problems Involving Perimeter

You learnt to find the perimeter of simple and composite shapes in the previous lessons.



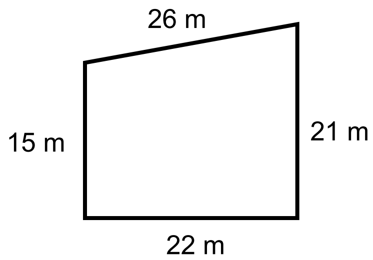
In this lesson, you will:

- solve problems in real life involving perimeter.

There are lots of problems that you face in real life that require your skills and knowledge in finding perimeters.

Example 1

Steve would like to install an electric fence on his plot of land to keep stray animals away. How much fencing is required if the dimensions of Steve's plot of land are as shown below?



Solution: All sides of the land are known. The perimeter can be found by the formula:

$$P = s_1 + s_2 + s_3 + s_4$$

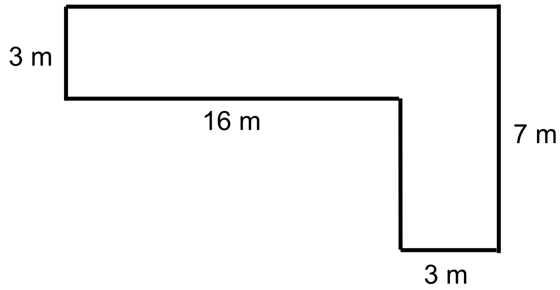
$$P = 15 \text{ m} + 26 \text{ m} + 21 \text{ m} + 22 \text{ m}$$

$$P = 84 \text{ m}$$

Hence, the required fencing would be 84 m.

Example 2

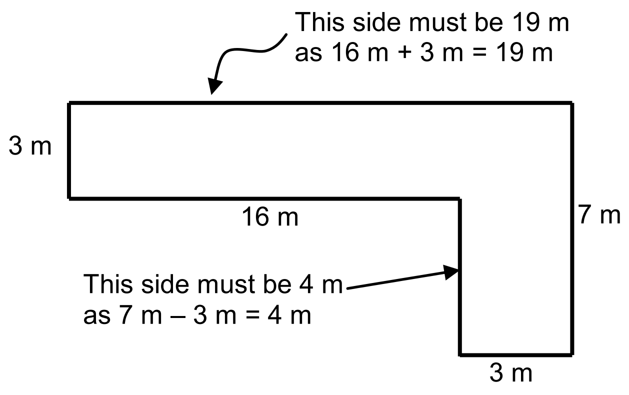
How much edging is needed for an L-shaped flower bed whose dimensions are shown below?



Solution:

To work out how much edging is needed for the L-shaped flower bed, you must get its perimeter. The measurements of two sides of the flower bed are missing. You need to find them.

First calculate the lengths of the missing sides.



To get the perimeter, add up all the lengths of all the sides.

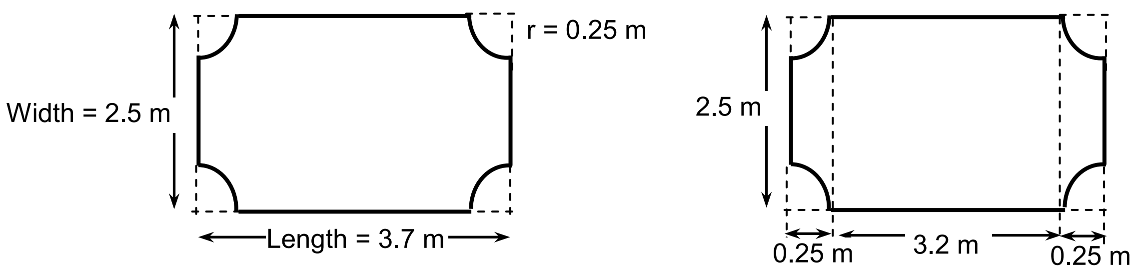
Perimeter = 3m + 16 m + 4 m + 3 m + 7 m + 19 m

= 52 m

Therefore, 52 m of edging is needed for the flower bed.

Example 3

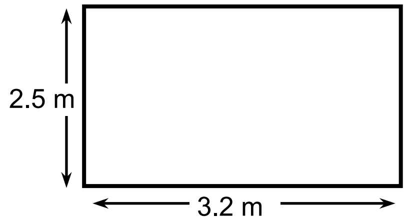
Find the perimeter of the table top which is in the form of a rectangle of length 3.7 m and width of 2.5 m. It has four decorative cutouts that are each $\frac{1}{4}$ of a circle with a radius 0.25 m as shown below. Write your answer to the nearest tenths.



Solution:

To find the perimeter of the composite shape or table top:

Step 1: Find the perimeter of the rectangle without the decorative ends.



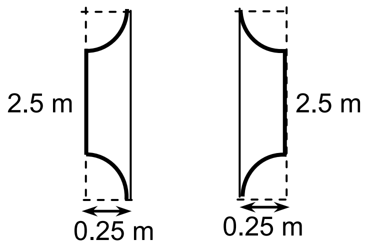
Perimeter = 2L + 2W

= 2(3.2 cm) + 2(2.5 cm)

= 6.4 cm + 5.0 cm

= 11.4 cm

Step 2: Find the perimeter of two decorative ends.



Each decorative end is a rectangle with two $\frac{1}{4}$ circle cutouts.

Hence, the Perimeter of the two decorative ends will be obtained by the formula:

$$P = 2(2L + 2W) - 2(\tfrac{1}{4}\,2\pi r)$$

Substituting the values in the figures on the formula, we have

$$P = 2\left[(2)(2.5) + (2)(0.25)\right] - 2\left[\frac{1}{4}(2)(3.14)(0.25)\right]$$

$$= 2(5 + 0.5) - 2(0.3925)$$

$$= 2(5.5) - 0.785$$

$$= 11 - 0.785$$

$$\mathbf{= 10.215\ m}$$

Step 3 Add the two perimeters found in steps 1 and 2 to find the perimeter of the table top.

Perimeter of the table top = 11.4 m + 10. 215 m

= 21.615 m

Therefore, the perimeter of the table top is 21.6 m rounded to the nearest tenth.

Example 4

A school yard is quadrilateral in shape. Its sides are 145 m, 151 m, 138 m and 142 m.

a) What is its perimeter?

b) If you walk 64 m per minute, about how many minutes will it take you to walk around the school once?

Solution:

a) Perimeter = 145 m + 151 m + 138 m + 142 m

= 576 m

b) To find how long it will take you to walk around the school, divide the perimeter by 64 m.

Length of time to walk around = $\frac{576\,\text{m}}{64\,\text{m}}$ = 9 min

Therefore, it will take you 9 minutes to walk around the school once.

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

Solve the following:

1. A corn field is pentagonal in shape. The lengths of the sides are 115 m, 153 m, 205 m, 187 m and 165m.

   a) If the owner wants to fence it with three strands of wire, how many meters of wire in all will be needed?

   Answer: ___________

   b) What fraction of a kilometer is the perimeter of the field?

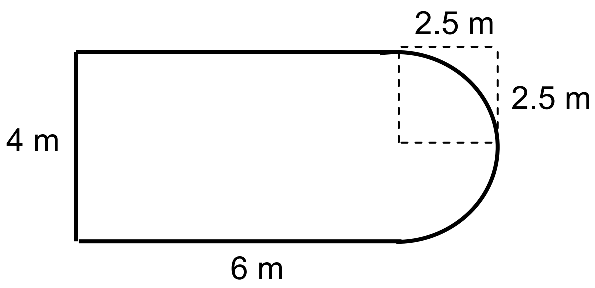
   Answer: ___________

   c) How many times should a long distance runner go around the field to make sure he has covered 4 km?

   Answer: ___________

---

2. How much edging is needed for a countertop with the following dimensions as shown below?



Answer: ___________

3. Mr. Kauna's orchard is shaped–like the figure below. The dimensions are as shown.

a) Find the perimeter of the orchard.

Answer: __________

b) Mr. Kauna would like to fence his orchard with three layers of wire. If a roll contains 50 m of barbed wire, estimate how many rolls he will need?

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

SUB-STRAND 1: SUMMARY

- **Length** is the measurement of how long an object or something is from one end to the other end.
- **Distance** is the length between two points or positions.
- The most commonly used units of length are the **millimetre**, **centimetre, metre and kilometre**. These are all standard units of measurements.
- **Estimation** means finding a value or number that is close enough to the right answer, usually with some thought or calculation involved. An **estimate** is an answer that should be close to an exact answer.
- **Estimating lengths and distances** is using your judgment to guess what the amount or measurement is without using the measuring instruments.
- **Measuring lengths and distances** is the actual measuring using the instruments to read the actual lengths.
- **Perimeter** means the measure or distance around a polygon.
- **Composite shapes** are the results of combining shapes, either by adding parts or taking parts away. They are figures made up of a number of shapes. They are figures that can be divided into more than one of the basic shapes like rectangles, squares, triangles, trapezoids (trapeziums) or circles.
- The **perimeter of a composite shape** is the total distance or length around the shape.
- The general formula for finding the perimeter of a polygon is:

$$\boxed{\textbf{Perimeter} = s_1 + s_2 + s_3 + \ldots + s_n}$$

For Triangle

$$\boxed{P = s_1 + s_2 + s_3}$$

For Square

$$\boxed{P = 4s}$$

For Quadrilateral

$$\boxed{P = s_1 + s_2 + s_3 + s_4}$$

REVISE LESSONS 1- 6 THEN DO SUB-STRAND TEST 1 IN ASSIGNMENT 3.

ANSWERS TO PRACTICE EXERCISES 1-6

Practice Exercise 1

1. (a) 3200 (b) 3700 (c) 15.7 (d) 3680 (e) 3.435

2. (a) 19 000 000 mm
   (b) 150 cm
   (c) 0.005 km
   (d) 0.00052 km

3. (a) 2.9 m
   (b) 149 mm
   (c) 1800 m
   (d) 0.005 km
   (e) 160 cm

Practice Exercise 2

1. (a) 40 mm (b) 20 mm (c) 70 mm (d) 64 mm (e) 120 mm

2. c

3. (a) 5 cm (b) 7 cm (c) 4 cm (d) 2.5 cm
   (e) 6.5 cm (f) 3 m

4. AB = 40 mm CD = 90 mm EF = 30 mm
   GH = 80 mm KL = 50 mm IJ = 70 mm
   MN = 60 mm

Practice Exercise 3

1. A = 0.5 cm; B = 6.5 cm; C = 8 cm
   X = 3.5 cm; Y = 11 cm; Z = 13 cm

2. AB = 6 cm; AC = 7.5 cm; AY = 10.5 cm; XY = 7.5 cm; BY = 4.5 cm

3. (a) 11 cm (b) 9 cm (c) 4 cm
   (d) 6 cm (e) 3 cm

Practice Exercise 4

1. (a) 6.85 m (b) 159 m (c) 32 cm (d) 10.2 cm
   (e) 91.35 cm (f) 37.6 cm

2. (a) s = 25 cm
   (b) Answers may vary eg: L= 35 cm; W = 15 cm
   L = 40 cm; W = 10 cm
   (c) $16\frac{2}{3}$ cm

3. (a) 585 m
   (b) 9 times

4. 36 rolls

Practice Exercise 5

1. (a) 170 m (b) 70 m (c) 850 cm (d) 25 m

2. (a) 380.96 m (b) 27.07 m (c) 56.52 cm

Practice Exercise 6

1. (a) 825 m of wire (b) $\frac{33}{40}$ (c) 5 times

2. 23.85 m of edging

3. (a) 1670.20 m (b) about 100 rolls

END OF SUB-STRAND 1

SUB-STRAND 2

AREA

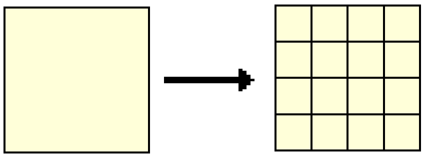
Lesson 7:	**Area of a Circle**
Lesson 8:	**Area of a Circle by Using Sectors**
Lesson 9:	**Area of a Circle by Counting Squares**
Lesson 10:	**Area of a Circle Using the Formula**
Lesson 11:	**Solving Problems Involving Area of Circle**

SUB-STRAND 2: AREA

Introduction

Area is defined as the number of square units that cover the surface of a closed figure.

In the example shown, the area of the square is 16 square units. That means, 16 square units are needed to cover the surface enclosed by the square.



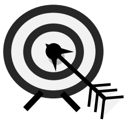
Formulas are defined to calculate the area of regular geometric figures like square, rectangle, circle, etc.

In this Sub-strand, you will learn to:

- compare the area of a circle to the area of inscribed squares and the square of the diameter
- determine the area of circles by counting squares
- relate the measured area of circles to the square of the radius
- understand and use the formula in finding the area of a circle.
- solve problems involving area of circles in real life situations.

Lesson 7: Area of a Circle

You learnt something about the circle and how it differs from other plane shapes in Strand 2. You also learnt the different properties of circles and their meanings.



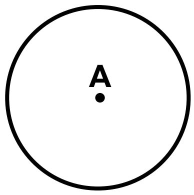
In this lesson, you will:

- revise concepts in circles and the parts of circles
- define the area of a circle
- compare the area and circumference of a circle.

First, let's revisit the definition of a circle.

A **circle** is a plane curve consisting of all points that are equidistant (have the same distance) from a special point in the plane. The special point is the **centre**.

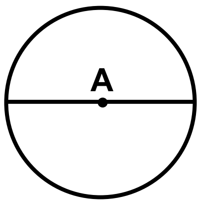
In the circle on the right, the center is point **A**. A circle is named according to the capital letter associated with the center. Thus, we have **circle A**.



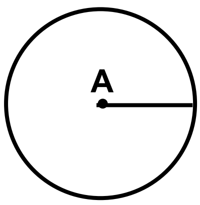
Parts of a circle

It is important to know what the different parts of a circle are called.

1. The distance across a circle through the centre is called the **diameter**.



2. The **radius** of a circle is the distance from the center of a circle to any point on the circle.



If you place two <u>radii</u> end-to-end in a circle, you would have the same length as one diameter. Thus, the diameter of a circle is twice as long as the radius.

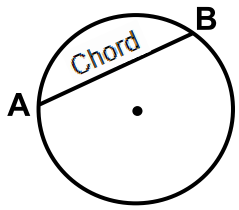
Thus we have:

2 x radius = 1 diameter or $2r = D$

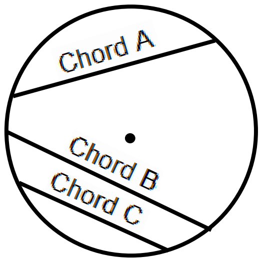
1 radius = $\frac{1}{2}$ diameter or $1r = \frac{D}{2}$

3. A **chord** is a line segment that joins two points on a curve. In geometry, a chord is often used to describe a line segment joining two endpoints that lie on a circle.

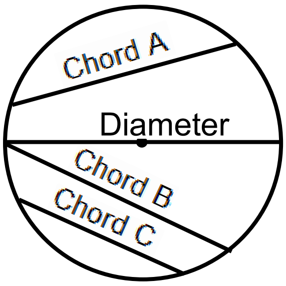
The circle on the right contains **chord AB.** If this circle was a pizza pie, you could cut off a piece of pizza along chord AB. By cutting along chord AB, you are cutting off a segment of pizza that includes this chord.



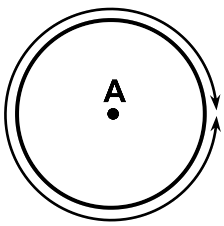
A circle has many different chords. Some chords pass through the centre and some do not. A chord that passes through the centre is called a diameter.



It turns out that the diameter of a circle is the longest chord of that circle since it passes through the centre. A diameter satisfies the definition of a chord however, a chord is not necessarily a diameter. This is because every diameter passes through the centre of a circle, but some chords do not pass through the centre. Thus, it can be stated, **every diameter is a chord, but not every chord is a diameter**.

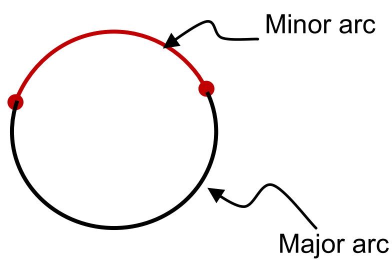


4. The **circumference** of a circle is the boundary line or the perimeter of the circle.

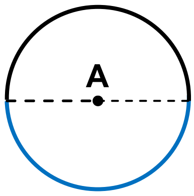


5. An **arc** is a part or portion of a circle.

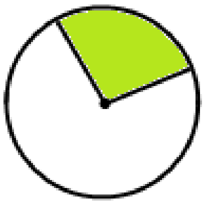
- A minor arc is less than a semicircle
- A major arc is greater than a semicircle.



6. **A semicircle** is an arc equals to half of a circle. The diameter separates a circle into two semicircles as shown below.



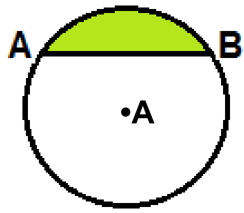
7. The **Sector of a circle** is the part of a circle enclosed by two radii of a circle and their intercepted arc.



As you can see from the figure above, a sector is a pie-shaped part of a circle. It has two straight sides (the two radius lines), the curved edge defined by the **arc**, and touches the centre of the circle.

8. **A segment of a circle** is the region between a chord of a circle and its associated arc.

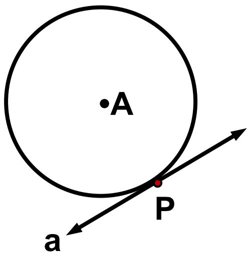
The shaded region is the segment of circle A. The chord AB in the figure defines one side of the segment.



9. The **Tangent** is a line that touches a circle at only one point.

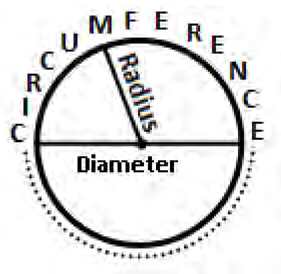
The point where the line touches the circle is called the **point of tangency.**

In the figure, point **P** is the point of tangency.



After revising all these properties of a circle, let us now compare the circumference of a circle to the area of a circle.

As we know, a circle is a curve consisting of all points that have the same distance from a fixed point called the centre. The common distance of the points on the curve from the centre is called the **radius**. The curve is referred to as the perimeter or the **circumference** of the circle. The distance across a circle through the centre is called the **diameter**.



The number π (Pi)

Pi (π) is the ratio of the circumference of a circle to its diameter.

Thus, for any circle, if you divide the circumference by the diameter, you get a value close to π. This relationship is expressed in the following formula:

$$\boxed{\frac{C}{d} = \pi}$$

where **C** is the circumference and **d** is the diameter. You can test this formula at home with a round dinner plate. If you measure the circumference and the diameter of the plate and then divide **C** by **d**, your quotient should come close to π. Another way to write this formula is:

$$\boxed{C = \pi \cdot d \text{ or } C = \pi d}$$

This second formula is commonly used in problems where the diameter is given and the circumference is not known.

We know earlier that the diameter of a circle is twice as long as the radius. With this relationship, we can also express the circumference in the following formula:

$$\boxed{C = 2\pi r}$$

The value of π is approximately 3.14159265358979323846... (as a decimal) or $\frac{22}{7}$ (as a fraction). We round π to 3.14 in order to simplify our calculation.

A circle is actually a closed curved line (a line that connects back to itself making a loop.
Imagine the circle to be a loop made from a piece of string. The string has no area, but the space inside the loop does. So, strictly speaking a circle has no area.

However, when we say the "area of a circle" we really mean the area of the space inside the circle. If you were to cut a circular shape from a piece of paper, the shape would have an area, and that is what we mean here. Thus we say, the region bounded by the circle is the **area of a circle.**

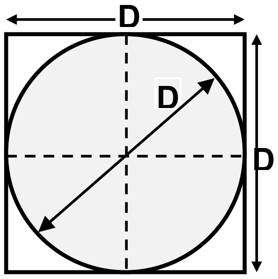
Compared to the circumference, the area of a circle is π times the radius squared, which is written:

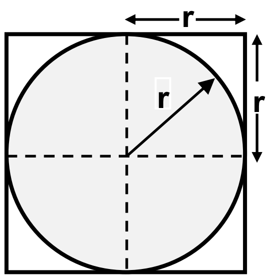
$$\boxed{A = \pi r^2}$$

or, in terms of the Diameter:

$$\boxed{A = \frac{\pi}{4} D^2}$$

It is easy to remember if you think of the area of the square that the circle would fit inside as shown in the diagram on the next page.





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Circle = $\frac{\pi}{4}$ x D^2
Square = D^2

Circle = π x r^2
Square = 4 x r^2

This comparison of the areas will be further discussed in the next lesson.

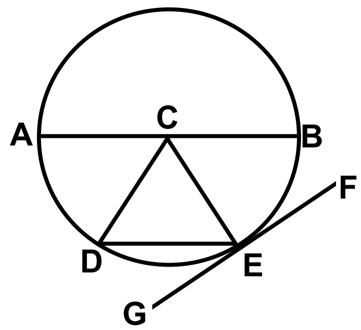
Now to recap what you learnt in this lesson, we can summarize the circle properties as follows:

A circle is a shape formed by points the same distance from its centre. A circle is named by its centre. The parts of the circle include the radius, diameter, arc and chord. All diameters are chords, but not all chords are diameters. The circumference of a circle is the distance around the edge of the circle. It is exactly Pi (π) times the diameter. The area of the circle is the region or space inside the circle. It is Pi (π) times the radius squared.

NOW DO PRACTICE EXECRISE 7

Practice Exercise 7

I. Study the figure below and encircle the correct answer from the options given.



1. BE is a/an
   a) diameter b) circumference c) arc d) radius

2. AB is a/an
   a) diameter b) chord c) arc d) radius

3. CE is a/an
   a) diameter b) circumference c) arc d) radius

4. C is the
   a) diameter b) centre c) chord d) radius

5. DE is a/an
   a) chord b) diameter c) arc d) radius

6. FG is a/an
   a) chord b) tangent c) diameter d) radius

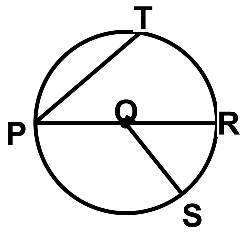
7. E is a/an
   a) chord b) point of tangency c) radius d) arc

8. The length of CE is the same as
   a) DE b) AB c) CB d) BE

9. The length of AB is
   a) 2 x DE b) 2 x AD c) 2 x BE d) 2 x CD

10. The length of AB is
    a) $\frac{CE}{2}$ b) 2 x CE c) CE^2 d) CE

II. Refer to the diagram to answer the questions below. Circle the letter of your answer from the options A, B, C or D.



1. Which of the following is a chord, but not a diameter?

   A. $\overline{PR}$
   B. $\overline{QS}$
   C. $\overline{PT}$
   D. $\overline{QR}$

2. Which of the following is a radius?

   A. $\overline{PQ}$
   B. $\overline{QS}$
   C. $\overline{QR}$
   D. All of the above.

3. Name the centre of this circle.

   A. Point P
   B. Point Q
   C. Point R
   D. Point S

4. What is $\overline{PR}$ or $\overline{PQR}$?

   A. the arc
   B. the centre
   C. the radius
   D. the diameter

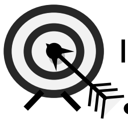
5. If $\overline{\mathbf{PQ}}$ is 4 cm long, then how long is $\overline{\mathbf{PR}}$?

   A. 4 cm
   B. 6 cm
   C. 8 cm
   D. 10 cm

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 8: Area of a Circle by Using Sectors

You revised the properties of the circle in the last lesson. You also defined the area of the circle and distinguished it from the circumference.



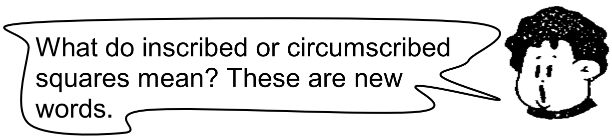
In this lesson, you will:

- define sectors and inscribed squares
- investigate the area of a circle using sectors
- relate the area of a circle to the area of an inscribed square and the square of the diameter.

We have learnt that when we find the area of a figure, we divide it up into square units, such as square centimetres. This is easy for figures or shapes which have straight sides like the rectangle.

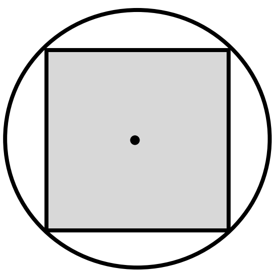
But how can we find the area of a circle?

As you know, the area of a circle is the region inside the circle. By working through the following investigation, you will discover a formula you can use to find the area of a circle. In this lesson, you will investigate the area of a circle using inscribed or circumscribed squares and using sectors.

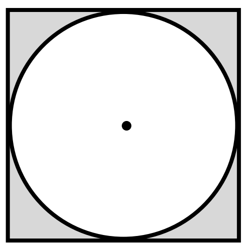


- An **Inscribed Square** is a square drawn inside the circle so that all its sides are chords of the circle. In this case, the circle is said to be **circumscribed** about the square.
- A **Circumscribed square** is a square drawn outside the circle so that all its sides are tangents to the circle. In this case, the circle is **inscribed** inside the square.

See the figures below:

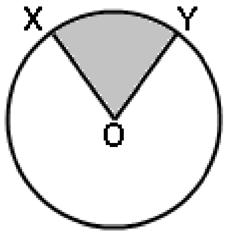


Inscribed Square



Circumscribed Square

The Sector of a circle is the part of a circle enclosed by two radii of the circle and their intercepted arc. It is a pie-shaped piece of a circle.



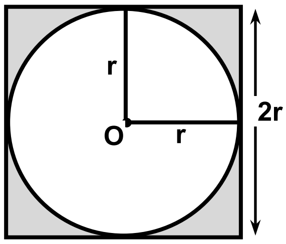
On the figure, the shaded region is the sector of a circle.

OX and OY are the radii

$\overset{\frown}{XY}$ is the intercepted arc.

Now let us try to investigate and approximate the area of a circle with a square.

Look at Figure 1.

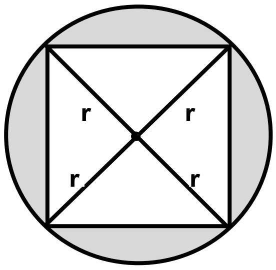


The circle has a radius of r units and has been drawn inside a square.

Can you see that the side of the square must be twice r (2r) units long?

Which is larger, the area of the square or the area of the circle?

Now look at Figure 2.



Inside the circle with radius r units, a square is drawn. The area of the square will be the same as two triangles that each has a base of 2r units and a height of r units.

What is the area of the square?

Which is larger, the area of the square or the area of the circle?

By studying the two figures, you should be able to see that the area of a circle lies between $\mathbf{4r^2}$ and $\mathbf{2r^2}$.

For example: $2r^2 <$ area of a circle $< 4r^2$

Hence you can have a reasonable approximation for the area of a circle which might be:

$$\mathbf{A \doteqdot 3r}$$

Let us now look at the circle in **Figure 1** whose area is composed of slices of the circle or cut the circle into sectors and then arrange them as shown in **Figure 2**.

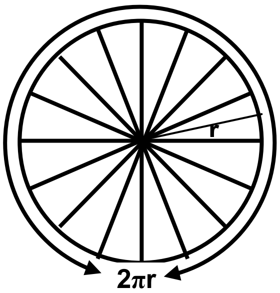


Figure 1

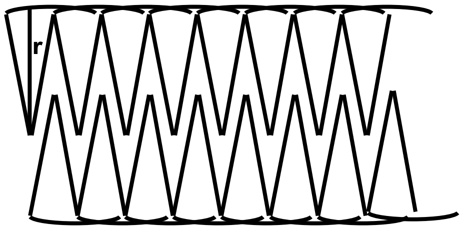


Figure 2

By taking half of the sectors on one end and placing it on the other end , we obtain a figure that looks very much like a rectangle, as shown in **Figure 4** below.

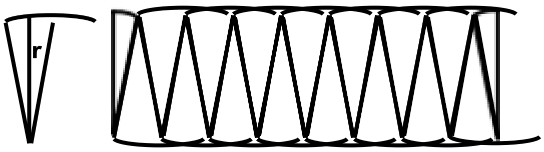


Figure 3

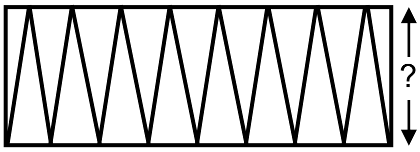


Figure 4

Now the length of this rectangle is half the length of the circumference of the circle. What is this length? What would be the breadth or width of the rectangle?

The circumference of the circle is $2\pi r$, so the length of the rectangle would be $\frac{1}{2}(2\pi r)$.

The radius of the sector of the circle is r, so the breadth or width of the rectangle would be r.

Since, the area of the rectangle is length x width, what would be the area of this rectangle, (which of course, would be the same as the area of the circle)?

Area of circle = Area of rectangle

$$A = \frac{1}{2}(2\pi r) \times r$$

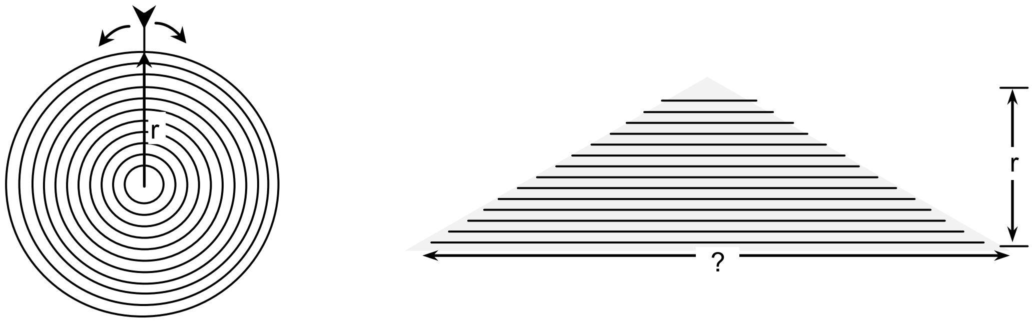
$$A = \pi r^2$$

NOW DO THE PRACTICE EXERCISE 8

Practice Exercise 8

Refer to the diagram below and answer the Questions that follow.

Imagine the area inside a circle to be a series of rubber rings. If the rings were cut along a radius and allowed to fold out flat, the layers would form a triangle as shown below.



Questions:

1. What is the formula for the area of a triangle?

2. The height of the triangle would be r, the radius of the circle. What would be the length of the base of the triangle?

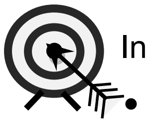
3. What then would be the area of the triangle?

4. What would be the area of the circle?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 9: Area of a Circle by Counting Squares

You learnt to approximate the area of a circle using sectors and relate it to the area of an inscribed square in the previous lesson and



In this lesson, you will:

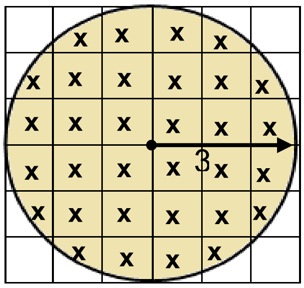
- find the area of a circle by counting squares and comparing it to the area found using the formula of the circle.

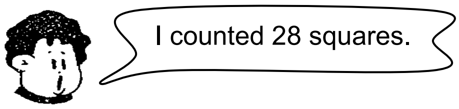
In lesson 7, you learnt that the **area of the circle** is the number of square units inside the circle.

To gain an approximate value of a circle, you could "count the squares", including those which are more than half of a square which lies inside the circle.

For example:

The circle at the right has a radius of 3 units. If you count the shaded squares marked **x**, what is the approximate value for the area of the circle?

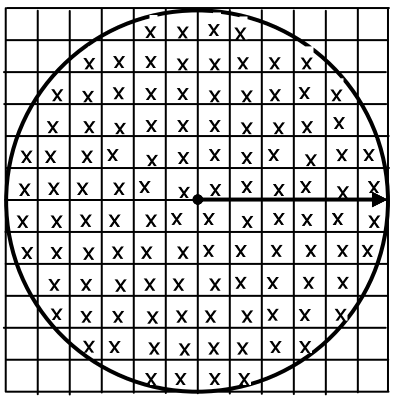




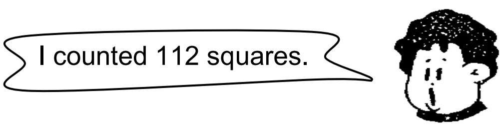
How does this compare with the result you get using the formula $A = \pi r^2$?

You will notice that it is approximately equal to the result which comes from using the formula $A = \pi r^2$ which is 28.26 square units.

This method can be made more accurate by using smaller square units.



If we carefully count the squares marked in the circle at the left, what is the approximate area of this circle in square units?



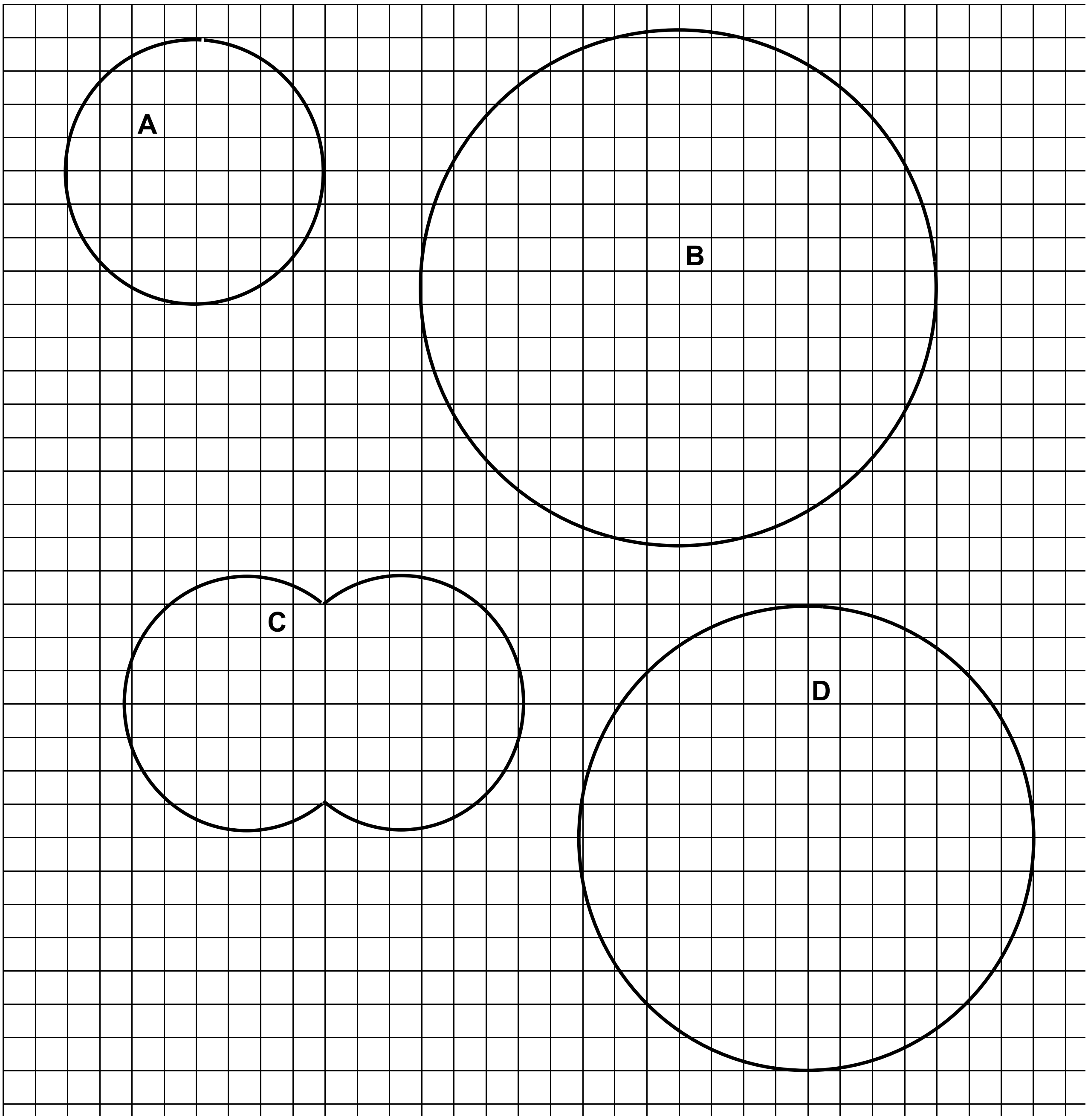
How does this compare with the result from the formula $A = \pi r^2$?

You will notice that it is almost approximately equal to the result from the formula $A = \pi r^2$ which is 113.04 square units.

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Carefully count the squares in the circle below and write what is the approximate area of each circle in square units.

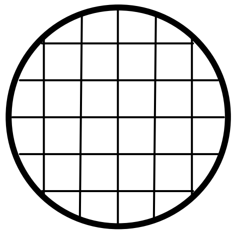


CORRECT YOUR WORK, ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 10: Area of a Circle by Using Formula

You learnt to approximate the area of a circle by counting squares.

In this lesson, you will:
- calculate the area of a circle by using the formula.



The **area of the circle** is the number of square units inside the circle. If each square in the circle to the left has an area of 1 cm^2, you could count the total number of squares to get the area of this circle. Thus, if there were a total of 28.26 squares, the area of this circle would be 28.26 cm^2. However, to calculate the area of a circle, it is easier to use the following formulas:

1. In terms of radius:

$$\boxed{A = \pi r^2}$$

Where: A = area of circle
r = the radius of the circle
$\pi = 3.14$ or $\frac{22}{7}$

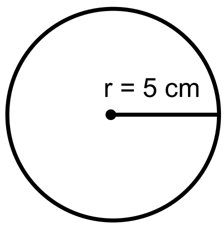
Or, in terms of diameter:

$$\boxed{A = \frac{\pi}{4} D^2}$$

Where: A =area of the circle
D = diameter of the circle
$\pi = 3.14$ or $\frac{22}{7}$

Let us look at some examples on the area of a circle. In each of these three examples below we will use $\pi = 3.14$.

Example 1 The radius of a circle is 5 cm. What is the area?



Solution: Given is the radius, so you use the formula $A = \pi r^2$

Substitute r = 5 cm and $\pi = 3.14$

$A = 3.14 \times (5 \text{ cm})^2$

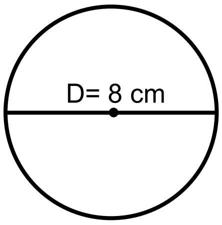
$= 3.14 \times 5 \text{ cm} \times 5 \text{ cm}$

$= 3.14 \times 25 \text{ cm}^2$

$= 78.5 \text{ cm}^2$

Therefore the area of the circle of radius 5 cm is 78.5 cm^2.

Example 2 The diameter of a circle is 8 cm. What is the area?



Solution: Given is the diameter. So, you use the formula:

$$A = \frac{\pi}{4}D^2$$

Substituting the given values for D and π, you have:

$$A = \frac{\pi}{4}D^2$$

$$A = \frac{3.14}{4}(8\ \text{cm})^2$$

$$A = \frac{3.14}{4} \times 8\ \text{cm} \times 8\ \text{cm}$$

$$A = \frac{3.14}{4}(64)\ \text{cm}^2$$

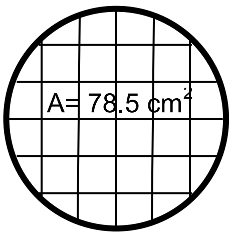
$$A = 3.14(16)\ \text{cm}^2$$

$$A = 50.24\ \text{cm}^2$$

Therefore, the area of a circle with a diameter of 8 cm is 50.24 cm^2.

Example 3

The area of a circle is 78.5 cm^2. What is the radius?



Solution: Given is the area. Using the formula $A = \pi r^2$, we substitute the value of A to find the radius.

So, $A = \pi r^2$

$78.5\ \text{cm}^2 = 3.14(r)^2$

$\frac{78.5\text{cm}^2}{3.14} = r^2$ ⟶ divide both sides by 3.14

$25\ \text{cm}^2 = r^2$ ⟶ find the square root of both sides.

$5\ \text{cm} = r$ ⟶ square root of 25 is 5and of r^2 is r.

or $r = 5\ \text{cm}$

Therefore, the radius of a circle whose area is 78.5 cm^2 is 5 cm.

Note: Given the radius or diameter of a circle, you can find its area. You can also find the radius (and diameter) of a circle given its area.

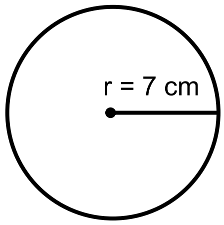
See some more examples on the next page.

Here are some more examples.

In each of these two examples below we will use $\pi = \mathbf{\frac{22}{7}}$.

Example 4

The radius of a circle is 7 cm. What is the radius?



Solution: Use the formula: $A = \pi r^2$

Substitute R = 7 cm and $\pi = \frac{22}{7}$

$A = \frac{22}{7}(7\text{ cm})^2$

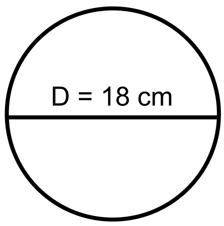
$A = \frac{22}{7}(49)\text{ cm}^2$

$A = 154\text{ cm}^2$

Therefore, the area of the circle of radius 7 cm is 154 cm^2.

Example 5

A circle has a diameter of 18 cm. What is the area of this circle?



Solution: Use the formula: $A = \frac{\pi}{4}D^2$

Substitute D =18 cm and $\pi = \frac{22}{7}$

$A = \frac{\frac{22}{7}}{4}(18\text{ cm})^2$

$A = \frac{\frac{22}{7}}{4}(18)(18)\text{ cm}^2$

$A = \frac{\frac{22}{7}}{4}(324)\text{ cm}^2$

$A = \frac{22}{7}(81)\text{ cm}^2$

$A \approx 254.57\text{cm}^2$

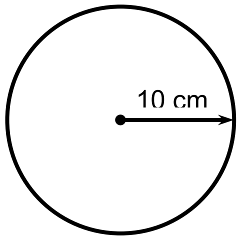
Therefore, the area of the circle of diameter 18 cm is approximately 254.57 cm^2.

NOW DO PRACTICE EXERCISE 10

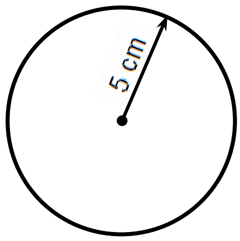
Practice Exercise 10

1. Using the approximation π =3.14, calculate the area of each of the following circles.

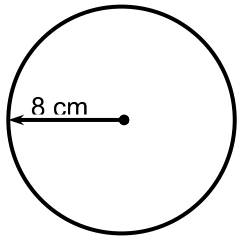
a)



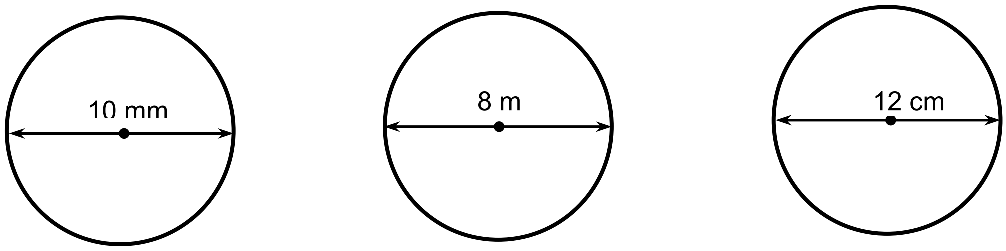
b)



c)

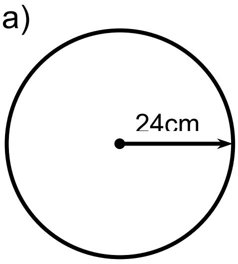


d)

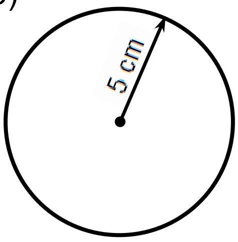


2. Using the approximation $\pi = \frac{22}{7}$, calculate the area of each of the following circles.

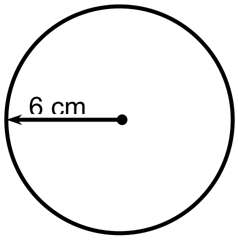
a)



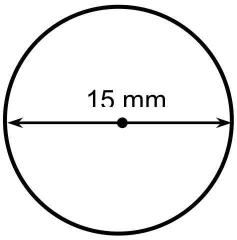
b)



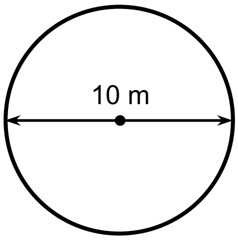
c)



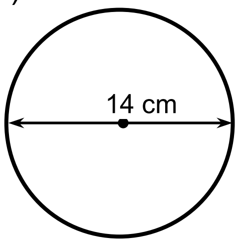
d)



e)



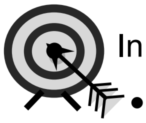
f)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 11: Solving Problems Involving the Area of a Circle

You learnt to work out the area of a circle using the formula $A = \pi r^2$ in the last lesson.



In this lesson, you will:

- solve word problems involving the area of a circle.

There are problems in real life that need to be solved that involve the area of a circle. Your knowledge and skills learnt in the previous lessons are required to work out the solutions to these problems.

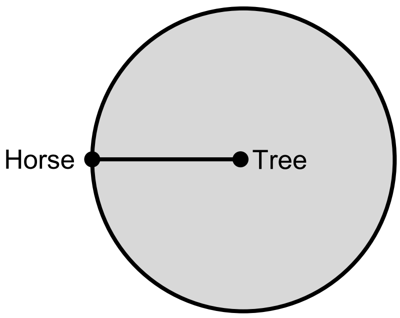
Here are some examples.

Example 1

George's horse is tied to a tree with an 8-metre rope as shown in the diagram below. Around how many square metres of gracing area does the horse have?

In this problem, you are required to find how large the gracing area the horse would have.

If you take the tree as the centre of the gracing area and the rope as the radius, we can figure out the size of the gracing area (shaded) as the horse moves around, as shown below.



Since it appears as a circle, we can now work out the total gracing area using the formula:

$A = \pi r^2$, where $\pi = 3.14$ and r = 8 m.

Substituting for π and r into the formula, you have

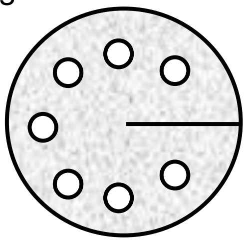
$A = 3.14 \times (8\text{m})^2$
$A = 3.14 \times 8 \text{ m} \times 8 \text{ m}$
$A = 3.14 \times 64 \text{ m}^2$
$\mathbf{A = 200.96 \text{ m}^2}$

Therefore, the horse would have 200.96 square metres of gracing area all around.

Example 2

In a circular park with a radius of 250 metres, there are 7 lamps whose bases are circles with a radius of 1 m. The entire area of the park has grass with the exception of the bases for the lamps. Calculate the lawn area.

First let us draw a representative diagram.



To find the lawn area, subtract the area of the bases of the seven lamps from the entire area of the park. So,

Lawn area = area of entire park – area of the seven bases of lamps

Applying the formula $A = \pi r^2$, we have Lawn area = $\pi r_1^{\,2} - 7\pi r_2^{\,2}$

$$\text{Lawn area} = \pi\,(250\text{ m})^2 - 7[(\pi)(1\text{ m})(1\text{m})]$$
$$= 3.14 \times 62\,500\text{ m}^2 - 7(3.14)(1\text{m}^2)$$
$$= 196\,250\text{ m}^2 - 21.98\text{ m}^2$$
$$= 196\,228.02\text{ m}^2$$

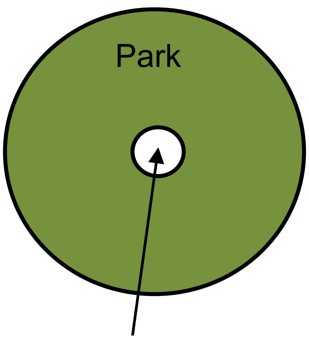
Therefore, the lawn area is 196 228.02 m^2.

Example 3

A circular fountain of 5 m radius lies alone in the centre of a circular park of 700 m radius. Calculate the total walking area available to pedestrians visiting the park.

Solution:

First, let us draw the figure.



Just looking at the figure, we have

Walking Area = Area of park – Area of fountain

Using the formula: $A = \pi r^2$, we have

$$\text{Walking area} = 3.14(700\text{ m})^2 - 3.14(5\text{ m})^2$$
$$= 3.14(490\,000\text{ m}^2) - 3.14(25\text{ m}^2)$$
$$= 1\,538\,600\text{ m}^2 - 78.5\text{ m}^2$$
$$= 1\,538\,521.5\text{ m}^2$$

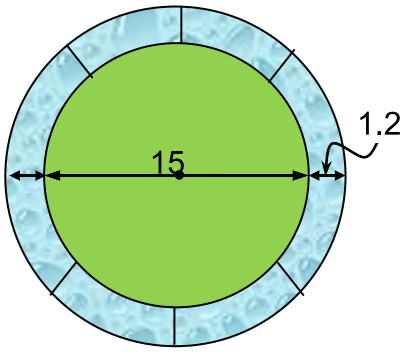
Therefore, the walking lawn available is 1 538 521.5 m^2.

Example 4

A new feature garden is to be built at the entrance to Mr. Kopi's house. The centre of the garden is a circular native plants garden 15 m in diameter. A path 1.2 m wide is going to be built completely surrounding the garden.

a) Calculate the area of the circular native plants garden.
b) The path around the garden is to be paved. How many square metres of paving will be required?

First, let us draw the figure.



For **Question a**

Since the diameter is given, use the formula in terms of the diameter.

$$A = \frac{\pi}{4}D^2$$

Solution:

Area of the garden $= \frac{\pi}{4}(15 \text{ m})^2$

$= \frac{3.14}{4}(225)\text{m}^2$

$= 176.625 \text{ m}^2$

For **Question b**

To find the number of square metres of paving required, find the entire area of the feature garden and subtract the area of the circular garden.

Solution:

Area of the paving = Area of feature garden – area of circular garden

$= 3.14(8.7 \text{ m})^2 - 176.625 \text{ m}^2$
$= 3.14(75.69)\text{m}^2 - 176.625 \text{ m}^2$
$= 237.666 \text{ m}^2 - 176.625 \text{ m}^2$
$= 61.04 \text{ m}^2$

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

Solve the following:

1. Yuri is riding a horse which is tied to a pole with a 3.5 m piece of rope and her friend Natasha is riding a donkey which is 2m from the same centre point.

   Calculate the area covered by each when they have gone around the centre.

Answer: ___________

2. The Manly Tigers run around the outside circular rugby field as part of their training. The diameter of the field is 145 m.

   What is the area of the rugby field?

Answer: ___________

3. Sebastian has been contracted to tile a verandah which is in the shape of a circle with a radius of 2.5 metres.

   How many square metres of tiles will he need?

Answer: ___________

4. Cleopatra imports floor rugs to sell in her furnishing shop. The rugs are circular in shape with a diameter of 2.4 m.

   Calculate the area a rug covers.

Answer: __________

---

5. Alex uses a lawn sprinkler to water the circular region of the lawn having a 30 ft diameter.

   Calculate the area covered by the sprinkler on the lawn.

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

SUB-STRAND 2: SUMMARY

In this summary you will find some of the important ideas and concepts to be remembered.

- A circle is a shape with all points the same distance from its centre.
- The parts of a circle include the circumference, radius, diameter, chord and an arc.
- The circumference is the distance around a circle.
- The diameter is the distance across a circle through its centre.
- The radius is the distance from the centre of a circle to any point on the circle.
- A chord is a line segment joining two points on a circle. All diameters are chords but not all chords are diameters.
- An arc is a part or portion of the circumference of the circle.
- We use the Greek letter π (pronounced Pi) to represent the ratio of the circumference of a circle to the diameter.
- Pi (π) has an approximate numerical value of **3.1416** or $\mathbf{\frac{22}{7}}$ as fraction.
- The area of a circle is the number of square units inside that circle.
- Given the radius, or diameter of a circle, we can find its area.
- The formula for the area of a circle are as follows:

  $\mathbf{A = \pi r^2}$, in terms of the radius and $\mathbf{A = \frac{\pi}{4} D^2}$, in terms of the diameter.

REVISE LESSONS 7 – 11. THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 3.

ANSWERS TO PRACTICE EXERCISES 7- 11

Practice Exercise 7

I.

(1)	c	(6)	b
(2)	a	(7)	b
(3)	d	(8)	c
(4)	b	(9)	d
(5)	a	(10)	b

II. (1) C (2) D (3) B (4) C (5) C

Practice Exercise 8

(1) $A = \frac{1}{2}ab$ (2) $2\pi r$ (3) $A = \frac{1}{2}(2\pi r)(r) = \pi r^2$ (4) $A = \pi r^2$

Practice Exercise 9

(1)
A. 52 square units
B. 192 square units
C. 80 square units
D. 156 square units

Practice Exercise 10

(1)	a)	314 cm^2	b)	78.5 cm^2	c)	200.96 cm^2
	d)	78.5 mm^2	e)	50.24 m^2	f)	113.04 cm^2
(2)	a)	1810.29 cm^2	b)	78.57 cm^2	c)	113.14 cm^2
	d)	176.79 mm^2	e)	78.57 m^2	f)	154 cm^2

Practice Exercise 11

(1) Yuri = 38.465 m^2
Natasha = 12.56 m^2

(2) 16504.625 m^2

(3) 19.625 m^2 or 20 m^2

(4) 4.5216 m^2

(5) 706. 5 ft^2

END OF SUB-STRAND 2

SUB-STRAND 3

SURFACE AREA

Lesson 12:	**Surface Area**
Lesson 13:	**Surface Area of Rectangular Prisms**
Lesson 14:	**Surface Area of Triangular Prisms**
Lesson 15:	**Surface Area of Cylinders**
Lesson 16:	**Surface Area of Pyramids**
Lesson 17:	**Solving Problems Involving Surface Area**

SUB-STRAND 3: SURFACE AREA

Introduction

We live in a three-dimensional world. Every object we can see or touch has three dimensions that can be measured: **length, width,** and **height**. The room you are sitting in can be described by these three dimensions.

Even you can be described using these three dimensions. In fact, the clothes you are wearing were made specifically for a person with your dimensions.

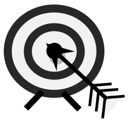
In the world around us, there are many three-dimensional geometric shapes. In these lessons, you'll learn about some of them. You'll learn some of the terminologies used to describe them, how to calculate their surface area, as well as learn about their mathematical properties.

In this Sub-strand, you will:

- define surface area and three-dimensional solid
- identify the parts of a three-dimensional solid
- find the surface area of the different three-dimensional solid using their nets and using formula.

Lesson 12: Surface Area

You learnt the meaning of area in your Grade 7 Strand 3 Sub-strand 2.

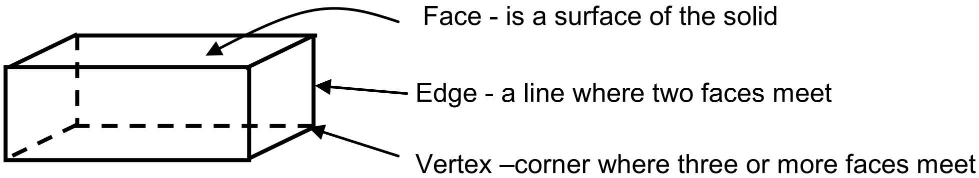


In this lesson, you will:

- define surface area for three-dimensional solids
- identify the edges, faces, vertices of a three-dimensional solid
- find the surface area of a three-dimensional solid given its net.

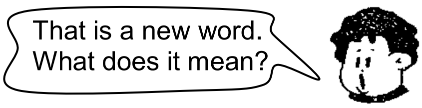
Solids shapes have thickness as well as length and breadth. We say they are three-dimensional (3-D).

Below is a diagram showing the parts of a solid.



There are many types of three-dimensional shapes. In Grade 7, you learnt that there are two main families of solids, the prisms and the pyramids. Other solids are either prisms or pyramids. The most common are the cylinder, cone and sphere.

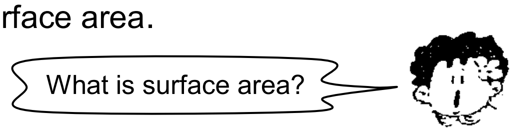
In this lesson, you shall study only the surface area of three-dimensional shapes known as **polyhedra**.



Polyhedra are three-dimensional shapes whose faces are plane shapes with straight edges. (The faces are flat). Polyhedra is the plural for polyhedron.

Have you ever wrapped a birthday gift? If so, then you've covered the surface area of a 3D shape called polyhedron with wrapping paper.

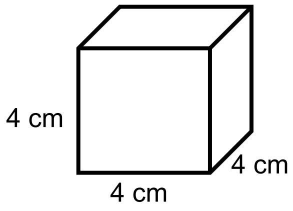
Surface area is exactly what it sounds like - the area of all of the outside surfaces of a three-dimensional object. In this lesson, you will learn more about this concept as well as how to compute the surface area.



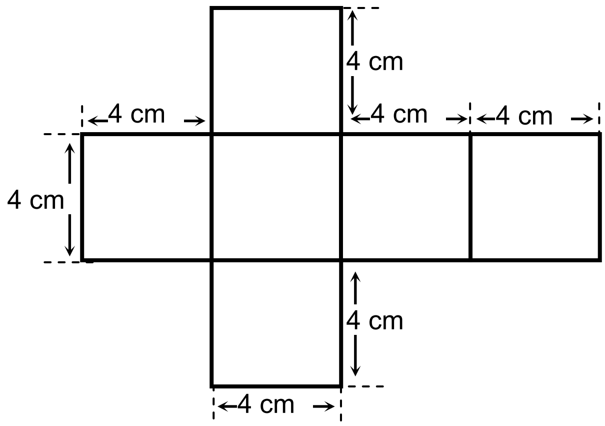
The surface area of any solid is equal to the sum of the area of all of its faces. Said another way, the surface area is the total area covered by the net of a solid.

To better understand the meaning of surface area, take a look at the cube and it's net.

Cube



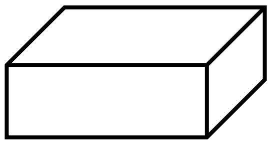
Net of the cube

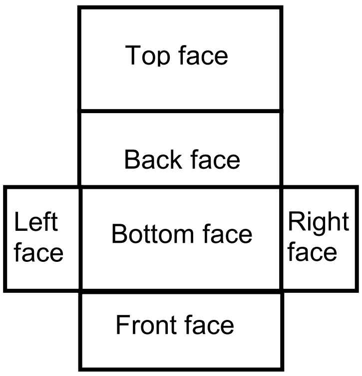


As you already know, a cube has six square faces. If each of those faces is 4 cm by 4 cm, then the area of each face is 4 × 4 = 16 square cm. And since there are six of them, the total surface area is 16 + 16 + 16+ 16 + 16 + 16 = 96 square cm.

Another example of three dimensional objects is a rectangular box. As you know, a box is constructed with four rectangular sides, a rectangular top, and a rectangular bottom.

The surface area of a rectangular box, then, is the sum of the areas of its four sides plus the area of the top plus the area of the bottom.





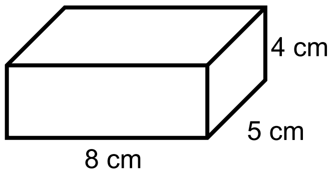
In general, to find the surface area of any shape, you can follow the process described below:

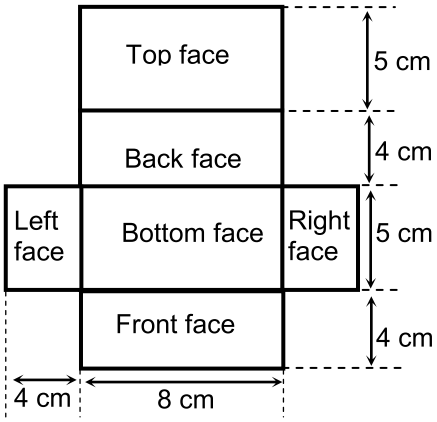
1. Draw a net of the shape.
2. Calculate the area of each face.
3. Add up the area of all the faces.

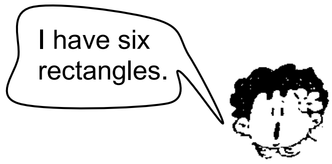
See the application example on the next page.

Example 1

A shoe box is 8 cm long, 5 cm wide and 4 cm high. Draw its net and hence find its surface area.







.
Solution:

Surface area = Area of six faces

$= A_{Top} + A_{Bottom} + A_{Back} + A_{Front} + A_{Left} + A_{Right}$

$= 2(8\text{ cm} \times 5\text{ cm}) + 2(8\text{cm} \times 4\text{ cm}) + 2(5\text{ cm} \times 4\text{ cm})$

$= 2(40\text{ cm}^2) + 2(32\text{ cm}^2) + 2(20\text{ cm}^2)$

$= 80\text{ cm}^2 + 64\text{ cm}^2 + 40\text{ cm}^2$

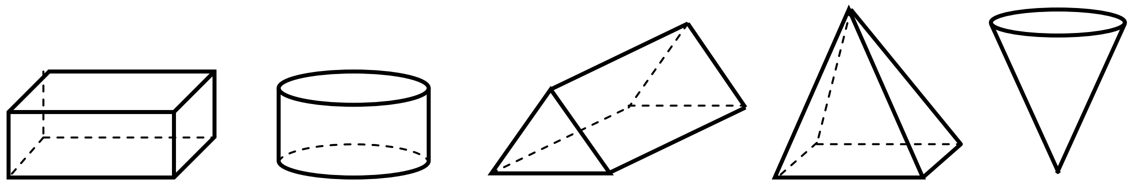
$= 184\text{ cm}^2$

Therefore, the total surface area of the shoe box is 184 cm^2.

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

1. Refer to the solids given below.



M N O P Q

a) Give the name of each solid.

M __________ **N** __________ **O** __________ **P** __________**Q** __________

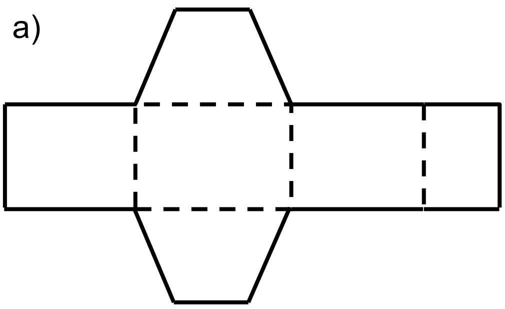
b) Which of these solid have curved surfaces?

c) For solid **M**, **O** and **P** complete the table by filling in the correct information

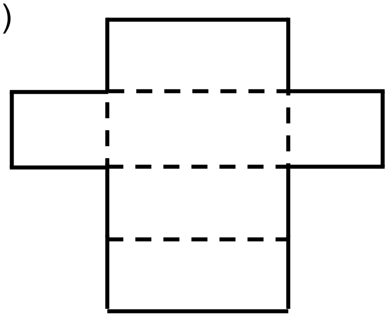
Solid	Number of Faces	Number of Edges	Number of Vertices
M			
O			
P			

2. Name the solid represented by each of the net drawn below when folded. Is it open or closed?

a)



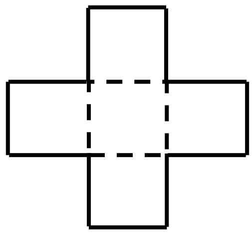
b)



Answer: __________________

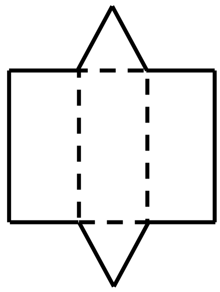
Answer: __________________

c)



Answer: ____________________

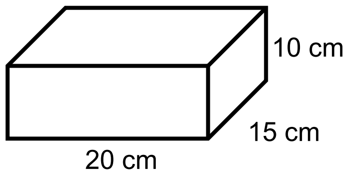
d)



Answer: ____________________

Refer to the information below and answer Questions 3 and 4.

Lorie is making a sewing box which is 20 cm long, 15 cm wide and 10 cm high as shown.



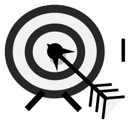
3) Draw the net of Lories sewing box showing all the six faces.

4) Lorie wants to cover the box with silk cloth. How many square centimetres of silk does she need?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 13: Surface Area of Rectangular Prisms

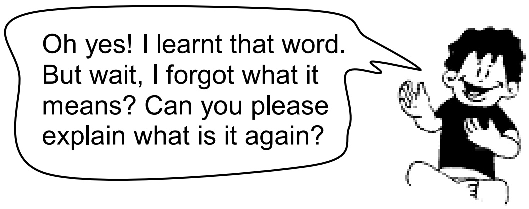
You learnt the meaning of three-dimensional solids and surface area in the previous lesson. You also learnt to find the surface area of these solids given their nets.



In this lesson, you will:

- define rectangular prism
- find the surface area of rectangular prisms.

You might have come across the word "**cuboid**" in your early study of shapes in mathematics.

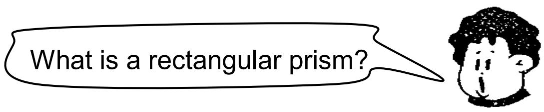


A **cuboid** is a box-shaped object.

It has six flat sides and all angles are **right angles.**

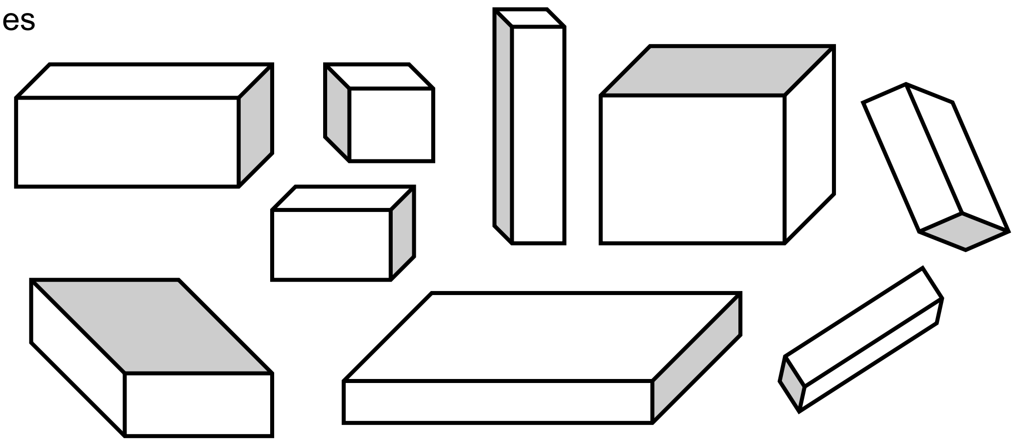
And all of its faces are **rectangles**.

It is also a **prism** because it has the same cross-section along a length. In fact it is a **rectangular prism**.

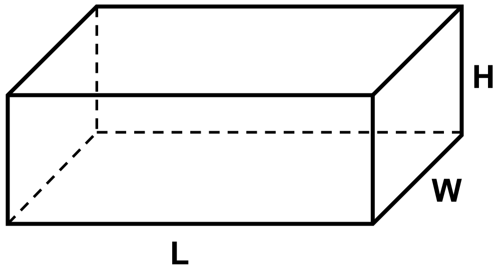


A rectangular prism is the simplest form of prism which has all six of its faces shaped like rectangles.

Examples



A rectangular prism has six flat surfaces called **faces**. The surface area of the rectangular prism is the sum of the areas of all six faces. Think of the six faces as three pairs of opposite, parallel faces. Since opposite faces have the same area, you find the surface area of one face in each pair of opposite faces then find the sum of these three areas and double the result.



The simplest rectangular prisms have all six of their faces as rectangles. These prisms look like boxes. The sum of all the faces of a rectangular prism is called the **total surface area (TSA).** The sum of the areas of the four faces that are not bases is called the **lateral surface area (LSA)**.

You can find the total surface area of a rectangular prism if you know its dimensions: length (L), width (W), and height (H).

See the steps below.

Step 1: Find the area of one face in each pair of opposite faces.

Area of the base = L x W

Area of front face = L x H

Area of side face = W x H

Step 2: Find the sum of the areas of the three faces.

Sum of areas = (L x W) + (L x H) + (W x H)

Step 3: Multiply the sum of the three areas by 2.

Surface area of Prism = $2\left[(L \times W) + (L \times H) + (W \times H)\right]$

So, in finding the surface area of a rectangular prism we use the formula:

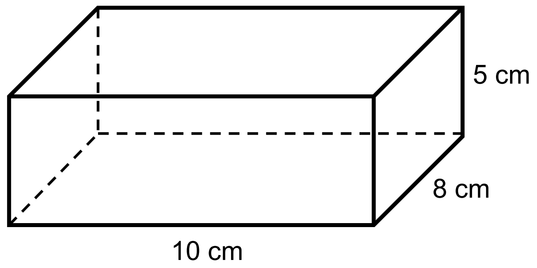
$$\boxed{\mathbf{TSA = 2\left[(L \times W) + (L \times H) + (W \times H)\right]}}$$

Where: **TSA** is the Total surface area
L is the length of the base
W is the width of the base
H is the height of the prism

Now look at the examples on the next page.

Example 1

Find the total surface area of the rectangular prism.



Solution: Use the formula TSA = $2[(L \times W) + (L \times H) + (W \times H)]$.
Length (L) = 10 cm
Width (W) = 8 cm
Height (H) = 5 cm

$$\begin{aligned} \text{TSA} &= 2[(10\text{ cm} \times 8\text{ cm}) + (10\text{ cm} \times 5\text{ cm}) + (8\text{ cm} \times 5\text{ cm})] \\ &= 2(80\text{ cm}^2 + 50\text{ cm}^2 + 40\text{ cm}^2) \\ &= 2(170\text{ cm}^2) \\ &= 340\text{ cm}^2 \end{aligned}$$

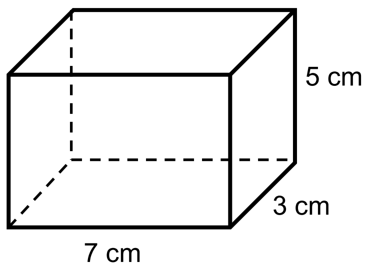
Therefore, the surface area of the rectangular prism is **340 cm^2**.

Example 2

Find the lateral area of a rectangular prism whose length, height and width are shown in the diagram.

Solution:

The top and bottom faces are designated as bases. We will only consider the front, back and side faces of the prism.



So, the area of back and front faces = 2(7 cm x 5 cm)
= 2(35 cm^2)
= 70 cm^2

Area of side faces = 2(3 cm x 5 cm)
= 2(15 cm^2)
= 30 cm^2

Lateral surface area (LSA) = 70cm^2 + 30 cm^2
= 100 cm^2

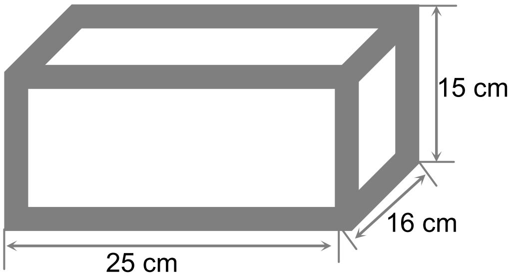
Therefore, the lateral surface area of the prism is 100 cm^2.

Example 3

A rectangular savings box is to be made of wood 1 cm thick. If the exterior dimensions are to be 25 cm by 16 cm by 15 cm,

a) What is the total area of the outside surface of the box?
b) What is the total area of the inside surface of the box?

To solve the problem, it is better to draw the box showing its dimensions.



Solution:

a) Find the total area of outside surface. Use the formula

$$\text{TSA} = 2[(L \times W) + (L \times H) + (W \times H)]$$

Length = 25 cm $= 2[(25\text{ cm} \times 16\text{ cm}) + (25\text{ cm} \times 15\text{ cm}) + (16\text{ cm} \times 15\text{ cm})]$

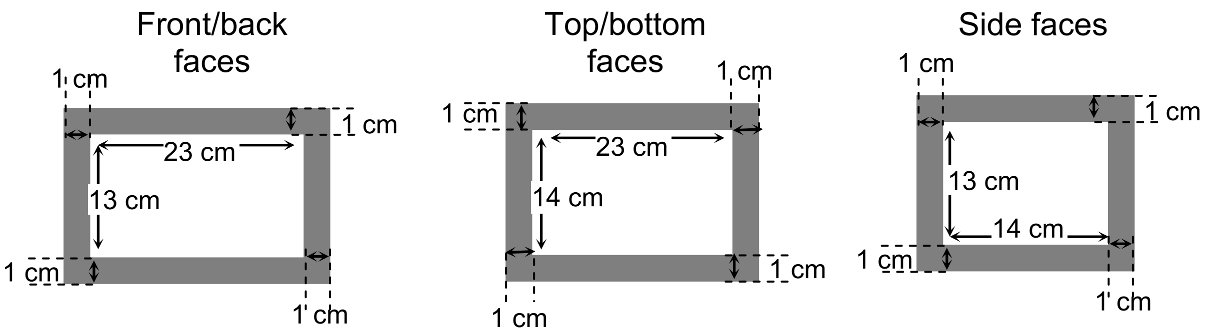
Width = 16 cm $= 2(400\text{ cm}^2 + 375\text{ cm}^2 + 240\text{ cm}^2)$

Height = 15 cm $= 2(1015\text{ cm}^2)$

$= 2030\text{ cm}^2$

Therefore, the total area of outside surface is 2030 cm^2.

b) Find the total area of inside surface.



Notice that in the diagram, the dimensions of the inside surface are reduced by 2 cm respectively. This is done because of the wood's thickness which is 1 cm.

Hence, the total area of the inside surface = TSA = $2[(L \times W) + (L \times H) + (W \times H)]$

Length = 23 cm $= 2[(23\text{ cm} \times 14\text{ cm}) + (23\text{ cm} \times 13\text{ cm}) + (14\text{ cm} \times 13\text{ cm})]$

Width = 14 cm $= 2(322\text{ cm}^2 + 299\text{cm}^2 + 182\text{ cm}^2)$

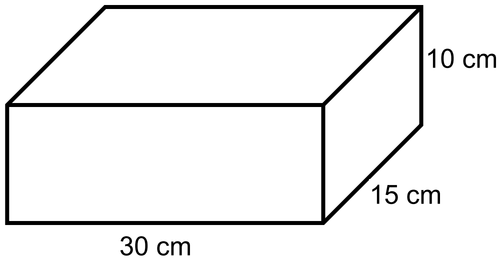
Height = 13 cm $= 2(803\text{ cm}^2)$

$= 1606\text{ cm}^2$

Therefore, the total area of inside surface is 1606 cm^2.

Example 4

How much paper is needed to wrap a box 30 cm long, 15 cm wide and 10 cm high as shown below?



Solution: Find the total surface area of the box using the formula.

$$TSA = 2[(L \times W) + (L \times H) + (W \times H)]$$

Length (L) = 30 cm
Width (W) = 15 cm
Height (H) = 10 cm

$$TSA = 2[(30\text{ cm} \times 15\text{ cm}) + (30\text{ cm} \times 10\text{ cm}) + (15\text{ cm} \times 10\text{ cm})]$$
$$= 2(450\text{ cm}^2 + 300\text{ cm}^2 + 150\text{ cm}^2)$$
$$= 2(900\text{ cm}^2)$$
$$= 1800\text{ cm}^2$$

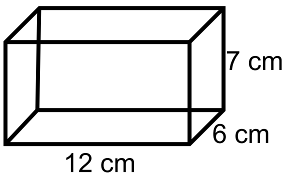
Therefore, the surface area of the rectangular prism is 1800 cm^2.

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. Find the surface area of each of the following rectangular prisms.

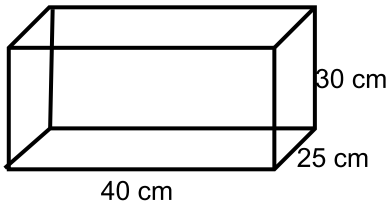
a)



Working out:

Answer: __________

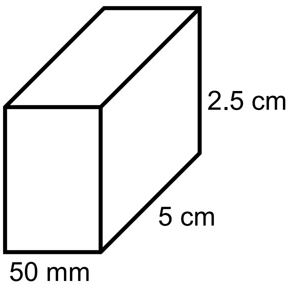
b)



Working out:

Answer: __________

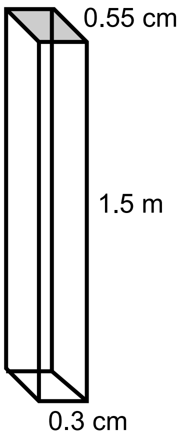
c)



Working out:

Answer: ___________

d)



Working out:

Answer: ___________

2. Komeng and Iamo built a glass rectangular fish tank 61 cm long, 34 cm wide and 29 cm high. The fish tank does not have a top.

Find the area of the glass used to build the fish tank.

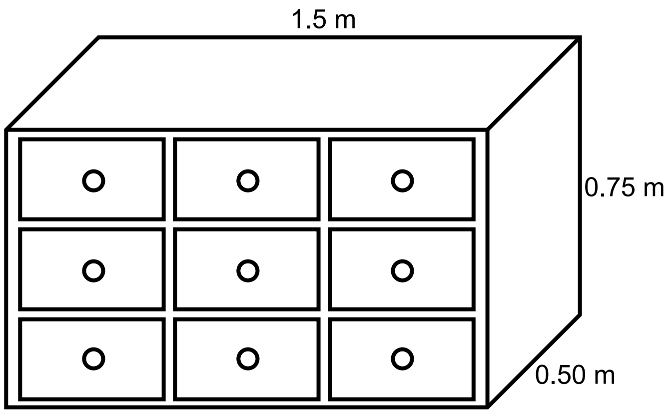
Working out:

Answer: __________

---

3. An unpainted clothes drawer is to be varnished on the top, the sides and the front.

How many square metres of surface will be varnished? Round off your answer to the nearest square metre.

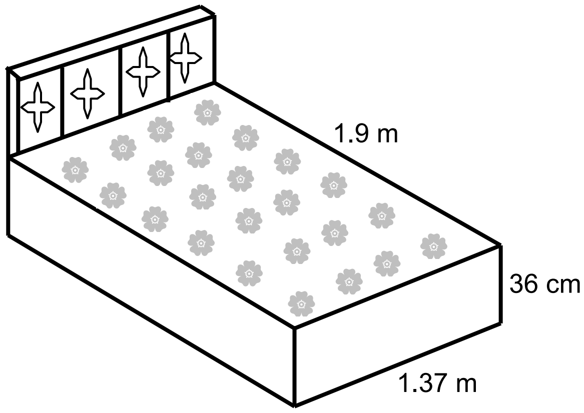


Working out:

Answer: ___________

4. Rova will make a bedspread for the bed shown below. The spread is to cover the top, sides and front of the bed.

   How much cloth will she need to make the bedspread? Round off your answer to the nearest square metres.



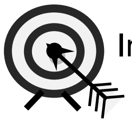
Working out:

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 14: Surface Area of Triangular Prisms

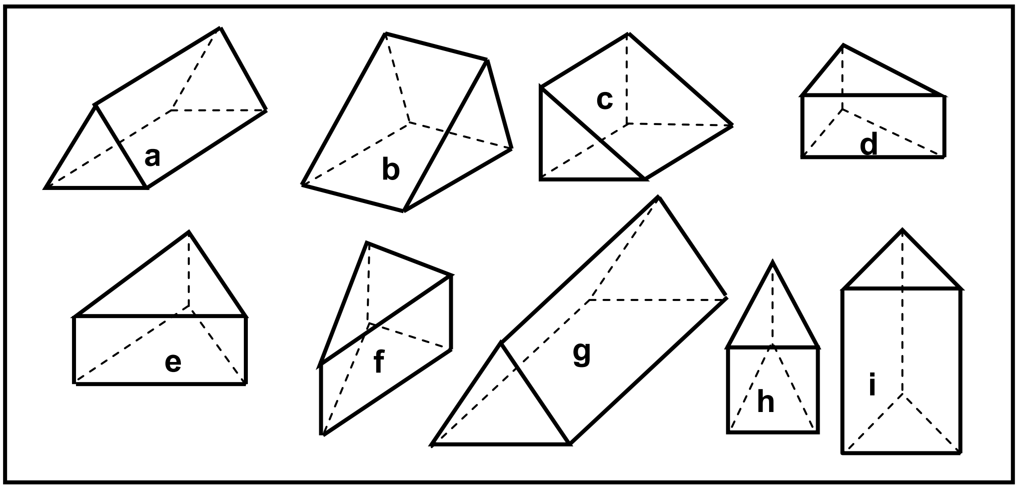
You learnt how to find the surface area of a rectangular prism in the previous lesson.



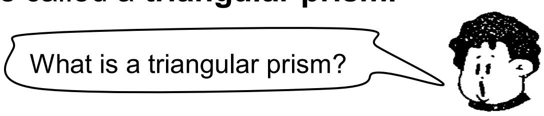
In this lesson, you will:

- describe a triangular prism
- find the surface area of a triangular prism

Look at the following solid shapes.

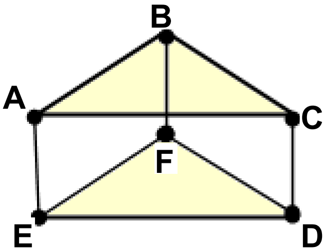


Each of these solid shapes is called a **triangular prism.**



A triangular prism is a prism composed of two triangular bases and three rectangular lateral faces. It has 6 vertices and 9 edges.

Here is a diagram showing all the parts of a triangular prism.



Bases - Δ **ABC** and Δ **EFD**

Lateral faces- □ **ABEF**, □**BCDF**,and □ **ACDE**

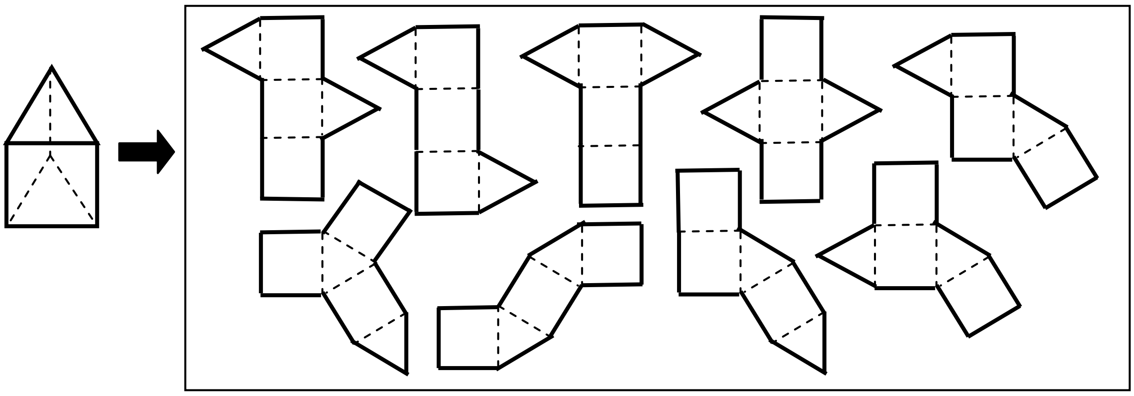
Vertices- (•) Points **A, B, C, D, E** and **F**

Edges - $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{DF}$, $\overline{AE}$, $\overline{BF}$, $\overline{EF}$

If the bases of the triangular prism are regular (means the triangles are both equilateral and equiangular) it is called a **regular right triangular prism.**

- Equilateral means all sides are of equal lengths.
- Equiangular means all angles are congruent.

A regular right triangular prism has nine distinct nets, as shown below.

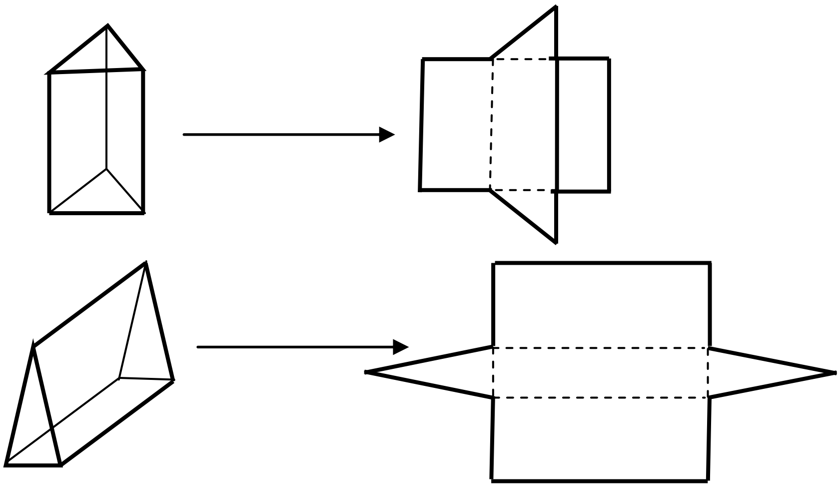


When the nets are folded along the dotted lines, each one of them will form a triangular prism with equal dimensions.

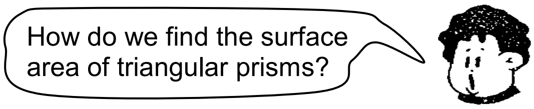
For irregular right triangular prisms the net always has two identical triangles and three rectangles of different sizes.

The bases of an irregular right prism are either a scalene, isosceles or right triangles.

Below are examples of irregular right triangular prisms and their nets.



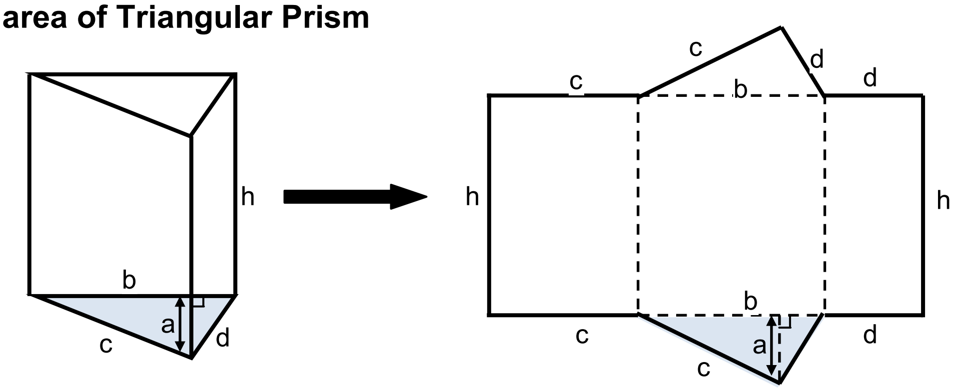
Whatever the sizes triangular prisms are, their nets should always have a pair of identical triangles and three rectangles, giving a total of five (5) faces. These five faces make up the surface area of a triangular prism.



Finding the surface area of a triangular prism follows the same method that you use to find the surface area of any prism.

Look at the following discussion on the next page to better understand how the surface area is obtained.

Surface area of Triangular Prism



The net of the triangular prism is made up of the area of two identical triangles (bases) and the three rectangles (lateral faces).

The surface area of any triangular prism equals the sum of the areas of its faces, which include the bases (top and bottom) and lateral faces. Because the top and the bottom bases have the same shape, the surface area can always be found as follows:

Method 1

Step 1: Find the area of the two bases. (2 Triangles)

$$\text{Area} = 2 \times \frac{1}{2}(a \times b)$$

Step 2: Find the area of lateral faces. (3 rectangles)

$$\text{Area} = (c \times h) + (b \times h) + (d \times h)$$

$$\text{Area} = (c + b + d) \times h$$

Step 3: Add the two areas to find the total surface area (TSA).

.TSA = Area of bases + Area of rectangles

$$\boxed{\begin{aligned} \text{TSA} &= 2 \times \frac{1}{2}(a \times b) + (c + b + d) \times h \\ &= (a \times b) + (c + b + d) \times h \end{aligned}}$$

Method 2

Look at the triangular prism and its net above. Notice that (c + b + d) is the perimeter (**P**) of a base of the triangular prism.

Thus, we also say that the total surface area of a triangular prism is equal to the area of its two bases plus the product of the perimeter of a base times its height.

TSA = 2 x area of base + perimeter of base x height.

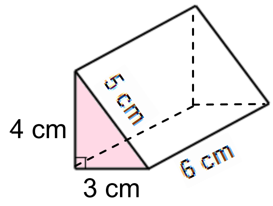
In Symbol,

$$\boxed{\mathbf{TSA = 2 \times \frac{1}{2}(a \times b) + P \times h}}$$

Now look at the examples on the next pages.

Example 1

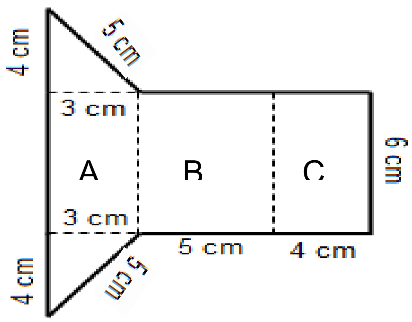
Find the surface area of the triangular prism.



Solution:

Step 1: Draw the net.

Net →



Step 2: Find the area of the two triangles. Use the formula A = $\frac{1}{2}$(b x h).

Total Surface Area = Area of 2 Triangles + Area of 3 Rectangles (different sizes)

Hence, TSA = 2 x $\frac{1}{2}$(b x h) + (A + B + C)

$= 2 \times \frac{1}{2}(3\text{ cm} \times 4\text{ cm}) + [(3\text{ cm} \times 6\text{ cm}) + (5\text{ cm} \times 6\text{ cm}) + (4\text{ cm} \times 6\text{ cm})]$

$= 2 \times \frac{1}{2}(12\text{ cm}^2) + (18 + 30 + 24)\text{ cm}^2$

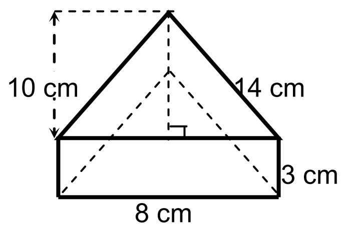
$= 2 \times 6\text{ cm}^2 + 72\text{ cm}^2$

$= 12\text{ cm}^2 + 72\text{ cm}^2$

$= 84\text{ cm}^2$

Therefore, the total surface area of the triangular prism is 84 cm^2.

3. Find the surface area of the triangular prism with dimensions shown below.



Solution: TSA = Area of 2 bases + Area of 3 side faces

$= 2 \times \frac{1}{2}(8 \times 10)\text{ cm}^2 + [(3 \times 14) + (3 \times 8) + (3 \times 14)]\text{cm}^2$

$= 2 \times \frac{1}{2}(80)\text{ cm}^2 + [(42) + (24) + (42)]\text{ cm}^2$

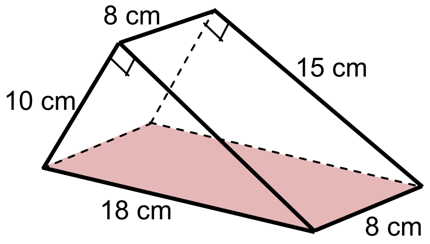
$= 80\text{ cm}^2 + 108\text{ cm}^2$

$= 188\text{ cm}^2$

Therefore, the surface area of the triangular prism is 188 cm^2.

Example 2

Find the surface area of the triangular prism whose dimensions are shown below.



Solution:

TSA = Area of 2 triangles + Area of 3 rectangles

a) Find the areas of the two triangles.

$A = 2 \text{ x } \frac{1}{2}(10 \text{ cm x } 15 \text{ cm})$

$= 2 \text{ x } \frac{1}{2}(150 \text{ cm}^2)$

$= 2 \text{ x } 75 \text{ cm}^2$

$= 150 \text{ cm}^2$

b) Find the areas of the three rectangles.

$A = (10 \text{ x } 8) \text{ cm}^2 + (15 \text{ x } 8) \text{ cm}^2 + (18 \text{ x } 8) \text{ cm}^2$

$= 80 \text{ cm}^2 + 120 \text{ cm}^2 + 144 \text{ cm}^2$

$= 344 \text{ cm}^2$

c) Find the total surface area of the triangular prism by adding the results in (**a**) and (**b**).

Total surface area $= 150 \text{ cm}^2 + 344 \text{ cm}^2$

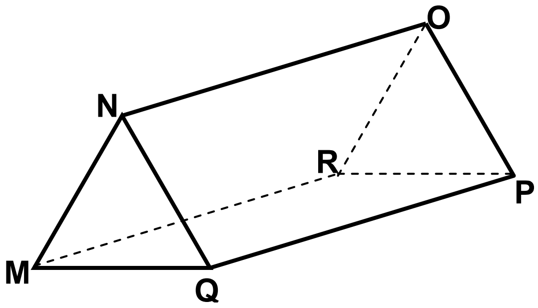
$= 494 \text{ cm}^2$

Therefore, total surface area of the prism is 494 cm^2.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Refer to the triangular prism below and answer the questions that follow.



Identify and name the following:

a) Bases **Answer**: ________________

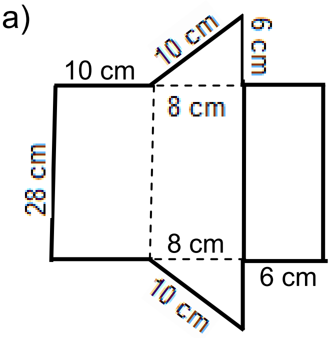
b) Lateral faces **Answer**: ________________

c) Edges **Answer**: ________________

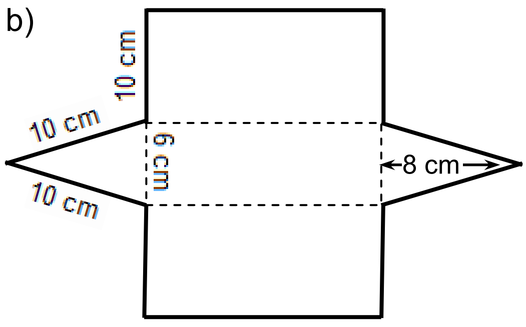
d) Vertices **Answer**: ________________

2. Find the surface area of each of the following triangular prisms given their nets. (Note: The figures are not drawn to scale.)

a)



b)



Working out:

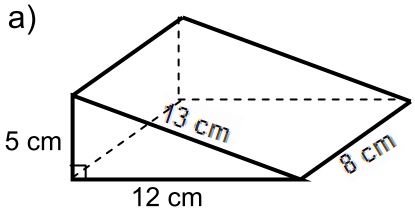
Working out:

Answer: ___________

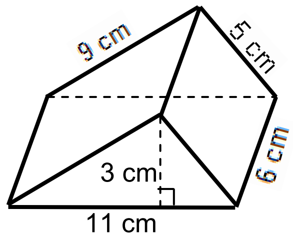
Answer: _____________

3. Find the surface area of each triangular prism.

a)



b)



Working out:

Working out:

Answer:_________

Answer: _________

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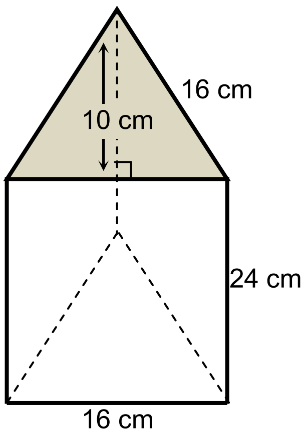
4. A glass prism has an equilateral triangle as base. Its measurements are given in the figure.

What is its surface area?



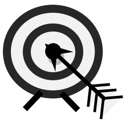
Working out:

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 15: Surface Area of Cylinders

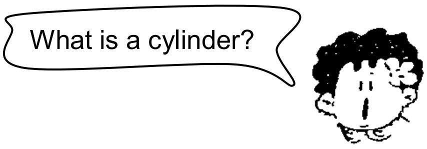
You defined triangular prism and identified its properties in the last lesson. You also learnt to draw the net of a triangular prism and to calculate its surface area.



In this lesson, you will:

- describe a cylinder and identify its properties
- find the surface area of cylinders.

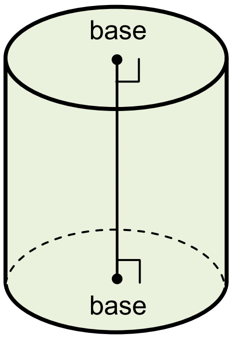
Some solids are neither prisms nor pyramids. One of these is the cylinder.



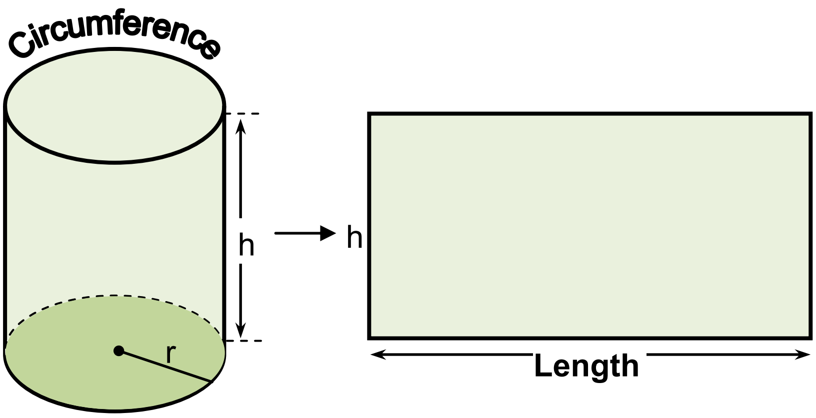
A cylinder is a 3-dimensional shape that has two circular bases that are parallel and congruent and are connected by a curved surface.
A curved surface is a surface that is round rather than flat.

The simplest cylinders look like food cans and are called **right cylinders**.
Their bases are perpendicular to the line joining the centres of the bases.

To find the area of the curved surface of a right cylinder, imagine a can of milk with a label. If you cut the label perpendicular to the top and bottom of the can, peel it off, and lay it flat on a surface, you will get a rectangle.



Right Cylinder



The length of the rectangle is the same as the circumference (**C**) of a base of the cylinder. The width of the rectangle is the same as the height (**h**) of the can.

Therefore, the area of the curved surface (called the lateral surface area) is the product of the circumference of the base and the height of the can.

To find the lateral surface area, we must find the circumference of the base since it represents the length of the curved surface.

Circumference of base = 2 x π x r or πD

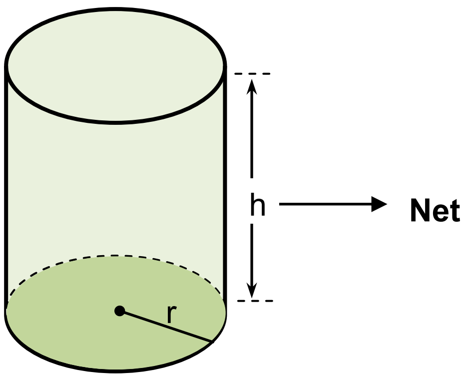
C = 2πr or πD

Therefore, Area of curved surface (LSA) = circumference of base x height of the can

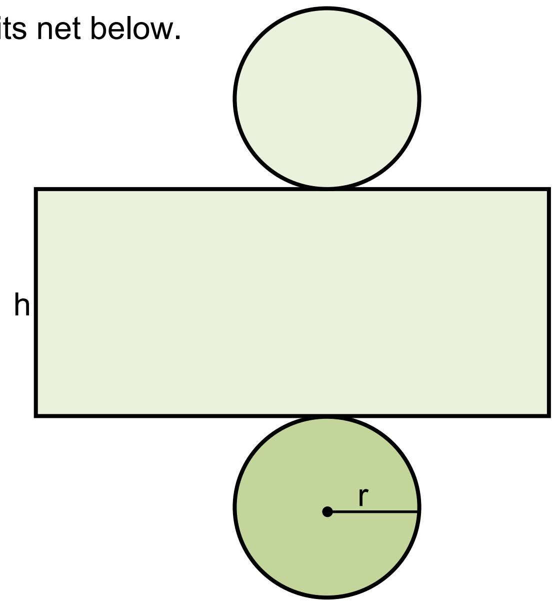
Area of curved surface (LSA) = C x h = (2 x π x r) x h

LSA = C x h or 2πrh

Now look at the right circular cylinder and its net below.



Right circular cylinder



Looking at the net, we can say that the surface area of a cylinder is the sum of the areas of the two bases (circles) and the Area of the curved surface or LSA (rectangle).

In symbols, Surface area = Area of 2 bases + Area of Curved surface (LSA)

$$\boxed{S = 2(\pi r^2) + 2\pi rh}$$

Where **S** is the surface area

r is the radius of the base

h is the height of the cylinder

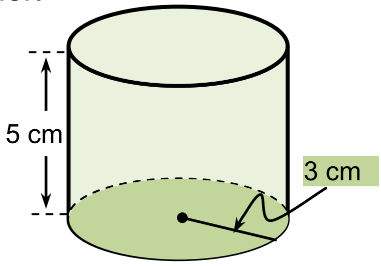
Note:

When working out problems related to the surface area of a cylinder, we need to know if the cylinder is open or closed.

Now study the examples on the next page.

Example 1

Find the surface area of the closed right cylinder.



Solution:

Use the formula **S = 2(πr²) + 2πrh**

- radius of base (r) = 3 cm
- height (h) = 5 cm

Use either $\frac{22}{7}$ or 3.14 as an approximate value for π.

Surface area (S) = (2 x 3.14 x 3 cm x 3 cm) +[(2 x 3.14 x 3 cm) x (5 cm)]

S = (3.14 x 18 cm^2) + (3.14 x 30 cm^2)

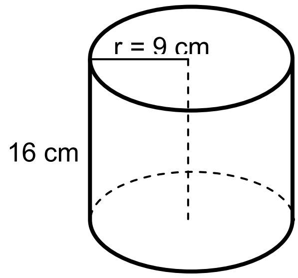
S = 56.52 cm^2 + 94.2 cm^2

S = 150.8 cm^2 rounded to the nearest tenth of a cm^2

Therefore, the surface area of the cylinder is 150.8 cm^2.

Example 2

A closed cylindrical box is shown on the right.



Find its (a) lateral surface area and (b) total surface area.

Solution:

a) Find the lateral surface area. Use the formula **LSA = 2πrh.**

LSA = 2πrh

= 2 x 3.14 x 9 cm x 16 cm

= 2 x 3.14 x 144cm^2

= 904.32 cm^2

Therefore, the lateral surface area is 904.32 cm^2.

b) Find the total surface area. Use the formula **S = 2(πr²) + 2πrh**

S = (2 x 3.14 x 9 cm x 9 cm) + 904.32 cm^2

S = (2 x 3.14 x 81cm^2) + 904.32 cm^2

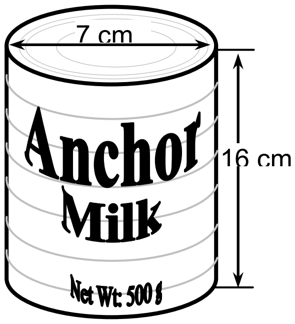
S = 508.68 cm^2 + 904.32 cm^2

S = 1413 cm^2

Therefore the total surface area of the closed cylindrical box is 1413 cm^2.

Example 3

How much paper is needed for the label to cover the can of milk whose dimensions are shown below? Give an allowance of 1 cm for the overlap when pasting the paper.



Solution:

Find the lateral surface area of the can.

To find the lateral surface of the can we first find the circumference of the base since it represents the length of the body. Use the formula **C = πD.**

$$C = \pi D$$
$$C = 3.14 \times 7 \text{ cm}$$
$$C = 21.98 \text{ cm}$$

Since, a 1 cm allowance for the overlap is given, the length of the body is C + 1 cm, which is 21.98 + 1 cm = 22.98 cm.

Now find the lateral surface area. Use the formula **LSA = C x h**

$$\begin{aligned} LSA &= C \times h \\ &= 22.98 \text{ cm} \times 16 \text{ cm} \\ &= 367.68 \text{ cm}^2 \end{aligned}$$

Therefore, the lateral area of the can is 367.68 cm^2. This represents the length of the body of the can to be covered by the paper label.

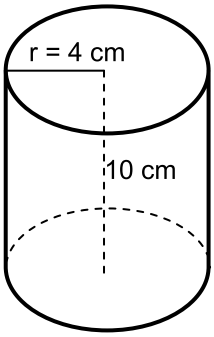
Hence, the amount of paper needed for the label is 367.68 cm^2.

NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. Find the lateral surface area of the following cylinders. (The figures are not drawn to scale).

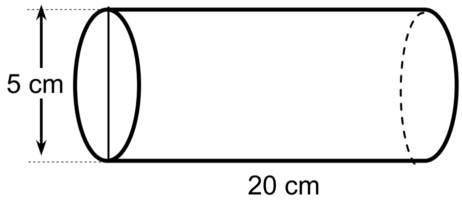
a)



Working Out:

Answer: __________

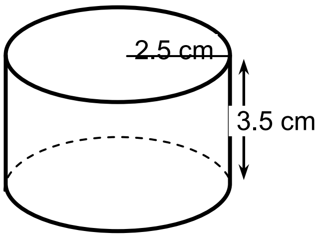
b)



Working Out:

Answer: __________

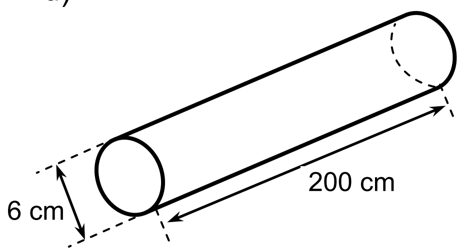
c)



Working Out:

Answer: __________

d)

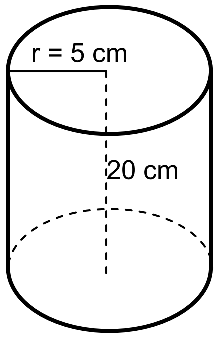


Working Out:

Answer: __________

2. Find the total surface area of each of the cylinders below.

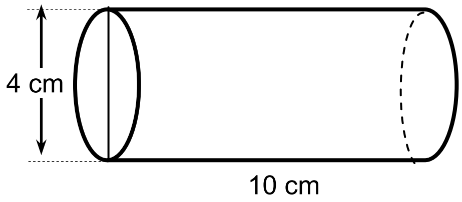
a)



Working Out:

Answer: __________

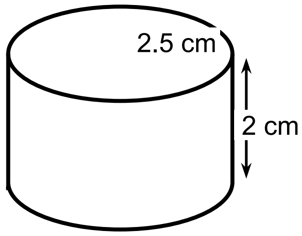
b)



Working Out:

Answer: __________

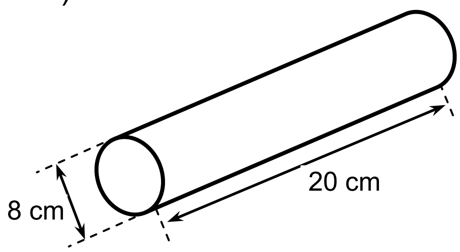
c)



Working Out:

Answer: __________

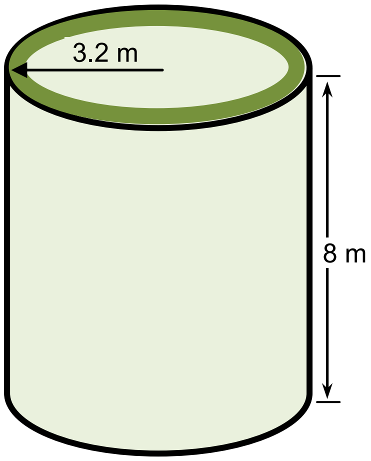
d)



Working Out:

Answer: __________

3. A closed circular water tank is made of concrete. Its outer radius is 3.2 m and its height is 8 m as shown below. If it is painted on the exterior, how much total surface is to be painted? Use $\pi = 3.14$. Round off your answer to 2 decimal places.

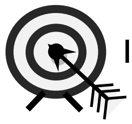


4. How many square centimetres of material will be needed to make an open tin can of radius 3 cm and height of 10 cm? Use $\pi = 3.14$. Round off your answer to 2 decimal places.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 16: Surface Area of Pyramids

In the previous lesson, you learnt to identify the properties of the cylinder, such as the base, the height and the lateral faces. You also learnt how to find the lateral surface area and total surface area of cylinders.



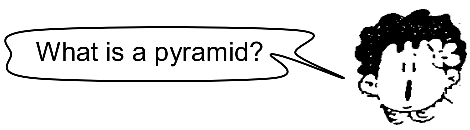
In this lesson, you will:

- Describe a pyramid
- compare the pyramid to a prism
- find the surface area of a pyramid.

Let us start the lesson by looking at the picture below.

When we think of pyramids we think of the **Great Pyramids of Egypt**.

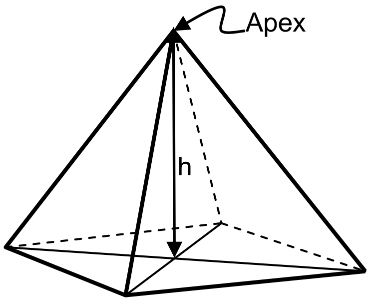
They are actually **Square Pyramids**, because the base of a pyramid is a Square.



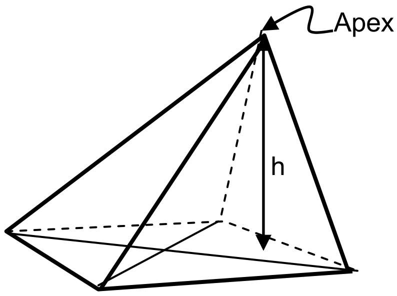
A pyramid is a solid that has a **base** in the shape of a polygon, each vertex of which is joined to a single point in a plane other than that of the base**.** This point is known as the **vertex** of the pyramid which is also called the **apex** of the pyramid. The sides of the pyramids, known as its **lateral faces**, are all triangular in shape and meet at the vertex. The segment where the lateral faces meet are called the **lateral edges**. The **altitude** of the pyramid is the segment perpendicular from the vertex to the base. The **slant height** of the pyramid is the segment perpendicular from the vertex to the base of a lateral face.

Pyramids are classified according to the position of the apex and the shape of the base.

- If the apex is directly above the centre of the base, then it is a **right pyramid**, otherwise it is an **Oblique pyramid**.



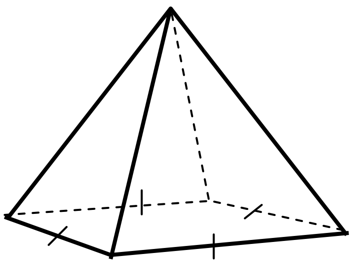
Right Pyramid



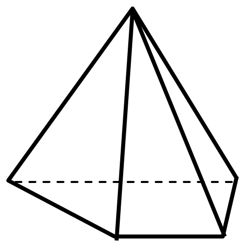
Oblique Pyramid

Earlier in your study of polygons, you learnt that a regular polygon has congruent sides and congruent angles.

- If the base is a regular polygon, then it is a **regular pyramid**, otherwise, it is an **Irregular pyramid**.

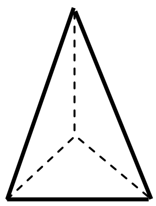


Regular Pyramid

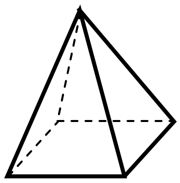


Irregular Pyramid

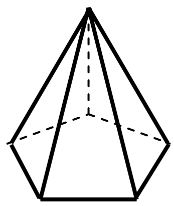
Below are some more examples of pyramids.



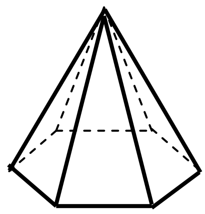
a



b



c



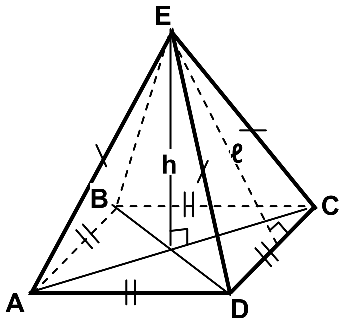
d

From the left, **a** is a regular triangular pyramid, **b** is a regular square pyramid, **c** is a regular pentagonal pyramid and **d** is a regular hexagonal pyramid. Note that a pyramid is named by the shape of its base. We shall consider only pyramids with quadrilateral and triangular bases.

A tetrahedron is considered regular if the lateral faces are all equilateral triangles.

In a regular pyramid, the sides are all congruent isosceles triangles. The altitude of any of those triangles is the slant height of the pyramid, not to be confused with the altitude of the pyramid.

Here is a pyramid with it parts.



E is the vertex or apex of the pyramid

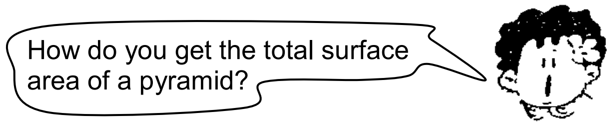
h is the altitude of the pyramid

ℓ is the slant height

Square ABCD is the base

△ABE, △BCE, △CDE and **△ADE** are lateral faces

$\overline{\textbf{AE}}$, $\overline{\textbf{BE}}$, $\overline{\textbf{CE}}$, and $\overline{\textbf{DE}}$ are lateral edges.



Just like the other solids you have looked at, pyramids have lateral surface area and total surface area.

The Surface Area of a pyramid has two parts: the area of the base (the **Base Area**), and the area of the side faces (the **Lateral Surface Area**).

The **Base Area** depends on the shape. There are different formulas for the triangle, square, etc.

For the **Lateral Surface Area** when all the side faces are the same:

- Just multiply the perimeter by the "slant length" and divide by 2. This is because the side faces are always triangles and the triangle formula is *"base times height divided by 2"*

In symbols we have $\mathbf{LSA = \frac{1}{2} P\ell}$

Where **LSA** is the lateral surface area
P is the perimeter of the base
ℓ is the slant height

But if the side faces are different (such as in an "irregular" pyramid) then add up the area of each triangular shape to find the lateral surface area.

So, to find the Total Surface Area of a Pyramid, the general formula is:

When all side faces are the same,

Total Surface Area (TSA) = [Base Area] + $^1/_2$ × Perimeter × [Slant Length]

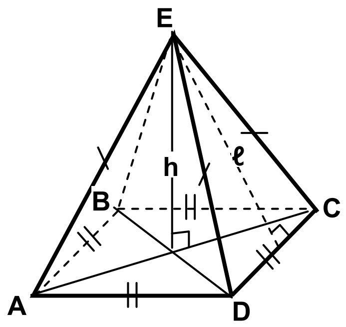
$$\boxed{\mathbf{TSA = B + \frac{1}{2} P\ell}}$$

When side faces are different

Total Surface area = [Base Area] + [Lateral Surface Area]

Example 1

The base of the regular pyramid below is a square. Given that AD = 14 cm, DE = 8 cm, h = 6 cm and ℓ = 7 cm.



a) Find the lateral surface area of the regular pyramid.

b) Find the total surface area of the regular pyramid.

Solution:

a) Find the lateral surface area using the formula LSA = $\frac{1}{2}P\ell$

Since the base is a square, the perimeter (P) = 4 x AD

P = 4 x 14 cm

= 56 cm

Write the formula: LSA = $\frac{1}{2}P\ell$

Substitute: LSA = $\frac{1}{2}$ x 56 cm x 7 cm

= 196 cm^2

Therefore the lateral surface area of the pyramid is 196 cm^2.

b) Find the total surface area.

First find B (Area of the square base). B = $(AD)^2$

Substitute: B = $(14\text{ cm})^2$

= 196 cm^2

Use the formula: TSA = B + LSA

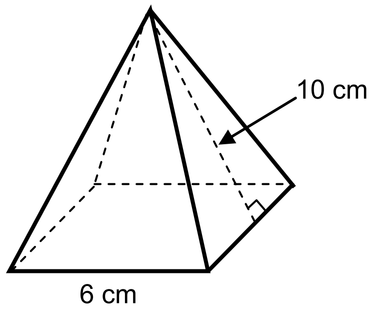
Substitute: TSA = 196 cm^2 + 196 cm^2

= 392 cm^2

Therefore the total surface area of the pyramid is 392 cm^2.

Example 2

The base of a given pyramid is a square. If the side of the square base measures 6 cm while the slant height of each of the congruent triangle faces is 10 cm, find the total surface area of the pyramid.



Solution:

To find the Total surface area of the pyramid, use the formula TSA = B + LSA.

First find the Area of the base (B).

$$\begin{aligned} B &= (\text{side})^2 \\ &= (6\text{ cm})^2 \\ &= 36\text{ cm}^2 \end{aligned}$$

Find Lateral Surface Area (LSA)

$$\begin{aligned} LSA &= \frac{1}{2}P\ell \\ &= \frac{1}{2}(4 \times 6\text{ cm} \times 10\text{ cm}) \\ &= \frac{1}{2}(240\text{ cm}^2) \\ &= 120\text{ cm}^2 \end{aligned}$$

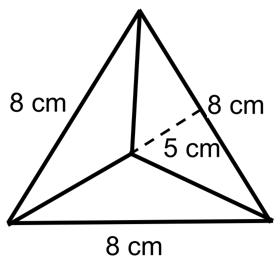
Find the total surface area by adding the two areas obtained.

$$\begin{aligned} TSA &= B + LSA \\ &= 36\text{ cm}^2 + 120\text{ cm}^2 \\ &= 156\text{ cm}^2 \end{aligned}$$

Therefore, the total surface are of the pyramid is 156 cm^2.

Example 3

Find the lateral surface area of a regular pyramid with a triangular base if each edge of the base measures 8 cm and the slant height is 5 cm.



Solution:

Find the perimeter of the base which is the sum of the sides.

P = 3(8) = 24 cm

Find the lateral surface area using the formula; LSA = $\frac{1}{2}P\ell$

$$LSA = \frac{1}{2}(24\text{ cm x }5\text{ cm})$$

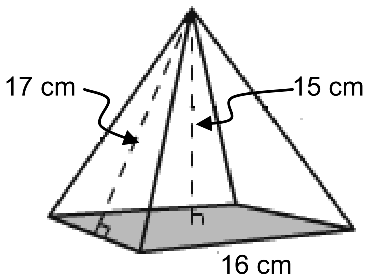
$$LSA = \frac{1}{2}(120\text{ cm}^2)$$

$$= 60\text{ cm}^2$$

Therefore, the lateral surface area of the triangular pyramid is 60 cm^2.

Example 4

Find the total surface area of a regular pyramid with a square base if each edge of the base measures 16 cm, the slant height of a side is 17 cm and the altitude is 15 cm.



Solution:

Since it is a square, the perimeter of the base is 4 x length of a side.

$$P = 4(16) = 64\text{ cm}$$

The area of the base is (side)^2 or s^2.

$$B = 16^2 = 256\text{ inches}^2$$

Total surface area = Area of base + Lateral surface area

$$T.\ S.\ A. = B + \frac{1}{2}P\ell$$

$$= 256 + \frac{1}{2}(64\text{ X }17)$$

$$= 256 + 544$$

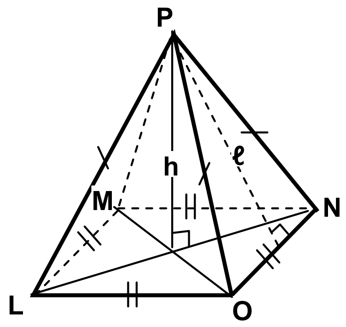
$$= 800\text{ cm}^2$$

Therefore, the total surface area of the regular pyramid is 800 cm^2.

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. Refer to the figure below and identify and name the following.



a) Apex or vertex

b) Lateral edges

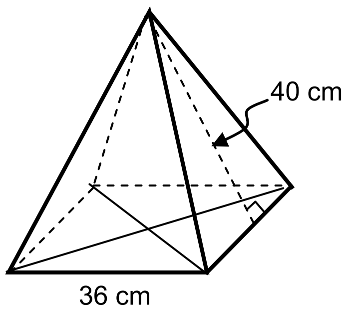
c) Lateral faces

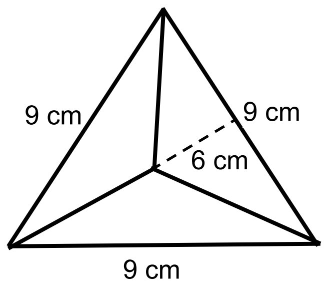
d) Altitude of the pyramid

e) Slant height

f) Base

2. Find the lateral surface area of each of the following regular pyramids.





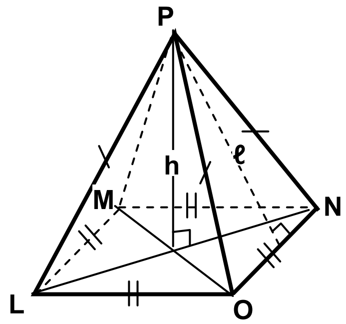
Working out:

Working out:

Answer: __________

Answer: __________

3. For the figure below, LO = 12 cm, OP = 10 cm, h = 8 cm and ℓ = 9 cm.



a) Find the lateral surface area of the regular square pyramid.

b) Find the total surface area of the regular square pyramid.

Answer:

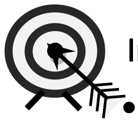
---

4. What is the surface area of a wooden pyramid on a square base of a side 15 cm if the altitude to the base of each triangular face is 18 cm?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 17: Solving Problems Involving Surface Area

You learnt about the meaning and properties of prisms and pyramids in the previous lessons. You also learnt how to find the surface area of prisms and pyramids.



In this lesson, you will:

- solve problems involving surface area in real life situations.

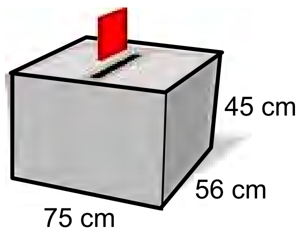
Have you ever wrapped a birthday gift or a Christmas gift? If so, then you have covered the surface area of a prism with wrapping paper.

The knowledge and skills of surface area plays an important role in our daily lives as it is necessary for placement. For example, have you ever placed furniture in your house? The placement of furniture depends very much on the available area to place it in, the less the area, the less the amount of furniture that will fit into the available space.

We need to be able to measure the surface areas of these objects to determine how much material like paint, furniture, wrapping paper, etc. will be needed to cover or fill them.

Example 1

How much coloured paper will be needed to cover the outside surface of a raffle box 75 cm long, 56 cm wide and 45 cm high?



Solution:

Find the surface area of the box.

Use the formula TSA = 2 x [(L x W) + (L x H) + (W x H)]

Step 1: Find the area of one of each pair of opposite faces.

Area of base = L x W = 75 cm x 56 cm = 4200 cm^2

Area of front face = L x H = 75 cm x 45 cm = 3375 cm^2

Area of side face = W x H = 56 cm x 45 cm = 2520 cm^2

Step 2 Substitute the three areas in the formula.

TSA = 2 x (4200 cm^2 + 3375 cm^2 + 2520 cm^2)

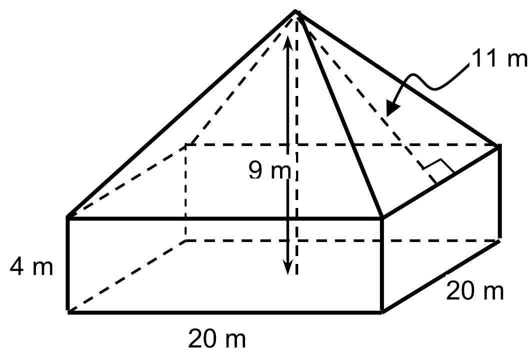
= 2 x 10095 cm^2

= 20190 cm^2

Therefore, 20190 cm^2 of coloured paper will be needed to cover the outside surface of the box.

2. A circus tent has the shape and dimensions given in the figure below. The height from the floor to the top of the tent is 9 m.

   How much canvas was used in making the tent?



Solution: The tent is made up of two parts: Square pyramid and rectangular prism.

To find how much canvas was used to make the tent, you find the sum of the lateral surface areas of these two parts.

Step 1 Find the lateral surface area of the square pyramid.

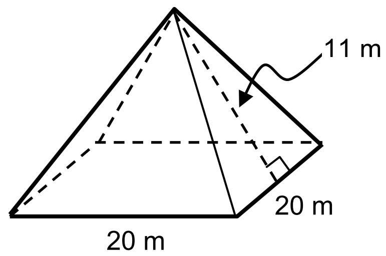
The lateral surface area of the square pyramid is;

Use the formula LSA = $\frac{1}{2}P\ell$

$$\text{LSA} = \frac{1}{2}(4\text{x } 20 \text{ m x } 11 \text{ m})$$

$$= \frac{1}{2}(880)\text{m}^2$$

$$= 440 \text{ m}^2$$



Step 2 Find the lateral surface area of the rectangular prism.

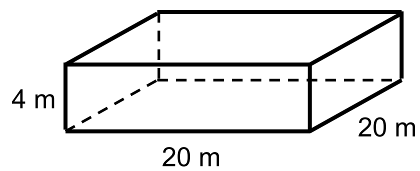
The lateral surface area of the rectangular prism is the sum of the four rectangle faces.

Use the formula LSA= 4x(Lx W)

$$= 4 \text{ x } (20 \text{ m x } 4 \text{ m})$$

$$= 4 \text{ x } 80 \text{ m}^2$$

$$= 320 \text{ m}^2$$



Step 3 Add the two lateral surface areas to get the lateral surface area of the tent.

LSA of the tent = 440 m^2 + 320 m^2

$$= 760 \text{ m}^2$$

Therefore, 760 m^2 of canvas was used to make the circus tent.

Example 3

If r = 5 cm and h = 11 cm, calculate the total surface area of:

a) the evaporated milk can shown on the right.

b) the paper label on the can, assuming that it does not overlap at the ends.

Use $\pi = 3.14$.

Solution:

a) To find the total surface area of the can.

Total surface area = $2(\pi r^2) + 2\pi rh$ Where r = 5 cm, and h = 11 cm

$= 2 \times 3.14 \times (5\text{ cm})^2 + 2 \times 3.14 \times 5\text{ cm} \times 11\text{ cm}$
$= 6.28 \times 25\text{ cm}^2 + 6.28 \times 55\text{ cm}^2$
$= 157\text{ cm}^2 + 345.5\text{ cm}^2$
$= 502.5\text{ cm}^2$

Therefore the total surface area is 502.5 cm^2.

b) Find the area of the curved surface to find the area of the paper label.

Curved surface area = $2\pi rh$ Where r = 5 cm, and h = 11 cm

$= 2 \times 3.14 \times 5\text{ cm} \times 11\text{ cm}$
$= 6.28 \times 55\text{ cm}^2$
$= 345.5\text{ cm}^2$

Therefore the area of the paper label is 345.5 cm^2.

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

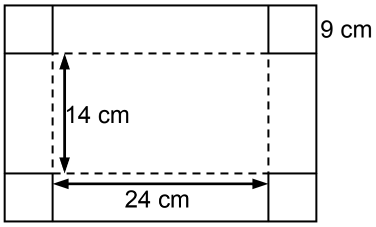
1. Below is a tomato sauce can with the radius and the height shown.

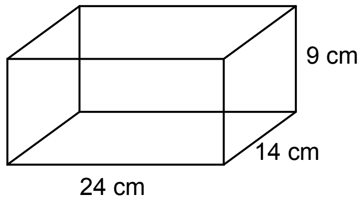
a) Calculate the total surface area of the sauce can. Use $\pi = 3.14$.

b) Find the surface area of the paper label assuming that it does not overlap at the ends.

c) If the sauce company received an order of 1020 tomato sauce cans, how much paper label will be used to label all the cans?

2. The body of a shoe box is 24 cm by 14 cm by 9 cm. It is made from a continuous piece of cardboard, the corners, as shown in the diagram below, are cut off and thrown away.





a) How much cardboard is used in making one box?

b) How much cardboard is wasted?

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3. A school classroom is 8.5 m long, 7.2 m wide and 3 m high. The walls and ceiling are to be painted. Allowing 10 m^2 for one door and windows, how many square metres of area is to be painted?

4. A tent has a square base of side 7.5 m. If each of the 4 triangular faces forming sides is an isosceles triangle and has an altitude of 6 m, how much canvas is used in making its sides?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

SUB-STRAND 3: SUMMARY

- The **surface area of any solid** is equal to the sum of the area of all its faces. It is the total area covered by the **net of a solid**.
- A **Rectangular prism** is the simplest prism which has all its faces shaped like rectangles. Its **total surface area (TSA)** is found by using the formula

$$\textbf{TSA = 2}\left[(l \times w) + (l \times h) + (w \times h)\right]$$

- A **Triangular prism** is a prism composed of two triangular bases and three rectangular lateral faces. Its **total surface area** is found by using the formula

$$\textbf{TSA} = 2 \times \frac{1}{2}(a \times b) + P \times h$$

- A **cylinder** is a 3-dimensional shape that has two circular bases that are parallel and congruent and are connected by a curved surface. The simplest types look like food cans called **right circular cylinders**. The **total surface area** is found by using the formula

$$\textbf{TSA} = 2(\pi r^2) + 2\pi rh$$

- A **pyramid** is a solid that has a base in the shape of a polygon. Each vertex of which is joined to a single point in a plane other than that of the base. This point is called the **apex or vertex** of the pyramid. The sides of the pyramid, known as **lateral faces** are all triangular in shape and meet at the vertex. The segments where the lateral faces meet are called **lateral edges**. The **altitude** of the pyramid is the segment perpendicular from the vertex to the base. The **slant height** of the pyramid is the segment perpendicular from the vertex to the base of a lateral face.
- The **total surface area** of a pyramid is the sum of its polygonal base and its lateral surface area. It is calculated by using the formula

$$\textbf{TSA} = B + \frac{1}{2}P\ell$$

REVISE LESSONS 12-17. THEN DO SUB-STRAND TEST 3 IN ASSIGNMENT 3.

ANSWERS TO PRACTICE EXERCISES 12 - 17

Practice Exercise 12

1. a) M = rectangular prism N = cylinder O = triangular prism
   P = pyramid Q = cone

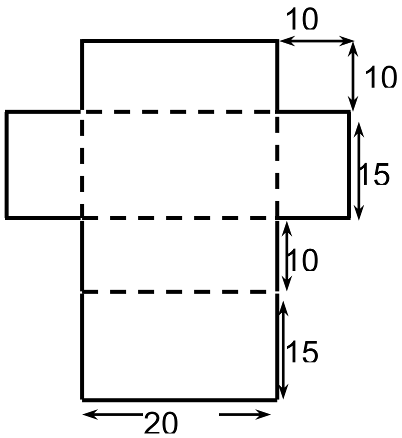
   b) N and Q

   c)

Solid	Number of Faces	Number of Edges	Number of Vertices
M	6	12	8
O	5	9	6
P	5	8	5

2. a) trapezoidal prism; closed b) rectangular prism; closed
   c) cube; open d) triangular prism; open

3.



4. 1300 cm^2

Practice Exercise 13

1. a) 396 cm^2 b) 5900 cm^2
   c) 100 cm^2 d) 2.88 cm^2
2. 7584 cm^2
3. 3 m^2
4. 5 m^2

Practice Exercise 14

1. a) ΔMNQ and ΔROP
   b) Rectangles MNOR, NOPQ, and MRPQ
   c) Segments $\overline{MN}$, $\overline{NQ}$, $\overline{MQ}$, $\overline{QP}$, $\overline{PR}$, $\overline{PO}$, $\overline{NO}$, $\overline{OR}$ and $\overline{MR}$
   d) Points M, N, Q, R, O, and P

2. a) 720 cm^2 b) 360 cm^2
3. a) 300 cm^2 b) 183 cm^2
4. 1312 cm^2

Practice Exercise 15

1. a) 251.20 cm^2 b) 314 cm^2
   c) 54.95 cm^2 d) 3768 cm^2
2. a) 768 cm^2 b) 150.72 cm^2
   c) 70.65 cm^2 d) 602.88 cm^2
3. 225.08 m^2
4. 216.66 cm^2

Practice Exercise 16

1. a) Point P
   b) $\overline{LO}, \overline{ON}, \overline{NP}, \overline{OP}, \overline{LP}, \overline{MP}, \overline{LM}, \overline{MN}$
   c) ΔLPO, ΔOPN, ΔMPN and ΔLPM
   d) h
   e) ℓ
   f) □LMNO
2. a) 2880 cm^2 b) 81 cm^2
3. a) 216 cm^2 b) 360 cm^2
4. 765 cm^2

Practice Exercise 17

1) a) 678.24 cm^2 b) 452.16 cm^2 c) 461 203. 20 cm^2
2. a) 1020 cm^2 b) 324 cm^2
3. 206 m^2
4. 90 m^2

END OF SUB-STRAND 3

SUB-STRAND 4

VOLUME AND CAPACITY

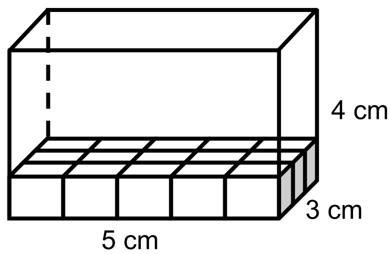
Lesson 18:	**Volume of Prisms**
Lesson 19:	**Volume of Cylinders**
Lesson 20:	**Volume of Pyramids**
Lesson 21:	**Volume of Cones**
Lesson 22:	**The Litre**
Lesson 23:	**Solving Problems Involving Volume and Capacity**

SUB-STRAND 4: VOLUME AND CAPACITY

Introduction

You learnt that the amount of space that a solid contains is called **volume**. Volume is measured in **cubic units**. The international standard for the metric unit used to measure volume is the cubic centimetre or the cubic metre. For the cubic centimetre, instead of cm^3, **cc** is commonly used.

For example: Think of filling the box on the right with 1-cm cubes. The box is 5 cm long, 4 cm high and 3 cm wide.



The number of cubes in the bottom layer = 5 x 3 = 15
The total number of cubes = 4 x 15 = 60

The volume of the box is 60 cm^3.

Note that the area of the base is 15 and the number of cubes in the bottom layer is also 15. The height of the box is 4 cm which represents the number of layers needed to fill the box. Thus, the volume of the box is 60 cm^3.

A solid with the same dimensions is called a rectangular solid.

To find the volume of a rectangular solid, we get the product of the length, width and height. Thus, we have

Volume = Length x Width x Height or Volume = Area of Base x Height

In symbol, we have:

$\boxed{V = l \times w \times h}$ or $\boxed{V = Bh}$

If the solid or box has Length = Width = Height, it is called a cube. Its dimension is called an **edge** and the volume is

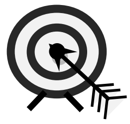
$$\boxed{V = e^3}$$

In this sub-strand, you will:

- Investigate the volumes of some other common simple solids to determine the rules
- Apply the volume and capacity measurement in problem solving.

Lesson 18: Volume of Prisms

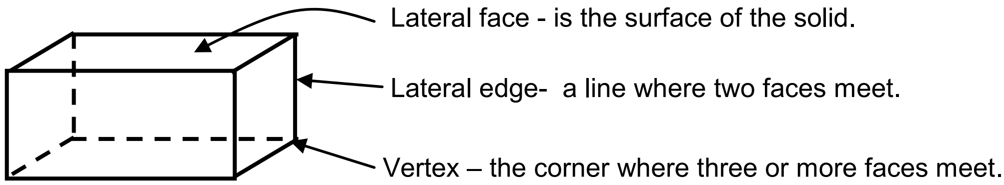
You learnt about volume in your Grade 7 Mathematics Strand 3 Sub-strand 3.



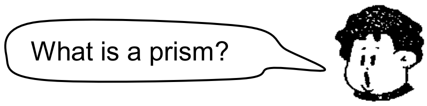
In this lesson, you will:

- describe prisms
- identify the types of prisms
- identify the base, height and uniform cross section of prisms
- calculate the volume of prisms.

In Grade 7 Mathematics you learnt about a solid or three dimensional (3D) shape and its parts as shown below.



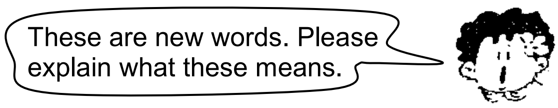
Solids are grouped into two main families. One of these groups is the prisms.



A prism is a solid with two congruent polygonal bases which are parallel and whose other faces called the lateral faces are parallelograms.

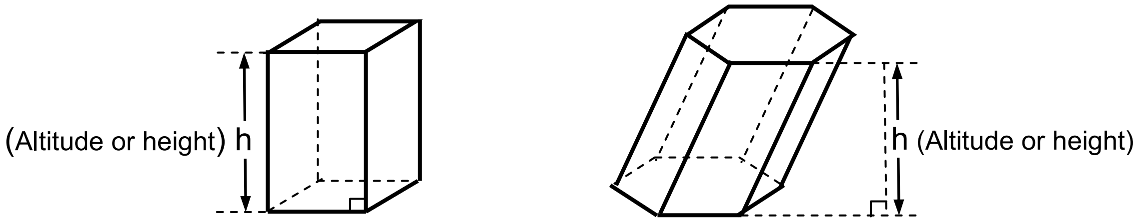
All prisms have a special pair of parallel faces. These faces are the only two faces that need not be parallelogram or rectangular in shape. They are called the **bases** of the prisms.

There are two types of prisms. These are the **right prisms** and the **oblique prisms**.



- **A right prism is a prism whose lateral edges are perpendicular to the bases. All the lateral faces are rectangles. The altitude or height of the prism is a segment perpendicular to the planes of the bases and having an endpoint in each base.**
- **An oblique prism is a prism whose lateral edges are not perpendicular to the bases. All the lateral faces are parallelograms. The altitude or the height is the horizontal distance between its bases.**

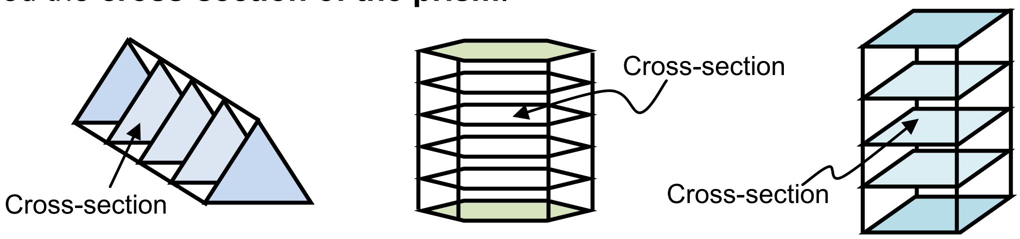
Look at the diagram.



Right Prism **Oblique Prism**

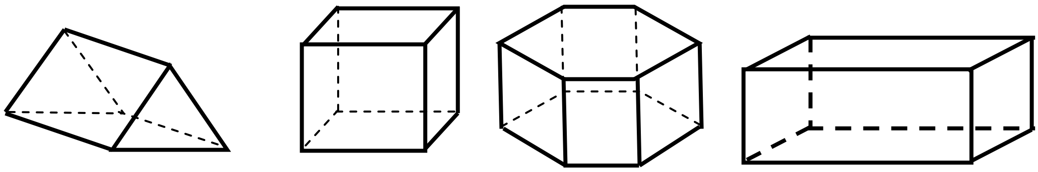
In this lesson, we shall study only prisms whose lateral faces are rectangles.

If a prism is cut parallel to these faces, the same shape always results. This shape is called the **cross-section of the prism**.



A prism is named according to the shape of its base or its cross-section.

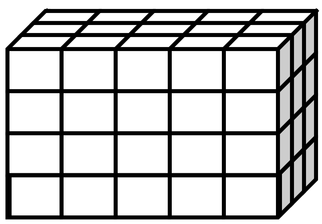
Below are some examples of prisms and their names.



Triangular prism Cube Hexagonal prism Rectangular prism

Let us consider finding the volume of a prism.

You have learnt that the volume of any solid is found by the number of unit cubes it takes to fill it completely: The volume of the rectangular solid shown below is equal to the number of 1 cm cubes that fill it. It is 5 cm long, 3 cm wide and 4 cm high.



Its volume may be found by multiplying those dimensions, 5 x 3 x 4 = 60 cm^3.

By Formula:

$$V = l \times w \times h = B \times h \quad \text{where } B = l \times w\text{, the area of the base.}$$

This formula holds for other solid figures like prisms and cylinders. In general, we can find the area of the base and we know the height of a solid figure, we can find the volume.

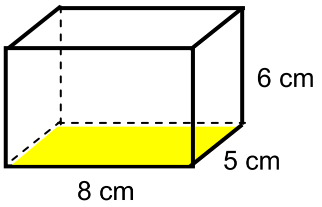
Hence, the volume of any prism with a base area **B,** and a height **h,** is given by the equation

$$\boxed{V = B \times h}$$

Where **B** is the area of the base or cross section
h is the height of the prism.

Example 1

Find the volume of the rectangular prism.



Solution:

Step 1: Find the area of the base (B). Use the formula A = l x w.

Length of the rectangular base (l) = 8 cm
Width of the rectangular base (w) = 5 cm
Area of the Base (B) = 8 cm x 5 cm = 40 cm^2

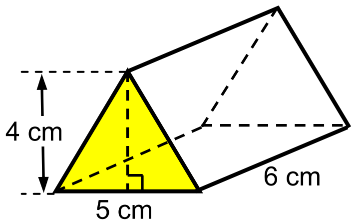
Step 2: Multiply the area of the base by the height of the rectangular prism. Use the formula V = B x h.

Area of base (B) = 40 cm^2
Height of prism (h) = 6 cm
Volume (V) = 40 cm^2 x 6 cm = 240 cm^3

Therefore, the volume of the rectangular prism is 240cm^3.

Example 2

Find the volume of the triangular prism below.



Solution:

Step 1: Find the area of the base (B). Use the formula A = $\frac{1}{2}$b x h.

Length of the triangular base (b) = 5 cm
Height of the triangular base (h) = 4 cm
Area of the Base (B) = $\frac{1}{2}$(5 cm x 4 cm) = 10 cm^2

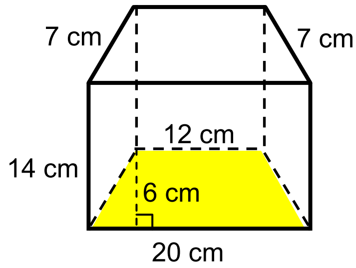
Step 2: Multiply the area of the base by the height of the triangular prism. Use the formula V = B x h.

Area of base (B) = 10 cm^2
Height of prism (h) = 6 cm
Volume (V) = 10 cm^2 x 6 cm = 60 cm^3

Therefore, the volume of the triangular prism is 60cm^3.

Example 3

Find the volume of the right trapezoidal prism. Its dimensions are shown below.



Solution:

Step 1: Find the area of the base. The prism has a trapezoidal base, use the formula Area = $\frac{1}{2}(a + b)h$.

The Length of the shorter base of trapezoidal base (a) = 12 cm

The Length of the longer base of trapezoidal base (b) = 20 cm

The Height of the trapezoidal base (h) = 6 cm

Area of the trapezoidal base (B) = $\frac{1}{2}$(12cm + 20 cm)(6 cm)

$B = \frac{1}{2}(32)(6)\text{ cm}^2$

$B = \frac{1}{2}(192)\text{ cm}^2$

$\mathbf{B = 96\ cm^2}$

Step 2: Multiply the area of the base by the height of the trapezoidal prism. Use the formula V = B x h.

Area of the base (B) = 96 cm^2

Height of the prism (h) = 14 cm

Volume of the prism (V) = B x h

= 96 cm^2 x 14 cm

= 1344 cm^3

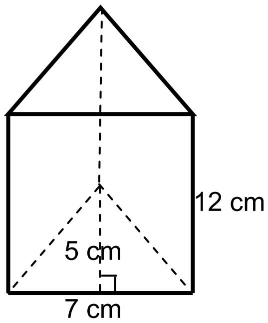
Therefore, the volume of the trapezoidal prism is 1344 cm^3.

Notice that the dimensions 7 cm and 7 cm had nothing to do with the problem.

Now look at the other examples on the next page.

Example 4

A glass prism has an isosceles triangle as its base. Its measurements are given as shown in the figure below. What is its volume?



Solution:

The glass prism is triangular.

Step 1: Find the area of the triangular base. Use the formula $A = \frac{1}{2}(b \times h)$.

Length of the triangular base (b) = 7 cm
Height of the triangular base (h) = 5 cm
Area of the Base (B) = $\frac{1}{2}(7 \text{ cm} \times 5 \text{ cm}) = 17.5 \text{ cm}^2$

Step 2: Multiply the area of the base by the height of the triangular prism. Use the formula V = B x h.

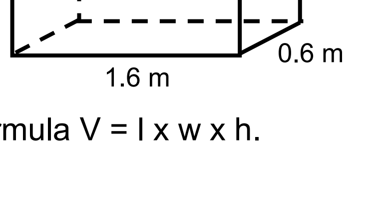
The Area of the base (B) = 17.5 cm^2
The Height of the prism (h) = 12 cm

Volume = Base x height
$= (17.5 \times 12) \text{ cm}^3$
$= 210 \text{ cm}^3$

Therefore, the volume of the glass prism is 210 cm^3.

Example 5

A flower box is 1.6 m long, 0.6 m wide and 0.5 m high. How much soil will the flower box hold if it is filled to the top?



Solution:

The box is a rectangular prism.

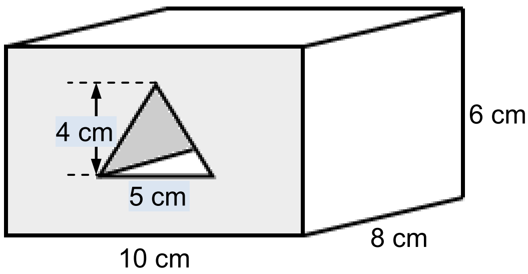
Find the volume of the flower box using the formula V = l x w x h.

Length of the box (l) = 1.6 m
Width of the box (w) = 0.6 m
Height of the box (h) = 0.5 m
Volume (V) = $1.6 \text{ m} \times 0.6 \text{ m} \times 0.5 \text{ m} = 0.48 \text{ m}^3$

Therefore, the flower box will hold 0.48 m^3 of soil.

Example 6

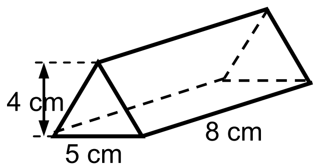
Calculate the volume of the prism below which has a triangular hole cut out of it.



Solution:

Step 1: a) Find the volume of the triangular hole

Area of base of the triangular hole (B) = $\frac{1}{2}$(b x h).



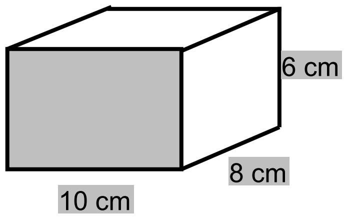
Length of the triangular base (b) = 5 cm
Height of the triangular base (h) = 4 cm

Area of the triangular base (B) = $\frac{1}{2}$(5 cm x 4 cm)

B = 10 cm^2

b) Multiply the area of the base by the height of the triangular hole.
Use the formula V = B x h.
= 10 cm^2 x 8 cm
= 80 cm^3

Step 2: a) Find the volume of the rectangular prism.
Use the formula V = l x w x h



Length of the rectangular base (l) = 10 cm

Width of the rectangular base (w) = 6 cm

Height of the rectangular base (h) = 8 cm

Volume of the rectangular prism = 10 cm x 6 cm x 8 cm

= 480 cm^3

Step 3: Find the volume of the prism. Subtract the Volume of the triangular hole from the volume of the rectangular prism.

Volume of the prism = 480 cm^3 – 80 cm^3
= 400 cm^3

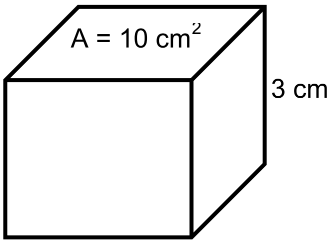
Therefore, the volume of the prism is 400 cm^3.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Calculate the volume of each prism, given the area of the cross-section or base and the height.

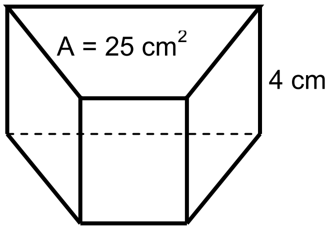
a)



Working out:

Answer: __________

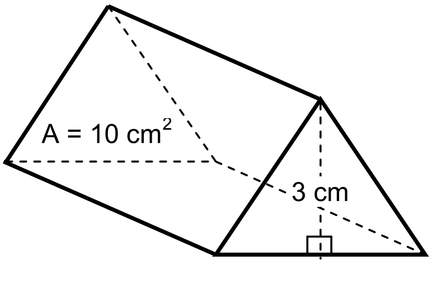
b)



Working out:

Answer: __________

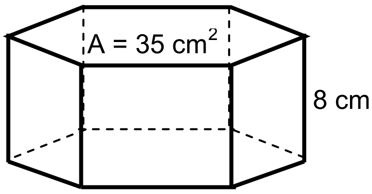
c)



Working out:

Answer: __________

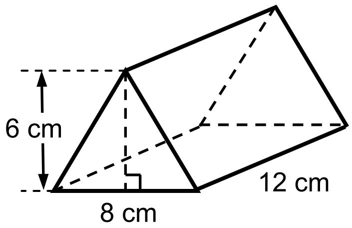
d)



Working out:

Answer: __________

2. Find the volume of the triangular prism below.



Working out:

Answer: __________

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3. A room is 12 m long, 7 m wide and 3.5 m high. How many cubic metres of airspace is in the room?

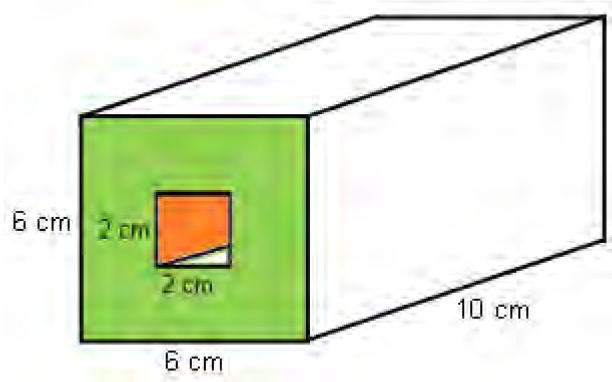
Answer: __________

---

4. Gravel and sand used in construction is sold by cubic metres. How much sand does a truck bed contain if its dimensions are 5 m by 0.8 m by 3 m?

Answer:__________

5. Below is a square prism which has a hole cut out of it as shown.



.

a) Calculate the area of the shaded cross-section.

Working out:

Answer: __________

b) Calculate the volume of the prism.

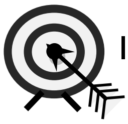
Working out:

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 19: Volume of Cylinders

You defined prisms and identified their types and properties. You also learnt to calculate their volumes using rules.



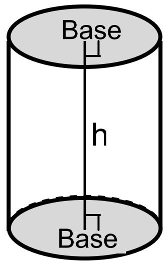
In this lesson, you will:

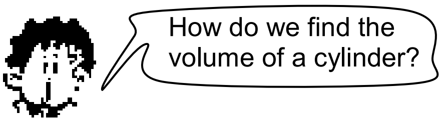
- identify the base and the height of a cylinder
- calculate the volume of a cylinder.

You learnt in Sub-strand 3, Lesson 15, that the simplest cylinders look like food cans and are called **right cylinders**. Their bases are perpendicular to the line joining the centers of the bases.

Here again is a cylinder. The base and height are shown.

The **height** of a cylinder is the shortest distance between its bases.





You learnt that the volume of any prism could be calculated by the formula

$$\boxed{\mathbf{V = B \times h}}$$

Where **V** is the volume

B is area of the base

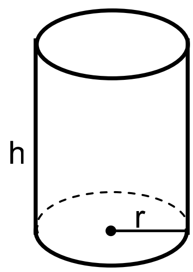
h is the height of the prism.

A cylinder can be thought of as a prism whose cross-section is a circle. The formula is also true for any cylinder.

Since the base of a cylinder is a circle, **B = area of circle =** $\boldsymbol{\pi r^2}$ **.**

So, a cylinder with radius **r** units and height **h** units has a **volume** of **V** cubic units given by the formula

$$\boxed{\mathbf{V = \pi r^2 h}}$$



where: V = volume of the cylinder

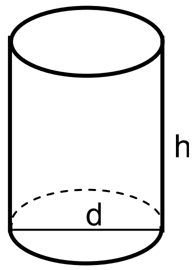
π = 3.14

r = the radius of the cylinder

h = the height of the cylinder

Since another formula for the area of a circle is $\mathbf{A = \frac{\pi d^2}{4}}$, then another formula for the volume of a cylinder is

$$\boxed{\mathbf{V = \frac{\pi d^2}{4} h}}$$

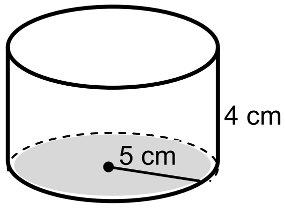


where **d** = diameter of the cylinder
h = height of the cylinder

Now look at the examples.

Example 1

Find the volume of the cylinder on the right.



Solution:

Step 1: Find the area of the base (B). Use the formula $\mathbf{A = \pi r^2}$.

- radius of base (r) = 5 cm
- area of base (B) = πr^2

Use either the π key or the approximate value of π = 3.14.

- area of base (B) = 3.14 x 5 cm x 5 cm
  = 78. 5 cm^2

Step 2: Multiply the area of the base by the height of the cylinder.
Use the formula **V = B x h**

- area of base (B) = 78.5 cm^2
- height of the cylinder (h) = 4 cm
- volume of the cylinder (V) = 78.5 cm^2 x 4 cm
  = 314 cm^3

Therefore, the volume of the cylinder is 314 cm^3.

Or you can find the volume of the cylinder by using directly the formula $\mathbf{V = \pi r^2 h}$.

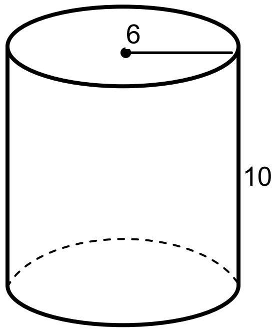
Hence, the volume of the cylinder is V = 3.14 x (5 cm)^2 x 4 cm
= 3.14 x 25 cm^2 x 4 cm
= 3.14 x 100 cm^3
= 314 cm^3

Example 2

Find the volume of a cylinder with base of 6 cm and height of 10 cm.

Solution:

The area of the base is not given, hence, we solve for it first.



Area of the base (B) = πr^2

$B = 3.14 \times (6\text{ cm})^2$

$= 3.14 \times 6\text{ cm} \times 6\text{ cm}$

$= 113.04\text{ cm}^2$

Volume = B x h

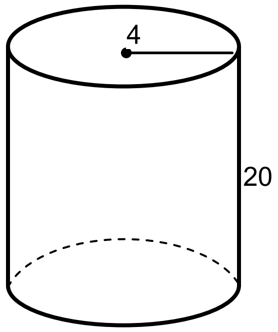
$= 113.04\text{ cm}^2 \times 10\text{ cm}$

$= 1130.4\text{ cm}^3$

Therefore, the volume of the cylinder is 1130.4 cm^3.

Example 3

Find the volume of the cylinder on the right.



Solution:

Since $B = \pi r^2$, then $V = \pi r^2 h$

$V = 3.14 \times (4\text{ cm})^2 \times 20\text{ cm}$

$= 3.14 \times 16\text{ cm}^2 \times 20\text{ cm}$

$= 1004.8\text{ cm}^3$

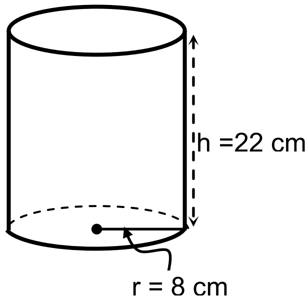
Therefore, the volume of the cylinder is 1004.8 cm^3.

Example 4

A condensed milk can has a radius of 8 cm and a height of 22 cm. When empty it can be used for storing water. What volume of water can it hold?

Solution:

Given: radius (r) = 8 cm; height (h) = 22 cm



Volume = $\pi r^2 h$

$= 3.14 \times (8\text{ cm})^2 \times 22\text{ cm}$

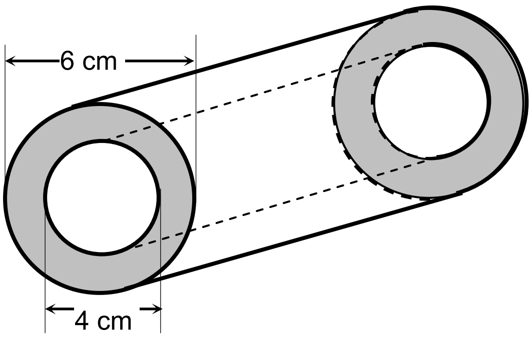
$= 3.14 \times 8\text{ cm} \times 8\text{ cm} \times 22\text{ cm}$

$= 4421.12\text{ cm}^3$

Therefore, the volume of water it can hold is 4421.12 cm^3.

Example 5

Find the volume of metal in the pipe below whose inside and outside diameters are 4 cm and 6 cm respectively and whose height is 50 cm.



Solution:

Step 1: Find the volume of the inner cylinder.

$$V = \frac{\pi d^2}{4} h$$
$$= \frac{3.14 \times (4\text{ cm})^2}{4} \times 50\text{ cm}$$
$$= 3.14 \times 4\text{ cm}^2 \times 50\text{ cm}$$
$$= 3.14 \times 200\text{ cm}^3$$
$$= 628\text{ cm}^3$$

Step 2: Find the volume of the outer cylinder

$$V = \frac{\pi d^2}{4} h$$
$$= \frac{3.14 \times (6\text{ cm})^2}{4} \times 50\text{ cm}$$
$$= 3.14 \times 9\text{ cm}^2 \times 50\text{ cm}$$
$$= 3.14 \times 450\text{ cm}^3$$
$$= 1413\text{ cm}^3$$

Step 3: Find the volume of the metal in the pipe

Volume of metal in the pipe = volume of outer cylinder – volume of inner cylinder

$$V = 1413\text{ cm}^3 - 628\text{ cm}^3$$
$$V = 785\text{ cm}^3$$

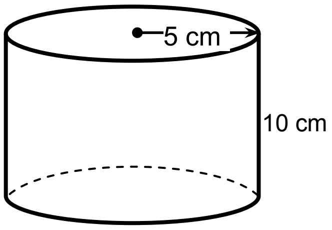
Therefore, the volume of metal in the pipe is 785 cm^3.

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. Calculate the volume of each of the following cylinders, correct to 1 decimal place. Use $\pi = 3.14$.

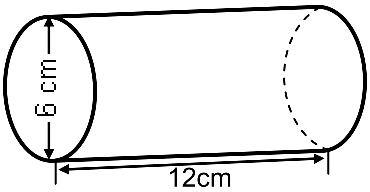
a)



Working out:

Answer: _________

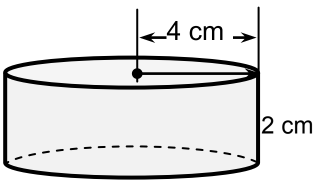
b)



Working out:

Answer: __________

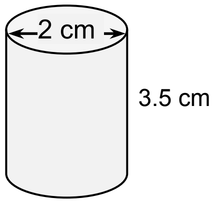
c)



Working out:

Answer: _________

d)

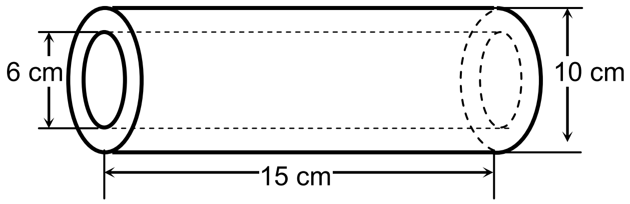


Working out:

Answer: __________

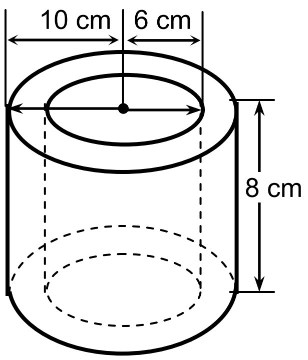
2. Calculate the volume of each of the following cylindrical solids, to the nearest cm^3. Use $\pi = 3.14$.

a)



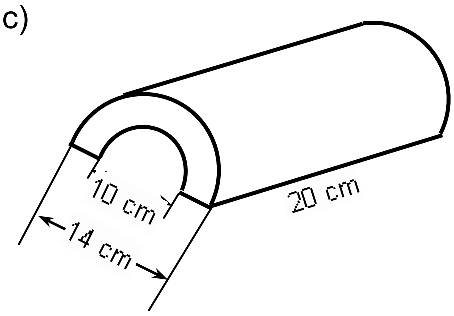
Answer: _________

b)



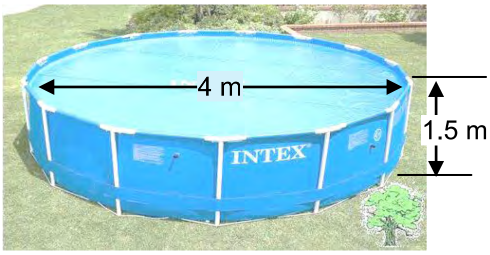
Answer: _________

c)



Answer: _________

3. A circular swimming pool has a diameter of 4 m and a depth of 1.5 m. calculate the volume of the pool.



Answer: __________

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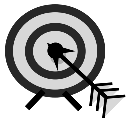
4. A flag pole is 10 m high and has a diameter of 36 cm. How many cubic metres of timber are there in the pole?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 20: Volume of Pyramids

In Lesson 16, you learnt how to find the surface area of pyramids.

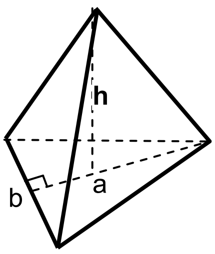


In this lesson, you will:

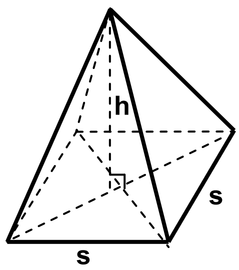
- recognise that the volume of a pyramid depends on the area of its base and height.
- relate the volume of prisms to the volume of a pyramid
- calculate the volume of a pyramid.

You know that a pyramid is a solid with a polygonal base and whose lateral faces are triangles with common vertices. In this lesson, we will only consider the volume of pyramids with quadrilateral or triangular bases. If a pyramid has a triangular base it is usually called a **tetrahedron**. If the base is a square it is called a **square pyramid**. If the base is a rectangle it is called a **rectangular pyramid**.

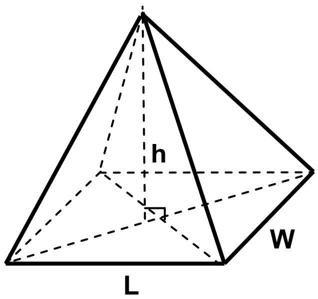
Tetrahedron

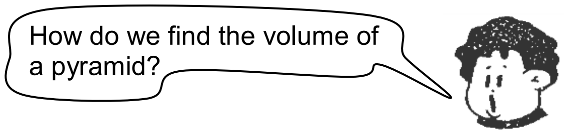


Right Square Pyramid



Rectangular Pyramid





If you are someone who wants to know why things work, you can experiment to find the formula for the volume of a pyramid by knowing how the volume of a pyramid is related to the volume of a prism.

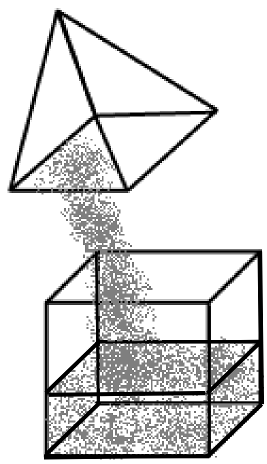
For example

The pyramid and the prism on the figure on the right have congruent bases and congruent heights. Think of filling the pyramid with sand or water.

at part of the prism is filled?

You will find that you need to fill the pyramid 3 times before you can fill the prism.

Thus, the volume of a pyramid of any polygonal base is equal to one-third of the volume of a prism.



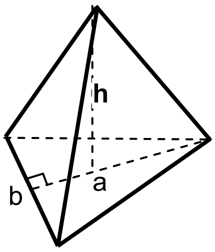
Hence, we have the following formula:

In the right prism, $\boxed{V = Bh}$

For the pyramid, $\boxed{V = \frac{1}{3} Bh}$

Since, there are different types of pyramids the volume of a pyramid depends on the area of base and its height.

So, for triangular pyramid: $V = \frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]$



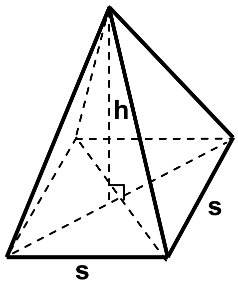
where V= volume of the pyramid

a = altitude of the triangle base

b = base of the triangle base

h = height of the pyramid

For a Square pyramid: $V = \frac{1}{3}s^2h$

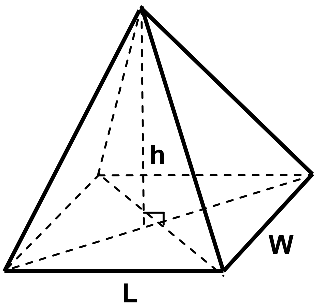


where V = the volume of the pyramid

s = a side of the square base

h = the height of the pyramid

For rectangular pyramids: $V = \frac{1}{3}(L \times W) \times h$



where V = the volume of the pyramid

L = the length of the rectangle base

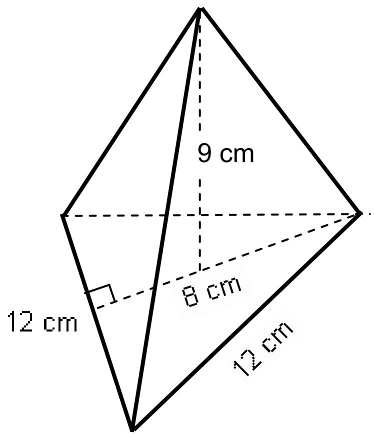
W = the width of the rectangle base

h = the height of the pyramid

Now look at the following examples on the next page.

Example 1

Find the volume of the triangular pyramid on the right.



Solution:

Find the volume using the formula:

$$V = \frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} \text{ x } 12 \text{ cm x } 8 \text{ cm}\right) \text{x } 9 \text{ cm}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} \text{ x } 96 \text{ cm}^2\right) \text{x } 9 \text{ cm}\right]$$

$$= \frac{1}{3}\left[48 \text{ cm}^2 \text{ x } 9 \text{ cm}\right]$$

$$= \frac{1}{3}(432 \text{ cm}^3)$$

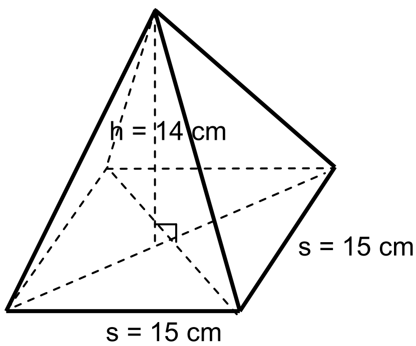
$$= 144 \text{ cm}^3$$

The volume of the triangular pyramid is 144 cm^3.

Example 2

A pyramid has a square base of side 15 cm and a height of 14 cm. Find its volume.

Solution:



Use the formula: $V = \frac{1}{3}s^2h$

$$= \frac{1}{3}[(15 \text{ cm})^2 \text{ x } 14 \text{ cm}]$$

$$= \frac{1}{3}[(15 \text{ cm x } 15 \text{ cm}) \text{ x } 14 \text{ cm}]$$

$$= \frac{1}{3}(225 \text{ cm}^2 \text{ x } 14 \text{ cm})$$

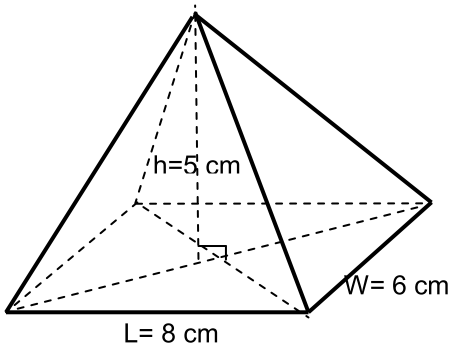
$$= \frac{1}{3}(3150 \text{ cm}^3)$$

$$= 1050 \text{ cm}^3$$

Therefore, the volume of the square pyramid is 1050 cm^3.

Example 3

Find the volume of a rectangular pyramid whose base is 8 cm by 6 cm and whose height is 5 cm.



Solution:

Use the formula: $V = \frac{1}{3}(L \times W) \times h$

$= \frac{1}{3}(8 \text{ cm} \times 6 \text{ cm} \times 5 \text{ cm})$

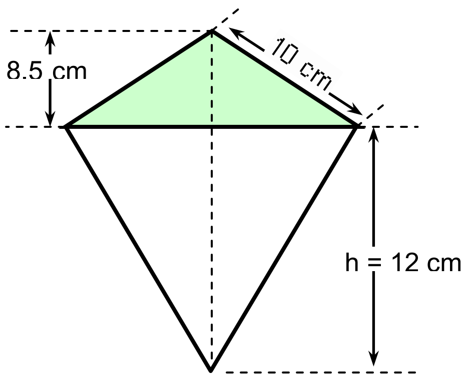
$= \frac{1}{3}(240 \text{ cm}^3)$

$= 80 \text{ cm}^3$

Therefore, the volume of the rectangular pyramid is 80 cm^3.

Example 4

Find the volume of a container which is in the form of an inverted triangular pyramid having an equilateral triangle as its top horizontal surface as shown.



Solution:

Use the formula:

$$V = \frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} \times 10 \text{ cm} \times 8.5 \text{ cm}\right) \times 12 \text{ cm}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} \times 85 \text{ cm}^2\right) \times 12 \text{ cm}\right]$$

$$= \frac{1}{3}\left[42.5 \text{ cm}^2 \times 12 \text{ cm}\right]$$

$$= \frac{1}{3}(510\text{cm}^3)$$

$$= 170 \text{ cm}^3$$

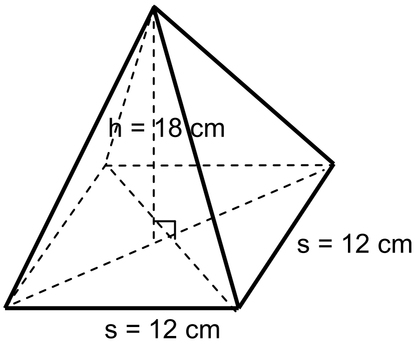
Therefore the volume of the container is 170 cm^3.

NOW DO PRACTICE EXERCISE 20

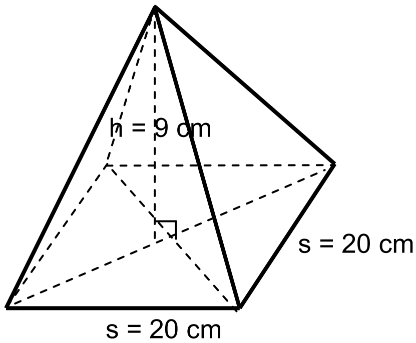
Practice Exercise 20

1. Find the volume of each of the following square pyramids. Note that the figures are not drawn to scale.

a)



b)



Working out;

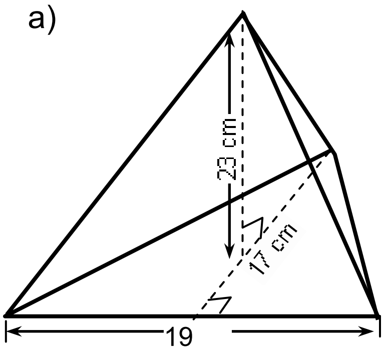
Answer: __________

Working out:

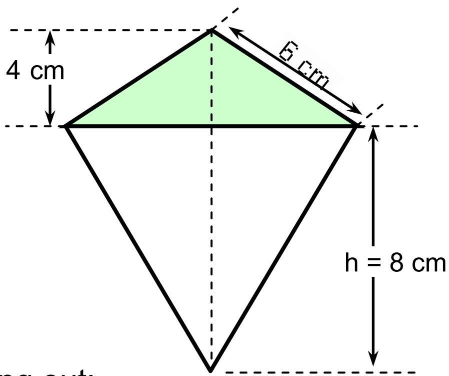
Answer: __________

2. Find the volume of each of the following triangular pyramids.

a)



b)



Working out;

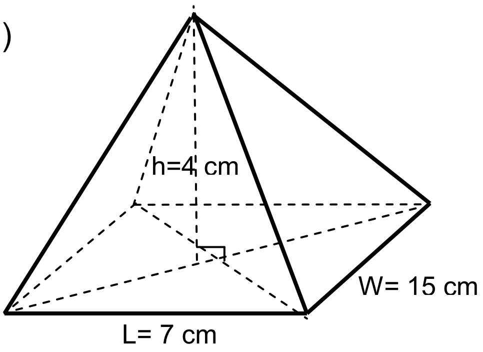
Answer: __________

Working out:

Answer: __________

3. Calculate the volume of each of the following rectangular pyramids.

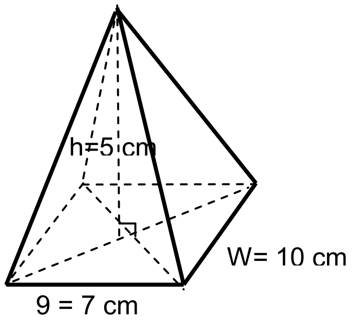
a)



Working out:

Answer: __________

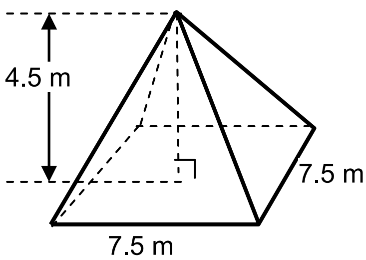
b)



Working out:

Answer: __________

4. What volume of air is enclosed by a circus tent with a square base of side 7.5 m and a height of 4 .5 m?



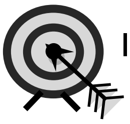
Working out:

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 21: Volume of Cones

You learnt to find the volume of pyramids in the last lesson.



In this lesson, you will:

- describe a cone
- identify the base and the height of a cone
- relate the volume of a cone to the volume of a cylinder.
- calculate the volume of a cone.

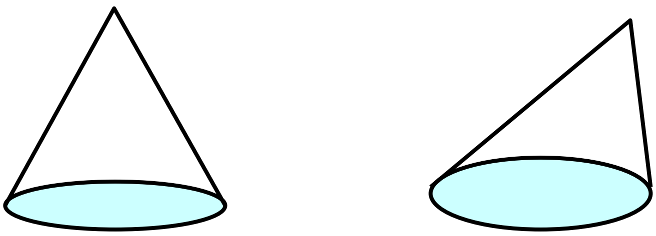
You defined cone in your Gr 7 Mathematics Strand 2 Lesson 16.

Here again is the definition of the cone.

A cone is a special solid figure with a plane circular base bounded by a conical surface. It has a net which is a quarter of a circle.

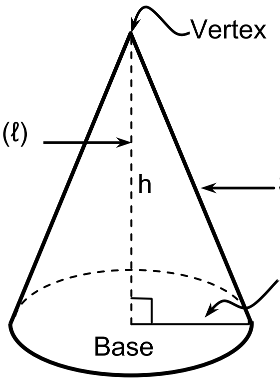
.
Example

The pictures below are two different views of a cone.



A right circular cone is a look-alike for a regular pyramid, but the base is a circle instead of polygon.

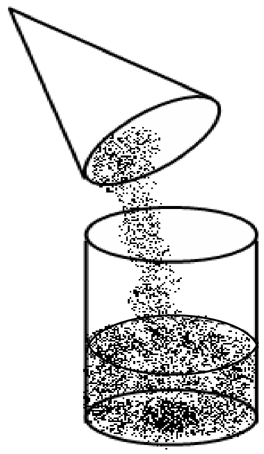
Look at the following diagram, and you will see how similar the two are.



Notice the names of the relevant parts. The rules and formula in finding the total surface area and volume seems quite familiar too.

In the previous lesson, you found out the relationship of the volume of a pyramid to the volume of a prism. Is this relationship also true for a cone and a cylinder?

Let us find out by doing the same experiment you did for a pyramid and prism.



Make a cone and a cylinder of congruent diameters and congruent heights using a piece of stiff cardboard. Think of filling the cone with sand or water and pour the sand or water into the cylinder. What part of the cylinder is filled?

You will find that you need to fill the cone three times before you can fill the cylinder.

Thus, the volume of a cone is equal to one-third of the volume of a cylinder.

Thus, we have the following formula:

In the right circular cylinder, $\mathbf{V = \pi r^2 h}$

For the cone, $\boxed{\mathbf{V = \frac{1}{3}\pi r^2 h}}$

Where V = volume of the cone
r = radius of the base
h = height of the cone

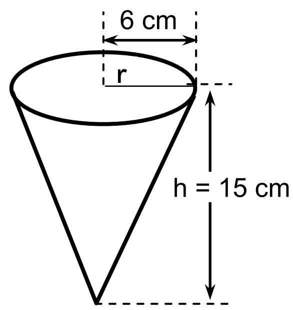
In terms of the diameter, $\boxed{\mathbf{V = \frac{1}{3} \times \frac{\pi d^2}{4} \times h}}$

V = volume of the cone
d = diameter of the base
h = height of the cone.

Example 1

Find the volume of a cone having a radius of 6 cm and a height of 15 cm. Use π = 3.14.

Solution:



To find the volume use the formula $V = \frac{1}{3}\pi r^2 h$.

- radius (r) = 6 cm
- height (h) = 15 cm

Substitute in the formula: $V = \frac{1}{3} \times 3.14 \times (6\text{ cm})^2 \times 15\text{ cm}$

$= \frac{1}{3} \times 3.14 \times 36\text{ cm}^2 \times 15\text{ cm}$

$= \frac{1}{3} \times 1695.6\text{ cm}^3$

$= 565.2\text{ cm}^3$

The volume of the cone is 565.2 cm^3.

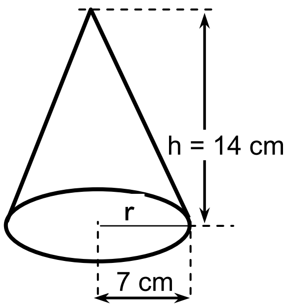
Example 2

Find the volume of each cone with the following dimensions. Use π = 3.14 and give your answers to the nearest whole of the given unit.

a) Radius (r) = 7 cm
Height (h) = 14 m

Solution:

First let us draw the diagram. (The diagram is not drawn to scale.)



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \text{ x } 3.14 \text{ x } (7 \text{ cm})^2 \text{ x } 14 \text{ cm}$

$= \frac{1}{3} \text{ x } 3.14 \text{ x } 49 \text{ cm}^2 \text{ x } 14 \text{ cm}$

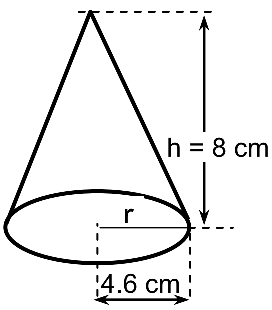
$= \frac{1}{3} \text{ x } 2154.04 \text{ cm}^3$

$= 71\ 8.\ 0013 \text{ cm}^3$

$= 718 \text{ cm}^3$ rounded to the nearest whole cm^3.

The volume of the cone is 718 cm^3.

b) Radius (r) = 4.6 m
Height (h) = 8 cm



Solution:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \text{ x } 3.14 \text{ x } (4.6 \text{ cm})^2 \text{ x } 8 \text{ cm}$

$= \frac{1}{3} \text{ x } 3.14 \text{ x } 21.16 \text{ cm}^2 \text{ x } 8 \text{ cm}$

$= \frac{1}{3} \text{ x } 531.539 \text{ cm}^3$

$= 177.1797 \text{ cm}^3$

$= 177 \text{ cm}^3$ rounded to the nearest whole cm^3.

The volume of the cone is 177 cm^3.

NOW DO PRACTICE EXERCISE 21

Practice Exercise 21

1. Find the volume of the following cones. Use π = 3.14 and give the answer to the nearest tenth of the given unit. Draw a diagram to help you.

a) radius (r) = 3 m
height (h) = 3.5 m

Draw diagram here.

Working out:

Answer:_________

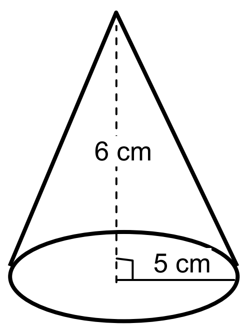
b) radius (r) = 5 m
height (h) = 7 m

Draw diagram here.

Working out:

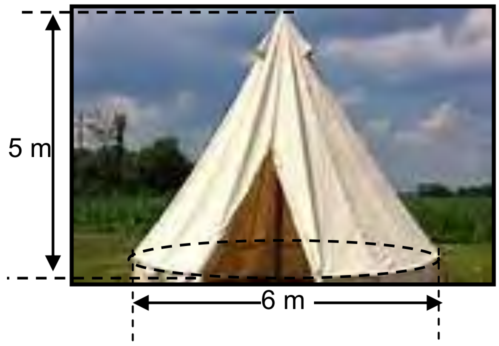
Answer:_________

2. Find the volume of a right circular cone with radius = 5 cm and height of 6 cm.



Answer: _________

3 A conical tent has a base diameter of 6 m and height of 5 m. What volume of air does it enclose?

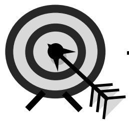


Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 22: The Litre

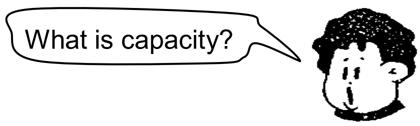
You learnt about volume of different solid shapes in the previous lessons



.In this lesson, you will:

- define capacity
- compare volume and capacity
- convert between volume and capacity measures.

You learnt in your early study of mathematics that when you measure the amount of liquid that a container can hold, you are measuring its capacity.



Capacity is the amount of matter, usually liquid, which can be put into a container. It is the volume of liquid.

We might say that an amount of a liquid is so many cupfuls or spoonfuls, but as with volume of solids, we need some standard units.

Litre is the basic unit of capacity in the metric system and is equivalent to a cubic decimetre. It is denoted by the capital letter (L) as its symbol.

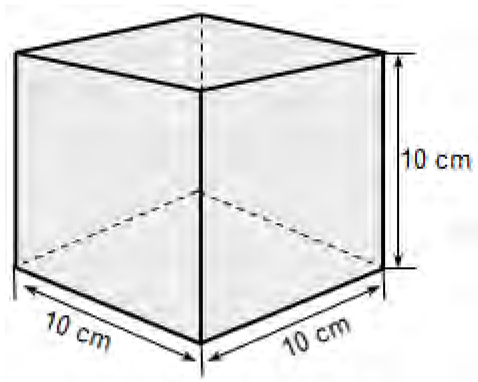
Other units of capacity are:

10 milliltres (mL)	=	1 centilitre (cL)
10 centilitres (cL)	=	1 decilitre (dL)
10 decilitres (dL)	=	1 litre (L)
10 litres (L)	=	1 decalitre (daL)
10 decalitres (daL)	=	1 hectolitre (hL)
10 hectolitres(hL)	=	1 kilolitre (kL)

This means that: 1000 millilitres (mL) = 1 litre (L)
1000 litres (L) = 1 kilolitre (kL)

The capacity of a container is known as the volume of the liquid or gas that it can hold. Thus capacity and volume have the same meaning when what we mean is the amount of something that can be put in a container. Capacity and volume are equivalent.

Look at the cube on the right.



One litre is equivalent to this cube.
Each side is 10 cm.

We say that, $\boxed{\mathbf{1\ L = 1000\ cm^3}}$

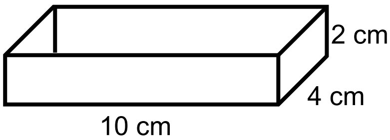
This also means that: $\boxed{\mathbf{1\ m^3 = 1\ kL}}$

and $\boxed{\mathbf{1\ mL = 1\ cm^3}}$

Looking at the link between the capacity of liquids and the volume of solids, you can now convert capacity measures into volume measures.

Example 1

How many mL of liquid would this prism hold?



Solution: V =L x W x H
V = 10 cm x 4 cm x 2 cm
V = 80 cm^3

Since 1 mL = 1 cm^3,

Therefore, the prism would hold 80 mL of liquid.

Example 2

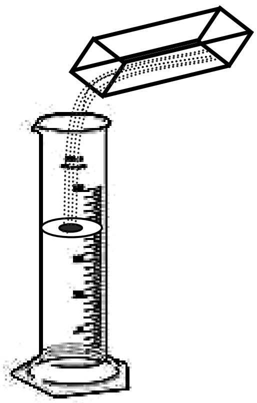
A measuring cylinder has a capacity of 500 millilitres (mL). What must its volume be in cubic centimetres?

Solution: 1 mL = 1 cm^3

Since the capacity is in mL, convert the mL units to cm^3 by multiplying the given unit by the equivalence factor which is 1.

Thus, you have 500 mL = 500 x 1
= 500 cm^3

Therefore, the volume in cm^3 of the cylinder must be 500.



Example 3

A petrol can is in the form of a rectangular prism with dimensions of 20 cm by 15 cm by 25 cm.

How many litres (L) of petrol will it hold?

Solution: Find the volume of the can. Use V = L x W x H.

- Length (L) = 20 cm
- Width (W) = 15 cm
- Height (H) = 25 cm

$$V = L \times W \times H$$
$$= 20 \text{ cm} \times 15 \text{ cm} \times 25 \text{ cm}$$
$$= 7500 \text{ cm}^3$$

Since, 1 L = 1000 cm^3, divide the volume found by 1000 to find the amount of petrol the can will hold.

$$V = \frac{7500}{1000} \text{L} = 7.5 \text{ L}$$

Therefore, the can will hold 7.5 L of petrol.

4. The capacities of car engines are sometimes quoted in litres and sometimes in cubic centimetres .

   a) A car's engine is said to be 2.4 L. How many cubic centimetres is this?
   b) Another car engine has a capacity of 1600 cubic centimetres. What is this in litres?

Solution:

a) 1 L = 1000 cm^3, so 2.4 L = 2.4 x 1000 cm^3

$$\mathbf{= 2400 \text{ cm}^3}$$

b) 1 L = 1000cm^3, so 1600 cm^3 = $\frac{1600}{1000}$ L

$$\mathbf{= 1.6 \text{ L}}$$

NOW DO PRACTICE EXERCISE 21

Practice Exercise 22

1. Complete the following.

a) 35 L = __________ cm^3

b) 5000 cm^3 = __________L

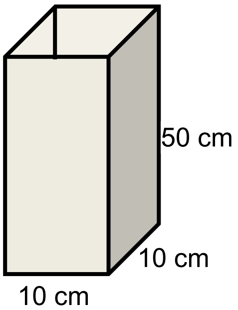
c) 8.2 m^3 = __________kL

d) 3.5 L = __________cm^3

e) 186 mL = __________cm^3

2. How many mL of liquid would each of these prisms hold?

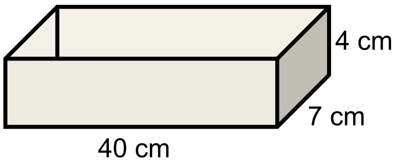
a)



Working out:

Answer: _________

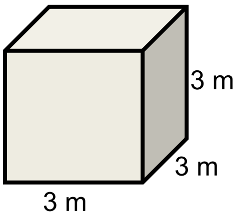
b)



Working out:

Answer: __________

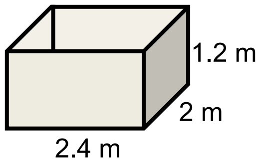
c)



Working out:

Answer: _________

d)



Working out:

Answer: __________

3. Calculate the capacity, in mL, of each of the following containers with the following volumes.

a) 24 000 cm^3

b) 6750 cm^3

c) 3000 cm^3

d) 1595 cm^3

4. Calculate the space occupied, in cm^3, by the following amounts of liquid.

a) 4000 L

b) 0.8 kL

c) 38 540 L

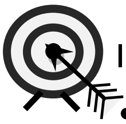
d) 5.2 kL

5. A big can of oil has a square base of 25 cm on each side. If it is 40 cm high, how many litres of oil does it contain? How many bottles each containing 250 mL can be filled from its contents?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 23: Solving Problems on Volume and Capacity

You learnt about the meaning of capacity and how it is related to volume in the last lesson. You also learnt how to convert between capacity measures and volume measures.



In this lesson, you will:

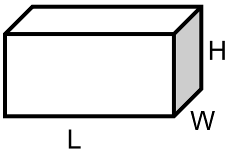
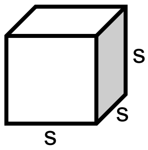
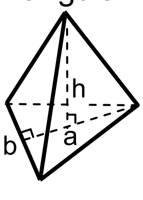
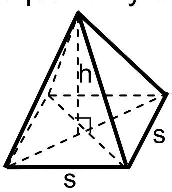
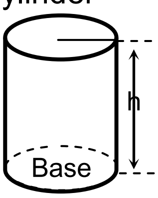
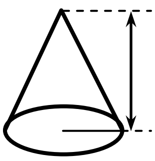
- solve problems involving volume and capacity in real life situations.

In real life situations, understanding volume or capacity is especially important in solving problems such as in the fields of medicine, chemistry, engineering, business and commerce when one is dealing constantly with space and liquid measurements.

For example, when one is measuring the space occupied by an amount of oil or gas in a tank, amount of annual milk production in a dairy farm, amount of water in a swimming pool and so on.

Understanding volume and capacity should be crystal clear to you with these real life examples of volume.

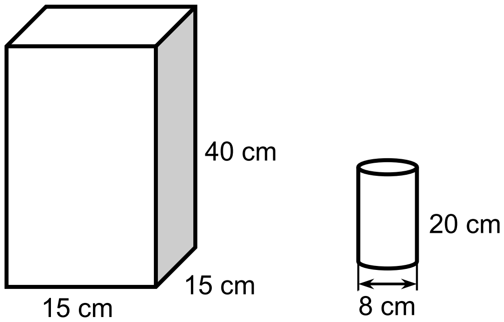
Now, you will be examining a variety of "real-world" problems that can be solved by referring to familiar facts and applying the skills you learnt. These problems will usually require that you compute the volume of one or more simple geometric figures, such as a rectangular solid, cylinder, pyramid or cone. The formulas for computing the volume of these solids are shown below.

Rectangular Prism  $V = L \times W \times H$	Cube  $V = s^3$
Triangular Pyramid  $V = \frac{1}{3}abh$ $V = \frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]$	Square Pyramid  $V = \frac{1}{3}Bh$ $V = \frac{1}{3}s^2h$
Cylinder  $V = Bh$ $V = \pi r^2h$ or $V = \frac{\pi d^2}{4}h$	Cone  $V = \frac{1}{3}Bh$ $V = \frac{1}{3}\pi r^2h$

Now look at the examples.

Example 1

Amos has a box of birdseed that is 15 cm x 15 cm x 40 cm. Estimate how many cylindrical bird feeders with a diameter of 8 cm and height of 20 cm he can fill?



Solution:

Step 1: Find the volume of the box of birdseed. Use the formula: V = L x W x H

- Length (L) = 15 cm
- width (W) = 15 cm
- Height (H) = 40 cm
- Volume (V) = 15 cm x 15 cm x 40 cm
  = 9000 cm^3

Step 2: Find the volume of the cylindrical bird feeder.

Use the formula: $V = \frac{\pi d^2}{4} h.$

- Diameter (d) = 8 cm
- height (h) = 20 cm
- $\pi = 3.14$
- Volume (V) = $\frac{3.14 \times (8\,cm)^2}{4} \times 20$ cm
  = 3.14 x 64 cm^2 x 5 cm
  = 1004.8 cm^3

Step 3: Divide the volume of the box by the volume of a cylindrical bird feeder to find the number of feeders to be filled.

Number of cylindrical bird feeders = $\frac{\text{Volume of box}}{\text{Volume of a cylinder Feeder}}$

$= \frac{9000\,cm^3}{1004.8\,cm^3}$

= 8.95

Step 4: Round 8.95 to the nearest whole number. 8. 95 = 9

Therefore, Amos can fill approximately 9 cylindrical bird feeders.

Example 2

A can of milk has a radius of 10 cm and a height of 20 cm. Use $\pi = 3.14$.

a) How much material was used in making it?

b) What volume of milk does it contain if $\frac{1}{20}$ of the can is left empty?

Solution:

a) You need to find the total surface area of the can to know how much material was used.

Step 1: Use the formula: $TSA = 2\pi r^2 + 2\pi rh$

$$= [2 \times 3.14 \times (10\text{ cm})^2] + [2 \times 3.14 \times 10\text{ cm} \times 20\text{ cm}]$$
$$= (6.28 \times 100\text{ cm}^2) + (6.28 \times 200\text{ cm}^2)$$
$$= 628\text{ cm}^2 + 1256\text{ cm}^2$$
$$= 1884\text{ cm}^2$$

Therefore, 1884 cm^2 of material was used to make the can of milk.

b) To find the volume of milk the can contains if $\frac{1}{20}$ of it is left empty, do the following steps:

Step 1: Find the volume of can when full. Use the formula:

$$V = \pi r^2 h$$
$$= 3.14 \times (10\text{ cm})^2 \times 20\text{ cm}$$
$$= 3.14 \times 100\text{ cm}^2 \times 20\text{ cm}$$
$$= 3.14 \times 2000\text{ cm}^3$$
$$\mathbf{= 6280\text{ cm}^3}$$

Step 2: Find $\frac{1}{20}$ of the volume of the can.

$$\frac{1}{20} \text{ of } 6280\text{ cm}^3 = \frac{1}{20} \times 6280\text{ cm}^3$$
$$\mathbf{= 314\text{ cm}^3}$$

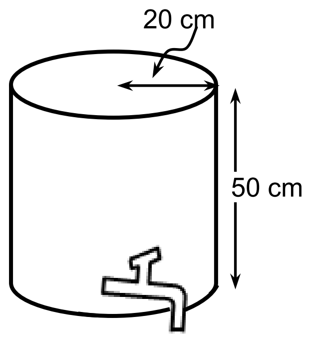
Step 3: Subtract the results in Step 2 from the result in Step 1 to find the volume of milk the can contain if $\frac{1}{20}$ of it is left empty

$$V = 6280\text{ cm}^3 - 314\text{ cm}^3$$
$$\mathbf{= 5966\text{ cm}^3}$$

Therefore, the can contains 5966 cm^3 of milk when $\frac{1}{20}$ of it is left empty.

Example 3

A coffee urn in a coffee shop is a cylinder with a radius of 20 cm and a height of 50 cm. as shown on the right.



a) Find the volume of coffee in the urn when it is full. Use $\pi = 3.14$.

b) Each cup of coffee served has a volume of 157 cm^3. How many cups of coffee can be served from a full urn?

Solution:

a) Find the volume of the urn. Use the formula $V = \pi r^2 h$.

$$V = \pi r^2 h$$

$$V = 3.14 \times 20 \text{ cm} \times 20 \text{ cm} \times 50 \text{ cm}$$

$$= 3.14 \times 20\,000$$

$$= 62\,800 \text{ cm}^3$$

The volume of the urn is 62 800 cm^3 when full.

b) Find the number of cups of coffee that can be served from a full urn.

$$\text{Number of cups} = \frac{\text{volume of urn}}{\text{Volume of each cup}}$$

$$= \frac{62\,800 \text{ cm}^3}{157 \text{ cm}^3}$$

$$= 400$$

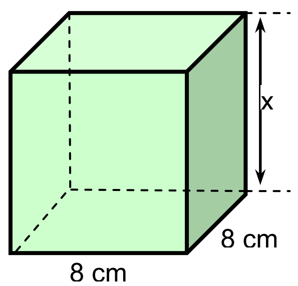
Therefore, 400 cups can be served from a full urn.

Example 4

A container has a square base of 8 cm. What is the height of the box if its volume is 384 cm^3?

Solution:

$$V = Bh$$



B = Area of square base = 8×8
$= 64 \text{ cm}^2$

$$\text{Height of container(h)} = \frac{\text{Volume}}{\text{Base}}$$

$$= \frac{384 \text{ cm}^3}{64 \text{ cm}^2}$$

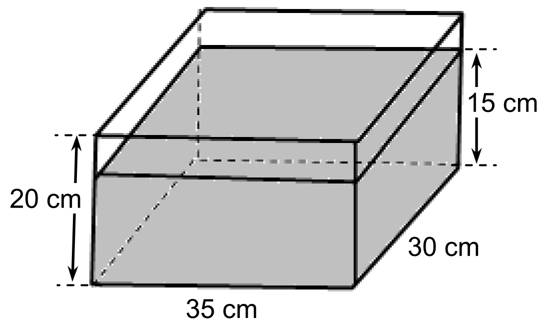
$$= 6 \text{ cm}$$

The height of the container is 6 cm.

Example 5

A rectangular tank 35 cm by 30 cm by 20 cm contains water to a height of 15 cm.

Find the volume of water in the tank. (Give your answer in litres).



Solution:

Step 1 Find the volume of water in the tank. Use V = L x W x H.

- length = 35 cm; width = 30 cm; height = 15 cm

Substitute in the formula Volume = L x W x H

You have, V = 35 cm x 30 cm x 15 cm

= 15 750 cm^3

Step 2 Convert the volume from cm^3 to litres

- 1 L = 1000 cm3

Thus, 15 750 cm3 = $\frac{15\ 750}{1000}$

=15.75 litres

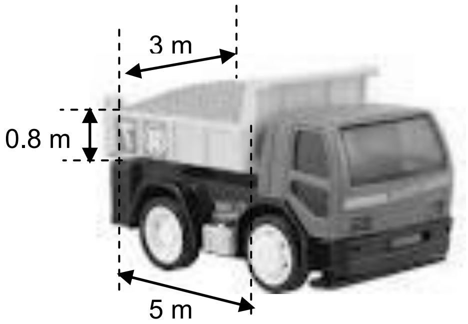
Therefore, the volume of water is 15.75 L.

NOW DO PRACTICE EXERCISE 23

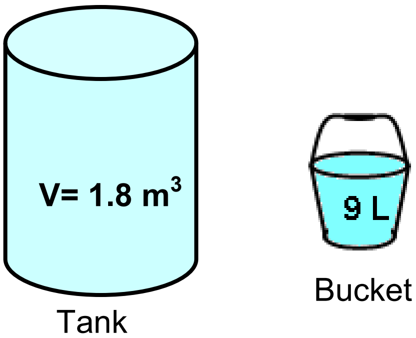
Practice Exercise 23

Solve each word problem.

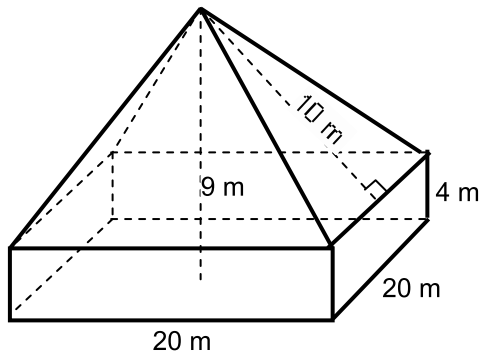
1. Gravel and sand used in construction is sold in cubic metre. How many cubic metres of sand does a truck bed contain if its dimensions are 5 m by 0.8 m by 3 m?



2. Vincent is watering his garden using a bucket that has a capacity of 9 L. he fills the bucket from a tank that has a volume of 1.8 m^3. How many bucketsful of water are in the tank?



3. A circus tent has the shape and dimensions given in the figure. The height from the floor to the top of the tent is 9 m. What volume of air does it enclose?



4. The swimming pool shown in the picture is 16 m long, 8 m wide and 2.3 m deep. How many litres of water does it contain when it is $\frac{4}{5}$ full?

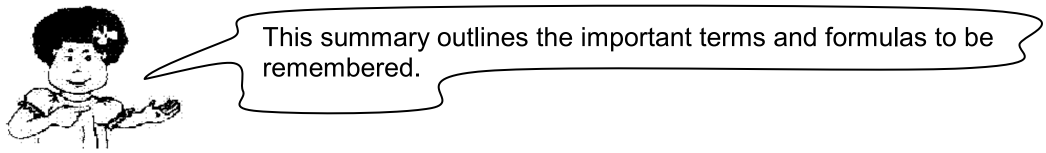
5. A big can of oil has a square base of 250 cm on each side.

   a) If it is 40 cm high, how many litres of oil does it contain?

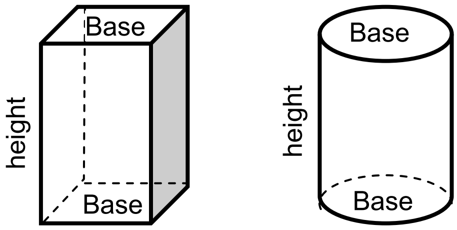
   b) How many bottles each containing 250 mL can be filled from its oil content?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

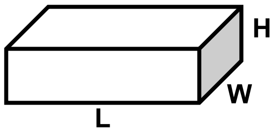
SUB-STRAND 4: SUMMARY



- In a **prism** or a **cylinder** the cubes can be arranged in layers that contain the same number of cubes each layer or fraction of cubes.
- The **height** of a prism or a cylinder is the shortest distance between its **bases**.

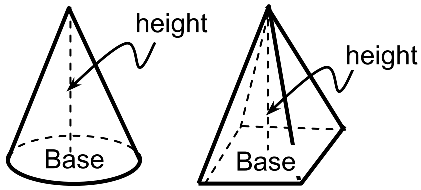


- The volume of a prism or cylinder is the product of the area of the base (the number of cubes in one layer) multiplied by its height (the number of layers).
- The general formula for the volume of a prism and a cylinder is **$V = B\,h$**, where **B** is the area of the Base, **h** is the height of the prism or cylinder.

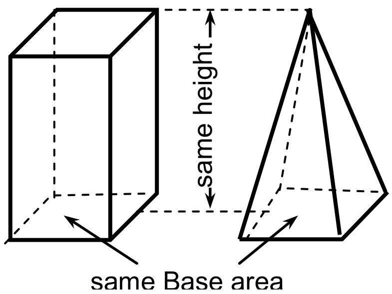


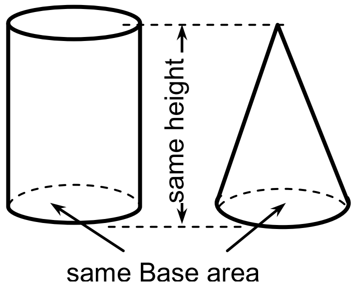
- For right rectangular prisms: **$V = (L \times W) \times h$.**

  For right cylinders: **$V = \pi r^2 h$**



- The height of a pyramid or cone is the shortest distance between its base and the vertex opposite its base.
- If a prism and a pyramid have the same base and height, then the volume of the pyramid is one-third the volume of the prism. **$V = \frac{1}{3}Bh$**
- If a cylinder and a cone have the same base and height, then the volume of the cone is one-third the volume of the cylinder. **$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$**





- **Capacity** is the amount of matter, usually liquid, which can be put into a container. It is the volume of liquid.
- **Litre** is the basic unit of capacity in the metric system and is equivalent to a cubic decimetre. It is denoted by the capital letter **(L)** as its symbol.

REVISE LESSONS 18-23.THEN DO SUB-STRAND TEST IN ASSIGNMENT 4.

Answers to Practice Exercise 18 - 23

Practice Exercise 18

1. a) 30 cm^3 b) 100 cm^3 c) 30 cm^3 d) 280 cm^3
2. 288 cm^3
3. 294 m^3 of airspace
4. 12 m^3
5. a) 32 cm^2 b) 320 cm^3

Practice Exercise 19

1. a) 785 cm^3 b) 339.1 cm^3 c) 113.04 cm^3 d) 10.8 cm^3
2. a) 3014 cm^3 b) 1608 cm^3 c) 3014 cm^3
3. 75.36 m^3
4. 1.02 m^3

Practice Exercise 20

1. a) 684 cm^3 b) 1200 cm^3
2. a) 1238.17 cm^3 b) 32 cm^3
3. a) 140 cm^3 b) 116.7 cm^3
4. 253.125 m^3

Practice Exercise 21

1. a) 33.0 cm^3 b) 183.2 m^3
2. 157 cm^3
3. 47.1 m^3

Practice Exercise 22

1. a) 35 000cm^3 b) 5 L c) 8.2 kL
   d) 3500 cm^3 e) 186 cm^3

2.	a)	5000 mL	b)	1120 mL
	c)	9 000 000 mL	d)	5 760 000 mL
3.	a)	24 000 mL	b)	6750 mL
	c)	3000 mL	d)	1595 mL
4.	a)	4 000 000 cm^3	b)	38 540 000 cm^3
	c)	800 000 cm^3	d)	5 200 000 cm^3
5.	100 bottles			

Practice Exercise 23

1 12 m^3

2. 2000 buckets

3. 160000 m^3

4. 235.52 m^3

5. a. 250 L b) 1000 cans

END OF STRAND 3

NOW YOU MUST COMPLETE ASSIGNMENT 3.
RETURN IT TO THE PROVINCIAL COORDINATOR.

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